Measuring Phase Synchronization of Superimposed Signals

Frank C. Meinecke,[1,*] Andreas Ziehe,[1,†] Jürgen Kurths,[3] and Klaus-Robert Müller[1,2,‡]

[1]*Fraunhofer FIRST.IDA, Kekuléstrasse 7, D-12489 Berlin, Germany*

[2]*Institut für Informatik, Universität Potsdam, August-Bebel Straße 89, D-14482 Potsdam, Germany*

[3]*Institut für Physik, Universität Potsdam, Am Neuen Palais, PF 601553, D-14415 Potsdam, Germany*

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Phase synchronization is an important phenomenon that occurs in a wide variety of complex oscillatory processes. Measuring phase synchronization can therefore help to gain fundamental insight into nature. In this Letter we point out that synchronization analysis techniques can detect spurious synchronization, if they are fed with a superposition of signals such as in electroencephalography or magnetoencephalography data. We show how techniques from blind source separation can help to nevertheless measure the true synchronization and avoid such pitfalls.

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Phase synchronization (PS) is a ubiquitous phenomenon in nature [1]. It can occur in many oscillatory processes and has been studied in a wide variety of systems such as mechanical oscillators, electrical circuits, and chemical or biological systems (e.g., [2–7]).

So far in order to measure synchronization all methods implicitly use the assumption that the data analyzed do not consist of a superposition of signals. However, if synchronization between superimposed signals is measured one can find high synchronization although, in fact, the underlying signals might *not* be synchronized at all. For example, in electroencephalography (EEG) recordings on the scalp, since the underlying brain sources cannot be measured separately, but only superpositions of their signals are accessible, brain sources might erroneously appear to be synchronized, which could ultimately lead to wrong biological conclusions. The same reasoning applies for many multichannel measuring devices. So it is of high importance to devise a test that can decide whether a high synchronization can be explained by superposition effects or not. Recently, Dolan *et al.* [8] developed a surrogate data test that addresses this problem. In this Letter we approach the problem from a different perspective by using modern techniques from the field of blind source separation (BSS).

In the last decade, there has been an increasing interest in the development of BSS methods (see, e.g., [9–13]). These methods allow us to reconstruct the original source signals from a given set of superpositions. The term *blind* refers to the fact that these methods do not use any additional information about, e.g., time courses or spectra of these unknown sources. However, they all presuppose certain conditions on the data. The most widely used class of BSS methods, known as independent component analysis (ICA), assumes that the given data are a linear and instantaneous mixture of mutually independent source signals. In order to test for synchronization on mixed data, it seems to be quite straightforward, to apply ICA to the data to obtain the true source signals and use them to estimate a synchronization index. However, if we assume the sources to be coupled or synchronized, the independence condition of ICA will be violated. So there is a fundamental dilemma: mixing effects cause the synchronization analysis to fail; synchronization effects in turn might harm the ICA. We show how BSS-related techniques can nevertheless be used for synchronization analysis of superimposed signals.

PS between two oscillators is typically measured by examining the difference between their instantaneous phases $\Delta\phi(t) = \phi_2(t) - \phi_1(t)$. Practically, the instantaneous phase of a measured oscillatory signal can be obtained by using the analytical signal approach (i.e., the Hilbert transform [14,15]) or wavelets. In the following, we use the phase locking value

$$\gamma = |\langle \exp[i\Delta\phi(t)]\rangle_t| \tag{1}$$

as a synchronization index. This index ranges from 0 (no phase synchronization, i.e., the phase difference $\Delta\phi(t)$ is uniformly distributed) to 1 (perfect phase synchronization, i.e., $\Delta\phi(t)$ is constant). Please note that the following considerations apply as well to more sophisticated synchronization measures like those based on stroboscopic phase observations (see, e.g., [16]).

If it is not possible to measure the source signals $s_j(t)$, which are produced by the oscillators, directly, but only linear superpositions $x_i(t)$ of them, i.e.,

$$x_i(t) = \sum_j A_{ij} s_j(t), \tag{2}$$

calculating the above mentioned synchronization index from the mixtures $x_i(t)$ will generally lead to the wrong results. As a simple example consider two sine waves of different frequencies (see the Appendix). Although there is no phase locking between them (in fact, they do not even stem from coupled dynamical systems), the mixtures will reveal an arbitrary high synchronization index, depending on the mixing coefficients A_{ij}. So, ideally, the mixing process should be undone before applying the synchronization measure.

BSS methods allow us to estimate the mixing coefficients A_{ij} and to invert Eq. (2) under some technical assumptions (i.e., the number of mixtures $x_i(t)$ must be greater or equal to the number of sources $s_j(t)$ and the columns of A must be linearly independent [9]). However, these methods typically assume the *source signals* $s_i(t)$ to be mutually independent (ICA, e.g., [10,11]) or temporally uncorrelated (see, e.g., [13,17]), which is not true for coupled oscillators, especially not, if the oscillators are in a synchronous regime. In this case, we cannot expect that ICA algorithms will recover the original source signals.

But, fortunately we do not really need to find the basis of the true source signals. To estimate the synchronization index, it suffices to find a transformation that removes any spurious synchronization but does not destroy real synchronization.

In the following, we see that under mild assumptions, ICA-BSS methods provide such transformations. Here we use the temporal decorrelation separation method (TDSEP [13]), which is a powerful BSS strategy for signals with a strong (and distinct) temporal structure. TDSEP—a generalization of [12]—determines the mixing matrix by performing a joint approximate diagonalization of several time-lagged covariance matrices; i.e., TDSEP minimizes temporal cross correlations between the output signals. To estimate the source signals, the algorithm first removes all instantaneous linear correlations by a whitening transformation (i.e., rescaling along the principal component directions such that the covariance matrix becomes the identity matrix). The remaining rotation is then achieved by a simultaneous diagonalization [18,19] of a set of time-lagged covariance matrices $E[\mathbf{x}(t)\mathbf{x}^T(t-\tau_k)]$, where the τ_k are arbitrary numbers. (A more detailed description of TDSEP can be found at [13]; the MATLAB source code is available at [20].)

To show that synchronized signals will stay synchronized if they are linearly transformed in the signal space (i.e., mixed or demixed), consider two general phase-synchronous oscillatory time series

$$s_i(t) = Q_i(t)\cos[\phi_i(t)], \qquad i = 1, 2 \tag{3}$$

with positive, slowly varying amplitude functions $Q_i(t) > 0$ and monotonous phase functions $\phi_i(t)$ with $\phi_2(t) = \phi_1(t) + \delta$. After applying a linear transformation $x_i(t) = \sum_{j=1,2} A_{ij} s_j(t)$ in the signal space, the transformed signals (i.e., mixtures) read

$$x_i(t) = B_i(t)\cos[\phi_1(t) + \psi_i(t)] \tag{4}$$

with

$$B_i(t) = \sqrt{(A_{i1}Q_1 + A_{i2}Q_2\cos\delta)^2 + (A_{i2}Q_2\sin\delta)^2},$$

$$\psi_i(t) = -\arctan\frac{A_{i2}Q_2\sin\delta}{A_{i1}Q_1 + A_{i2}Q_2\cos\delta}.$$

Since $B_i(t) > 0$, these functions can be interpreted as amplitudes; the superpositions are still phase synchronized [21]. This ensures that even though in the synchronous case ICA does not succeed in finding the original source signals, the ICA estimates are still synchronized. If, on the other hand, the original source signals are independent (not synchronized), ICA will be able to find them and by this remove any spurious synchronization. This reasoning justifies us to apply BSS as a preprocessing step before estimating a synchronization index of superimposed signals. The following simulations show the practical usefulness of this approach.

As a toy example we look at a Rössler oscillator that is driven by a periodic external force:

$$\begin{aligned} \dot{x} &= -y - z + cy\cos(\nu t), \\ \dot{y} &= x + 0.2y - cx\sin(\nu t), \qquad \dot{z} = 1 + (x-9)z. \end{aligned} \tag{5}$$

The driving frequency ν is varied between 1 and 1.03 and the coupling strength c between 0 and 0.1. For each combination of these two parameters, the system Eq. (5) is integrated and the synchronization index γ between the driving sine wave and $x(t)$ from the Rössler system [Fig. 1 (left column)], between random mixtures of them [i.e., the mixing coefficients are drawn from a normal distribution (middle column)], and the TDSEP source estimates (right column) is calculated.

The first row of Fig. 1 shows surface plots of the synchronization index, the lower row the results after setting a threshold at $\gamma_0 = 0.6$. For the original sources, the synchronization region (white) reveals the typical shape of an Arnold tongue (i.e., the frequency band $\Delta\nu$ that allows PS increases with the coupling strength c). Although this structure is invisible in the mixed signals (in fact, most time series are classified as synchronized), the TDSEP solutions again reveal the synchronization region.

However, although none of the time series is spuriously classified as phase synchronized on the TDSEP data set,

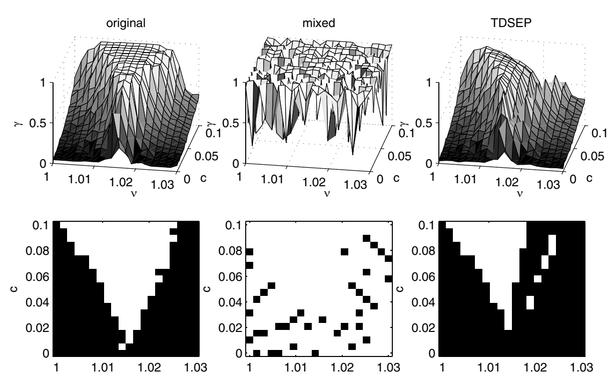

FIG. 1. The synchronization index γ at different driving frequencies ν and different coupling strengths c of the driven Rössler system [Eq. (5)] calculated on the original "sources" [i.e., $x(t)$ and $\sin(\nu t)$, (left column), the mixtures (middle column), and the TDSEP estimates (right column)]. The upper row shows a surface plot of the synchronization index and the lower row the result after setting a threshold at $\gamma_0 = 0.6$. (The white regions indicate PS.)

the right column of Fig. 1 shows that there is a part of the Arnold tongue missing. This is because in this region $x(t)$ is not just phase synchronized, but exactly in phase with $\sin(\nu t)$ and almost identical to the driving force. In fact, identical signals due to identical synchronization might as well be caused by strong superpositions. Since these two cases are not distinguishable, our test does not recognize synchronization in the case of identical signals. So, performing TDSEP ensures that signals which are perceived as synchronized are synchronized indeed. It avoids the problem of spurious synchronization. The reverse, however, is not always true: If the synchronization index on TDSEP data is low, this does not mean that there cannot be synchronization. It just means that mixing effects cannot be ruled out.

Consider now two coupled Rössler systems

$$\begin{aligned} \dot{s}_{1,2} &= -\omega_{1,2}u_{1,2} - v_{1,2} + c(s_{2,1} - s_{1,2}), \\ \dot{u}_{1,2} &= \omega_{1,2}s_{1,2} + 0.15u_{1,2}, \\ \dot{v}_{1,2} &= 0.2 + v_{1,2}(s_{1,2} - 10) \end{aligned} \tag{6}$$

with different eigenfrequencies $\omega_1 = 1.015$, $\omega_2 = 0.985$ and a piecewise constant coupling function $c = c(t)$, jumping between two values $c_{\mathrm{off}} = 0$ and $c_{\mathrm{on}} = 0.04$. The Rössler systems are integrated between $t = 0$ and $t = 30\,000$, with a step width of $\Delta t = 0.5$. The synchronization index γ has been calculated on sliding windows of length 400 with an overlap of 375, i.e., shifted in steps of 25 on (i) the original source signals $s_j(t)$, (ii) on the mixtures $x_i(t)$, which are generated according to Eq. (2) with a mixing matrix $A = [1, 0.95; 0.9, 1]$ plus some additional white Gaussian δ-correlated noise ($snr \approx 4$), and (iii) on the TDSEP-source estimates.

The upper panel of Fig. 2 shows that the Rössler systems switch between a synchronized and an unsynchronized state, depending on the coupling. Calculating the synchronization index on the mixtures (middle panel) yields uniformly high values in both the synchronous and the asynchronous regimes of the time series. In the TDSEP basis (lower panel) these two states can be well differentiated.

Real-world examples of synchronization phenomena that are masked by strong superpositions are, for example, EEG measurements. It is believed that synchronization effects play an important role in information processing of the human brain [1]. In the experimental setup that is considered here, a subject is asked to press a key with the left or the right index finger at self-chosen time instances (with intervals of at least 2 s). For the left keystrokes, the synchronization index between two adjacent electrodes (C2 and C4) over the motor cortex of the contralateral hemisphere has been computed [Fig. 3 (upper curve); for a detailed experimental setup, see [22]]. To obtain this high temporal resolution, we evaluate the mean in Eq. (1) as an average over trials rather than over time. The synchronization index γ shows uniformly high values that are spurious. In contrast, the TDSEP solutions reveal a pronounced synchronization peak lasting approximately from 500 ms before the keypress until 300 ms after it. The dashed lines show 95% quantiles with respect to background brain activity without movement. This finding allows us to observe the well-known movement preparation (see, e.g., [23]) as a synchronization effect in the *mu*-band (for this subject ≈11 Hz).

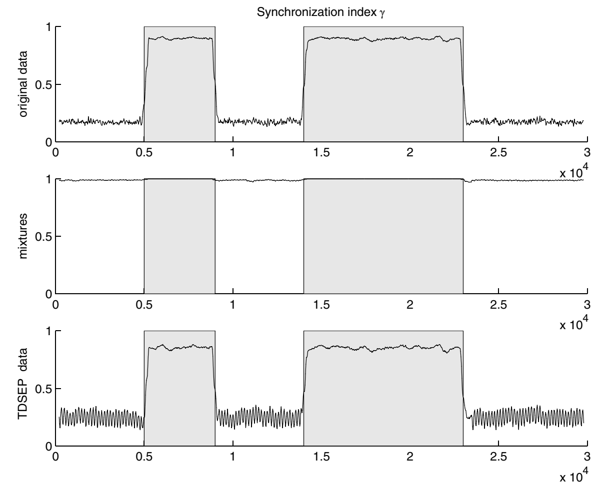

FIG. 2. Synchronization index calculated in shifting windows on the original data set [true Rössler time series (upper panel)], the mixtures (middle panel), and the TDSEP solutions (bottom panel). The gray boxes indicate regions of nonzero coupling.

This demonstrates that BSS methods are a suitable preprocessing step in the synchronization analysis of superimposed real-world signals. Superpositions are inevitable in every experimental setup or physical system where one has access only to surface measurements, while the interesting dynamical processes are hidden underneath this surface. Scalp measurements of EEG or magnetoencepha-

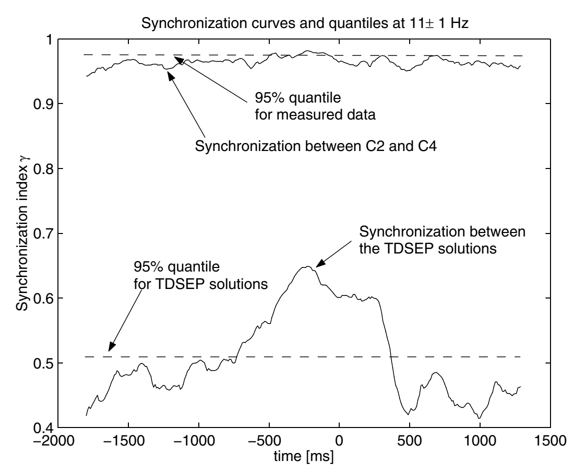

FIG. 3. Synchronization between EEG signals during finger movements measured by electrodes located over the motor cortex. The high phase locking values of the originally measured signals (upper curve) do not reveal any interesting temporal structure and can in large part be explained by superposition effects. The TDSEP-source estimates show a significant synchronization peak directly before the movement.

lography are very important examples, but the same reasoning applies to the study of active stars, to some experiments in materials science research, or to seismology. So synchronization phenomena that are hidden in multivariate time series of this kind will be masked by these mixing effects. The proposed method is applicable even for time-varying coupling (which is typical for many physical processes) and in the presence of observational noise. Even though in general the assumptions of the BSS algorithms are violated in the case of coupled oscillators, and they do not succeed in finding the original source signals, they can still transform the data into a basis that allows a more reliable estimation of synchronization indices. Instead of comparing the synchronization index of the measured data with those of suitably chosen surrogates (as proposed in [8]), we *directly* remove any spurious synchronization due to linear mixtures while maintaining the true synchronization.

In conclusion, we have devised a test to decide whether a high sychronization index stems from the measurement of intrinsically mixed signals or whether "true" sychronization is observed. Thus, our work contributes to a better understanding of complex physical systems.

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Appendix: Spurious Synchronization due to Mixing Effects.—To see how a linear superposition can lead to spurious synchronization, consider two simple oscillatory time series of the form $s_i(t) = \cos(\omega_i t)$, $i = 1, 2$ with $\omega_1 \neq \omega_2$. These time series are independent and consequently the synchronization index vanishes. If we apply a linear transformation $x_i(t) = \sum_{j=1,2} A_{ij} s_j(t)$ in the signal space, the transformed signals (i.e., mixtures) read $x_i(t) = B_i(t)\cos[\omega_1 t + \psi_i(t)]$ with $B_i(t) = \sqrt{A_{i1}^2 + 2A_{i1}A_{i2}\cos(\Delta\omega t) + A_{i2}^2}$, $\psi_i(t) = \arctan\frac{A_{i2}\sin(\Delta\omega t)}{A_{i1}+A_{i2}\cos(\Delta\omega t)}$, and $\Delta\omega = \omega_2 - \omega_1$. Since $B_i(t) > 0$ (except for the case of $|A_{i1}| = |A_{i2}|$), these functions can be interpreted as instantaneous amplitudes, while the instantaneous phases $\phi_i(t)$ are given by the argument of the cosine.

We now consider the one-parametric group of transformations of the form $A_{11} = A_{22} = 1$, $A_{12} = 0$, and $A_{21} = \alpha$. The phase term of the first mixture then simplifies to $\phi_1(t) = \omega_1 t$, and the phase difference between the two mixtures is given by $\delta(t) = \psi_2(t) = \arctan\frac{\sin(\Delta\omega t)}{\alpha+\cos(\Delta\omega t)}$. With rising α the phase difference concentrates around zero and therefore the synchronization index rises as well. In the extreme case of $\alpha \to \infty$, the observed phase values are all the same (i.e., spurious perfect PS). So linear mixing can lead to false detection of synchronization, even if the original signals are not synchronized at all.

*Electronic address: meinecke@first.fhg.de
†Electronic address: ziehe@first.fhg.de
‡Electronic address: klaus@first.fhg.de

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