

Mechanical performance of a dental composite : probabilistic failure prediction

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Mechanical Performance of a Dental Composite: Probabilistic Failure Prediction

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In clinical situations, the mechanical performances of dental structures—for example, composite restorations—depend on many factors. Most of them have a probabilistic character. Because composites are brittle materials, their strength should also be considered as a probabilistic quantity. For successful prediction of mechanical failure of structures consisting of these materials, a probabilistic approach is indispensable, and a suitable definition of equivalent stress must be introduced. An equivalent stress facilitates the transfer of strength data of laboratory specimens to situations where the stress state is much more complicated.

The tensile and compressive strengths of composites differ considerably. Of two equivalent stress definitions that potentially describe this experimental fact (the Drucker-Prager and the Modified von Mises equivalent stress), the predictive capacity was investigated for a microfine composite. In a probabilistic approach to failure, use of the Drucker-Prager equivalent stress appeared to be superior, because the average failure load of notched beams was predicted with an error smaller than 8%.

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Introduction.

The mechanical strength of dental materials has always been an important issue in dentistry. Standardized tests are performed to determine the various types of strength—for example, tensile or compressive strength. Also, more sophisticated methods, such as those applied in fracture mechanics, are used to characterize mechanical failure (Lloyd and Mitchell, 1984; Draughn, 1985; Goldman, 1985; Pilliar *et al.*, 1986; Davis and Waters, 1987; Al-Mulla *et al.*, 1988; De Groot *et al.*, 1988; Ferracane, 1988), and fatigue studies are gaining more and more importance (Asmussen and Jørgensen, 1982; Draughn, 1985; Soltész *et al.*, 1988; Zardiackas *et al.*, 1989; McCabe *et al.*, 1990; Huysmans *et al.*, 1992). Recently, the probabilistic approach to the mechanical testing of materials was introduced in dentistry, mainly as a method for statistical analysis of data (De Rijk and Tesk, 1986; McCabe and Carrick, 1986; Drummond and Miescke, 1991).

The probabilistic approach (Kittl and Diaz, 1988) has been developed to account for the effects (size effects, variability, processing and machining influences) of statistically distributed damage (porosities, micro cracks,

and inclusions) on the initial (quasi-static) and the fatigue strengths of structures. For successful prediction of structural failure, a probabilistic procedure is necessary, particularly for brittle engineering materials such as glass or ceramics. Dental composites and, albeit to a lesser extent, amalgams can be considered as brittle materials. So it is logical to use the probabilistic approach to failure prediction also in dentistry. Van der Varst *et al.* (1991) outlined a computational and experimental method for prediction of long-term mechanical performance, *i.e.*, the mechanical lifetime of dental structures. In this method, the probabilistic approach is not limited to material behavior but also includes the stochastic character of the masticatory loading. Huysmans (1992) used the method for evaluation of the long-term behavior of direct post-and-core restorations with quasi-static and fatigue data. Basically, the method described by Van der Varst *et al.* (1991) can be used for pre-clinical testing, for more efficient planning of clinical research, or for purposes of material comparison.

The idea is to establish a failure probability p as a function of some load variable. With the so-called Weibull three-parameter function, the basic equation for the failure probability of a uni-axially and uniformly stressed tensile specimen is:

$$p(\sigma) = \begin{cases} 0 & \text{if } \sigma < \sigma_0 \\ 1 - \exp\{-[(\sigma - \sigma_0)/\sigma_r]^m\} & \text{if } \sigma \geq \sigma_0 \end{cases} \quad (1)$$

in which σ , the load variable mentioned above, is the tensile stress applied to the specimen, and σ_0 , σ_r , and m , the Weibull parameters, are the threshold stress, the reference stress, and the Weibull modulus, respectively. In practice, the stress state is mostly multi-axial and also non-uniform, and the Weibull formula (1) has to be generalized. Usually, the expression

$$p(F) = 1 - \exp\left\{-\frac{1}{V_r} \int_{vol} \left[\frac{(\sigma_{eq}(\chi, F) - \sigma_0(\chi))}{\sigma_r(\chi)} \right]^m dV \right\} \times H(\sigma_{eq}(\chi, F) - \sigma_0(\chi)) \quad (2)$$

is proposed. In this formula, F is the load, V_r is a reference volume, the Weibull parameters are as above, χ is the position of the material points of the structure, H is the Heaviside step function ($H = 1$ if its argument is equal to or greater than zero and $H = 0$ if its argument is smaller than zero), and σ_{eq} is the so-called equivalent stress. By allowing the Weibull parameters to depend, in a suitable manner, on the position χ , eq. (2) can also be adapted for application to compound structures, including interfaces, or for a description of failure dominated by surface effects.

Formula (2) can be used in two ways: first, for estimation of the Weibull parameters on the basis of laboratory data; and second, once these parameters are known, for prediction of the failure probability of other structures. In

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the latter case, the analysis is usually preceded by a finite element analysis (Dortmans and De With, 1990; Van der Varst *et al.*, 1991) for determination of the stress state in the structure.

The concept of equivalent stress expresses the idea that widely different local stress states lead to the same failure probability for a small volume or surface element of the material. So, these stress states are considered to be equivalent, and each complete set of equivalent stress states is characterized by a scalar quantity called equivalent stress. In view of the central role of equivalent stress in formula (2), the question of how to define an equivalent stress for a particular material is important. The equivalent stress is a scalar representation of the three-dimensional stress state which is defined by the three principal stresses and associated directions. Obviously, this mapping is dependent on material characteristics such as the initial damage. For dental composites, it is reasonable to assume that the initial damage is isotropic. In that case, the equivalent stress depends only on the principal stresses.

For ceramic materials, equivalent stress definitions are derived from empirical criteria or from a variety of principles such as independent action, normal stress averaging, coplanar energy release rate, and maximum non-coplanar energy release rate (Dortmans and De With, 1992). It is typical of these approaches that the starting point is an oriented crack subjected to normal and shear stresses and that next a summation over all possible directions of the crack normal is performed. Therefore, a classification of these approaches as 'micro fracture mechanics' methods is justified. In this paper, a phenomenological approach is pursued. A set of equivalent multi-axial stress states is displayed as a surface in the three-dimensional space spanned by the principal stresses (Fig. 1). Each surface is characterized by a constant value of the equivalent stress. Because the equivalent stress is uniquely related to the failure probability, these surfaces may be referred to as 'equi-probability of failure' surfaces. The dependence of the equivalent stress on the principal stresses σ_1 , σ_2 , and σ_3 is mathematically expressed as:

$$\sigma_{eq} = \sigma_{eq}(\sigma_1, \sigma_2, \sigma_3). \quad (3)$$

The right-hand side should be invariant when the three principal stresses are interchanged. This can be achieved by formulating σ_{eq} in terms of I_1 , I_2 , and I_3 , the three invariants of the stress tensor (Timoshenko and Goodier, 1970). Although there are several possibilities to do so, it is common practice to apply the formulation:

$$\sigma_{eq} = \sigma_{eq}(I_1, J_2, I_3), \quad (4)$$

with:

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3, \quad (5)$$

$$J_2 = 2I_1^2 - 6I_2 \\ = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2, \quad (6)$$

$$I_3 = \sigma_1\sigma_2\sigma_3. \quad (7)$$

It is well-known that for dental composites the uniaxial compressive strength is about 5 to 10 times the uni-

axial tensile strength. This vast difference between compressive and tensile strengths implies that the equivalent stress at least should depend on I_1 and J_2 . To keep matters as simple as possible, it makes sense to assume that the equivalent stress is independent of I_3 , because this only implies that the surfaces of equi-probability of failure possess rotational symmetry with respect to the hydrostatic axis.

The problem of how a suitable equivalent stress for dental composites should be defined was considered some years ago by De Groot (1986). Instead of a probabilistic approach, he proposed to use a critical value of the so-called Drucker-Prager (DP) stress:

$$\sigma_{dp} = \frac{k-1}{2k} I_1 + \frac{k+1}{2k} \sqrt{\frac{1}{2} J_2} \quad (8)$$

or of the Modified von Mises' (MVM) stress:

$$\sigma_{mvm} = \frac{k-1}{2k} I_1 + \frac{1}{2k} \sqrt{(k-1)^2 I_1^2 + 2J_2} \quad (9)$$

as failure criterion (k is the ratio of compressive strength to tensile strength). This approach was tested for two types of dental composites. The results were rather disappointing in the sense that the critical values of σ_{dp} and also those of σ_{mvm} , which should have been equal for two different and independent experiments, differed in fact by a factor of approximately 2.5. However, a rough analysis of the experimental data of De Groot (1986) suggests that this large difference was probably not caused by a completely wrong definition of the equivalent stress but might be attributed to the fact that a deterministic approach was used.

Both the DP and the MVM stress can serve as appropriate definitions of equivalent stress in a probabilistic approach and can take the difference in compressive and tensile strengths of composites into account. So, the aim of our study was to investigate, within the setting of a probabilistic theory of failure, the predictive capabilities of each of these definitions of equivalent stress. First, the Weibull parameters σ_0 , σ_r , and m were estimated from failure data (three-point bend tests) of rectangular beams. Next, the average failure loads of differently shaped specimens (notched beams), also tested in three-point bending, were predicted. These predictions were compared with experimental data of notched specimens fractured in a three-point bend test.

Materials and methods.

Three-point bend tests with rectangular beam (RB) specimens.—Specimens (16 x 2 x 2 mm) of composite filling material for anterior or posterior use (Silux[®], a BisGMA resin with colloidal silica particles, average size 0.04 μ m; filler content 51% by weight, according to the manufacturer, batch nos. 060884, 5502U, 4Y1 and 121384 5502U, 4BC1, 3M Co., St. Paul, MN) were produced by insertion of the paste into a brass mold, followed by polymerization with a visible-light source (Translux[®], Kulzer & Co. Bereich Dental, D-6382 Friederichsdorf 1, Germany) through a glass cover. Each specimen was illuminated three times, each time for 40 s. Before being tested, all specimens were stored in tap water at 37°C for one day. The specimens

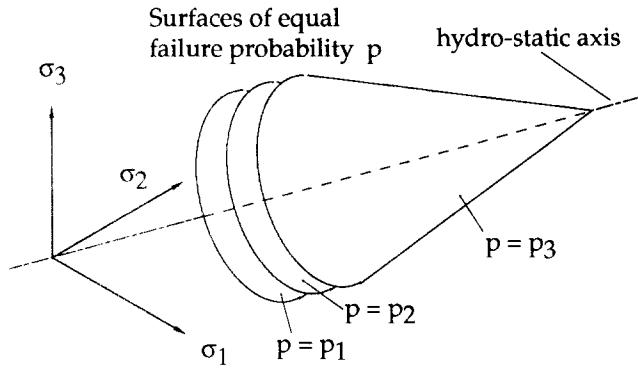


Fig. 1—Example of surfaces of equal failure probability p in the space spanned by the principal stresses σ_1 , σ_2 , and σ_3 . Equivalent stress definition according to Drucker-Prager. Surfaces are cones with: $p_1 < p_2 < p_3$.

were fractured in a three-point bend test (Fig. 2) with an Instron testing machine (cross-head speed, 0.5 mm/min), and load-deflection (F,u) curves were recorded. Young's modulus E was determined according to (Williams, 1973):

$$E = \frac{F s^3}{4 b u h^3} \left\{ 1 + 3 \left(1 + \frac{\nu}{2} \right) \frac{h^2}{s^2} \right\}, \quad (10)$$

in which F and u are the load and the corresponding deflection, respectively, during linear elastic behavior. The height and width of the specimens are denoted by b and h , while s is the span (Fig. 2). Poisson's ratio ν was taken to be 0.3, a value well within the range found in the literature (Whiting and Jacobsen, 1980).

For an RB specimen, loaded with a force F , the maximum bend stress σ equals

$$\sigma = \frac{3Fs}{2bh^2}. \quad (11)$$

Application of the basic equation (2) for the failure probability, assuming that the Weibull parameters are constant throughout the material and applying simple beam theory, with the choice of either σ_{dp} or σ_{mvm} as the definition of the equivalent stress gives the same failure probability p

$$p(\sigma; \alpha, \sigma_o, m) = 1 - \exp \left\{ -\alpha \psi(\sigma/\sigma_o, m) H(\sigma - \sigma_o) - \alpha \psi(\sigma/(k \sigma_o), m) H(\sigma - k \sigma_o) \right\}, \quad (12)$$

with α and the function ψ as defined by Kittl and Diaz (1988):

$$\alpha = \frac{bhs}{2v_r} \left(\frac{\sigma_o}{\sigma_r} \right)^m, \quad (13)$$

$$\psi(\xi, m) = \frac{1}{\xi(m+1)} \int_1^\xi (t-1)^{m+1}/t dt. \quad (14)$$

Note that instead of the fracture load F , the maximum bend stress σ is used as the load-related variable in the failure probability. The reference volume v_r can be chosen arbitrarily as $v_r = bhs = 48 \text{ mm}^3$. The value of k was derived from values of tensile and compressive strength as given by the manufacturer, $k = 8$. From the failure data of the RB

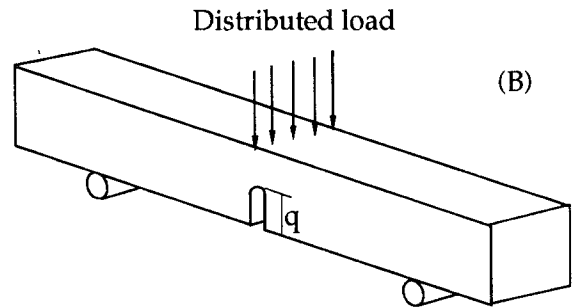
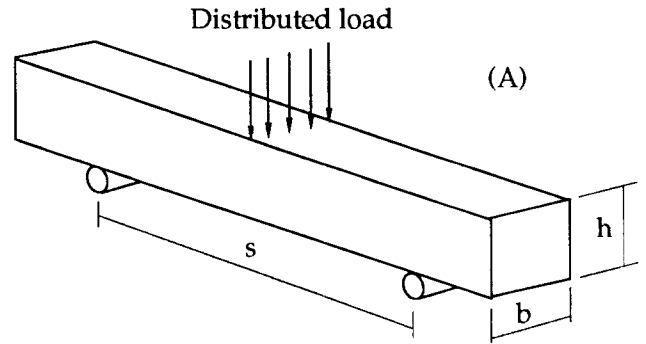


Fig. 2—Experimental set-up (three-point bend test of simply supported beams) for the RB specimens (A) and the NB specimens (B). The span $s = 12 \text{ mm}$, height $h = 2 \text{ mm}$, width $b = 2 \text{ mm}$, and the notch depth q (various sizes) are also indicated. Drawing not to scale.

specimens, estimation of the parameters α , σ_o and m was performed according to the maximum likelihood method (Dudewicz and Mishra, 1988). This implies that, for known failure data σ_i ($i = 1, 2, \dots, n$), the parameters α , σ_o , and m were determined by maximization of the likelihood function:

$$\prod_{i=1}^n \{ \partial p(\sigma; \alpha, \sigma_o, m) / \partial \sigma \} \sigma = \sigma_i. \quad (15)$$

To ensure that the estimates for α , σ_o and m have physical meaning, the estimation was performed under the constraints $0 < \alpha$, $0 < \sigma_o < \min\{\sigma_1, \dots, \sigma_n\}$, and $m > 1$. With the results for α , σ_o , and m , the estimate for σ_r was derived from eq. (13).

Three-point bend tests with notched beam (NB) specimens.—From the same type of composite filling material, rectangular beam specimens were first made following the procedure as described above. Next, the set of specimens was divided into 3 groups. In each specimen of each group, a notch [depth $q \approx 0.5 \text{ mm}$ (group I), 1 mm (group II), or 1.5 mm (group III)] was machined with a 0.15-mm-thick diamond disk (537/220 H superdiaflex Horico*, Hopf Ringleb & Co. GmbH, Berlin, Germany) under water coolant (Fig. 2). Storage and testing equipment and conditions were the same as for the RB specimens. The specimens were fractured in a three-point bend test (Fig. 2), and the loads at fracture were recorded. The notch depth q was measured with a measuring microscope.

Sample mean and sample variation of the load at fracture and the notch depth were determined for each of the groups.

Prediction of the failure probability of the NB specimens: general analysis.—Because of the limited accuracy with which the notch could be machined, the notch depth should also be considered as a random variable, since the strong influence of notch depth on the strength of the specimens cannot be neglected. The failure probability p , determined by application of the basic formula (2) to a notched specimen, is in fact a conditional probability $p(F|q)$, the condition being that the notch depth has a known value. Let $\phi(q;g)$, $\mu_q(g)$, and $V_q(g)$ denote the probability distribution density, expected value, and variance of the notch depth, respectively, of the specimens of group g :

$$\mu_q(g) = \int q \phi(q;g) dq, \tag{16}$$

$$V_q(g) = \int [q - \mu_q(g)]^2 \phi(q;g) dq. \tag{17}$$

The expected value $\mu_F(g)$ and variance of the failure load $V_F(g)$ of the specimens of group g is then

$$\mu_F(g) = \int \mu_{F|q}(q) \phi(q;g) dq, \tag{18}$$

$$V_F(g) = \int [V_{F|q}(q) + \{\mu_{F|q}(q) - \mu_F(g)\}^2] \phi(q;g) dq. \tag{19}$$

Here, the functions $\mu_{F|q}(q)$ and $V_{F|q}(q)$ are the conditional expected value and conditional variance, respectively, of the failure load F :

$$\mu_{F|q}(q) = \int_0^{\infty} F \frac{\partial p(F|q)}{\partial F} dF = \int_0^{\infty} \{1 - p(F|q)\} dF, \tag{20}$$

$$\begin{aligned} V_{F|q}(q) &= \int_0^{\infty} \{F - \mu_{F|q}(q)\}^2 \frac{\partial p(F|q)}{\partial F} dF \\ &= 2 \int_0^{\infty} F \{1 - p(F|q)\} dF - \{\mu_{F|q}(q)\}^2. \end{aligned} \tag{21}$$

Sample mean and sample variance are unbiased estimators of $\mu_F(g)$ and $V_F(g)$. Agreement of the experimental

results with the predictions can be examined by comparison of the sample mean and the sample variance of the data with the predicted values for $\mu_F(g)$ and $V_F(g)$. Moreover, the standard deviation of the mean of a sample of size n is $\sqrt{V_F(g)/n}$. For calculation of the predicted values of $\mu_F(g)$ and $V_F(g)$, data about $\phi(q;g)$ and the conditional failure probability $p(F|q)$ are required.

The probability distribution density $\phi(q;g)$ of the notch depth.—It was assumed that the notch-depth distributions differed only by a translation. Particularly, it was assumed that $\phi(q;g)$ was normally distributed, with $\mu_q(g)$ as the expected value, and that the variance $V_q(g)$ was the same for all three groups. Provided the last assumption is true, the arithmetic mean of the sample variances of the three groups is an unbiased estimator of the variance $V_q(g)$.

The conditional failure probability $p(F|q)$.—To obtain $p(F|q)$, a finite element (FE) stress analysis of the NB specimens was performed. The material was assumed to be isotropic, homogeneous, and linear elastic. Young's modulus E was determined from the experimental data for the RB specimens, and Poisson's ratio ν was taken to be 0.3. For reasons of symmetry, only a quarter of the specimens need to be considered, and a finite element mesh of this part of the specimens was generated (Fig. 3) by use of 20-node isoparametric elements with 8 integration points (Gauss-Legendre integration). Symmetry conditions were enforced with appropriate boundary conditions. Four different values of q were considered. For each value of q , the FE analysis needs to be performed only once, since by taking either $\sigma_{eq} = \sigma_{dp}$ or $\sigma_{eq} = \sigma_{mvm}$ it can be shown that:

$$\sigma_{eq}(\text{at load } F) = \frac{F}{F_n} \sigma_{eq}(\text{at load } F_n) \tag{22}$$

for $F > 0$ and $F_n > 0$. Application of eq. (2) to the NB specimens and discretization of the volume integrals using 8-point Gauss-Legendre integration yield:

$$\begin{aligned} p(F|q) &= 1 - \exp\left[-\frac{4}{V_r} \sum_{e=1}^{ne} \left\{ \sum_{i=1}^8 w_{ei} \{(\sigma_{eq}^{(ei)}(F,q) - \sigma_o)/\sigma_r\}^m \right. \right. \\ &\quad \left. \left. \times H(\sigma_{eq}^{(ei)}(F,q) - \sigma_o) \right\} \right], \end{aligned} \tag{23}$$

in which the reference volume v_r is as before ($v_r = bhs = 48 \text{ mm}^3$), ne is the total number of elements, σ_o , σ_r , and m are the estimated values for the Weibull parameters, $\sigma_{eq}^{(ei)}(F,q)$ is the value of the equivalent stress in integration point i of element number e of a beam with notch depth q and loaded with a total force of F . The quantities w_{ei} are the volume fractions associated with the integration points of the elements. The results of a FE analysis are primarily the values of the principal stresses in every integration point of every element, and from this the values of $\sigma_{eq}^{(ei)}(F,q)$. In every FE analysis, the computation of the stiffness matrix involves numerical integration over the volume of each element, a procedure for which the volume fractions w_{ei} must be evaluated anyway. The FE program was adapted to output these quantities. For each value of the notch depth q , from the results of the FE analysis and the DP and the MVM equivalent stress as definition for σ_{eq} , the failure probabilities $p(F|q)$ were determined. The results for $p(F|q)$ were used to determine numerically the conditional av-

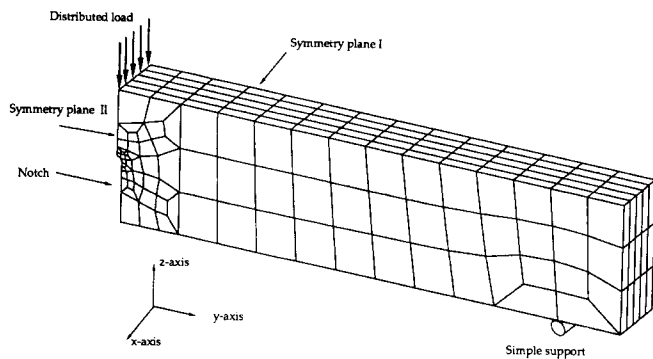


Fig. 3—Mesh of a quarter NB specimen. Load on the mesh is one-fourth of the total load. Planes of symmetry: (I) backside of the mesh and (II) left-hand side of the mesh (above notch). Symmetry enforced by suppression of appropriate displacements (in the x direction for the points of plane I and the y direction for plane II). Shown is the mesh for $q = 1.00$ mm. Specimens were analyzed for notch depth values $q = 0.50, 1.00, 1.50,$ and 1.65 mm, with a mesh consisting of, respectively, $ne = 395, 403, 385,$ and 500 elements.

TABLE 1
RESULTS OF THE BEND TESTS
WITH THE RB SPECIMENS

Number of specimens, n	18
Young's modulus E ± SD (GPa)	5.11 ± 0.36
Average bend strength ± SD (MPa)	68 ± 8
Threshold stress σ_0 (MPa)	40.5
Reference stress σ_r (MPa)	5.3
Weibull modulus m (-)	3.05

average $\mu_{F,q}(q)$ and variance $V_{F,q}(q)$, again for each of the four values of the notch depth. With the aid of numerical fitting procedures, these values for $\mu_{F,q}(q)$ and $V_{F,q}(q)$ were used to establish polynomials representing the conditional average $\mu_{F,q}(q)$ and variance $V_{F,q}(q)$ as continuous functions of the notch depth q. These polynomials, together with $\phi(q;g)$, were used to determine numerically the predicted group means $\mu_p(g)$ and group variances $V_p(g)$, according to eqs. (18) and (19).

Results.

For both the RB and the NB specimens, the relation between load and deflection was found to be linear up to the onset of (sudden) failure.

Experiments with the RB specimens.—Table 1 contains the number of specimens, Young's modulus, average bend strength [bend strength is related to the failure load according to equation (11)] and the estimated values for the Weibull parameters. In Fig. 4, the empirical failure probability, defined as $i/(n+1)$ (n is sample size and i is rank number), has been plotted as a function of

the ranked (in increasing order of magnitude) bend strength data. Fig. 4 also shows a graph of the failure probability according to the Weibull approach [formula (12)] with use of the estimated values of σ_0 , σ_r , and m.

Notch depth of the NB specimens.—Fig. 5 illustrates the probabilistic character of the notch depth. For each of the three groups, the differences between the specimen notch depths and the group sample mean $S\mu_q(g)$ were calculated. The empirical notch depth probability—the ratio between rank number and sample size plus 1—was plotted against the (ranked) difference between the specimen notch depth and the group sample mean. Sample size n, sample mean $S\mu_q(g)$, and sample variation $SV_q(g)$ of the three groups are given in Table 2. When the F-test with the ratios of the group sample variations are used as test statistics, the null hypothesis that the variance of the notch depth is the same for each group was tested (confidence level 95%) against the alternative that the group variances differ. The null hypotheses could not be rejected (Table 2). Therefore, the random variable q was considered to be normally distributed with $S\mu_q(g)$ as an estimate of $\mu_q(g)$ and with equal population variance for all groups. Consequently, the average 0.00357 of the sample variances of the groups is an estimate of the population variance. Fig. 5 also shows the cumulative probability distribution applied for prediction of $\mu_p(g)$ and $V_p(g)$ by use of eqs. (18) and (19).

Experimental failure data of the NB specimens.—The results (sample size, sample mean, and sample standard deviation) are displayed in Table 3.

Predictions of the failure probability of the NB specimens.—Prior to the final computations, several test calculations were made to establish an optimal number of elements. For the system under consideration, the quarter beam, this number was approximately 400 (Fig. 3). The predictions concerning the conditional average $\mu_{F,q}(q)$ and variance $V_{F,q}(q)$ could be fitted by the following polynomials:

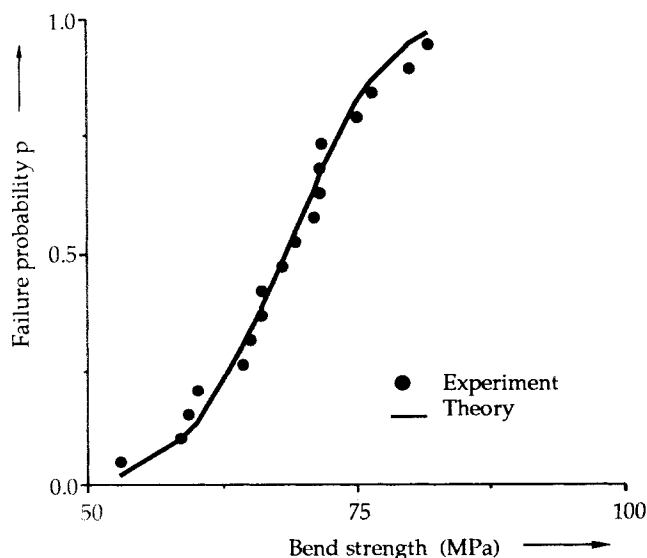


Fig. 4—Experimental and derived theoretical failure probability as function of the bend stress for the RB specimens. Average values (± SD) are: 68 ± 8 MPa (experiment) and 68 ± 7 MPa (theory).

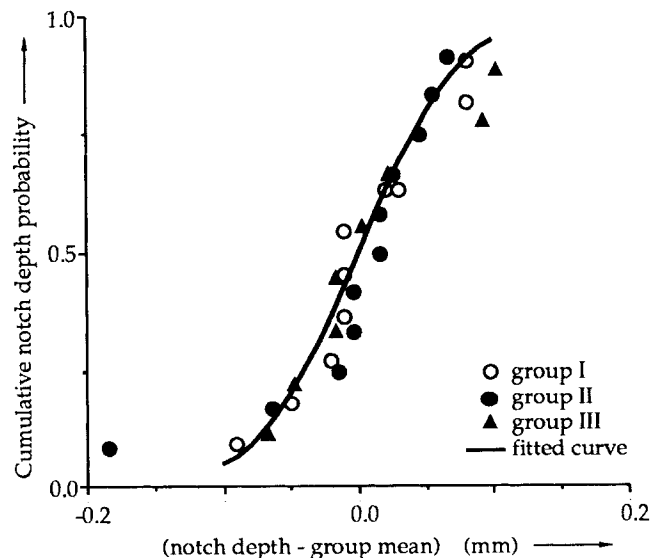


Fig. 5—Experimental data of the cumulative notch depth probability and the derived probability curve. This curve is a normal distribution with zero mean and 0.06 mm standard deviation.

TABLE 2
DATA OF THE NOTCH DEPTH q AND THE RESULTS OF THE F TEST

g	n	$S\mu_q(g)$ (mm)	$SV_q(g)$ (mm ²)	Compared Groups	Test Statistic Type and Value	95% Confidence Interval for Test Statistic
I	10	0.52	0.00289	I and II	$SV_q(I)/SV_q(II)$ = 0.892	0.236-3.536
II	11	1.02	0.00324	II and III	$SV_q(II)/SV_q(III)$ = 0.709	0.149-2.800
III	8	1.64	0.00457	III and I	$SV_q(I)/SV_q(III)$ = 0.632	0.131-2.654

$$\mu_{Fq}(q) = \begin{cases} 16.90 - 13.37 q + 2.30 q^2 & \text{(DP)} \\ 13.90 - 11.31 q + 2.07 q^2 & \text{(MVM)} \end{cases} \quad (24)$$

$$V_{Fq}(q) = \begin{cases} 14.00 - 22.41 q + 12.46 q^2 - 2.43 q^3 & \text{(DP)} \\ 8.69 - 13.94 q + 7.75 q^2 - 1.50 q^3 & \text{(MVM)} \end{cases} \quad (25)$$

Note that the constants in these equations have different dimensions because the dimensions of q , $\mu_{Fq}(q)$, and $V_{Fq}(q)$ are mm, N, and N², respectively. Table 3 contains the results of the theoretical predictions when either the DP or the MVM equivalent stress was used. The predicted values were calculated with use of the formulae (18), (19), (24), and (25) and the results for $\phi(q;g)$ as described above. Also, the values of the standard deviation of the mean were added to the table.

Discussion.

Because during the experiments linear material behavior up to failure was observed and failure occurred rather suddenly, the composite can be considered as brittle. So, the material falls within the class for which a probabilistic approach is deemed necessary (Kittl and Diaz, 1988).

TABLE 3
EXPERIMENTAL AND PREDICTED RESULTS
FOR THE NB SPECIMENS

Group g	I	II	III
Sample size n	10	11	8
Sample mean (N)			
• Experiments	11.5	5.9	1.2
• Predicted \pm SD mean			
* DP	10.6 \pm 0.8	5.6 \pm 0.4	1.2 \pm 0.1
* MVM	8.6 \pm 0.6	4.5 \pm 0.3	0.9 \pm 0.1
Standard deviation (N)			
• Experiments	0.7	0.8	0.5
• Predicted			
* DP	2.4	1.3	0.4
* MVM	1.9	1.0	0.3

Moreover, the linearity ensures that a FE analysis of the type that was performed here is capable of predicting the stresses at the onset of failure.

The value of Young's modulus (Table 1) is in the upper range of the values (Silux: 3.3-5.3 GPa) given by Reinhardt and Vahl (1983), and the bend strength of the RB specimens (Table 1) compares well with the value (Silux: 67.1 \pm 3.1 MPa) found by Boyer *et al.* (1984).

Estimation of the Weibull parameters was performed under the constraints that $m > 1$ and $0 < \sigma_0 < \min\{\sigma_1, \dots, \sigma_n\}$. The first condition is necessary because the hazard function can be expected to be an increasing function of the stress (Batdorf, 1984), whereas a value of m smaller than 1 would lead to a decreasing hazard function for the material. The second condition stems from the observation that, within the setting of the theory, a failure stress smaller than σ_0 is absolutely impossible. So, for a consistent estimation procedure, σ_0 should be smaller than the smallest failure stress (bend strength) actually recorded during the experiments.

Often, the threshold stress σ_0 is assumed to be zero in advance. The advantage of this assumption is that parameter estimation is considerably simplified. However, setting the threshold value to zero is tantamount to restricting the set of admissible parameters, and from mathematics it is known that the maximum of any function over a restricted set is smaller than or, at best, equal to the maximum over the non-restricted set. Consequently, to ensure that the maximum likelihood estimates are actually found, the threshold stress should not be set to zero in advance. The smallest failure stress (bend strength) recorded during the experiments with the RB specimens was approximately 52 MPa. Compared with this value, the estimate of 40.5 MPa for the threshold stress cannot be considered as negligibly small.

The maximum likelihood principle for parameter estimation is intuitively appealing, but for small sample sizes the estimates may be biased. No information is available about the bias of the MLH estimates of the parameters of the probability distribution (12) for the RB specimens. However, assuming that known results about the bias of the two-parameter Weibull distribution [a special case of the distribution (1)] also apply (at least qualitatively) to the present study, the reference value is expected to be biased only slightly, and the Weibull modulus m is underestimated by some 10% (Bain and Engelhardt, 1991). Whatever the bias of the estimates may be, the curve of

the estimated failure probability of the RB specimens fits the experimental data quite well (Fig. 4).

Since the manner in which the notches were made was the same for all three groups, the notch depth variance is expected to be the same for each group. This expectation is supported by the data, because the F-test revealed that the null hypothesis (notch depth variance is the same for each group) cannot be rejected (Table 2). The application of the F-test implies that the underlying population should be normal. Theoretically, this cannot be true, because the notch depth cannot be smaller than zero nor greater than 2 mm (the beam height), whereas a normally distributed random variable may range from $-\infty$ to $+\infty$. However, the standard deviation is much smaller than the difference between the average notch depth values and the notch depth limits. Therefore, the range: average \pm 4.5 times standard deviation, a range that contains more than 99.99% of the probability mass, does not exceed the limits of the notch depth. So, for all practical purposes, the normal distribution could be considered as a suitable candidate for approximating the notch depth distribution. Fig. 5 shows that a normal distribution fits the data rather well. Since a particular distribution is needed anyway, because otherwise the predictions $\mu_F(g)$ and $V_F(g)$ cannot be computed, a normal distribution was used for this purpose. As a corollary, it was possible to use the F-test instead of a nonparametric test such as Wilcoxon's test to decide whether the notch depth distributions of the groups differ only by a translation.

Table 3 contains the final and most basic results of this study. Comparison of the average failure loads showed that, when the DP equivalent stress was used, the difference between prediction and experiment was 8% for group I, 5% for group II, and virtually zero for group III. Only for group I was the observed sample mean just outside the predicted interval (average \pm standard error of the mean) for the sample mean. The difference between predicted and experimental average, when the MVM equivalent stress was used, was larger, and the observed sample means fell outside the predicted intervals. Therefore, it is concluded that the DP definition for the equivalent stress is to be preferred over the MVM definition and, as far as average values are concerned, that the use of the DP equivalent stress leads to sufficiently accurate predictions. For materials that are comparable (in composition and structure) with the material investigated in this paper, it can be expected that the DP definition of equivalent stress will also be appropriate. However, for other types of composites, the DP definition of equivalent stress may not be valid, because there is no reason to assume that a single definition applies to such a large class of materials. After all, the question of how an equivalent stress for a particular material should be defined can be settled only by experimentation.

For both equivalent stress definitions, larger differences occurred between predictions and observations of the standard deviation (SD). While the experimental data suggest more or less constant values, the predicted SD's decreased with increasing notch depth. For group I, the predicted SD was ca. 3.5 (DP) to 2.5 (MVM) times the experimental value. For group III, the predicted and experimental values were of comparable magnitude. These differences might be attributed to an increase of the actual value of the Weibull modulus m caused by the machining

of the notch. Generally, the machining of the notches is expected to have a damage-increasing effect. Particularly in the region near the notch tip, the region where the stresses are highest anyway, it is conceivable that the machining of the notches increased locally the density of the most probable flaw sizes. This would mean that the dispersion of the flaw size distribution decreased, and since this dispersion is thought to be proportional to $1/m$ (Freudenthal, 1968), the machining of the notches may have led to a value of m for the specimens that was higher than the value used for calculation of the predictions.

The DP definition of the equivalent stress seems presently the best measure for assessment of mechanical performance of dental structures consisting of composites of the type considered here. Indeed, used in conjunction with the Weibull approach (3) for the failure probability and using one set of material parameters k , σ_0 , σ_c , and m , failure of specimens in quite different experimental situations (uni-axial tension, uni-axial compression, three-point bending of RB and NB specimens) can be predicted. The accuracy of the average values is, for all practical purposes, sufficient. Combined with a probabilistic approach to short- and long-term mechanical failure [the predictive pre-clinical testing method outlined by Van der Varst *et al.* (1991)], it is possible to model mechanical aspects of the clinical situation more realistically. The manner in which the random character of the notch depth was incorporated is basically the same as the manner of including clinical shape differences, for example, those between restorations of the same type. Fatigue properties can be taken into account by determination of the Weibull modulus, the threshold and reference stresses, and the ratio of compressive to tensile strength during fatigue experiments. The fatigue behavior of the materials also determines the influence of the stochastic nature of masticatory and para-functional loading. If damage growth is independent of the sequence of the load cycles in a given load history, Van der Varst *et al.* (1991) described how to include this influence. Note that the probabilistic approach is complementary to the methods which use more traditional material parameters. While the latter are based on a local description of material behavior, the former focuses on the performance of materials in dental structures operating in normal circumstances.

For the future, it would be interesting to investigate how the Weibull parameters depend on clinically important factors of a more qualitative nature, such as, for example, the dexterity of the dentists, location and accessibility of a restoration, or the handling characteristics of the materials.

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