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Measuring the impact of efficient household travel decisions on potential travel time savings and accessibility gains

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ABSTRACT

Using the conceptual framework of time-space geography, this paper incorporates both spatio-temporal constraints and household interaction effects into a meaningful measure of the potential of a household to interact with the built environment. Within this context, personal accessibility is described as a measure of the potential ability of individuals within a household not only to reach activity opportunities, but to do so with sufficient time available for participation in those activities, subject to the spatio-temporal constraints imposed by their daily obligations and transportation supply environment. The incorporation of activity-based concepts in the measurement of accessibility as a product of travel time savings not only explicitly acknowledges a temporal dimension in assessing the potential for spatial interaction but also expands the applicability of accessibility consideration to such real-world policy options as the promotion of ride-sharing and trip chaining behaviors. An empirical application of the model system provides an indication of the potential of activity-based modeling approaches to assess the bounds on achievable improvements in accessibility and travel time based on daily household activity patterns. It also provides an assessment of roles for trip chaining and ride-sharing as potentially effective methods to facilitate transportation policy objectives.

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1. INTRODUCTION

Travel demand theory, whether derived from consumer demand theory or direct demand principles, is intrinsically rooted in the notion that travel time is a commodity to be saved. To the extent that travel time is not merely just a surrogate for the actual economic cost of travel, the implication is that the time savings can and will be transformed by the traveler into something of intrinsic value – ostensibly either in more time spent on performing activities of economic, or other, value, or in increasing the "capture space" of alternative locations for such activities. In the first instance, the time savings is implicitly assumed to be generally transferable to any other activity, including travel, that may be on the traveler's agenda – in traditional trip-based demand analysis, while the saving of travel time is an explicit consideration of the modeling process, the allocation of the savings is virtually always an exogenous consideration not addressed. In the second instance, the travel time savings is "mapped" into a spatial measure of "reachability," commonly referred to as accessibility.

Although springing form the same well, concepts of accessibility and travel demand have largely been treated separately. With the growth and standardization of the travel demand forecasting process and the associated focus on trip making, concepts of accessibility were primarily associated with the land use forecasts fronting the process and, to some degree, the policy evaluation which followed the process. Between Hansen's (1959) early model development and seminal review works twenty years later by Pirie (1979) and Morris *et al*. (1979), most development and application of accessibility concepts were conducted quite independent of developments and applications of demand forecasting. Early accessibility studies are almost exclusively concerned with the spatial aspects of travel decisions (Ingram, 1971; Dalvi and Martin, 1976). Limitations on the definition and estimation of traditional accessibility measures in transportation planning have restricted applications to the use of simple aggregate indices, temporally independent, which can not reflect the potential for connectivity between activities. Applications of accessibility concepts have, therefore, been limited to such aggregate analyses as land use and facility location studies (for example, see Hansen, 1959). Consequently, the impacts of changes of transportation policy cannot be expressed easily in terms of accessibility-related effects, although the provision of accessibility is typically an important goal in the implementation of new transportation policy options.

The time-space geography paradigm Hagerstrand (1970) offered the potential to better integrate the spatial and temporal components of travel-related decisions that underpin the concepts of travel demand and its relationship to accessibility. The approach held potential as a basis for deductive accessibility measurement and as one of the foundations for inductive diary analysis. It is commonly referred to as the "constraint-based approach," given the defining role that spatial and temporal constraints in the formulation; space is typically expressed as a two-dimension plane, while time is depicted via a third, vertical axis. Within this three-dimensional space, so-called time-space prisms define the limits of what is accessible. Lenntorp (1976, 1978) extended Hagerstrand's approach by developing a model that calculated the total number of space-time paths an individual could follow given a specific activity program (i.e., the set of desired activities and durations) and the urban environment, as defined by the transportation network and the spatial-temporal distribution of activities. Lenntorp's PESASP (Program Evaluating the Set of Alternative Sample Paths) model, noteworthy since it represented the first attempt to operationalize the theoretical framework advanced by Hagerstrand in a manner that would allow meaningful policy evaluation, was followed by two parallel lines of work. The first, extending Lenntorp's full pattern generation approach, began with CARLA (see Jones *et al.*, 1983) and STARCHILD (Recker *et al*., 1986a, 1986b). The second, focussed on the accessibility aspects of the time-geography approach, is exemplified by Burns (1976, 1979) who examined constraints on individual behavior through a methodological study of accessibility. Burns viewed accessibility as the freedom of individuals to participate in different activities and, with the aid of the space-time prism, investigated the dependence of accessibility on its transportation, temporal, and spatial components. Accessibility benefit measures were constructed based on different assumptions about how individuals value the opportunities available to them, and these were used to analyze and compare the accessibility implications of a variety of transportation, temporal, and spatial strategies.

Both Morris *et al*. (1979) and Pirie (1979) provided comprehensive reviews of the state-of-the-art. Morris *et al*.(1979) classified accessibility approaches as conventional (e.g. Hansen, 1959), axiomatic (e.g., Weibull, 1976), and user benefit (Koenig), but did not include the constraint-based approach. Twenty years later, Miller (1999) reconciled these complementary approaches, as well as the constraints-oriented approach of time-geography, deriving accessibility and benefit measures that extended the utility formulation of Burns (1979) and were consistent with Weibull's (1976, 1980) axiomatic framework. Kwan (1999) provided some empirical assessment of a range of accessibility measures.

Pirie (1979) extended his review with a proposal that accessibility be portrayed "as a condition (a vacancy) in an activity routine which, either deliberately created or formed as a residual, permits travel to and from and participation in one or more activities." Accessibility, he suggests, could be measured "in terms of the cost of creating the vacancy" and that these costs could reflect the degree of "activity, role, space, and time substitutions incurred." Pirie suggests that such time-geography approaches as Lenntorp's PESASP model could provide the framework for such accessibility measurement.

Adaptations and applications of Pirie's proposal are numerous (Arentze *et al*., 1994a, 1994b; Niishi and Kondo, 1992); however, operationization of Hagerstrand's framework to meaningful policy evaluation has proven extremely difficult, principally because of the complexity of the analytics embedded in the time-space approach. Most often, studies have either been restricted to descriptive analysis or have involved computer-oriented model systems that produce the enumeration of feasible time-space paths. Because of the complicated dimensionality of the problem, these latter efforts have relied principally on exhaustive enumeration or the application of sequential procedures to reduce the complexity in forming the activity pattern (Kitamura and Kermanshah, 1983, 1984; Recker et al., 1986a, 1986b). In a few cases, structural equation systems and simple multivariate data analysis techniques have been employed to determine the relationship between activity patterns and socio-demographic attributes (Golob, 1985, 1986; Golob and Meurs, 1987). With few exceptions (Koppelman and Townsend, 1987; Golob and McNally, 1997), prior research has focused on the activity patterns of individual members of households; work addressing the collective travel behavior by household members is quite limited.

Herein, we address Pirie's proposal with a conceptual framework that incorporates both spatio-temporal constraints and household interaction effects into a more meaningful measure of the potential of a household to interact with the built environment. Taken together, the components of the framework can be described by the Household Activity Pattern Problem, or HAPP (Recker, 1995); the solution patterns reveal personal travel behavior and activity participation within a household context, while preserving the concept that accessibility originates from participation in activities, that travel constitutes the linkage between activities, and in which all of the required components are contained in the activity scheduling problem.

Within this context, personal accessibility is described as a measure of the *potential ability of individuals within a household not only to reach activity opportunities, but to do so with sufficient time available for participation in those activities, subject to the spatio-temporal constraints imposed by their daily obligations and transportation supply environment.* The specific issue being addressed is not personal accessibility *per se*, but rather how travel decisions can improve accessibility. The intent is to identify and measure the potential for interaction that remains after accounting for the consumption of time-space needed to accomplish an individual's daily demand for in- and out-of-home activities.

The spatial location and temporal availability of activity sites, together with the maximum speed an individual can travel between sites, establishes the individual's space-time prism; the volume of this prism encompasses the full range of possible locations at which an individual can participate. Once an individual initiates a pattern, the potential action space for subsequent activities is reduced as a function of characteristics of prior activity (e.g., duration). The aggregation of changes the volume of the time-space prism in Hagerstrand's 3-dimensional space for each individual brought on by any particular travel/activity decision protocol can be interpreted as an assessment of macroscopic accessibility accruing to the household as a result of that particular decision regime.

Figure 1 provides an example to illustrate the calculation of such an accessibility measurement for a hypothetical member of a household, if the associated activity diary is known. In the case illustrated, the individual has three out-of-home activities that are to be completed, as well as two scheduled activities to be completed in the home. The spatio-temporal path, reconstructed from the activity diary of this hypothetical individual, is shown by the solid line; its projection onto the spatial plane is represented by the directed line segments. In the figure, the shaded prisms encompass the physical space that can be reached during the periods for which no specific activities are scheduled. The volumes of these prisms thus offer a measure of the potential to engage in additional activities. The "packaging" of the activity/travel decisions obviously impact the nature of such prisms contained in any activity pattern; in general, efficient travel decisions (e.g., trip chaining) tend to increase the potential to engage in additional activities should either the need or desire arise. An example of this is provided by the pattern shown in Figure 2, in which Activity 2 has been chained with Activity 1 rather than with Activity 3. When compared to the original pattern, the pattern in this figure results in a net gain in accessibility (at the home location), as well as a "transference" of accessibility to a different spatio-temporal location centered about the spatial location of Activity 2. Although not shown in this example, it is easy to imagine a similar transference that could take place between individuals in a particular household if, for example, another member of this particular household assumed responsibility for completing Activity 3. It is precisely these sorts of considerations that the approach described herein is designed to capture.

The approach illustrated above is of general use in the estimation of the transference of accessibility under a range of relevant policy scenarios. The addition of a temporal dimension provides the ability to measure the variation of accessibility by time-of-day, while the analysis of personal accessibility facilitates a better understanding of transference of accessibility among household members, both in terms of ride-sharing and trip-chaining behaviors. After a presentation of the overall model formulation, the potential utility of this approach in policy analysis is demonstrated using travel survey data.

4. Activity Pattern Representation

The household travel-activity decision-making process underlying the proposed activity-based measure of accessibility is based on an extension of the mathematical programming approach offered by Recker (1995) in which the household activity pattern problem (HAPP) is posed as a network-based routing model incorporating vehicle assignment, ride-sharing behavior, activity assignment and scheduling, and time window constraints. The general approach involves treating the HAPP as an analogy to the so-called Pickup and Delivery Problem with Time Windows (PDPTW).

In the analogy to the PDPTW, activities are viewed as being "picked up" by a particular household member at the location where performed and, once completed (requiring a specified service time) are "logged in" or "delivered" on the return trip home. Multiple "pickups" are synonymous with multiple sojourns on any given tour. The scheduling and routing protocol relative to some household objective produces the "time-space diagram" commonly referred to in travel/activity analysis.

The problem is defined by a network graph G=(V, A), where V is the set of all vertices, and A is the set of all arcs in the network. Physically, V can be a set of demand nodes, and A can be explained as the connections between these demand nodes. The standard Vehicle Routing Problem (VRP), that is applied in numerous studies (Golden, 1984; Desrochers, *et al*., 1988; Solomon and Desrosiers, 1988) is defined on this graph as the visit to each node once and only once by a stable of vehicles with specific capacity constraints. The Household Activity Pattern Problem (HAPP) is described as: *Minimize a hypothetical objective function* (which generally expresses some "generalized cost" to the household in order to complete all of the activities needed to be performed by the household members) *subject to the constraints related to transportation supply, time windows, vehicle capacity, and logical connection between activity nodes*. The HAPP, which is more complex than a generic VRP, can be defined on an expanded graph with the addition of temporary returning home nodes, and the replacement of the activity nodes with drop-off and pick-up function nodes, which physically represent the same locations as those of the activity nodes, and logically are used to explain different purposes of that trip. The requirements for the household members to complete all scheduled activities (visiting all activity nodes), which could be performed either by some specific person or by anyone available, are sustained within this model. Each activity in the HAPP must be performed once and only once (equivalent to the definition that each vertex of the network in the VRP should be visited once and only once), and there is a limitation on the time period of performing the activity.

Inclusion of ride-sharing options greatly increases the complexity of the model structure over that required for a non-ride-sharing formulation. However, the ride-sharing case can be treated by enumerating all possible ride-sharing options of the non-ride-sharing formulation, which is a VRPTW-based problem with budget constraints. Employing a relaxation on the number of available vehicles within a household, the problem with ride-sharing can be reduced to a VRPTW. Furthermore, assuming that the triangle inequality is satisfied, the solution of VRPTW can be shown to be a lower bound to the original problem.

The resulting HAPP formulation is in the form of a Mixed Integer Linear Programming (MILP) model. Details of the original formulation can be found in Recker (1995). The results quoted in this paper are based on a modified version of the mathematical program developed in that work (in terms of both the definition of variables and specification of problem constraints); however, the same general considerations apply. In the present study, the form of the HAPP mathematical program formulation of the travel/activity decisions for a particular household, say i, during some time period is represented by:

$$\text{Minimize } \Phi(\mathbf{Z}_i) = \mathbf{B}_i' \cdot \mathbf{Z}_i$$

$$\text{subject to :} \tag{1}$$

$$\mathbf{A}\mathbf{Z}_i \leq \mathbf{0}$$

where

$$\mathbf{Z}_i = \begin{bmatrix} \mathbf{V} \\ \mathbf{X} \\ -- \\ \mathbf{U} \\ \mathbf{Y} \\ -- \\ \mathbf{T} \\ \mathbf{H} \\ \mathbf{R} \end{bmatrix}, \ \mathbf{V} = \left[V_u^{\upsilon} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \right], \mathbf{X} = \left[X_{u_e w_f}^{\upsilon\, r} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \right], \ \mathbf{U} = \left[U_u^{\alpha} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \right], \mathbf{Y} = \left[Y_{u_e w_f}^{\alpha} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \right],$$

$$\mathbf{T} = \left[T_{u_e} \geq 0 \right], \ \mathbf{H} = \left[H_{u_1}^{\upsilon\alpha} \geq 0 \right], \ \mathbf{R} = \left[R_{u_e}^{\upsilon\alpha} \geq 0 \right]$$

The outputs $\mathbf{Z}_i$ of the optimization for each household i are specified by the following decision variables[1]:

V_u^{υ} binary decision variable equal to unity if vehicle $\upsilon \in V$ is used to perform activity $u \in A$, and zero otherwise.

$X_{u_e w_f}^{\upsilon\, r}$ binary decision variable equal to unity if r ($r = 1, \ldots, |\gamma|$) household members use vehicle υ to travel from a function $e \in F$ performed at activity node u to a function $f \in F$ performed at activity node w, and zero otherwise.

U_u^{α} binary decision variable equal to unity if activity u is performed by household member $\alpha \in \eta$, and zero otherwise.

$Y_{u_e w_f}^{\alpha}$ binary decision variable equal to unity if household member " travels from a function e performed at activity node u to a function f performed at activity node w, and zero otherwise.

T_{u_e} the time at which performance of function e begins at activity location u; $\forall u_e \ni \exists$ a value $Y_{u_e w_f}^{\alpha} \neq 0$, for some $\alpha \in \eta, w \in P, f \in F$.

$H_{u_1}^{\upsilon\alpha}$ the time at which household member " returns to home from activity u using vehicle υ; $V_u^{\upsilon}, U_u^{\alpha} \neq 0$.

$R_{u_e}^{\upsilon\alpha}$ the time at which household member " arrives at node u to perform function e by using vehicle υ; $\forall u_e, \alpha, \upsilon \ni \exists$ a value $Y_{u_e w_f}^{\alpha} \neq 0$, for some $w \in P, f \in F$ and a value $X_{u_e w_f}^{\upsilon\, r} \neq 0$, for some $w \in P, f \in F, r = 1, \ldots, |\gamma|$.

The various sets referenced in the above are defined by the following notation:

$A = \{1, 2, \ldots, j, \ldots, n\}$ the set of out-of-home activities scheduled to be completed by travelers in the household.

[1] The relationships among these decision variables that form the resulting mathematical program are presented in the appendix.

$\eta = \{1, 2, \ldots, \alpha, \ldots, \mid \eta \mid\}$	the set of all potential travelers in the household.
$V = \{1, 2, \ldots, \upsilon, \ldots, \mid V \mid\}$	the set of vehicles available to travelers in the household to complete their scheduled activities.
$P = \{1, 2, \ldots, u, \ldots, n)$	the set designating location at which each activity is performed.
$F = \{1, 2\}$	the set of functions that may be performed at an activity node, with "1", representing the eventual "return home" portion of a trip associated with a particular activity (a "delivery" in the VRP problem), and "2", the trip to a particular activity location (a "pickup" in the VRP problem). Physically, these functions are performed at the locations of the associated activity nodes, and logically, these functions are used to represent those expanded nodes associated with the original activity nodes.

In Equation (1) $\mathbf{B}_i$ is a vector of coefficients that defines the relative contributions of each of the decision variables to the overall disutility of the travel regime to the household. Descriptively, the constraint sets $\mathbf{AZ}_i \leq \mathbf{0}$ for this MILP are classified into six groups: (a) routing constraints that define the allowable spatial movement of vehicles and household members in completing the household's activity agenda; (b) scheduling constraints specify the relationship of arrival time, activity begin time, and waiting time, and continuity condition along the temporal dimension; (c) assignment constraints that are applied to match the relations between activity participation and vehicle usage as well as activity performers (household members); (d) time window constraints that are used to specify available schedules for activity participation; (e) coupling constraints that define the relations between vehicle-related variables and member-related variables; and (f) side constraints including budget, capacity, and rules for ride-sharing behavior.

The solution vector, $\mathbf{Z}_n^*$, to Equations (1) represents the household's utility maximizing behavior with regard to completing its activity agenda. The solution patterns reveal personal travel behavior and activity participation within a household context, while preserving the concept that the need for travel originates from participation in activities, that travel constitutes the linkage between activities, and in which all of the required components are contained in the activity scheduling problem.

5. SOLUTION PROCESS

The use of a mathematical programming approach offers some distinct advantages to activity-based analyses. In addition to its ability to verify the existence and correctness of the complex solutions arising from activity-travel path formulations, it is arguably the only approach that inherently accommodates the continuity and consistency requirements governing the series of activities and travel links that comprise a traveler's movement through the time and space continuum during a given period of analysis. Despite these strong conceptual advantages, the approach offers some serious challenges that must be overcome to enable its practical implementation as a tool for policy analysis. Such formulations typically contain a large number of constraints and variables, and the corresponding computational effort increases exponentially with the number of constraints and variables.[2] In general, it is extremely cumbersome to solve large-scale problems.

Such problems arise principally from the feature that the formulation is of the mixed integer programming type, which belongs to the so-called NP-Hard (non-deterministic polynomial time) class of problem; the number of fundamental computations increases exponentially with the size of the problem. Consequently, it is practically impossible to solve a large-scale problem in a limited amount of time, using formal solution algorithms. Because of these factors, a heuristic method was developed specifically to solve the HAPP.

The solution procedure that was employed is divided into a two-stage process. At the first stage, a VRPTW is solved to generate initial feasible solutions; at the second stage, an enumeration of tours with the ride-sharing option is generated subject both to the number of available vehicles and to a series of filters to screen out all illogical connections between activities. Since the feasible solution space of the first stage is larger than that of the second stage and the objective functions are the same; the solution generated from the first stage is feasible if any solution of the second stage is

[2] In the particular formulation of the HAPP used herein, an augmented network is constructed with nodes that number four times that of the original related vehicle routing problem. Moreover, additional complexity is associated with the included indices representing both the vehicles and persons as well as additional constraints on temporal feasibility and resource budgets.

feasible. This offers a way of reducing the enumeration by eliminating all infeasible initial solutions.

In the first stage of the solution procedure, a VRPTW-based problem with resource constraints is constructed with the introduction of a vehicle relaxation concept. The solution procedure for the first stage is derived from an analysis of constrained shortest path problems and the concept of opportunities from travel time savings. By adopting a shortest path algorithm, it is possible to find the locally least-cost tours. Application of relative savings information enables efficient comparisons among the possible candidate links to be inserted in the least cost tours. The combination of a constrained shortest path algorithm and the "savings method" produces a hybrid heuristic that integrates tour construction and tour improvement. Given a multipartite network with N x N nodes, the algorithm is:

Step 0: Set: k = 0

Step 1: Calculate the savings matrix **S**:

$$\mathbf{S} = [s_{ij}]; \quad s_{ij} = (t_{oi} + t_{oj} - t_{ij}) - w_{ij}$$

where:

$$w_{ij} = \max\{a_j - (T_i + s_i + t_{ij}),\ 0\}$$

and:

w_{ij} = the minimal required waiting time to go from node *i* to *j*

t_{ij} = travel time from node *i* to node *j* (o = current location)

a_j = the early time window at node *j*

T_i = activity begin time at node *i*

s_i = activity duration at node *i*

Step 2: Calculate opportunity cost matrix O:

$$\mathbf{O} = [o_{ij}]; \quad o_{ij} = \max_{m,n}\{s_{mn}\} - s_{ij}.$$

Step 3: Define the initial condition and the recurrence equations:

$$f_0(o) = 0$$

$$f_{k+1}(j) = \min\{f_k(i) + o_{ij} + t_{ij} + w_{ij}\},\ \forall j \in \mathbf{V}\ .$$

Step 4: Check feasibility of including the next node; if yes, go to 5; otherwise, go to Step 6.

Step 5: Set k=k+1; calculate $f_k(n)$ for each node n until all stages are finished.

Step 6: Calculate the cost for the whole network based on the sum of the constructed tours and all remaining single vertex tours.

Step 7: If no nodes remain, then terminate; otherwise, extract the selected tour from the network and go back to step 0 with the remaining network.

The algorithm begins with an initialization, and then calculates the general savings matrix from spatial and temporal variables. The opportunity cost is defined as the cost of not utilizing least-cost links. As a basis for dynamic programming, the recurrence equation is defined from opportunity cost. The feasibility of connecting the next node into the tour is tested for adherence to time window constraints, and compliance to person-node pairs (which mate an activity node with the person to perform that activity). The scheduling is handled by "moving forward", in which the arrival time at the activity node and the activity begin time are adjusted by advancing the time frame to find the first available time point for node insertion. The maximum time to shift forward and backward are stored for subsequent manipulation of the time window for the next node. If inserting the next node into the tour is infeasible or if the resulting savings is negative, a new defined cost is calculated that accounts for the spatial and temporal cost of the current and all other single-node tours. Otherwise, combination of nodes continues until the end of the stage.

In the second stage, the heuristic described above is modified by an insertion algorithm designed to deal with the large dimensionality of the problem. The algorithm is a modification of the savings method and based on the concept of "wasting," defined as the additional travel time needed to utilize ride-sharing. The resulting algorithm can be described as follows:

Step 1: Calculate the "wasting" of ride-sharing and "savings" of trip chaining.

Step 2: Sort the wasting and savings of each node-pair in ascending order.

Step 3: Select the minimal wasting node-pair to be the ride-sharing option and then apply the savings method to insert the other nodes into the first node-pair.

Step 4: Check the side constraints regarding the time window, budget, load, and vehicle-node-pair constraints. If violation of the side constraints is detected, select the node-pair with the next minimal wasting and go to Step 3.

Step 5: Repeat until all of the nodes are exhausted.

In combination, these algorithms take particular advantage of the characteristic that available matching patterns are quite selective and regular, which greatly simplifies the ride-sharing matching process through a combination of "wasting" and "savings" for all available nodes. For a typical problem involving a household with five members and twenty activities, the solution process is less than ten seconds in a standard personal computing environment.

6. EMPIRICAL APPLICATION

Observed activity patterns are the result of decisions made by household members subject to restrictions placed either by transportation supply or resulting from a variety of unobserved factors that influence travel behavior. The collection of individual travel/activity patterns provides a description of revealed household travel demand. Optimal travel/activity patterns derived from solution of the HAPP represent normative behavior that is expected to exist under conditions constrained *only* by spatio-temporal conditions. The gap between the *observed* and *optimal* provide a measure of the potential benefit that may be achieved through elimination of these unobserved restrictions. As such, solution to the HAPP when compared to a base scenario defined by the existing, or observed, activity/travel patterns can be used as a performance measure to evaluate the limits of the potential of specific policy alternatives that derive their benefit from either imposing or encouraging changes on daily activity/travel patterns.

To demonstrate its potential use in policy evaluation, upper-bounds of two widely-promoted TDM goals, increased ride-sharing and trip chaining, on accessibility and travel time gains are examined for a sample of residents drawn from the Portland, Oregon region of the United States. Specifically, the HAPP model system is used to obtain distributions of potential travel time savings and changes in an accessibility measure, defined by the volume of Hagerstrand space-time prisms, that would result from optimal activity scheduling, trip chaining, and ride-sharing. These potential gains are related to a variety of life-style, socio-demographic, and activity diary variables through ordinary least squares regression.

6.1 Data Specification

Since the HAPP model represents activity/travel behavior of the collective members of a household, detailed information is required on travel and activity participation for each member, as well as transportation supply information (including household vehicle holdings and network travel times). These data were available from the Southwest Washington and Oregon Area 1994 Activity and Travel Behavior Survey, which contains sufficiently detailed information, including comprehensive travel/activity diaries (with mode availability) and a regional transportation network model, for an application of the HAPP model.

The sample drawn for the survey included 2,232 households and 5,125 persons with a total of 67,016 activities and 37,965 trips (each split fairly equally between two consecutive survey days). Available household information includes: household size, household income, type of dwelling unit, and the number of available vehicles. The survey also provides person-level data including age, gender, employment status, occupation, student status, and driver license status. For this analysis, the sample was first segmented according to household size, and two-member, three-member, and four-member households that were headed by adult male-female couples were retained for further analysis.[3] As a further simplification, households having non-auto related travel activities were excluded, and in-home activities were assumed to be completely flexible in terms of their scheduling. Based on these criteria, the results presented here are for sub-samples of 402, 193, and 153 two-member, three-member, and four-member households, respectively. Summary statistics relating to household and travel characteristics are provided in Tables 1 and 2, respectively.

Network information, in the form of a household-specific matrix of travel times between all locations named in the household's travel diaries, was estimated by generating skim trees based on non-peak-hour travel times based on the coded network of 1,260 Traffic Analysis Zones, 21,868 links, and 9,703 nodes. A comparison between network and survey travel times indicated tendencies of respondents to round off reported travel time (to 5 or 10 minute categories) and, in most cases, to report a substantially longer travel time for trips with network travel times that were less than 5 minutes. A two-stage adjustment procedure was used to establish congruence between network and reported travel times. The first stage involved adjusting the network times, categorized in discrete time intervals, by the ratio of the mean reported time to network time for each of the three sub-samples analyzed. This global adjustment of network travel time is then further adjusted to account for any differences in perception of individual households by multiplying the adjusted network times associated with that household's activity patterns by the ratio between the household's total reported travel time and total adjusted network travel time for their patterns.[4] Following this adjustment, the sum of the reported travel times will be equal to the sum of the corresponding network travel times for each household, providing a basis for comparison between revealed and normative behavior.

6.2 Achievable Gains in Accessibility and Travel Time with Optimal Scheduling, Ride-sharing and Trip Chaining

Summary distributions of the potential savings in household travel time and changes in accessibility for each of the three household samples and the total sample are provided in Figures 3 through 7. Figures 3 and 4 describe the distributions of both the relative and absolute improvements in travel time associated with optimal scheduling, ride-sharing and trip chaining. The distributions of absolute travel time savings (Figure 3) are seen to be similar for the three household groupings and generally resemble a negative exponential distributions. While a large portion of the households can be expected to have only minor relative improvement in daily travel time with optimal scheduling, ride-sharing and trip chaining (almost forty percent of the total sample were projected to have a maximum savings of less than 20 minutes), almost thirty percent of sampled households exhibit the potential for gains in travel time that are in excess of one hour (Figure 7). The general positive relationship in these figures between the percentage of households achieving a specific potential improvement and household size is speculated as an artifact of a

[3] This stratification simplifies analysis and facilitates interpretation of results.

similar general tendency between the size of the household and the complexity of the related activity pattern (i.e., greater household size leads to more activities and trips, which lead to more complex time-space paths, which lead to a greater opportunity for finding efficiencies). The distributions of relative travel time savings (Figure 4), as measured by percent improvement in travel time, are generally flatter than their absolute counterparts; a quarter of the total sample would be expected to reduce their daily travel time by at least a fifty percent were they to fully exploit available scheduling, ride-sharing and trip chaining options.

Just what can be inferred from these results? It is emphasized that no claim is made that the full extent of travel time savings indicated here are achievable in practical terms. Under usual assumptions of rational choice, the observed travel/activity patterns are "optimal" in some sense relative to the collection of aspects that form any particular household's utility function. To the extent that the minimization of travel time is an important component of such utility, the results give some indication of the impact of considerations other than those strictly involving the travel aspects (e.g., travel time, auto availability) of activity participation on the household's decision structure. Such considerations likely include the appropriateness of grouping/ordering certain activities, personal tastes on when certain activities are best performed, and uncertainties involving activity duration, or even when the need/desire to participate in the activity at all arises. When taken in this light, the results give some measure of the infringement of these latter factors (and, likely, a host of other such factors) on the commonly held principle of travel time minimization that is presumed to govern travel decisions. For example, it may be inferred from the results that, for approximately twenty-five percent of the total sample, considerations other than travel time stand in the way of potential travel time savings of as much as fifty percent in executing their daily activity patterns.

Clearly, the potential travel time savings alluded to above have been weighed by the sample against other factors and a determination made that the utility gain from these "unobserved" factors outweighs the travel time savings. One tangible indicator of the magnitude of the tradeoff represented by this observation is captured in Hagerstrand's measure of accessibility defined by the cumulative volume of the space-time prisms generated by the travel time savings. To demonstrate this, potential gains in both absolute and relative accessibility, as measured by the Hagerstrand

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[4] For a related adjustment, see McNally and Recker (1986)

index, were developed based on optimal scheduling, ride-sharing and trip chaining behavior. The distributions of potential gains (Figures 5 and 6) generally follow the same tendencies noted for the corresponding travel time distributions. The corresponding cumulative distributions (Figure 7) indicate, for example, that approximately half the total sample could achieve increases in their accessibility index of 1,000 square mile-hours or more; forty percent, at least 2,000 square mile-hours; twenty-five percent, at least 3,000 square mile-hours; twenty per cent, at least 4,000 square-mile-hours; and fifteen percent, at least 5,000 square mile-hours. By way of perspective, over half the sample could have, at the extreme, either spent an additional twenty minutes in activities at their observed locations by virtue of travel time savings or, if they chose to maximize accessibility, expanded their search space either for new activities or for alternative locations for the activities that they performed by as much as 1800 square miles.[5] The actual utility of increased accessibility gained for each household under these circumstances depends on the positioning of the accessibility increases (i.e., the added volume(s) of the space–time prism(s) due to optimal scheduling and travel) within the context of each household's prescribed activity sequence. Under the view of accessibility represented by the Hagerstrand construct, the ability to access the locations of potentially performable activities is a necessary, but not sufficient, condition for realizing the utility that the location affords. Rather, there must also be sufficient time within the prism at that location to actually perform the activity. For example, an activity with the specified duration shown in Figure 8 can only take place within the shaded portion of the space-time prism displayed in that figure. In this example, the added utility of increased spatial accessibility is restricted to the area encompassed by the shaded disc, and further qualified by there being an acceptable location within that disc for performance of the activity.

To provide a more quantitative assessment of the potential value of the accessibility utility gains that could be achieved by the sample, the construct represented in Figure 8 was applied to "average" characteristics of some of the more frequent activity/travel decisions exercised by the sample. Table 3 presents summary statistics on the average travel time (one way) and duration for the most frequent discretionary activities participated in by the sample, together with the percent of the total sample that could potentially add any of these common discretionary activities (assuming

[5] calculated as the area of the circle defining the meridian of the space-time prism (optimal vs. observed) under the prevailing average network speed (approximately 30 mph).

their average conditions) to their daily activity patterns within the expanded space-time prisms generated by the travel time savings afforded by "optimal" scheduling and trip chaining. These greatly simplified results indicate that fifteen percent of the sample would have gained sufficient accessibility to perform an additional personal business or casual entertainment activity; twenty one percent could have performed an additional general shopping activity. Put another way, for the great majority of the sample (approximately eighty percent), the added accessibility that could be gained through the most efficient of travel/activity decisions generally would not be sufficient to translate into their participation in any additional out-of-home activity.

6.3 Relative Roles of Instrumental and Socio-Economic Variables in Enabling Changes in Accessibility and Travel Time

To gain a better understanding of just how much of a household's potential to achieve gains in travel time savings and accessibility is rooted in demographic structure (e.g., such variables as the presence of small children, the number of driver's license holders, and other household socio-demographic variables) versus such instrumental variables as the practice of ride-sharing and trip chaining, a series of regression analyses relating the outcomes of the prescribed optimal scheduling and travel behavior to sets of instrumental and socio-demograghic variables were performed. The notation and definition of the variables used in the regression analyses are listed in Table 4. The estimation results are provided in Tables 5 and 6 for accessibility and travel time improvements, respectively.

The results indicate that, although significant in reducing absolute travel time for the households (measured in units of total time spent traveling, and not in person-time units), ridesharing generally has no effect on increasing accessibility. Rather, it is the introduction of more complex trip chaining of activities that positively impacts the generation of additional accessibility. The positive effect of ridesharing on generating travel time savings tends to be enhanced by socio-demographic characteristics that mirror greater flexibility in meeting activity participation and travel (e.g., having more vehicles at the disposal of the household, or a flexible work schedule). Gains in accessibility also track such variables, but are seen to be limited more by such constraining factors as the presence of children or full-time students in the household or holding more than one job; these latter factors all tend to place “immovable pegs” in the space of feasible activity patterns.

7. Summary and Conclusions

Although accessibility measures have played an important role in transportation planning since the 1960's, their application is still largely confined to the use of aggregate data and average spatial distance to construct home-based measurements. Such measurements exclude activity-based constructs such as trip chaining and the role of temporal and spatial constraints on accessibility. As such, existing measures are responsive principally to changes in land use and transportation infrastructure. Activity-based analyses provide a framework with which to explore alternative structures for accessibility measurements that ameliorates these deficiencies.

The incorporation of activity-based concepts in the measurement of accessibility not only explicitly acknowledges a temporal dimension in assessing the potential for spatial interaction but also expands the applicability of accessibility consideration to a variety of real-world policy options. For example, increased accessibility resulting from trip chaining behavior can be easily accounted for in this context; such results can not be achieved with traditional accessibility measurements.

The empirical application of the HAPP model system provides an indication of the potential of activity-based modeling approaches to assess the bounds on achievable improvements in accessibility and travel time based on daily household activity patterns. It also provides an assessment of roles for trip chaining and ride-sharing as potentially effective methods to facilitate transportation policy objectives. The core element of HAPP is extremely complex. Standard mathematical programming methods do not take advantage of the characteristics of the problem, thus, solution methods are both cumbersome and impractical. A new solution process based on dynamic programming methods and modified by developing opportunity costs from the matrix of potential savings in travel and waiting time was developed and used in this application. Although the solution process simplifies the complex formulation and also accelerates solution time, it undoubtedly can be improved substantially.

There is room for substantial improvement in the scope of application of the modeling approach. Some strict assumptions made in the analysis, if relaxed, would make the approach more useful in general. In the future, the modeling process should more properly reflect behavioral aspects, uncertainty aspects, and a full GIS-based environment. The first aspect is imperative to fully reflect real activity interaction situations, the second is necessary to accommodate the inherent stochastic nature of activity participation and travel time variation, and the third would supply a better analysis environment. Although the data were processed in a GIS, the program ran independently. A full GIS environment would facilitate model application, in general, and would enable certain aspects such as destination choice to be more accurately specified. Herein, homogeneous behavior was assumed, however, the behavioral aspects which deal with user's response behavior and habit are probably quite different. Further research on user adaptation and habit would benefit the activity model. The stochastic nature of both activity participation and travel time should be considered.

Some of these limitations may reflect uncertainty regarding the very core of the choice protocol assumed. For example, the disparity between the so-called "optimal behavior' and that observed suggests that unmeasured constraints are present. The analysis assumes a simultaneous choice structure. Yet, it is apparent that sub-optimality undoubtedly results when patterns are formed in real-time – for example, the occurrence in the evening of an activity that could have been optimally scheduled in the morning *had it been planned*. Casting the approach in a more dynamic (and stochastic) framework would certainly be more realistic; however, the challenges involved are formidable.

In conclusion, the results presented here indicate that sizable time savings could result from urban trip makers optimizing their travel. To what extent such optimization *could* reasonably occur remains a major unanswered question. Our method merely enables the estimation of potential changes in accessibility via Pirie's "vacancy creations," and not the actual measurement of accessibility (which in any case is a relative, and not absolute, concept). The constraint-based approach utilized merely defines the limits on creating these "vacancies." It is not a theory, but rather, places physical bounds on whatever theory is valid.

8. Acknowledgements

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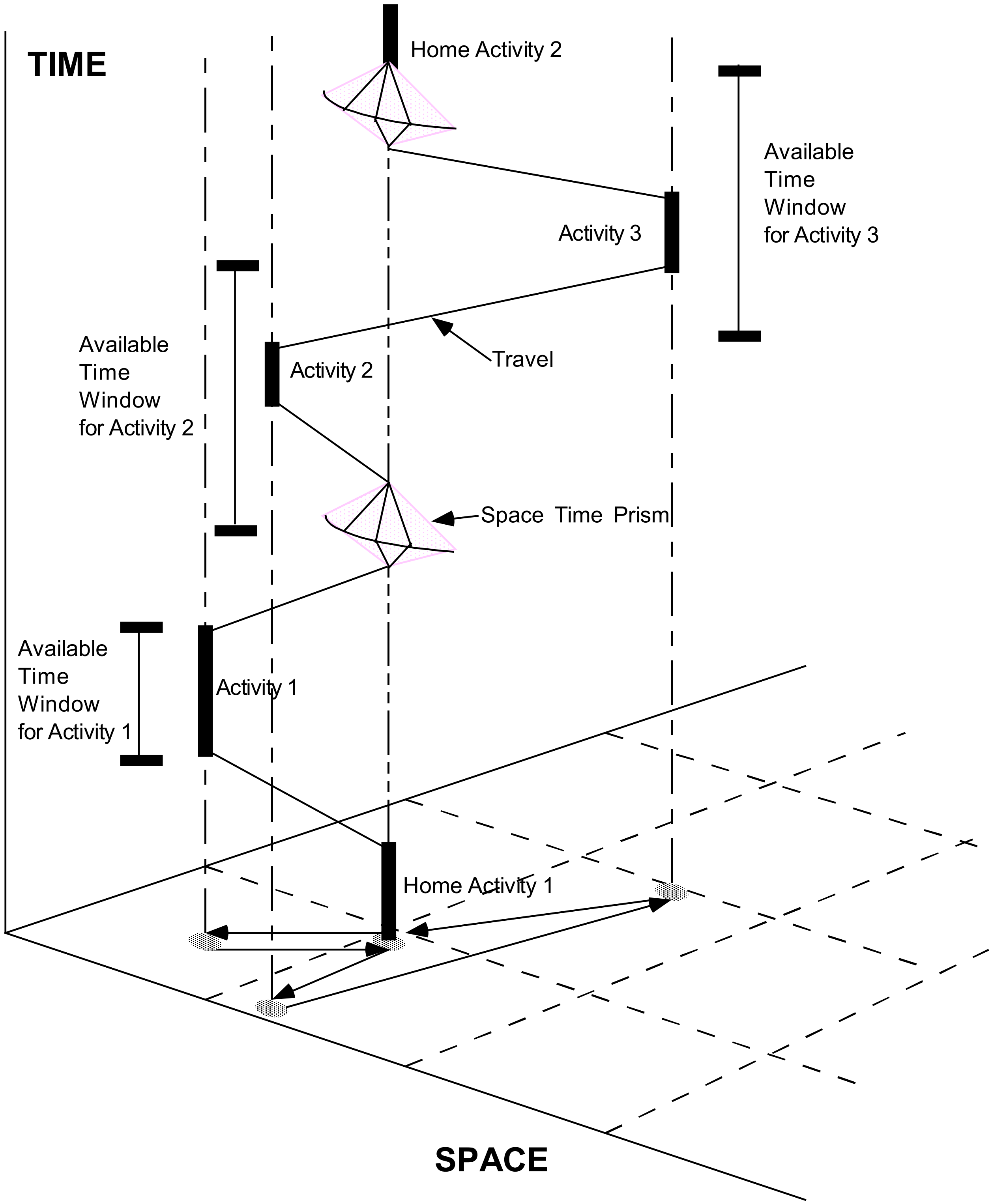

Figure 1. Example of Individual's Travel/Activity Pattern and Accessibility Prisms

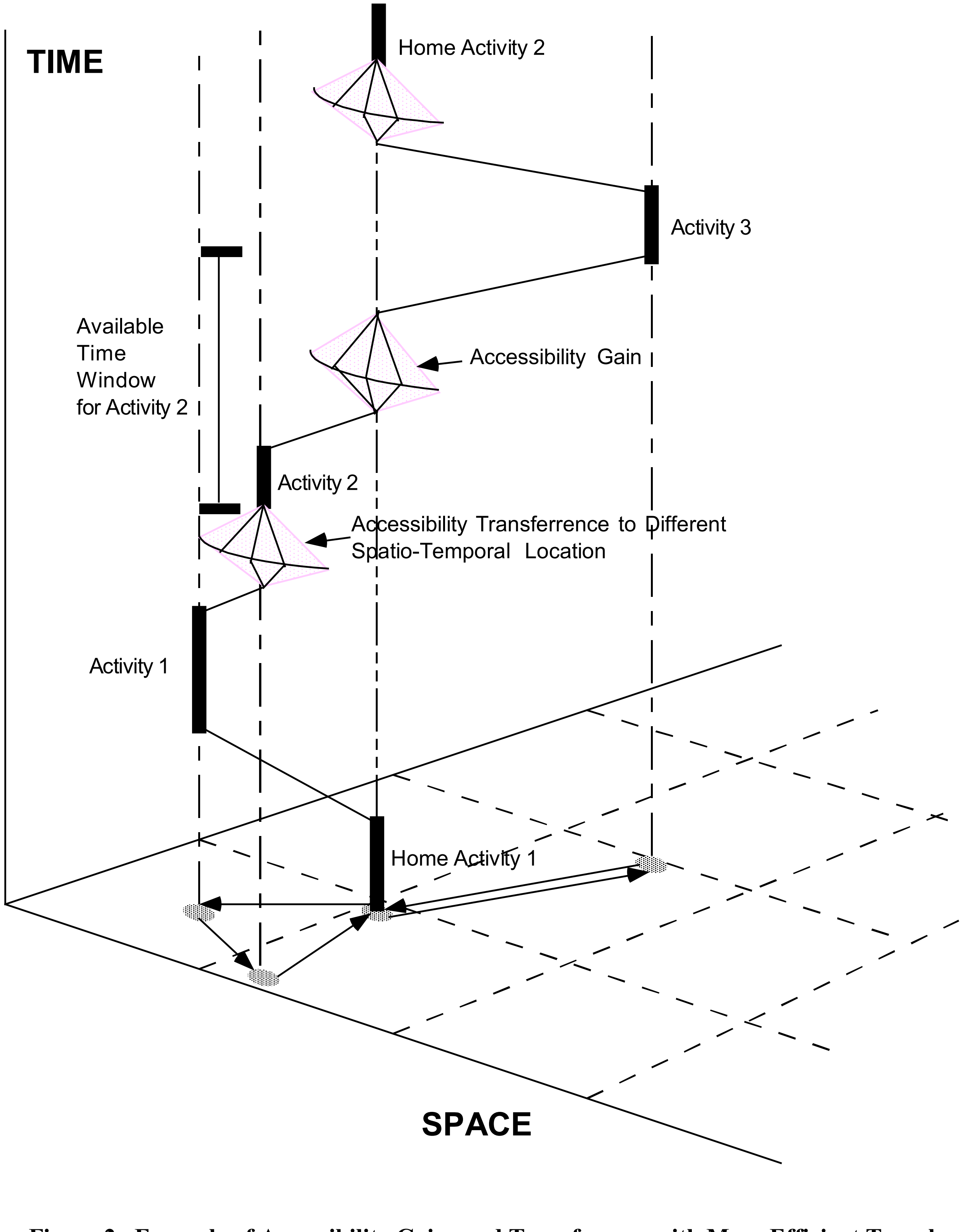

Figure 2. Example of Accessibility Gains and Transference with More Efficient Travel Behavior

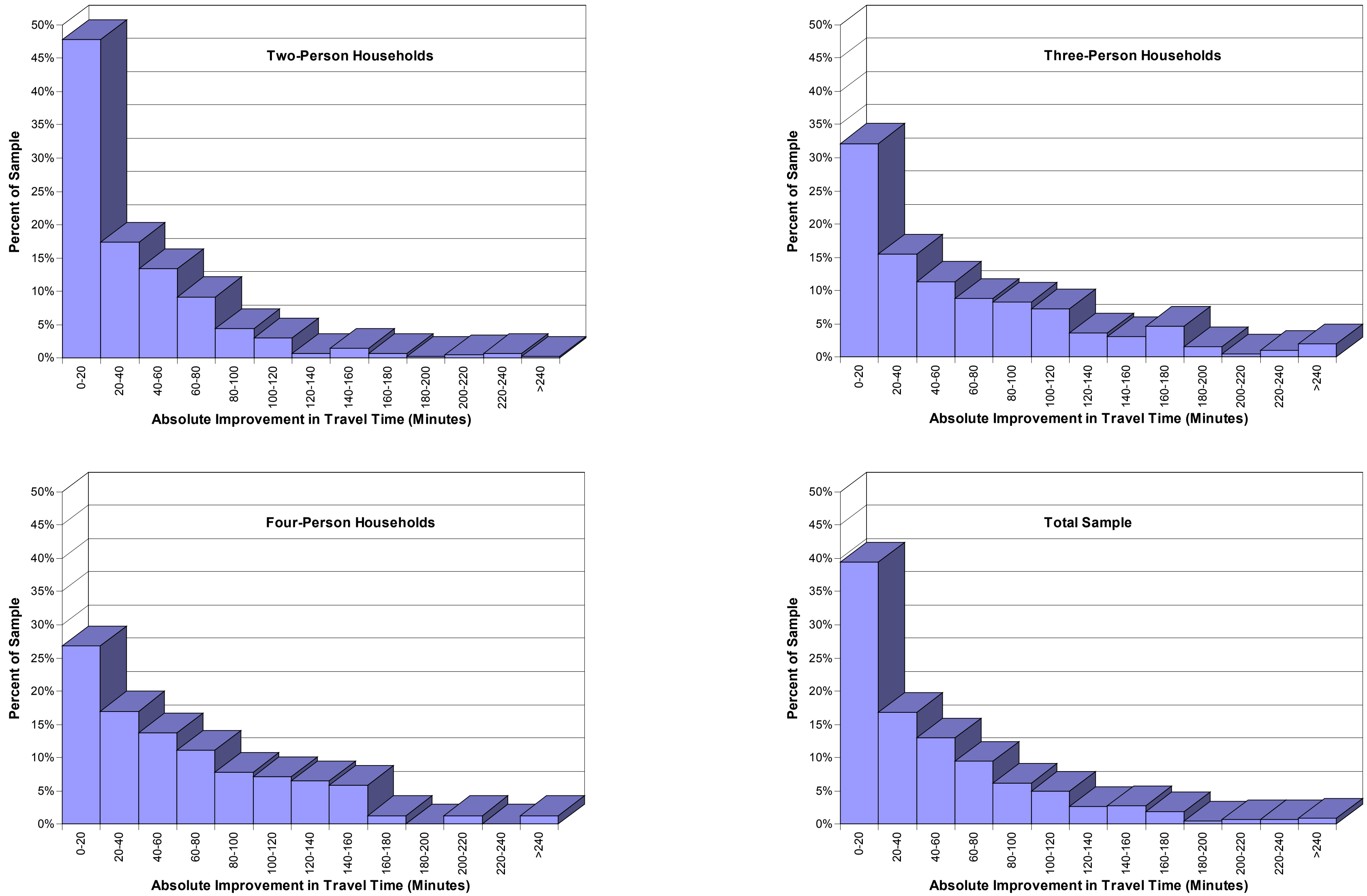

Figure 3. Distribution of Potential Household Travel Time Improvements with Ride-Sharing and Trip Chaining

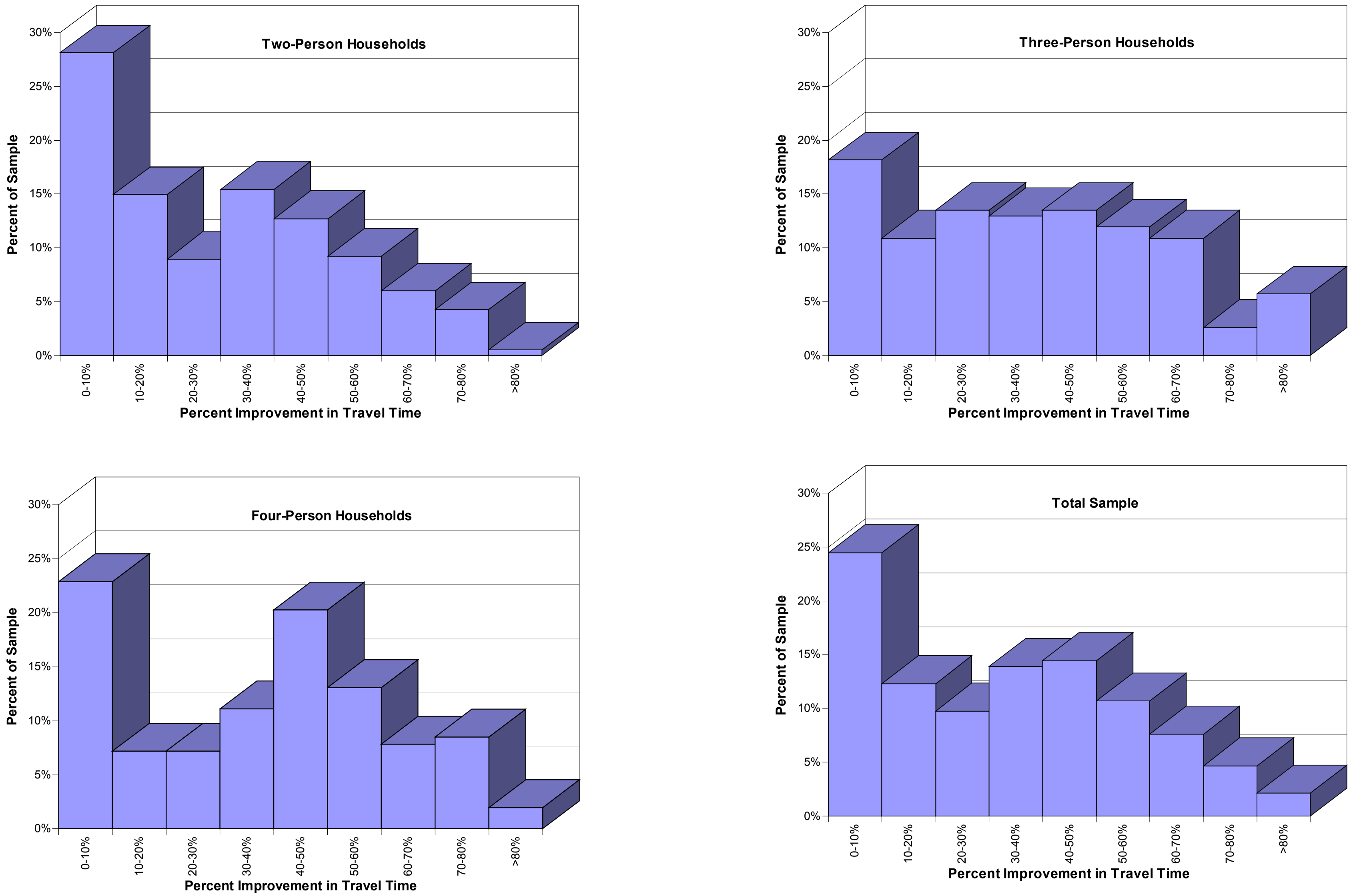

Figure 4. Distribution of Potential Relative Travel Time Improvements with Ride-Sharing and Trip Chaining

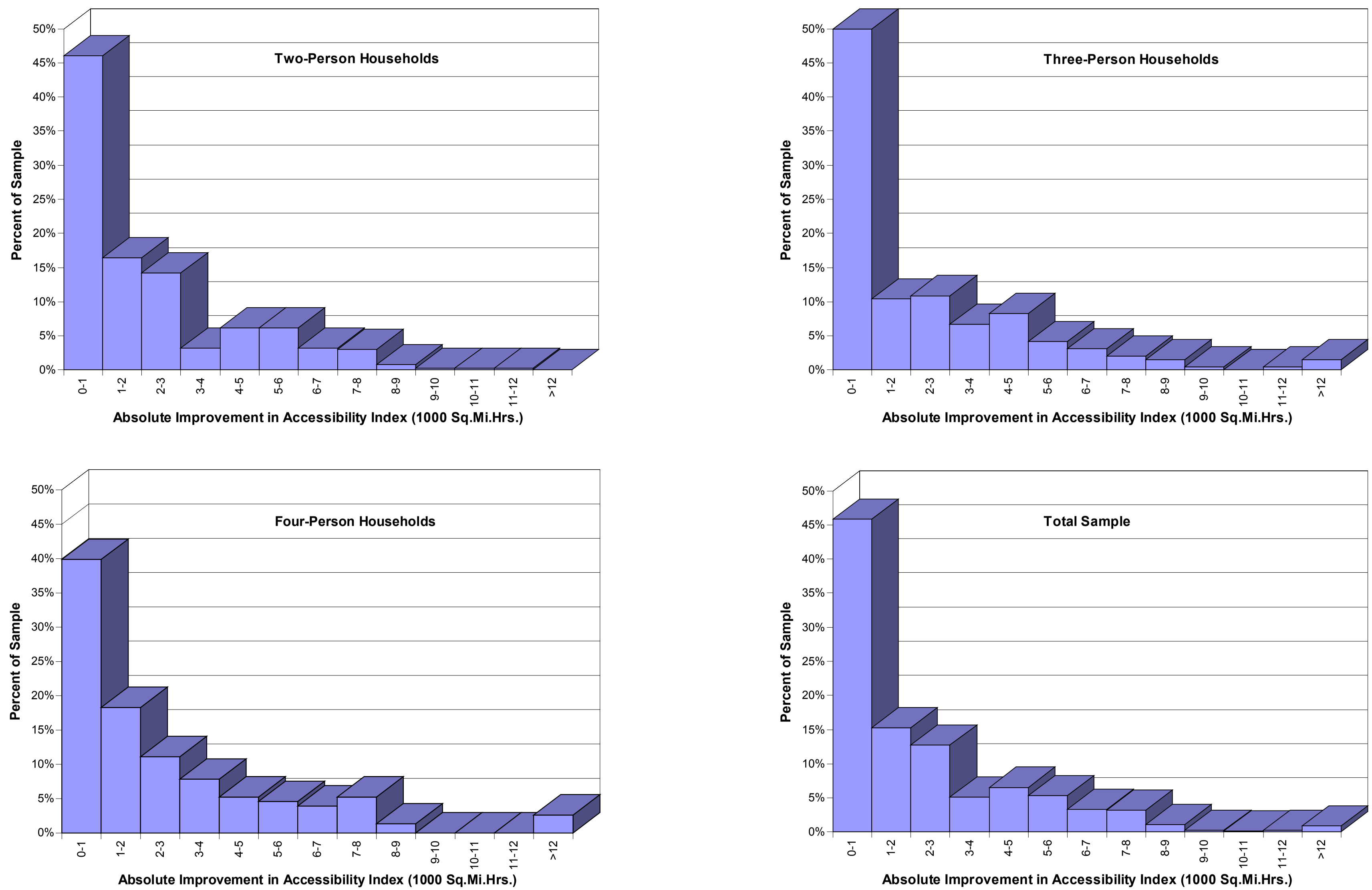

Figure 5. Distribution of Potential Household Accessibility Improvements with Ride-Sharing and Trip Chaining

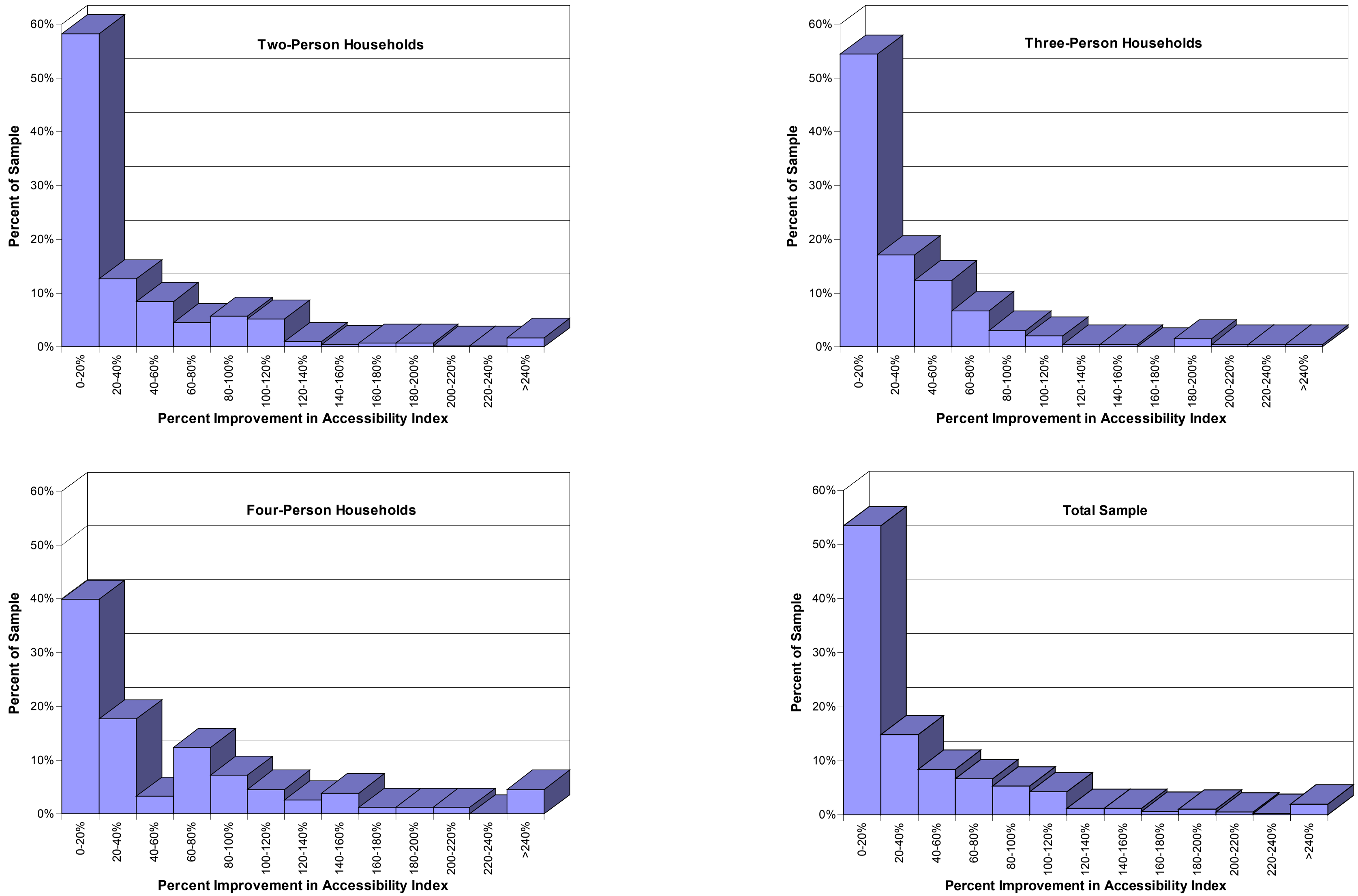

Figure 6. Distribution of Potential Relative Accessibility Improvements with Ride-Sharing and Trip Chaining

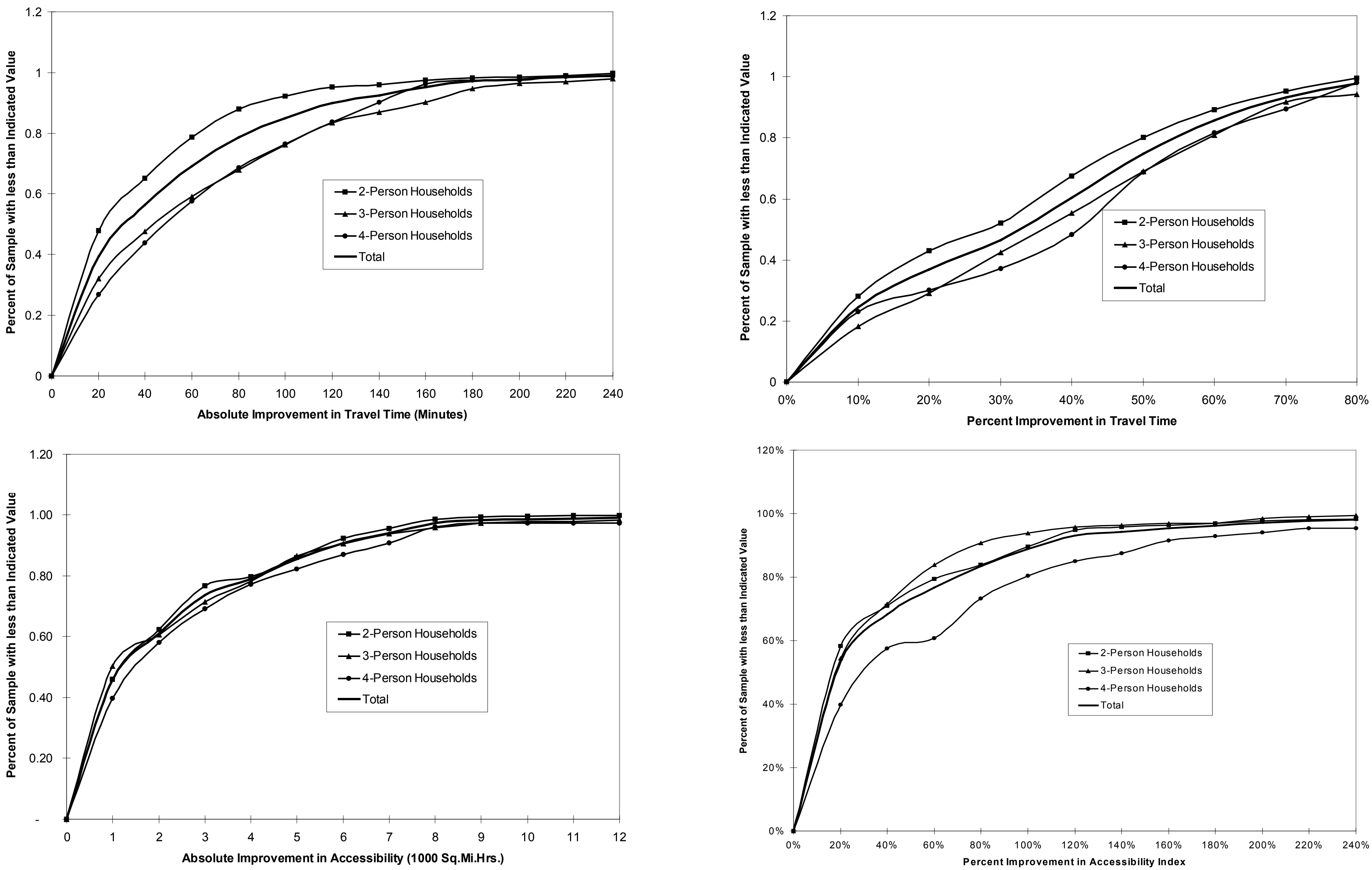

Figure 7. Cumulative Distributions of Potential Savings

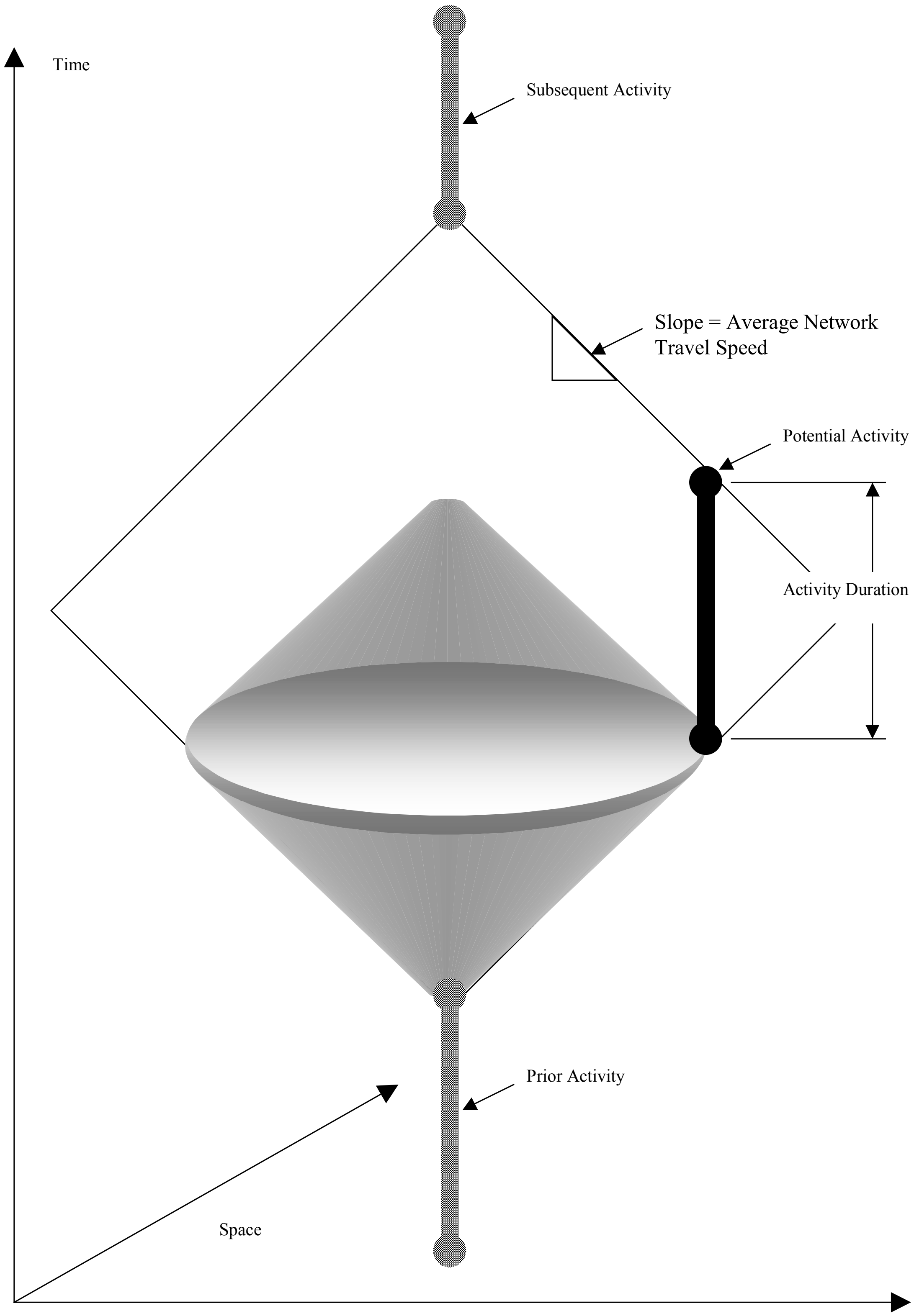

Figure 8. Accessibility Prism

Table 1. Demographic Statistics for Sample Groups

Household Attribute	Household Size		
	2 Person (N=402)	3 Person (N=197)	4 Person (N=153)
Median Household Income	$40-$45K	$50-$55K	$40-$45K
Mean No. of Vehicles Owned	2.0	2.5	2.3
Mean No. of Licensed Drivers	1.9	2.3	2.2
Mean No. of Employed Persons	1.2	1.8	1.8
Mean Age of Adults in Household	53	42	39

Table 2. Selected Mean Travel and Activity Characteristics

Characteristic	Household Size		
	2 Person	3 Person	4 Person
Number of Out-of-Home Household Activities per Person	2.8	2.7	2.9
Number of Trips per Person	4.0	3.8	4.2
Travel Time per person for Out-of-Home Activities (Min.)	67.7	67.7	67.7
Travel Time per Trip (Min.)	19.1	19.5	18.0
Number of Out-of-Home Activities per Trip Chain	1.88	1.86	1.79
Percent of Chains with One Activity	57.6	57.1	60.4
Percent of Chains with Two Activities	17.5	18.5	16.4
Percent of Chains with Three Activities	13.4	13.3	14.8
Percent of Chains with Four or More Activities	11.5	11.1	8.4

Table 3. "Usability" of Potential Travel Time Savings

Activity Type	Mean Duration (Hours)	Mean Travel Time (Hours)	Percent of Total Sample That Could Potentially Add Activity Within Expanded Prism
General Shopping	0.78	0.22	21%
Major Shopping	1.02	0.34	1%
Personal Services	1.32	0.24	1%
Professional Services	1.11	0.30	1%
Personal Business	0.85	0.24	15%
Casual Entertainment	0.79	0.25	15%

Table 4. Definition of Variables Used in Regression Analyses

VARIABLE	DESCRIPTION
HHSIZE	Number of persons in household
INCOME	Annual household income
CHILDREN	Number of children under 12 years of age in household
FLEX-TIME WORK	Presence of flexible work schedule
MULTI-JOB	Holds more than one job
# STUDENTS	Number of full-time students in household
LATE ACTIVITY	Presence of activities performed from 12am to 6am
# LICENSED DRIVERS	Number of household members with drivers license
# VEHICLES	Number of household vehicles
# ACTIVITIES	Total number of household activities
# TRIPS	Total number of household trips
Δ RIDESHARING	Ride-sharing incorporated in optimal pattern
Δ CHAIN COMPLEXITY	Difference between average trips per chain for optimal and observed patterns

Table 5. OLS Results: Absolute Accessibility Improvement

Variable	Household Size		
	2 Person (N=402)	3 Person (N=197)	4 Person (N=153)
CHILDREN		-971.50	
LATE ACTIVITY		1029.87*	
MULTI-JOB	-295.01		
# STUDENTS			-442.72*
# LICENSED DRIVERS	929.57		
# VEHICLES			709.40
# ACTIVITIES			65.31*
Δ RIDESHARING			
Δ CHAIN COMPLEXITY	769.86	626.92*	745.89*
Constant	-139.14*	1791.24	-693.51
R^2	0.24	0.11	0.15
Adjusted R^2	0.24	0.09	0.13
F statistic	43.01**	7.59**	5.06**

* Significant at 0.10 level. All other coefficients significant at 0.05.

** Significant at 0.05.

Table 6. OLS Results: Absolute Travel Time Improvement

Variable	Household Size		
	2 Person (N=402)	3 Person (N=197)	4 Person (N=153)
FLEX-TIME WORK		13.97*	
LATE ACTIVITY	12.34*		
# VEHICLES	3.69*		15.97
Δ RIDESHARING	35.14	53.03	27.42
Δ CHAIN COMPLEXITY	5.66	9.38	
Constant	14.34	22.08	22.61*
R^2	0.19	0.21	0.08
Adjusted R^2	0.18	0.20	0.07
F statistic	22.77**	16.85**	6.60**

* Significant at 0.10 level. All other coefficients significant at 0.05.

** Significant at 0.05.

APPENDIX

MODEL FORMULATION

The sets of activity nodes, household members, available vehicles and function for a given household are defined and described as follows:

$P = \{1, 2, \ldots, n - 1, n\}$
$P' = \{0, 1, 2, \ldots, n - 1, n, d\}$
$H = \{h_1, h_2, \ldots, h_{n-1}, h_n\}$
$H' = \{0, h_1, h_2, \ldots, h_{n-1}, h_n, d\}$
$N = \{1, 2, \ldots, n - 1, n, h_1, h_2, \ldots, h_{n-1}, h_n\}$
$N' = \{0, 1, 2, \ldots, n - 1, n, h_1, h_2, \ldots, h_{n-1}, h_n, d\}$
$\eta = \{1, 2, \ldots, |\eta|\}$
$\nu = \{1, 2, \ldots, |\nu|\}$
$\gamma = \{1, 2, \ldots, |\gamma|\}$
$F = \{1, 2\}$

where:

P	the set of all activity nodes scheduled to be completed by members in the household
P'	the set of all activity nodes including starting node and ending node
H	the set of all temporary returning home nodes associated with each activity node
H′	the set of all home nodes including starting node and ending node
N	the set of all activity nodes and return to home nodes associated with each activity node
N′	the set of N together with starting node and ending node
0	the set of all of the members in the household
<	the set of all of the available vehicles used by members in the household to complete their scheduled activities
(	the set of people in a vehicle (including the driver and the passengers)
F	the set of functions that could be performed at an activity node, with "1", representing delivery, and "2", pickup (physically, these functions are performed at the locations of

the associated activity nodes, and logically, these functions are used to represent those expanded nodes associated with the original activity nodes)

The following parameters are used in the formulation:

u, w node representing activity

0 departure depot node (starting node)

d final arrival depot node (ending node)

e, f function performed at node

v vehicle within the household

r number of people in a vehicle

" member within the household

a_{u^e} earliest available start time associated with activity node *u* while performing function *e*

b_{u^e} latest available start time associated with activity node *u* while performing function *e*

M an arbitrarily large number

Some additional sets are introduced to limit feasible connections among activity nodes, and they are:

ΩH_α the set of activities that can not be performed by household member "

$\Omega \mathrm{v}_v$ the set of activities that can not be reached using vehicle *v*

$\Psi_{u^e w^f}$ the feasible connection set between node *u* with function *e* and node *w* with function *f*

I the set of infeasible vehicle-person pairs (set of persons within the household without driver license)

The decision variables representing the routing aspects of vehicles and persons, temporal aspects regarding activity begin time and waiting time, assignment of vehicles and persons, and such other aspects as capacity are defined below:

$X_{u^e w^f}^{v\,r}$ binary decision variable equal to unity if vehicle v travels from activity node u after performing function e to activity node w to perform function f with r people in vehicle v, and zero otherwise.

$Y_{u^e w^f}^{\alpha}$ binary decision variable equal to unity if household member " travels from activity node u after performing function e to node w to perform function f, and zero otherwise.

T_{u^e} the time at which participation in activity u to perform function e begins.

$H_{u^l}^{v\alpha}$ the time at which household member " begins to participate in activity u by using vehicle v.

$R_{u^e}^{v\alpha}$ the time at which household member " arrives at node u to perform function e by using vehicle v.

$W_{u^e}^{v\alpha}$ waiting time for member " using vehicle v at activity node u to perform function e.

$AH_{u\alpha}$ binary decision variable equal to unity if activity node u is assigned to household member ", and zero otherwise.

Av_{uv} binary decision variable equal to unity if vehicle v is used to participate in activity u, and zero otherwise.

$L_{u^e}^{\alpha}$ cumulative load for household member " when participating in activity u and performing function e.

There are also some parameters associated with network traveling time and cost, and the associated household budget for these items, duration of an activity node and "load" of activity node, which is equivalent to a hypothetical index measuring some predefined cumulative aspect associated with performing that activity node as part of an existing chain:

t_{uw} travel time from the location of activity u to the location of activity w.

c_{uw} travel cost from the location of activity u to the location of activity w.

s_{u^e} duration of stay at activity node u to perform function e.

l_{u^e} load associated with activity node u while performing function e.

BC the household travel cost budget.

BT_α the travel time budget of household member ".

Using this notation, the mathematical formulation is expressed as follows:

$$\text{Minimize}\ \Phi(\mathbf{Z}_i) = \mathbf{B}'_i \cdot \mathbf{Z}_i$$

subject to :

$$\sum_{w\in P}\sum_{f\in F} Y^{\alpha}_{0^l w^f} \le 1, \alpha \in \eta \tag{A1}$$

$$\sum_{w\in P}\sum_{f\in F}\sum_{\alpha\in\eta} Y^{\alpha}_{0^l w^f} \ge 1 \tag{A2}$$

$$\sum_{u\in P}\sum_{e\in F} Y^{\alpha}_{u^e d^l} = \sum_{w\in P}\sum_{f\in F} Y^{\alpha}_{0^l w^f}, \alpha \in \eta \tag{A3}$$

$$\sum_{w\in P}\sum_{f\in F}\sum_{r\in\gamma} X^{v\,r}_{0^l w^f} \le 1, v \in \nu \tag{A4}$$

$$\sum_{w\in P}\sum_{f\in F}\sum_{v\in\nu}\sum_{r\in\gamma} X^{v\,r}_{0^l w^f} \ge 1 \tag{A5}$$

$$\sum_{u\in P}\sum_{e\in F}\sum_{r\in\gamma} X^{v\,r}_{u^e d^l} = \sum_{w\in P}\sum_{f\in F}\sum_{r'\in\gamma} X^{v\,r'}_{0^l w^f}, v \in \nu \tag{A6}$$

$$\sum_{u\in N'}\sum_{e\in F}\sum_{v\in\nu}\sum_{r\in\gamma} X^{v\,r}_{u^e w^l} = 1, w \in P \tag{A7}$$

$$\sum_{u\in N'}\sum_{e\in F} Y^{\alpha}_{u^e w^f} = \sum_{u'\in N'}\sum_{e'\in F} Y^{\alpha}_{w^f u'^{e'}}, w \in N, f \in F, \alpha \in \eta \tag{A8}$$

$$\sum_{u\in N'}\sum_{e\in F}\sum_{r\in\gamma} X^{v\,r}_{u^e w^f} \cdot r = \sum_{u' in N'}\sum_{e'\in F}\sum_{r'\in\gamma} X^{v\,r'}_{w^f u'^{e'}} \cdot r', w \in N, f \in F, v \in \nu \tag{A9}$$

$$\sum_{v\in\nu}\sum_{r\in\gamma} X_{u^e w^f}^{v\,r}\cdot r = \sum_{\alpha\in\eta} Y_{u^e w^f}^{\alpha}, \quad u,w\in P,\quad e,f\in F \tag{A10}$$

$$\sum_{v\in\nu}\sum_{r\in\gamma} X_{u^e d^l}^{v\,r}\cdot r + \sum_{v\in\nu}\sum_{r\in\gamma} X_{u^e w^l}^{v\,r}\cdot r = \sum_{\alpha\in\eta} Y_{u^e d^l}^{\alpha} + \sum_{\alpha\in\eta} Y_{u^e w^l}^{\alpha}\ , u\in P, w\in H, e\in F \tag{A11}$$

$$\sum_{u\in H'}\sum_{v\in\nu}\sum_{r\in\gamma} X_{u^l w^f}^{v\,r}\cdot r = \sum_{u\in H'}\sum_{\alpha\in\eta} Y_{u^l w^f}^{\alpha}, w\in P, f\in F \tag{A12}$$

$$\sum_{\alpha\in\eta} AH_{u\alpha} = 1\ , u\in P \tag{A13}$$

$$-(1-Y_{u^l u^2}^{\alpha})\cdot M \le AH_{u\alpha} - Y_{u^l u^2}^{\alpha} \le (1-Y_{u^l u^2}^{\alpha})\cdot M,\ u\in P, \alpha\in\eta \tag{A14}$$

$$\begin{aligned}-[(1-\sum_{u\in N'} Y_{u^l w^l}^{\alpha}) + (1-\sum_{u\in N'}\sum_{v\in\nu} X_{u^l w^l}^{v\,1})]\cdot M &\le AH_{u\alpha} - \sum_{u\in N'} Y_{u^l w^l}^{\alpha}\\ &\le [(1-\sum_{u\in N'} Y_{u^l w^l}^{\alpha}) + (1-\sum_{u\in N'}\sum_{v\in\nu} X_{u^l w^l}^{v\,1})]\cdot M,\ w\in P, \alpha\in\eta\end{aligned} \tag{A15}$$

$$A\nu_{wv} = \sum_{u\in N'}\sum_{e\in F}\sum_{r\in\gamma} X_{u^e w^l}^{v\,r}, w\in P, v\in\nu \tag{A16}$$

$$\begin{aligned}-(1-\sum_{u\in N'}\sum_{e\in F} X_{u^e w^l}^{v\,1})\cdot M &\le \sum_{u\in N'}\sum_{e\in F} X_{u^e w^l}^{v\,1} - \sum_{u'\in N'}\sum_{e'\in F} X_{w^l u'^{e'}}^{v\,1}\\ &\le (1-\sum_{u\in N'}\sum_{e\in F} X_{u^e w^l}^{v\,1})\cdot M,\ w\in P, w\ne u\ne u', v\in\nu\end{aligned} \tag{A17}$$

$$\begin{aligned}-(1-\sum_{u\in N'}\sum_{e\in F} X_{u^e w^l}^{v\,r'})\cdot M &\le \sum_{u\in N'}\sum_{e\in F} X_{u^e w^l}^{v\,r'} - \sum_{u'\in N'}\sum_{e'\in F} X_{w^l u'^{e'}}^{v\,r}\\ &\le (1-\sum_{u\in N'}\sum_{e\in F} X_{u^e w^l}^{v\,r'})\cdot M,\ w\in P, w\ne u\ne u', v\in\nu, r'=r+1\end{aligned} \tag{A18}$$

$$\begin{aligned}-(1-\sum_{u\in N'}\sum_{e\in F} X_{u^e w^2}^{v\,r})\cdot M &\le \sum_{u\in N'}\sum_{e\in F} X_{u^e w^2}^{v\,r} - \sum_{u'\in N'}\sum_{e'\in F} X_{w^2 u'^{e'}}^{v\,r'}\\ &\le (1-\sum_{u\in N'}\sum_{e\in F} X_{u^e w^2}^{v\,r})\cdot M,\ w\in P, w\ne u\ne u', v\in\nu, r'=r+1\end{aligned} \tag{A19}$$

$$\begin{aligned}-[(1-Y_{u^e w^f}^{\alpha}) + (1-\sum_{r\in\gamma} X_{u^e w^f}^{v\,r})]\cdot M &\le R_{u^e}^{v\alpha} + W_{u^e}^{v\alpha} + t_{uw} + s_{u^e}\cdot AH_{u\alpha}\\ &\quad - R_{w^f}^{v\alpha} \le [(1-Y_{u^e w^f}^{\alpha}) + (1-\sum_{r\in\gamma} X_{u^e w^f}^{v\,r})]\cdot M,\ u,w\in P, e,f\in F, \alpha\in\eta, v\in\nu\end{aligned} \tag{A20}$$

$$-[(1-\sum_{u\in H'} Y^{\alpha}_{u^1 w^f})+(1-\sum_{r\in\gamma} X^{vr}_{u'^1 w^f})]\cdot M \le H^{v\alpha}_{u^1}+t_{uw}-R^{v\alpha}_{w^f}$$
$$\le[(1-\sum_{u\in H'} Y^{\alpha}_{u^1 w^f})+(1-\sum_{r\in\gamma} X^{vr}_{u'^1 w^f})]\cdot M, \tag{A21}$$
$$u'\in H', w\in P, f\in F, \alpha\in\eta, v\in\nu$$

$$-[(1-Y^{\alpha}_{u^1 w^f})+(1-\sum_{u'\in H'}\sum_{r\in\gamma} X^{vr}_{u'^1 w^f})]\cdot M \le H^{v\alpha}_{u^1}+t_{uw}-R^{v\alpha}_{w^f}$$
$$\le[(1-Y^{\alpha}_{u^1 w^f})+(1-\sum_{u'\in H'}\sum_{r\in\gamma} X^{vr}_{u'^1 w^f})]\cdot M, \tag{A22}$$
$$u\in H', w\in P, f\in F, \alpha\in\eta, v\in\nu$$

$$-[(1-Y^{\alpha}_{u^e w^1})+(1-\sum_{w'\in H'}\sum_{r\in\gamma} X^{vr}_{u^e w'^1})]\cdot M \le T_{u^e}+t_{uw}+s_{u^e}\cdot AH_{u\alpha}-H^{v\alpha}_{w^1}$$
$$\le[(1-Y^{\alpha}_{u^e w^1})+(1-\sum_{w'\in H'}\sum_{r\in\gamma} X^{vr}_{u^e w'^1})]\cdot M, \tag{A23}$$
$$u\in P, w\in H', e\in F, \alpha\in\eta, v\in\nu$$

$$-[(1-\sum_{w\in H'} Y^{\alpha}_{u^e w^1})+(1-\sum_{r\in\gamma} X^{vr}_{u^e w'^1})]\cdot M \le T_{u^e}+t_{uw}+s_{u^e}\cdot AH_{u\alpha}-H^{v\alpha}_{w^1}$$
$$\le[(1-\sum_{w\in H'} Y^{\alpha}_{u^e w^1})+(1-\sum_{r\in\gamma} X^{vr}_{u^e w'^1})]\cdot M, \tag{A24}$$
$$u\in P, w'\in H', e\in F, \alpha\in\eta, v\in\nu$$

$$-[(1-Y^{\alpha}_{u^e w^2})+(1-\sum_{r\in\gamma} X^{vr}_{u^e w^2})]\cdot M \le R^{v\alpha}_{w^2}+W^{v\alpha}_{w^2}-T_{w^2}$$
$$\le[(1-Y^{\alpha}_{u^e w^2})+(1-\sum_{r\in\gamma} X^{vr}_{u^e w^2})]\cdot M,\ u,w\in P, e\in F, \alpha\in\eta, v\in\nu \tag{A25}$$

$$-[(1-AH_{u\alpha})+(1-A\nu_{uv})]\cdot M \le R^{v\alpha}_{u^1}+W^{v\alpha}_{u^1}-T_{u^1}$$
$$\le[(1-AH_{u\alpha})+(1-A\nu_{uv})]\cdot M,\ u\in P, \alpha\in\eta, v\in\nu \tag{A26}$$

$$-[(1-Y^{\alpha}_{u^e w^f})+(1-Y^{\alpha'}_{u^e w^f})+(1-X^{vr}_{u^e w^f})]\cdot M \le R^{v\alpha}_{w^f}-R^{v\alpha'}_{w^f}$$
$$\le[(1-Y^{\alpha}_{u^e w^f})+(1-Y^{\alpha'}_{u^e w^f})+(1-X^{vr}_{u^e w^f})]\cdot M \tag{A27}$$
$$u\in N', w\in P, e,f\in F, \alpha,\alpha'\in\eta, v\in\nu$$

$$-[(1-Y^{\alpha}_{u^e w^1})+(1-Y^{\alpha'}_{u^e w'^1})+(1-X^{vr}_{u^e w^1})]\cdot M \le H^{v\alpha}_{w^1}-H^{v\alpha'}_{w'^1} \le [(1-Y^{\alpha}_{u^e w^1})+(1-Y^{\alpha'}_{u^e w'^1})(1-X^{vr}_{u^e w^1})]\cdot M, \tag{A28}$$
$$w,w'\in H', u\in P, e\in F, \alpha,\alpha'\in\eta, v\in\nu$$

$$-[(1-X^{v1}_{u^e w^1})+(1-X^{v'1}_{u^e w'^1})+(1-Y^{\alpha}_{u^e w^1})]\cdot M \le H^{v\alpha}_{w^1}-H^{v'\alpha}_{w'^1}$$
$$\le [(1-X^{v1}_{u^e w^1})+(1-X^{v'1}_{u^e w'^1})+(1-Y^{\alpha}_{u^e w^1})]\cdot M,\ w,w'\in H',$$
$$u\in P, e\in F, \alpha\in\eta, v,v'\in \nu \tag{A29}$$

$$-[(1-AH_{u\alpha})+(1-Y^{\alpha}_{u^e w^f})]\cdot M \le L^{\alpha}_{u^e}+l_{u^e}-L^{\alpha}_{w^f}$$
$$\le [(1-AH_{u\alpha})+(1-Y^{\alpha}_{u^e w^f})]\cdot M,\ u,w\in N', e,f\in F, \alpha\in\eta \tag{A30}$$

$$a_{w^f}-T_{w^f}\le (1-\sum_{u\in P}\sum_{e\in F}\sum_{v\in\nu}\sum_{r\in\gamma}X^{vr}_{u^e w^f})\cdot M \ge -b_{w^f}+T_{w^f},\ w\in P, f\in F \tag{A31}$$

$$\sum_{u\in N'}\sum_{e\in F}\sum_{w\in N'}\sum_{f\in F}\sum_{v\in\nu}\sum_{r\in\gamma}X^{vr}_{u^e w^f}\cdot c_{uw}\le BC \tag{A32}$$

$$\sum_{u\in N'}\sum_{e\in F}\sum_{w\in N'}\sum_{f\in F}Y^{\alpha}_{u^e w^f}\cdot t_{uw}\le BT_{\alpha},\alpha\in\eta \tag{A33}$$

$$\sum_{v\in\nu}\sum_{r\in\gamma}X^{vr}_{u^e w^f}=0,\ (u^e, w^f)\notin \psi_{u^e w^f} \tag{A34}$$

$$X^{v1}_{u^2 w^2}=0, u,w\in N, v\in\nu \tag{A35}$$

$$X^{vr}_{u^1 u^2}=0, u\in P, v\in\nu, r\in\gamma, r\ge 2 \tag{A36}$$

$$X^{v1}_{u^1 w^1}+Y^{\alpha}_{u^1 w^1}\le 1, u,w\in P', \{v,\alpha\}\in I \tag{A37}$$

Equations (A1) and (A2) depict the origination of a person tour, and allow the possibility of a member to stay at home. Equation (A3) specifies that the end of a person tour must be associated with the corresponding beginning of the person tour. Equations (A4)-(A6) have the same functions as Equations (A1)-(A3) but are applied to the vehicle tour variable. Equation (A7) states that each activity node must be visited by one vehicle while performing the service of the activity. Equation (A8) ensures the flow conservation constraint between all activity nodes; that is, the number of people going into the activity node must be equal to the number of people going out of the activity node. The same situation can be formulated as Equation (A9) when applied to a vehicle tour. The coupling constraints for the vehicle routing variable and person routing variable are specified in Equations (A10) to (A12). Equation (A10) sets up the one-to-one relationship at activity nodes for vehicle-routing variable and person-routing variable. Equations (A11) and (A12) allow for the transference of vehicle and member when either arriving at or departing from home nodes. The association of activity and member can be described by Equations (A13)-(A15). Equation (A13) specifies that each activity must be performed by one, and only one, member. Equations (A14) and (A15) set the relation between assignment variable and person-routing variable. The relation between assignment variable of vehicle and vehicle routing variable is specified by Equation (A16). Equation (A17) addresses the flow conservation constraint for one-person vehicles (drive-alone vehicle). Equations (A18) and (A19) describe the change in the number of people inside a vehicle when stopping at a delivery node and a pickup node, respectively. Temporal constraints are listed from Equation (A20) to Equation (A29). Equation (A20) ensures the continuity of temporal space between activity nodes. Equations (A21) and (A22) match the temporal constraint for the links departing from different home nodes when person or vehicle transference is allowed. The same condition can be applied to Equations (A23) and (A24) for the links arriving at different home nodes. Equations (A25) and (A26) describe the relation between arrival time and activity begin time. The coupling constraints of the arrival time variable for different persons is matched by Equation (A27). Equations (A28) and (A29) ensure that the activity begin time of the home nodes be equal when transference of person or vehicle is allowed. The load budget constraint in Equation (A30) describes the accumulative load at each activity node. The time window constraint in Equation (A31) limits the freedom of performing an activity at certain time periods. Equations (A32) and (A33) specify the budget constraints of travel cost and travel time for each household member and the whole household , respectively. All illogical connections are listed in Equations (A34) to (A36). Finally, infeasible person-vehicle pairs are excluded to ensure that a vehicle is used by a qualified driver in Equation (A37).