Title	Mechanical Characteristics of Repair Welds in Thick Plate (Report I) : Distributions of Three-dimensional Welding Residual Stresses and Plastic Strains and Their Production Mechanisms(Welding Mechanics, Strength & Design)
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Citation	Transactions of JWRI. 15(2) P.359-P.368
Issue Date	1986-12
Text Version	publisher
URL	http://hdl.handle.net/11094/7141
DOI	
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Mechanical Characteristics of Repair Welds in Thick Plate (Report I)†

– Distributions of Three-dimensional Welding Residual Stresses and Plastic Strains and Their Production Mechanisms –

Yukio UEDA*, You Chul KIM**, Kazuteru GARATANI***, Teruhisa YAMAKITA****
and Han Sur BANG****

Abstract

Repair welding produces welding deformation, residual stresses, etc. which may newly produce repair welding cracks. Therefore, it is important to clarify the mechanical characteristics of repair welds in thick plates.

In this paper, three-dimensional unstationary thermal conduction analysis and three-dimensional thermal elasto-plastic analysis are performed. Based on the results, study is made on the characteristics of the distributions of three-dimensional welding residual stresses and plastic strains, and their production mechanisms. The main conclusions can be summarized as follows:

1) *Residual stresses and plastic strains in the base plate are largely affected by compressive mean normal stresses and compressive plastic strains produced in the heating stage, respectively. This is remarkable in the base plate near* HAZ.
2) *Irrespective of elastic or plastic state, severity of mechanical restraints in different directions can be known from the magnitude of produced stress components,* $\{\sigma_x, \sigma_y, \sigma_z\}^T$.
3) *Since plastic strains are assumed to be incompressive, the sign of plastic strain produced in the direction where maximum stress is produced (mechanical restraint is the severest) is opposite to that in the direction where minimum stress is produced (mechanical restraint is the weakest). Therefore, the signs of stress and plastic strain may not agree.*
4) *For evaluation of the mechanical damage of the material caused by welding, it is necessary to pay attention to the whole elasto-plastic behaviors, in addition to the magnitude of residual plastic strains. In this paper, the plastic work and the sum of the equivalent plastic strain increments are proposed as measures to evaluate the severity of mechanical damage throughout the entire history of thermal elasto-plastic behavior. Both measures become maximum in the base plate near* HAZ.

KEY WORDS: (Three-dimensional Thermal Elasto-Plastic Analysis) (Residual Stress) (Plastic Strain) (Equivalent Plastic Strain) (Plastic Work) (Repair Welding) (Welding Residual Stress)

1. Introduction

Since industrial machinery and welded structures are built on a large scale in recent years, various welding cracks and defects are apt to occur in the course of their construction and operation. It is very difficult to replace the defective structural members with new ones owing to economical problems, therefore, the so-called repair welding is applied to them after having removed cracks and defects. Repair welding produces welding deformation and residual stress, and weakens material, which may newly occur repair welding cracks and sometimes extremely harm the safety of structures and machinery. It is, therefore, very important to accurately predict the welding deformations and residual stresses before repair welding in order to guarantee the repair welds.

Repair welding is applied to a limited range, so that three-dimensional thermal elasto-plastic behavior is shown as the mechanical phenomenon in repair welding. Numerical analysis has been required since experimental analysis is impossible for three-dimensional thermal elasto-plastic behavior including the transient state in the weld metal of repair welding. Based on the theories of thermal elasto-plastic analysis, the authors developed an efficient and accurate method and have been carried out a series of analysis in order to clarify mechanical behavior during welding.$^{1)}$

But, three-dimensional elasto-plastic analysis, in which required large memory and long CPU-time, could not be performed from the financial reason. Therefore, elasto-plastic behavior and severity of three-dimensional restraint state for repair welds have not been clarified.

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† Received on Nov. 7, 1986

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Transactions of JWRI is published by Welding Research Institute of Osaka University, Ibaraki, Osaka 567, Japan

In this paper, first, three-dimensional unstationary thermal conduction analysis is performed on repair weld considering the moving effect of heat source. Later, three-dimensional thermal elasto-plastic stress analysis is carried out. Based on the results, the characteristics of distributions of three-dimensional welding residual stresses and plastic strains, and their production mechanisms are studied.

2. Modelling and Temperature Distribution

2.1 Model

The model for analysis of repair welding in thick plate and coordinates system are shown in **Fig. 1**. The single-pass submerged arc welding is carried out along x-direction at the slit ($l = 100, b = 8, d = 5$(mm)) on the middle of the upper surface ($y = 0, z = 0$ (mm)) of the model (height: $h = 90$, breadth: $B = 300$, length: $L = 600$(mm)). The welding conditions are: heat input $Q = 3.4$ kJ/mm, its efficiency $\eta = 0.9$ and welding speed $v = 5.5$ mm/sec.

Judging from the mechanical behavior of the model, the model may be regarded as the infinite plate.[2)]

The material of the model is mild steel. **Figures 2** and **3** show the mechanical properties and physical coefficients of the material, which are depending on temperature. The half of the model is chosen for the analysis of heat conduction and thermal stress because it is symmetry to the y-axis (breadth direction).

2.2 Three-dimensional temperature distributions

Heat conduction analysis of unstationary state including the moving effect of heat source is calculated by the finite element method. The distribution of transient temperature in the welded metal ($y = z = 0$) is shown in **Fig.4**. The change of location of maximum temperature after welding is a function of time and shown also in Fig. 4 by the dotted line. This position moves from the finishing end of welding toward the middle of weld metal. When the temperature at the center of specimen ($x = y = z = 0$) drops down to 700°C (at which the material is assumed to recover its mechanical stiffness[3)] in this paper) in 19 seconds after the finish of welding, isothermal contours on the upper surface ($z = 0$, xy-plane) is shown in **Fig. 5**. According to the results, though temperature is inclined to be high at the finishing end of welding, isothermal contours are elliptic symmetric with respect to the x-axis.

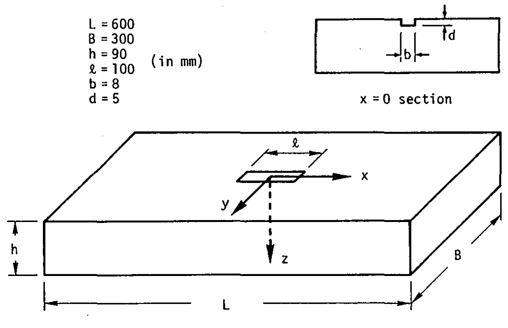


Fig. 1 Repair welding model for analysis.

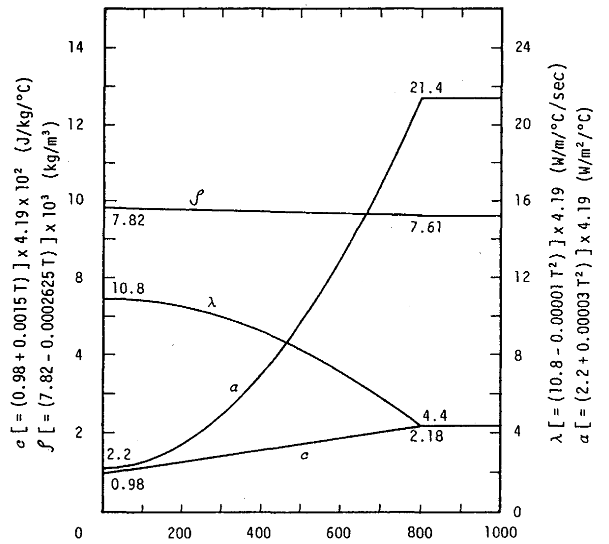


Fig. 2 Temperature dependency of physical properties used heat conduction analysis.

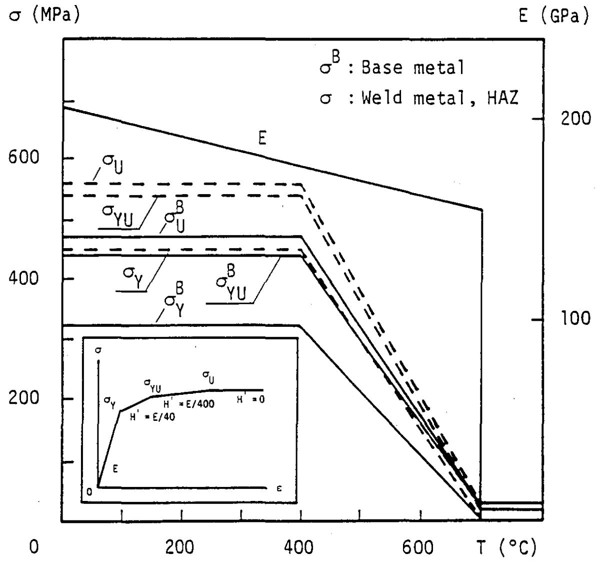


Fig. 3 Temperature dependency of mechanical properties used thermal stress analysis.

The isothermal curves of the cross sections (yz-plane) at $x = 0$ and $x = 30$(mm) are illustrated in Fig. 5. Temperature distribution is controlled mainly by heat conduction within the specimen rather than heat transfer from the surface of specimen, and isothermal contours become concentric circles on the yz-plane. Therefore, y and z-directions have the same thermal history during the heating and cooling stages. The isothermal curves at the xz-plane are semi-elliptic.

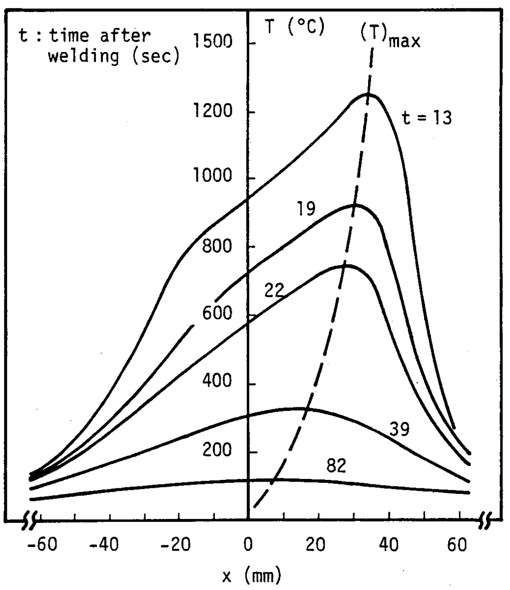


Fig. 4 Transient temperature distribution on weld metal.

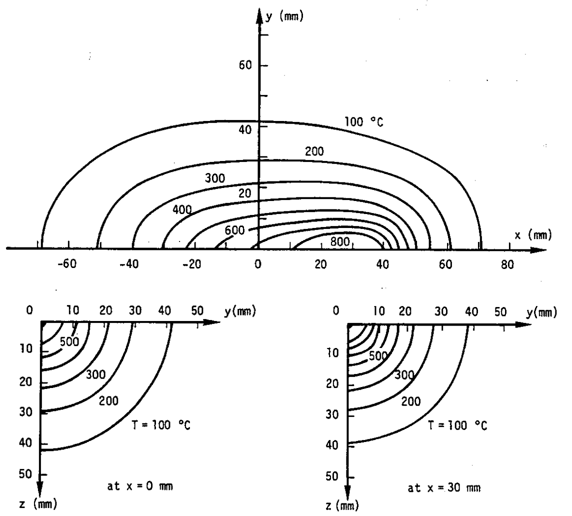


Fig. 5 Isothermal contours (time after welding $t = 19$ sec).

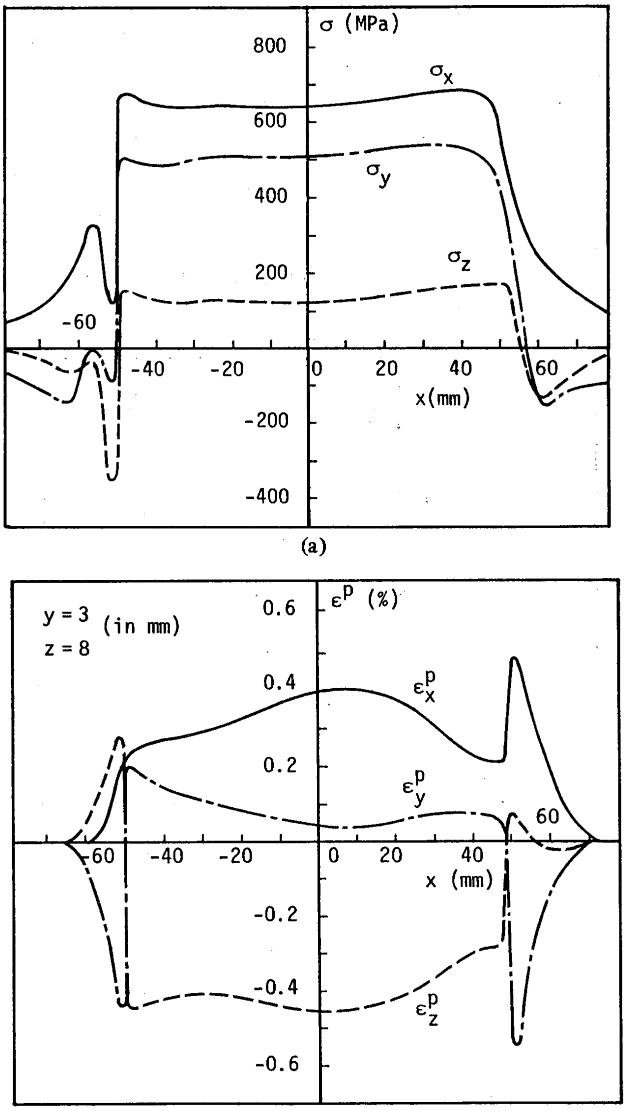
3. Characteristics of Distribution of Three-dimensional Residual Stresses and Plastic Strains, and Their Production Mechanisms

Three-dimensional thermal elasto-plastic stress analysis is performed by using the temperature distributions calculated in the previous Chapter. Based on the results, the characteristics of distributions of three-dimensional welding residual stresses and plastic strains are to be clarified respectively in this Chapter. In addition, the histories of transient stresses and plastic strains are investigated in detail in order to closely examine their production mechanisms.

3.1 Characteristics of distributions of welding residual stresses and plastic strains

The residual stresses and plastic strains are obtained from the three-dimensional thermal elasto-plastic analysis and those in the weld metal ($y = 3$, $z = 8$ mm) along the weld line (x-axis) are shown in **Fig. 6**. All residual stress components produced in the weld metal, σ_x, σ_y and σ_z are tensile, and their magnitudes are $\sigma_x > \sigma_y > \sigma_z$. These magnitudes may represent the severity of mechanical restraints in the order of x- $>$ y- $>$ z-direction.

Paying attention to the residual plastic strains, ϵ_x^p (plastic strain component along the weld line) is tensile all over the weld metal and becomes maximum at a small distance away from the center ($x = 0$) (refer to Fig. 6). ϵ_y^p (plastic strain component perpendicular to the weld line)



(a)

(b)

Fig. 6 Distributions three-dimensional residual stresses and plastic strains along weld line.

is also tensile all over the weld metal but its absolute value is less than the other two ones. Although the residual stress σ_z in the thickness direction of specimen is tensile, the plastic strain component ϵ_z^p (thickness direction) is largely compressive all over the weld metal and becomes maximum at the center. Various characteristics at the center ($x = 0$) of the repair welds will be discussed below.

For the first time, distributions of three-dimensional residual stresses and plastic strains in the z-direction at $y = 3$ mm of the mid-cross-section ($x = 0$, yz-plane) are shown in **Fig. 7**. Stress components, σ_x, σ_y and σ_z are generally tensile up to 20 mm inside from the upper surface of specimen and their maximum values of tension are measured at HAZ which is about *10* mm from the upper surface. The distribution of residual plastic strains is not so simple as that of residual stresses. Plastic strain component ϵ_x^p in the weld metal including HAZ is largely tensile and becomes maximum at HAZ. Strain component ϵ_y^p perpendicular to the weld line changes from compression to tension, and its absolute value is small. Strain component ϵ_z^p is largely compressive, and becomes maximum at HAZ. Whereas strain components, ϵ_x^p and ϵ_y^p are compressive in the base plate near HAZ, especially ϵ_y^p is much greater than ϵ_x^p.

Shown in **Fig. 8** are the distributions of three-dimensional residual stresses and plastic strains along the y-axis at the position $x = 0$, $z = 8$ mm in the weld metal. The components of residual stresses which are produced in the weld metal and HAZ, σ_x, σ_y and σ_z are tensile. The residual plastic strain component ϵ_x^p along the y-axis is tensile in the weld metal and HAZ, but compressive in the base plate. The component ϵ_y^p along the y-axis is small tensile in the weld metal and HAZ, but largely tensile in the base plate. Especially, the largest tension of ϵ_y^p occurrs near HAZ of the base plate. σ_z is tensile in the weld metal and HAZ, but the plastic strain ϵ_z^p is large in compression (Fig. 8).

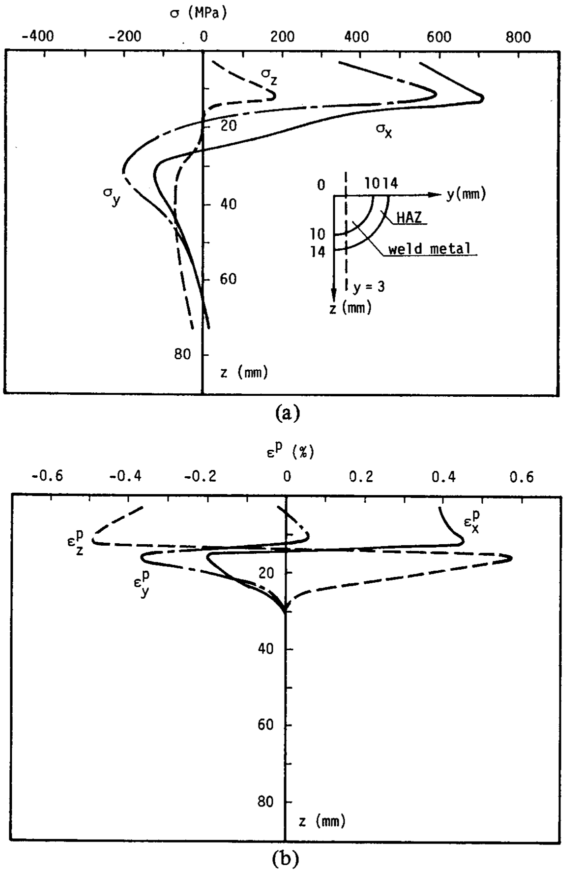


Fig. 7 Distributions three-dimensional residual stresses and plastic strains in z-direction (at middle section).

The signs may differ between the residual stress and plastic strain components in the same direction as mentioned above, which may be attributed to incompressibility of the plastic strain.

3.2 Production mechanisms of welding residual stresses and plastic strains

The production processes of residual stresses and plastic strains are closely investigated in order to clarify

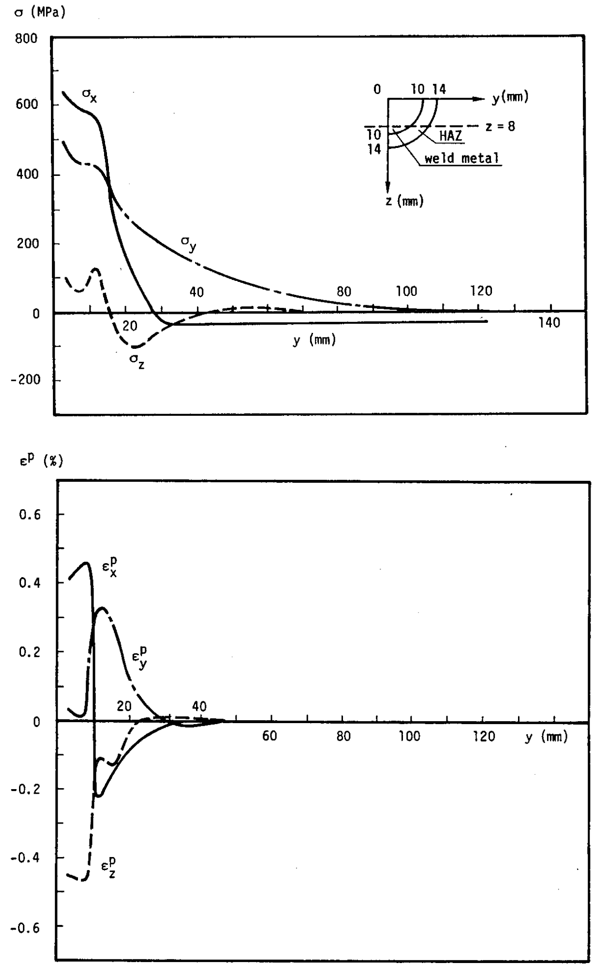


Fig. 8 Distributions three-dimensional residual stresses and plastic strains in y-direction (at middle section).

their mechanisms. As the weld metal undergoes only the cooling stage and the base plate and HAZ suffer both cooling and heating stages, they have different production mechanisms, so that they will be separately treated for clarifying the production mechanisms of residual stresses and plastic strains.

(a) Weld metal

The production processes of residual stresses and plastic strains which are produced at the middle section of the weld metal ($y = 3$, $z = 8$ mm) are shown in **Fig. 9**. First, equivalent stress $\bar{\sigma}$ and its components σ_x, σ_y and σ_z, in the transient state, are investigated. Even in the cooling process after welding, the weld metal stays in the so-called mechanically melting state and shrinks freely without producing any stress until it is cooled to the temperature T_m (*700*°C) at which the material recovers its mechanical stiffness. Stresses are produced in the weld metal which is cooled down below the temperature T_m, because the weld metal recovers its stiffness and its shrinkage is restrained by the base plate at low temperature. Due to the small yield stress σ_Y of the high temperature as depicted in Fig. 3, weld metal is easily plasticified even if the produced stress is small. Although σ_Y increase according to temperature drops, the stresses increase also and they retain in the plastic state. As shown in Fig. 3, equivalent plastic strain $\bar{\varepsilon}^p$ markedly increases in the weld metal in which equivalent stress $\bar{\sigma}$ is constant because yield stress of weld metal is constant below *400*°C. Mean normal stress σ_m which is tensile in the weld metal throughout the whole temperature history is as large as yield stress σ_Y at a room temperature.

In the next place, attention is paid to each component of stresses and plastic strains. Each stress component produced in the weld metal throughout the whole temperature history is tensile, and the order of their strength may be $\sigma_x > \sigma_y > \sigma_z$. Based on the results, the severity of mechanical restraint of the weld metal in each direction may be ranked as follows; x- > y- > z- direction. The plastic strain ε_x^p is large in tension as the stress σ_x, but the strain component ε_y^p is little produced in spite of the large tensile stress σ_y. Whereas, the strain component ε_z^p is compressive and large in spite of the tensile stress σ_z. This may be attributed to that large tensile plastic strain is produced in the x-direction in which mechanical restraint is the severest, so that large compressive plastic strain is produced in the z-direction in which mechanical restraint is the least in order to satisfy its incompressibility, i.e. the condition of constant volume. The sign of plastic strain increment is equal to that of deviated stress σ'.

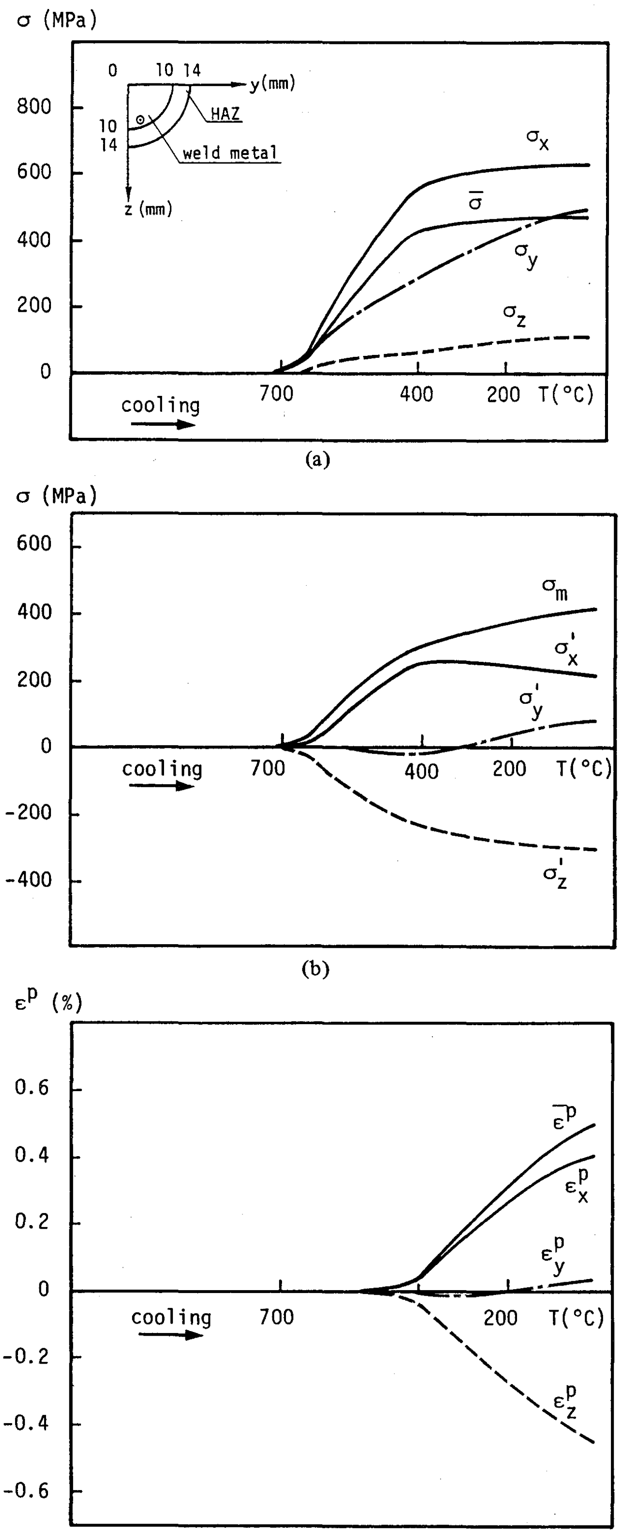


Fig. 9 Transient stresses $\{\sigma\}$, stress deviations $\{\sigma'\}$ and plastic strains $\{\varepsilon^p\}$ in weld metal (at middle section, $y = 3$, $z = 8$ mm).

(b) Heat affected zone (HAZ)

The production processes of residual stresses and plastic strains in HAZ ($y = 3$, $z = 12$ mm) of the mid-cross-section are shown in **Fig. 10**. This position is located in the mid-cross-section at the deepest HAZ in the thickness direction, at which tensile residual stresses become the maximum (refer to Fig. 7). It should be observed that this

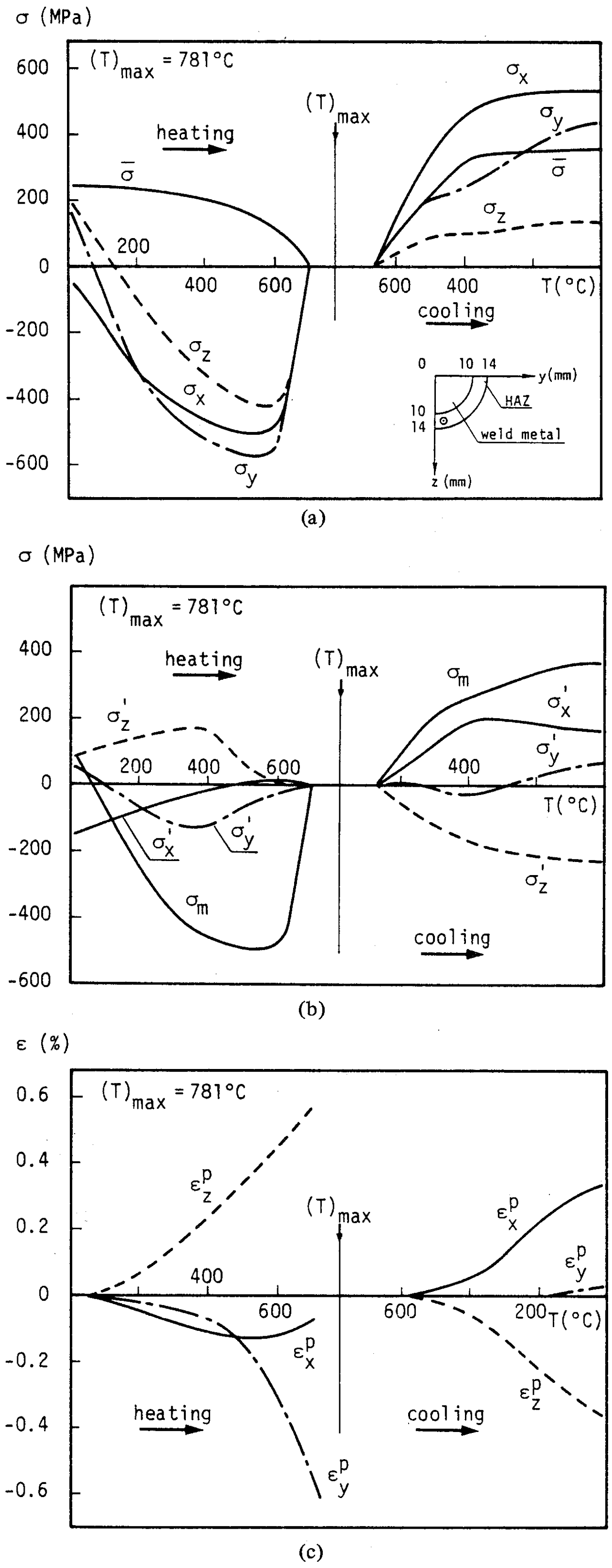


Fig. 10 Transient stresses { σ }, stress deviations { σ' } and plastic strains { ϵ^p } in HAZ (at middle section, $y = 3, z = 12$ mm).

point is heated up at the maximum temperature *781*°C and starts to be cooled down to a room temperature. The reason why σ_x is compressive but σ_y and σ_z are tensile at a room temperature (before the passing of arc) is that the relevant base plate restricts the thermal expansion of the welded portion. Hereinafter, the production mechanisms will be investigated.

Since the base plate at low temperature restricts the thermal expansion of HAZ at the heating stage after the passing of the welding arc, compressive stresses are produced in HAZ, of which absolute values tend to be large because of the compressive mean normal stress σ_m. Moreover, among the stress components, σ_x, σ_y and σ_z, only σ_y is remarkably increased.

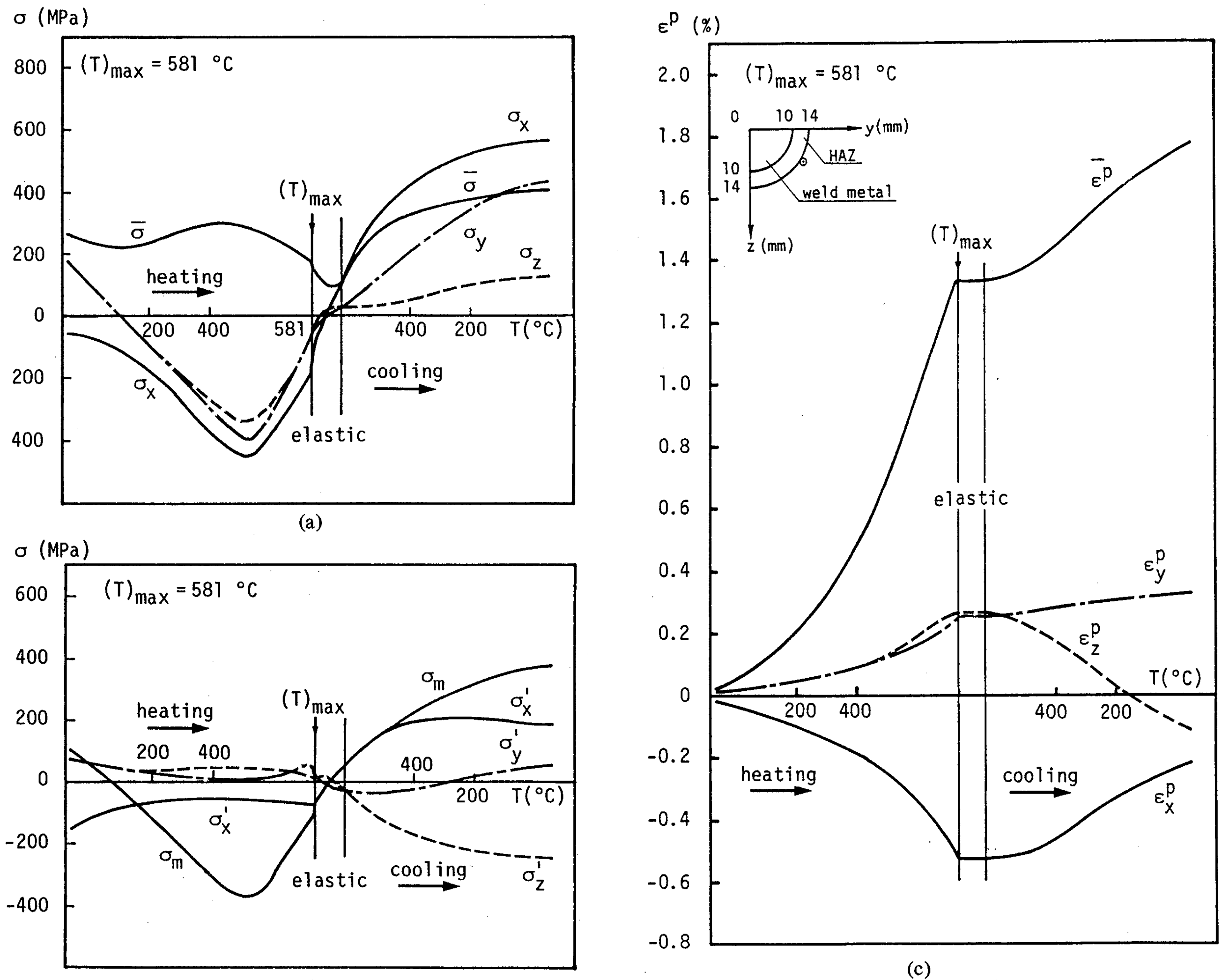
The degree of severity of mechanical restraint in each direction may be determined as $y\text{-} > x\text{-} > z$-direction from the magnitude of the produced stress components, σ_x, σ_y and σ_z. The restraint in the y-direction at the heating stage is the severest. Even if the weld metal is molted, the expansion of HAZ may be restricted by the base plate because the observing point is adjacent to the base plate. It may be considered that the least restraint is in the z-direction. The magnitude of stress components may be considered as $\sigma_y > \sigma_x > \sigma_z$ throughout the heating stage.

As for the plastic strains produced at the observing point in HAZ at the heating stage, the plastic strain component ϵ_y^p perpendicular to the weld line which is the severest of mechanical condition is largely compressive, and the strain component ϵ_x^p along the weld line is slightly compressive because the unwelded portion restricts the expansion at the observing point. On the contrary, the component ϵ_z^p becomes largely tensile in order to meet the condition of constant volume.

However, HAZ at the heating stage undergoes higher temperature than the mechanical melting temperature T_m (at which yield stress of material is extremely reduced; *700*°C, refer to Fig. 3) and releases once stresses and strains produced at the heating stage. Then, HAZ enters the cooling stage soon after having undergone the maximum temperature. HAZ is different to the weld metal and base plate since the stresses and strains produced in the heating stage are released in HAZ. Consequently, the residual stresses and strains in HAZ are produced only at the cooling stage below T_m similar to those in the weld metal. As the results, their production mechanisms are also the same as those in the weld metal.

(c) Base plate

Base plate around HAZ undergoes both heating and cooling stages in the wide temperature range. **Figure 11** shows the production processes of residual stresses and strains produced at the mid-cross-section of the base plate (y = *12*, z = *8* mm). Illustrated is the transient mean normal stress σ_m, which grows to be large in compression at the heating stage and greatly affects the three-dimensional residual stresses. Even though the base plate is at a room temperature (before the passing of arc), σ_x along



(a) (b) (c)

Fig. 11 Transient stresses $\{\sigma\}$, stress deviations $\{\sigma'\}$ and plastic strains $\{\varepsilon^P\}$ in base metal (at middle section, $y = 12, z = 8$ mm).

the weld line is compressive but both σ_y and σ_z are tensile, because the relevant base plate suppresses the thermal expansion of the welded portion.

Regarding the base plate concerned, the production processes will be considered, being divided into the heating and cooling stages.

First, consider the heating stage. The stress components $\left\{\sigma_x, \sigma_y, \sigma_z\right\}^T$ produced in the base plate by heating are entirely compressive because the expansion at the observing position is restricted by the surrounding base plate which stays at lower temperature, and their absolute values tend to be large. The increase in the absolute values of σ_x, σ_y and σ_z is attributed to the compressive mean normal stress σ_m mentioned above, which can increase without yielding.

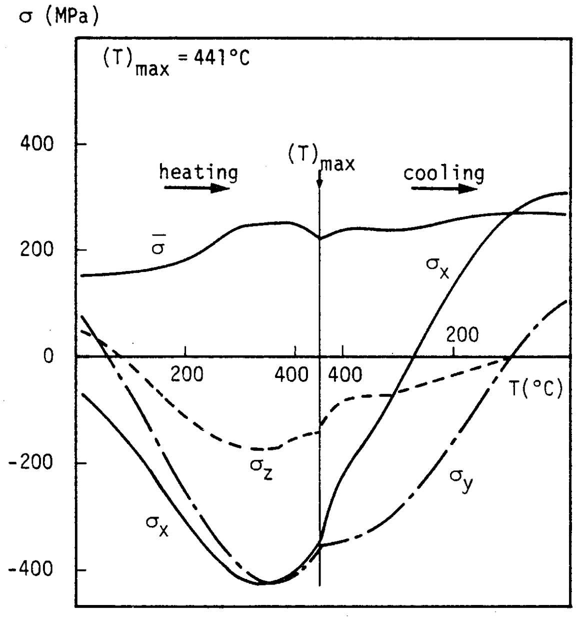
The magnitude of stress components at the heating stage may be ranged as follows: $\sigma_x > \sigma_y \simeq \sigma_z$. Accordingly, the x-direction is known as the severest direction of mechanical restrains at the observing point. As for the y and z-directions, the mechanical restraint of the base plate adjacent to the melting portion is considered almost the same in the both directions because the weld metal and HAZ melt at heating stage, therefore $\sigma_y \simeq \sigma_z$. Considering the plastic strains produced at the observing portion of the base plate ($y = 12, z = 8$ mm) by heating, a large compressive plastic strain ε_x^p is produced in the weld line which is the mechanically severest direction, and tensile strains are produced in other two directions. That is to say, compressive ε_x^p is produced in x-direction of the mechanically severest restraint, and tensile ε_y^p and ε_z^p are equally produced in y and z-directions of relatively smaller degree of restraint in order to satisfy the condition of incompressibility of plastic strains.

Secondly, the cooling stage will be considered. In the cooling stage after having attained the maximum temperature ($(T)_{max} = 581$°C), unloading begins to take place and the behavior is changed from the plastic range to the elastic. Thus attained elastic behavior (Fig. 11) is again changed to the plastic due to increase of stress and strain in the unloading direction. The weld metal and HAZ recover material stiffness at the cooling stage, and the severity of mechanical restraint in the y-direction differs from that in the z-direction near the upper surface of specimen. This may be explained by the fact that $\sigma_y > \sigma_z$. Different from the heating stage, the magnitude of stress components are $\sigma_x > \sigma_y > \sigma_z$ in the cooling stage.

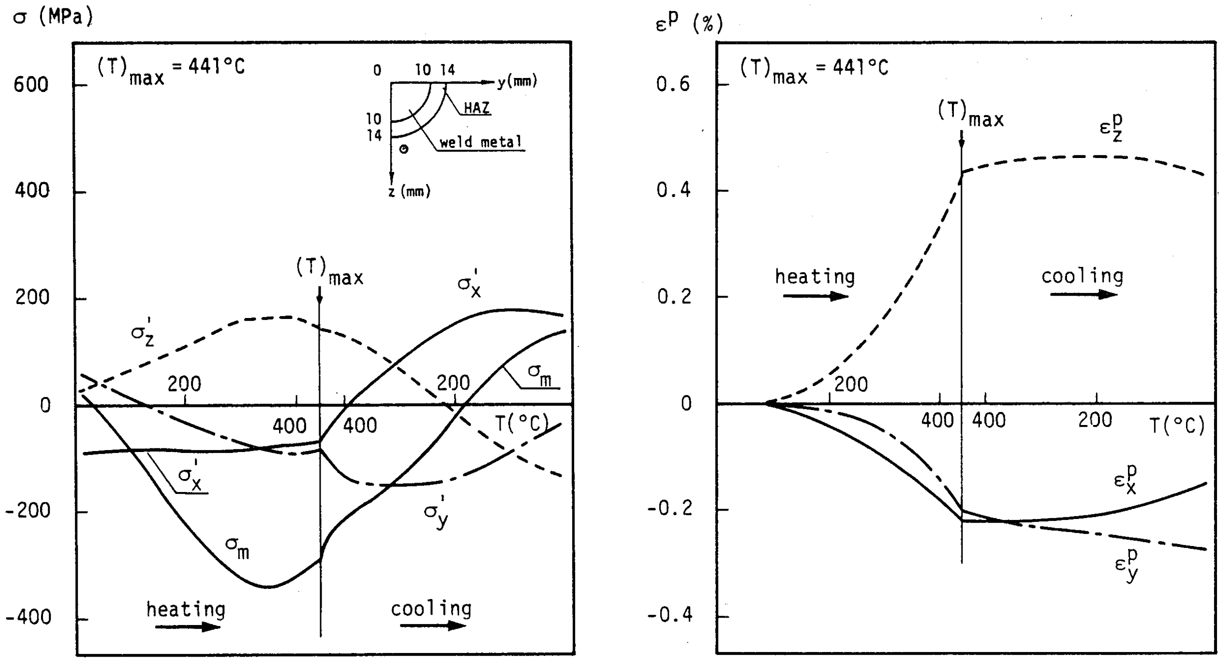
Large tensile plastic strain ϵ_x^p is produced by cooling in the weld line where the mechanical restraining condition is the severest. However, the total magnitude of this strain component ϵ_x^p is considered to be still compressive since the absolute value of compressive plastic strain produced by heating is very large. It may be also considered that the plastic strain in the z-direction, where the mechanical restraining condition is the least, changes largely under the influence of change of ϵ_z^p.

Residual stresses and plastic strains produced in the base plate ($y=3, x=16.5$mm) at the mid-cross-section are shown in **Fig. 12**. Due to the effect of large compressive stress produced by heating, residual stresses are not so large in the base plate near HAZ in spite of the large tensile stress produced by cooling. While the residual stress σ_z is scarcely produced, the reason why the large tensile plastic strain ϵ_z^p is produced in the thickness direction may be clarified by considering the severity of mechanical restraint in each direction at cooling and heating stages as in the following. It is predictable that the restraint in the z-direction is the weakest at the heating stage. Although the weld metal and HAZ recover the mechanical stiffness at the cooling stage, the mechanical restraint in the z-direction is still weaker than those in the other two directions due to the effect of the free surface of specimen. Accordingly, it is considered that, throughout the cooling and heating stages, the large residual plastic strain ϵ_z^p is produced in the z-direction where the mechanical restraint is the weakest.

In the base plate, differences between the absolute values of stresses and plastic strains produced by cooling and those by heating cause residual stresses and residual plastic strains at a room temperature. The residual stresses and plastic strains produced in the base plate near HAZ are largely influenced by heating. In other words, residual



(a)



(b) (c)

Fig. 12 Transient stresses $\{\sigma\}$, stress deviations $\{\sigma'\}$ and plastic strains $\{\epsilon^p\}$ in base metal (at middle section, $y = 3$, $z = 16.5$ mm).

stresses and plastic strains are largely influenced by the compressive mean normal stress which is indifferent with yielding and the compressive plastic strain produced by heating, respectively.

3.2 Consideration

The source of welding stresses and strains is the expansion and shrinkage due to the heating and cooling of the material. Stresses and plastic strains are produced by either internal or external restraint of these expansion and shrinkage. The magnitude and the direction of the produced stresses and plastic strains are determined depending on the distribution of temperature and the severity of mechanical restraints.

The maximum value of ϵ_x^p is observed near the center of specimen toward the finishing end of welding as seen in Fig. 6 which shows the distribution of residual strains in the weld metal along the weld line (refer to the Section 3.1). This point is near the position where temperature becomes maximum in the latter half of the cooling stage, and large shrinkage strains are restrained by a portion with lower temperature. This is predictable from the transient temperature curve shown in Fig. 5.

On the other hand, *y* and *z*-directional thermal histories are the same because the isothermal contours of *yz*-plane are concentric circles (Section 2.2), that is, the magnitude of expansion and shrinkage is same in the *y* and *z*-direction. The magnitude of residual stresses and plastic strains in the *y* and *z*-direction of the base plate near HAZ is different because the severity of mechanical restraint in each direction is varied in the cooling and heating stages.

Here, the reason why the maximum residual tensile stresses are produced at the bottom of HAZ in the mid-cross-section will be explained. HAZ does not receive any effect of the heating stage because it once entirely releases the stresses and strains produced at the heating stage. HAZ, which is located deep in the thickness direction of the plate and produces tensile stresses through the wide temperature range from the temperature to recover mechanical stiffness to a room temperature, is considered to hardly receive the effects of free surface.

In relation to the effect of plastic strains, according to other papers[4] on thermal strain embrittlement of steel welds, hot pre-plastic strains largely detract the ductility of steel.

As shown in Fig. 11, the residual plastic strain in the base plate near HAZ is small though the large plastic strain is produced around the maximum temperature. Considering the effect of hot pre-plastic strains, it is important to consider not only the magnitude of residual plastic strains but also the whole production history of plastic strains.

As a mechanical measure for the whole history of plastification, either the sum of the equivalent plastic strain increments or plastic work which is a scalar can be considered. In this paper, not only the sum of the equivalent plastic strain increments but also plastic work is proposed as a measure to quantitatively evaluate the severity of mechanical damages throughout the whole history of plastification. The equation of plastic work W^p is as follow:

$$W^p = \int dW^p = \int \{\sigma\}^T \{d\epsilon^p\}$$

where dW^p : increment of plastic work

$\{\sigma\}$: stress

$\{d\epsilon^p\}$: increment of plastic strain

Plastic work per unit volume at the mid-cross-section is calculated and shown along the thickness as in **Fig. 13**. The residual plastic strains become maximum in the HAZ but plastic work is maximum in the base plate near HAZ. The distributions of the sum of the equivalent plastic strain increments along the thickness tend to be equal to that of the plastic work.

Consequently, the base plate undergoes both histories of heating and cooling, and large plastic strain is produced in the heating stage, while HAZ undergoes only the

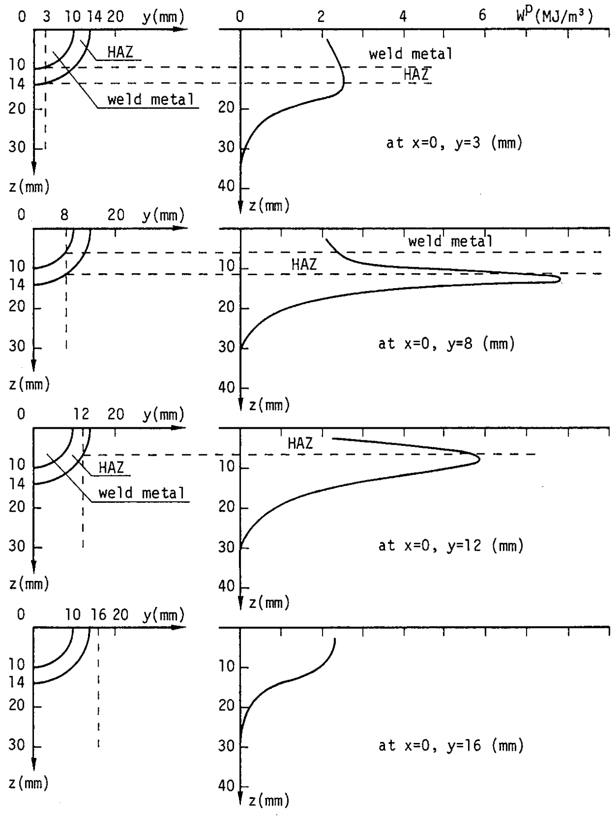


Fig. 13 Distributions of plastic work per unit volume (at middle section).

cooling history just like the weld metal. The weld metal, HAZ and base plate near HAZ are generally subjected to severe mechanical conditions.

4. Conclusion

In this paper, three-dimensional unstationary thermal conduction analysis and three-dimensional thermal elasto-plastic analysis are performed. Based on the results, study is made on the characteristics of the distributions of three-dimensional welding residual stresses and plastic strains, and their production mechanisms. The main conclusions can be summarized as follows:

(1) Residual stresses and plastic strains in the base plate are largely affected by compressive mean normal stresses and compressive plastic strains produced in the heating stage, respectively. This is remarkable in the base plate near HAZ.

(2) Irrespective of elastic or plastic state, severity of mechanical restraints in different directions can be known from the magnitude of produced stress components, $\left\{ \sigma_\sigma, \sigma_y, \sigma_z \right\}^T$.

(3) Since plastic strains are assumed to be incompressive, the sign of plastic strain produced in the direction where maximum stress is produced (mechanical restraint is the severest) is opposite to that in the direction where minimum stress is produced (mechanical restraint is the weakest). Therefore, the signs of stress and plastic strain may not agree.

(4) For evaluation of the mechanical damage of the material caused by welding, it is necessary to pay attention to the whole elasto-plastic behaviors, in addition to the magnitude of residual plastic strains. In this paper, the plastic work and the sum of the equivalent plastic strain increments are proposed as measures to evaluate the severity of mechanical damage throughout the entire history of thermal elasto-plastic behavior. Both measures become maximum in the base plate near HAZ.

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