## Measuring spatial coherence by using a reversed-wavefront Young interferometer

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Received November 14, 2005; accepted November 30, 2005; posted December 20, 2005 (Doc. ID 66013) A very simple optical setup for the measurement of the modulus and the phase of the two-point correlation function of a partially coherent light field is presented. The system consists of a slightly modified version of a Young interferometer and requires a single Young mask in order to determine the correlation function at any pairs of points. Experimental results are presented for the case of a synthesized partially coherent secondary source. © 2006 Optical Society of America

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Measuring the two-point field correlation function of electromagnetic fields is a task of great importance in optics, both from theoretical and applicative view-points. The standard device for the direct determination of the correlation function is, of course, the Young interferometer,<sup>1</sup> where the modulus and the phase of such a function are directly detected from the visibility and position, respectively, of the produced interference fringes. Despite its simple description, such an approach is actually quite cumbersome to implement, and ingenious systems have been conceived in the past to overcome one of the main drawbacks of the device,<sup>2</sup> namely, the huge number of required pinhole positions.

More complex and refined techniques, still based on a two-point interference experiment, have been developed. Most of them basically consist of detecting the intensity pattern produced by the superposition of a wavefront with a shifted (and possibly reversed) replica of it.  $^{3-13}$  Values of the modulus and the phase of the correlation function are typically retrieved by acquiring and postprocessing different patterns, obtained by varying the overall phase between the two replicas. In such a way, coherence between different pairs of points is displayed in a single interference pattern. Moreover, such systems are very efficient as far as the required light power is concerned. The main price to be paid in using wavefront-foldingbased techniques is the growing complexity of the experimental setups, which require very accurate and stable optical arrangements and precise phase shifters.

In this Letter we propose a simple device for the direct measurement of the modulus and the phase of the spatial coherence function of a quasimonochromatic optical field. It is a modified version of the classical Young interferometer but employs some peculiar features of reversed-wavefront interferometers. In particular, besides the elements of a standard Young interferometer (with a single mask), the proposed setup only requires the presence of a beam-splitter cube to produce a reversed replica of the incident wavefront. Experimental results carried out on a synthesized partially coherent source will be presented.

For the sake of clarity, we consider here a 2D case (i.e., field distributions depending on only one transverse coordinate), but the extension to the 3D one requires only slight modifications of the apparatus. The basic setup of the modified Young interferometer is sketched in Fig. 1. First, two replicas of an incident light beam are created by a beam-splitter cube (BS) and, because of the orientation of the cube, propagate along parallel directions. A similar arrangement was recently used in experiments concerning "ghost" imaging.<sup>14</sup> Because of the reflection that occurs at the internal surface of BS, one of the two replicas (the lower one in Fig. 1) turns out to be flipped with respect to such a surface. The two replicas then impinge onto a Young mask (Y), consisting of two pinholes with mutual distance d, in such a way that each of the holes of the mask samples points of one of the replicas. We shall refer to such an arrangement as a reversed-wavefront Young (RWY) interferometer. Unlike a classical Young interferometer, in which the correlation function must be sampled by varying the positions of both pinholes across the whole transverse plane, in the RWY interferometer the distance between the pinholes is kept fixed. To show this, we consider a given field distribution impinging on the BS, as sketched in Fig. 1. At a transverse plane beyond the BS, the field distribution is given by a sum of the form  $U(x) + U(x_0 - x)$ , where  $x_0$  represents the



Fig. 1. RWY interferometer.

distance between the replicas. Field values of the two replicas are then sampled by the mask Y, whose transverse position is specified by the coordinate ( $\xi$ ) of one of the holes in the *x* reference frame. The other hole, at  $\xi+d$ , samples a copy of the field that in the upper replica is located at  $\xi_e = x_0 - d - \xi$ . Then the sampling process is equivalent to taking field values at  $\xi$ and  $\xi_e$ . The intensity produced in the far field of the Young mask by a converging lens of focal length *f* can be written as<sup>1</sup>

$$I_{\text{tot}}(x_f) = I_1 + I_2 + 2\sqrt{I_1 I_2} |\gamma_{12}| \cos\left(\frac{2\pi d}{\lambda f} x_f + \phi_{12}\right), \quad (1)$$

where  $x_f$  denotes the transverse coordinate at the focal plane,  $\lambda$  is the wavelength,  $I_1 = \langle |U(x_0 - d - \xi)|^2 \rangle$  and  $I_2 = \langle |U(\xi)|^2 \rangle$  are the intensity values measured when holes 1 and 2 are open separately, and  $\gamma_{12}$  $=|\gamma_{12}|\exp(i\phi_{12})$  is the spectral degree of coherence<sup>1</sup> between points  $x_1 = x_0 - d - \xi$  and  $x_2 = \xi$ . Accordingly, the correlation function can be determined for all pairs  $(x_1, x_2)$  by varying the position of the Young mask and the distance between the replicas, which can be adjusted by moving the BS laterally. In particular, the mutual distance and average position of the two points turn out to be  $\delta_x = x_2 - x_1 = 2\xi - (x_0 - d)$ and  $\overline{x} = (x_2 + x_1)/2 = (x_0 - d)/2$ , respectively. Note that the above technique can be extended to the general 3D case without any conceptual difficulties. The only difference is that the replica of the incident field has to be flipped with respect to both axes, and the shifts of the reversed replica and of the Young mask have to be performed along both axes. Finally, it is important to notice that, since *d* is fixed, the period of the fringe pattern is independent of the coordinates of the sampled points, and the shift with respect to the origin of the reference frame depends only on  $\phi_{12}$ . This represents a key aspect of our arrangement, which makes the phase detection particularly simple and robust.

For the proposed technique to be tested experimentally, the spatial degree of coherence of a partially coherent secondary source has been measured by a RWY interferometer. The source has been synthesized starting from a laser beam and a rotating ground glass, following a procedure that was used in the past to produce Gaussian Schell-model sources.<sup>15</sup> A laser beam ( $\lambda = 532$  nm) with a Gaussian transverse profile is focused onto a rotating ground glass (GG) by a converging lens. The transverse extension of the radiation exiting GG, which can be thought of as being completely incoherent from the spatial point of view, is then limited by an iris diaphragm and is eventually sent into the RWY interferometer. The lateral position of the BS can be adjusted by a micrometric screw in order for the distance  $x_0$  between the two replicas to be changed. The Young mask Y, consisting of two circular holes having diameters of  $\sim$  300  $\mu$ m and a mutual distance of  $\sim$  9 mm, is placed at a distance z=30 cm from the exit face of the GG. The interference pattern is then observed across the back focal plane of a converging lens (100 mm focal

length), through a  $10 \times$  microscope objective, which images the pattern onto a  $256 \times 240$  CCD array.

The degree of coherence of the field at the mask plane can be evaluated by the van Cittert–Zernike theorem,<sup>1</sup> which yields

$$\gamma_{12} = \exp\left(-i\frac{2\pi}{\lambda z}\bar{x}\delta_x\right)\exp\left(-\frac{\pi^2 w^2}{\lambda^2 z^2}\delta_x^2\right), \quad (2)$$

where w denotes the spot size of the impinging Gaussian beam across the GG plane. It should be noted that, although the modulus of  $\gamma_{12}$  is shift invariant, its phase is not. The value of w was measured by directly imaging the intensity distribution of the incoherent source onto the CCD array, and it turned out to be  $w = 155 \pm 10 \ \mu m$ .

Measures of the modulus and phase of  $\gamma_{12}$  were performed in the following way. For a typical position of the mask Y, the intensity patterns  $I_1$ ,  $I_2$ , and  $I_{tot}$ were acquired. From the above three intensity distributions the matrix  $M = (I_{tot} - I_1 - I_2)/(2\sqrt{I_1I_2})$  was computed. Note that, as can be seen from Eq. (1), values of M are independent of the vertical coordinate, apart from noise effects. Averages over all the lines of Mhave then been used to reduce the noise. According to Eq. (1), the parameters of the fit, together with their associated spreads, provide the values of the modulus and phase of  $\gamma_{12}$  with the pertinent uncertainties. Figure 2 shows measured values of the modulus of the degree of coherence (dots) as functions of the dimensionless variable  $\delta_x/\sqrt{\lambda z}$ , together with the theoretical behavior (solid curve) obtained from Eq. (2). Experimental uncertainties turned out to be of the order of a few percent or less. Measured values of the phase of  $\gamma_{12}$  (in units of  $2\pi$ ) are shown in Fig. 3 as functions of  $\bar{x}/\sqrt{\lambda z}$  and  $\delta_x/\sqrt{\lambda z}$  on a (5×5)-point grid across the  $(\delta_r, \bar{x})$  plane. Experimental uncertainties for the phase values were approximately 1/10 of a rad or less. To quantitatively compare the experimental phase values to the theoretical ones given by Eq. (2), we evaluated the angular coefficients of the bestfitting lines corresponding to measured values of the



Fig. 2. Experimental (dots) and theoretical (solid curve) values of the modulus of the degree of coherence as functions of  $\delta_r/\sqrt{\lambda z}$ .



Fig. 3. Experimental values of the phase (in units of  $2\pi$ ) of the degree of coherence as functions of  $\bar{x}/\sqrt{\lambda z}$  and  $\delta_x/\sqrt{\lambda z}$ .



Fig. 4. Experimental values (dots) of the angular coefficients of the best-fitting lines corresponding to measured values of the phase, obtained on varying  $\delta_x$  for fixed values of  $\bar{x}$ , together with the theoretical behavior (solid line). The inset shows a typical set of phase values taken at  $\bar{x} = 100 \ \mu \text{m}$ .

phase, obtained on varying  $\delta_x$  for fixed values of  $\bar{x}$ . According to Eq. (2), in fact, the behavior of the phase as a function of  $\delta_x$  should be linear, with the angular coefficient given by  $-2\pi\bar{x}/(\lambda z)$ . The set of values corresponding to  $\bar{x} = 100 \ \mu m$  is shown as an example in the inset of Fig. 4 together with its best-fitting line (dashed). Angular coefficients evaluated for all the used values of  $\bar{x}$  are shown as dots in Fig. 4, where the normalized coordinates  $\bar{x}/\sqrt{\lambda z}$  and  $\delta_x/\sqrt{\lambda z}$  are used, together with their theoretical behavior (solid curve), which is a line with its slope given by  $-2\pi$  and

its bias depending on the choice of the origin of the *x* axis.

In conclusion, the RWY interferometer seems to be very attractive because of its extreme simplicity, compactness, stability, low cost, and the fact that it can be easily assembled in any optics laboratory. Furthermore, since no anisotropic devices or optical fibers are used, its use could be considered for joint coherence–polarization measurements.<sup>16,17</sup> An obvious drawback of the technique is that measures have to be taken sequentially, but acquisition and processing times can be made significantly shorter by suitably automatizing the procedures.

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## References

- 1. M. Born and E. Wolf, *Principles of Optics*, 7th ed. (Cambridge U. Press, 1999).
- 2. E. Tervonen, J. Turunen, and A. T. Friberg, Appl. Phys. B **49**, 409 (1989).
- M. Imai, Y. Ohtsuka, and S. Satoh, J. Opt. Soc. Am. A 3, 1059 (1986).
- M. G. Raymer, M. Beck, and D. F. McAlister, Phys. Rev. Lett. 72, 1137 (1994).
- C. Iaconis and I. A. Walmsley, Opt. Lett. 21, 1783 (1996).
- 6. H. Arimoto and Y. Ohtsuka, Opt. Lett. 22, 958 (1997).
- D. Mendlovic, G. Shabtay, A. W. Lohmann, and N. Konforti, Opt. Lett. 23, 1084 (1998).
- K. F. Lee, F. Reil, S. Bali, A. Wax, and J. E. Thomas, Opt. Lett. 24, 1370 (1999).
- 9. P. Ferraro, S. De Nicola, A. Finizio, and G. Pierattini, Appl. Opt. 41, 342 (2002).
- E. Mukamel, K. Banaszek, I. A. Walmsley, and C. Dorrer, Opt. Lett. 28, 1317 (2003).
- S. Chatterjee and O. Prakash, Opt. Laser Technol. 35, 133 (2003).
- J. Surville, L. Videau, S. Montant, C. Stenz, and C. Sauteret, Opt. Lett. 29, 2336 (2004).
- F. Pfeiffer, O. Bunk, C. Schultze-Briese, A. Diaz, T. Weitkamp, C. David, J. F. van der Veen, I. Vartanyants, and I. K. Robinson, Phys. Rev. Lett. 94, 164801 (2005).
- A. Gatti, M. Bache, D. Magatti, E. Brambilla, F. Ferri, and L. A. Lugiato, J. Mod. Opt. (to be published), arXiv: quant-ph/0504082.
- P. De Santis, F. Gori, G. Guattari, and C. Palma, Opt. Commun. 29, 256 (1979).
- F. Gori, M. Santarsiero, S. Vicalvi, R. Borghi, and G. Guattari, Pure Appl. Opt. 7, 941 (1998).
- 17. H. Roychowdhury and E. Wolf, Opt. Commun. **252**, 268 (2005).