1978

Mechanical Stresses of Valve Plates on Impact Against Valve Seat and Guard

J. Simonitsch

MECHANICAL STRESSES OF VALVE PLATES ON IMPACT AGAINST VALVE SEAT AND GUARD

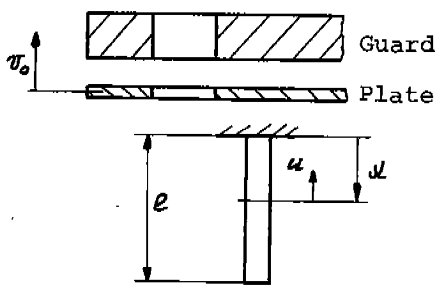
J. Simonitsch, Dir. of Research
Hoerbiger Ventilwerke AG
Vienna, Austria

1) Summary

The present study intends to make possible an estimation of the mechanical stresses which are to be expected in the operation of shocklike loaded valve plates. The charges of the plates are different for the stroke at the guard from those occurring at the impact on the seat. Therefore two different mechanical models are to be examined.

2) The opening conditions (guard-impact)

With most valves the profile of the guard is made equal to the profile of the valve plates in order to obstruct the gas flow as little as possible.



Thus with the opening stroke all points of the valve plate surface theoretically touch the points of the guard surface at the same time. All points of the plate which have the same distance to the contact surface will thus make identical movements at the same moment and therefore undergo identical strains irrespective of effects of transverse dilation. As a mechanical model system which deals with the opening stroke the elastic rod offers itself, which pushes in axial direction against a fixed chock and is excited to longitudinal vibrations.

This problem has often been dealt with in standard literature (f.i. TIMOSHENKO - "Vibration Problems in Engineerings"). His differential equation reads:

$$\frac{\partial^2 u}{\partial t^2} = c_s \frac{\partial^2 u}{\partial x^2} \qquad (1)$$

Under the for the guard shock actual end conditions:

$$x = 0: u = 0 \quad ; \quad x = \ell: \quad \varepsilon = \frac{\partial u}{\partial x} = 0$$

and the initial condition:

$$t = 0 \ : \quad u(x) = 0$$

it has the solution

$$u(x,t) = C \sin \frac{\lambda x}{\ell} \sin \omega t \qquad (2)$$

with $\lambda = \frac{\pi}{2}(2N-1)$ and $\omega = \lambda \frac{c_s}{\ell}$ (3)

where N is any complete, positive integer, c_s the sound velocity and C a constant which is defined by another initial condition.

In the present case the latter reads:

$$\frac{\partial u(x,0)}{\partial t} = v_0 \qquad (4)$$

The presentation of the velocity profile (4) result by superposition of the velocities of the single natural modes:

$$v_0 = \sum \omega_N \cdot C_N \cdot \sin \lambda_N \frac{x}{\ell} \qquad (5)$$

By using relations (3) for the eigenvalues, respectively for the angular natural frequencies we get:

$$v_0 = \frac{\pi \cdot c_s}{2\ell} \sum C_N (2N-1) \sin[(2N-1)\varphi] \qquad (6)$$

with $\varphi = \frac{\pi x}{2\ell}$

The mechanical stress according to Hook's law is given by:

$$\sigma = E \cdot \varepsilon = E \cdot \frac{\partial u}{\partial x} \qquad (7)$$

and can be calculated by a superposition of the single eigenformparts and taking (2) into account:

$$\sigma(x,t) = E \sum C_N \frac{\lambda_N}{\ell} \cos \lambda_N \frac{x}{\ell} \sin \omega_N t \qquad (8)$$

In the present case the mechanical stress at the point of impact is of particular interest:

$$\sigma(\phi,t) = \frac{E\pi}{2e}\sum C_N(2N-1)\sin[(2N-1)\psi] \qquad (9)$$

with $\psi = \frac{\pi \cdot c_s}{2e}\cdot t$

A comparison of equations (6) and (9) shows that for the period

$$\phi \le t \le \frac{2e}{c_s}$$

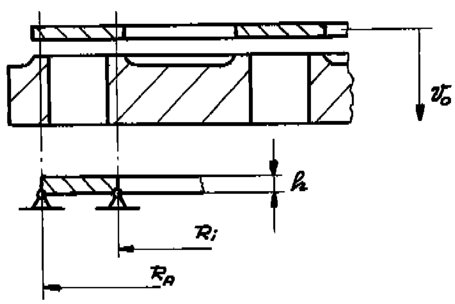
the strain at the point of impact becomes independent of time and is equal to

$$\sigma = E\,\frac{v_0}{c_s} \qquad (10)$$

This value agrees with the value which TIMOSHENKO derived for the rod charged with a single pulse.

3)The conditions when closing(seat-impact)

The valve plates usually represent a system of concentric circular rings which are connected by a number of bridges. When closing, these rings contact - theoretically at the same time - the seat ledges which have the form of narrow circular rings.



Neglecting the influence of the bridges the inwardly and outwardly flexibly suspended circular ring plate which is excited to transverse vibrations can be taken as a mechanical model system to examine the closing stroke.

This problem has also been dealt with in literature (f.i. SHOCK AND VIBRATION HANDBOOK). The following differential equation holds:

$$\Delta\Delta W(r,t) = -\frac{\gamma\cdot h}{g\cdot D}\cdot\frac{\partial^2 W(r,t)}{\partial t^2} \qquad (11)$$

with $D = \frac{E h^3}{12(1-\mu^2)}$ as plate stiffnes (12)

Under the presumption of Bernoulli's criterion

$$W(r,t) = W(r)\cdot W(t)$$

and of a rotationally symmetric load and suspension, equation (11) turns into

$$\frac{\partial^4 W}{\partial r^4} + \frac{2}{r}\frac{\partial^3 W}{\partial r^3} - \frac{1}{r^2}\frac{\partial^2 W}{\partial r^2} + \frac{1}{r^3}\frac{\partial W}{\partial r} - \kappa^4 W = \phi \qquad (13)$$

with $\kappa^4 = \frac{12(1-\mu^2)\omega^2}{h^2\cdot c_s^2}$ (14)

The solution of equation (13) - see f.i. MC LACHLAN:"Bessel Functions for Engineers" - reads:

$$W(r,t) = \{A J_0(\kappa r) + B Y_0(\kappa r) + C I_0(\kappa r) + D K_0(\kappa r)\}\sin\omega t \qquad (15)$$

With J_0, Y_0, I_0 and K_0 as regular, modified Bessel functions and of the 1 and 2 kind respectively and of ϕ order.

For the determination of the 4 constants A,B,C and D the 4 in our case valid end conditions are at our disposal which have to be satisfied at any time t:

$$r = R_A: \quad W = \phi,\; m_r = \phi$$
$$r = R_i: \quad W = \phi,\; m_r = \phi$$

With equation (15) and the equation for the radial moment of bending (wellknown from literature, f.i. GIRKMANN "Flächentragwerke")

$$m_r = -D\left(\frac{\partial^2 W}{\partial r^2} + \frac{\mu}{r}\frac{\partial W}{\partial r}\right) \qquad (16)$$

and after introduction of a dimension coordinate

$$\rho = r/R_A \qquad (17)$$

and with $\beta = R_i/R_A$ (18)

and $\alpha = \kappa\cdot R_A$ (19)

the above mentioned end conditions lead to the following in matrixform represented equations:

$$\begin{vmatrix} J_0(\alpha) & Y_0(\alpha) & I_0(\alpha) & K_0(\alpha) \\ J_1(\alpha) & Y_1(\alpha) & -I_1(\alpha)+\frac{2\alpha}{1-\mu}I_0(\alpha) & K_1(\alpha)+\frac{2\alpha}{1-\mu}K_0(\alpha) \\ J_0(\alpha\beta) & Y_0(\alpha\beta) & I_0(\alpha\beta) & K_0(\alpha\beta) \\ J_1(\alpha\beta) & Y_1(\alpha\beta) & -I_1(\alpha\beta)+\frac{2\alpha\beta}{1-\mu}I_0(\alpha\beta) & K_1(\alpha\beta)+\frac{2\alpha\beta}{1-\mu}K_0(\alpha\beta) \end{vmatrix} \cdot \begin{vmatrix} A \\ B \\ C \\ D \end{vmatrix} = \phi \qquad (20)$$

The equations (20) define the eigenvalues λ and the ratios of three out of four coefficients A to D to the forth. The equation for a normal mode of vibration is given by:

$$w(\rho,t) = C_1\{J_0(\lambda\rho) + C_2Y_0(\lambda\rho) + C_3I_0(\lambda\rho) + C_4K_0(\lambda\rho)\}\sin\omega t \tag{21}$$

This equation is determined except for the factor C_1 ($C_2=B/A$, $C_3=C/A$, $C_4=D/A$).

It is customary to represent the equations for the normal modes in normalised form, i.e. to choose the constant C_1 so that the following normalisation condition is satisfied: $\int w^2(\rho)\cdot\rho\cdot d\rho = 1$

With

$$\mathcal{E}(\rho) = C_1^*\{J_0(\lambda\rho) + C_2Y_0(\lambda\rho) + C_3I_0(\lambda\rho) + C_4K_0(\lambda\rho)\} \tag{22}$$

as natural vibration mode in normalised representation, the equation (21) can be written in the following form:

$$w(\rho,t) = A_0\cdot\mathcal{E}(\rho)\cdot\sin\omega t \tag{23}$$

In equation (23) the initial condition $w(t=\emptyset) = \emptyset$ has already been satisfied. For the determination of the amplitude A_0 another initial condition is at our disposal, namely the statement about the ratio of the initial velocities.

3.1) Superposition of the natural vibration modes

As with the examination of the contact on the guard here too a representation of the initial velocity profile

$$\dot{w}(\rho,\phi) = v_0$$

can be imagined by superposition of all the natural modes:

$$v_0 = \sum A_{0N}\cdot\omega_N\cdot\mathcal{E}_N(\rho) \tag{24}$$

An approximation of this initial velocity profile by a finite number of natural modes is connected with an error. If one demands that the coefficients A_{0N} have to be chosen so that sum of the squares of the errors in the total range becomes a minimum, one gets, taking the orthogonality properties of normal modes into account (literature f.i. COURANT-HILBERT: "Methoden der mathematischen Physik"):

$$A_{0_N} = \frac{v_0}{\omega_N}\int\mathcal{E}_N(\rho)\cdot\rho\cdot d\rho \tag{25}$$

Taking equations (14) and (25) into account the equation for the deformation of the plate by superposition of the normal modes reads:

$$w(\rho,t) = w_0\sum\frac{\alpha_N}{\lambda_N^3}\mathcal{E}_N(\rho)\sin\left(\frac{\lambda_N^2}{\lambda_1^2}\omega_1 t\right) \tag{26}$$

with $$W_0 = \frac{v_0}{c_s}\cdot\frac{R_A^2}{h}\cdot\sqrt{12(1-\mu^2)} \tag{27}$$

and $$\alpha_N = \lambda_N\int\mathcal{E}_N(\rho)\cdot\rho\cdot d\rho \tag{28}$$

The figures D1 and D2 show the representation of the eigenvalues λ_N and also the normal mode integrals α_N as a function of the ratio of the ring radii β in the range examined up to 25 order.

It should be noticed that the contribution of the overtones to the deformation pattern of the rings is very small because they are determined by amplitude factors of α_N/λ_N^3, i.e. they converge quickly.

The bending moments and shear forces we get accordingly to the laws of classical mechanics (see f.i. GIRKMANN) with equation (16) respectively

$$m_t = -D\left(\mu\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{29}$$

and $$q_r = -D\left(\frac{\partial^3 w}{\partial r^3} + \frac{1}{r}\frac{\partial^2 w}{\partial r^2} - \frac{1}{r^2}\frac{\partial w}{\partial r}\right) \tag{30}$$

equation (26) leads to

$$m_r(\rho,t) = \frac{-W_0 D}{R_A^2}\sum\frac{\alpha_N}{\lambda_N}\mathcal{R}_N(\rho)\cdot\sin\omega_N t \tag{31}$$

$$m_t(\rho,t) = \frac{-W_0 D}{R_A^2}\sum\frac{\alpha_N}{\lambda_N}\mathcal{T}_N(\rho)\cdot\sin\omega_N t \tag{32}$$

$$q_r(\rho,t) = \frac{-W_0 D}{R_A^3}\sum\alpha_N\,\mathcal{O}_N(\rho)\cdot\sin\omega_N t \tag{33}$$

Herein is

$$\mathcal{R}(\rho) = C_1^*\left\{\frac{1-\mu}{\lambda\rho}[J_1(\lambda\rho) + C_2Y_1(\lambda\rho) - C_3I_1(\lambda\rho) + C_4K_1(\lambda\rho)] - [J_0(\lambda\rho) + C_2Y_0(\lambda\rho) - C_3I_0(\lambda\rho) - C_4K_0(\lambda\rho)]\right\}$$

$$\mathcal{T}(\rho) = C_1^*\left\{\frac{\mu-1}{\lambda\rho}[J_1(\lambda\rho) + C_2Y_1(\lambda\rho) - C_3I_1(\lambda\rho) + C_4K_1(\lambda\rho)] - \mu[J_0(\lambda\rho) + C_2Y_0(\lambda\rho) - C_3I_0(\lambda\rho) - C_4K_0(\lambda\rho)]\right\}$$

$$\mathcal{O}(\rho) = C_1^*\{J_1(\lambda\rho) + C_2Y_1(\lambda\rho) + C_3I_1(\lambda\rho) - C_4K_1(\lambda\rho)\}$$

From equation (32) we get the contact reactions which interest particulary:

$$\begin{aligned} q_A(t) &= \frac{-W_0 D}{R_A^3}\sum Q_{A_N}\cdot\sin\omega_N t \\ q_i(t) &= \frac{-W_0 D}{R_A^3}\sum Q_{i_N}\cdot\sin\omega_N t \end{aligned} \tag{37}$$

with $Q_{A_N} = \alpha_N\,\mathcal{O}_N(1)$ and $Q_{i_N} = \alpha_N\,\mathcal{O}_N(\beta)$

The support functions Q_N for the outer respectively the inner edge have been represented as functions of the ratio of the radii β in the figures D3 and D4.

The equations (31) and (32) show, that the contributions of the waves of higher order to the course of the bending stresses are not irrelevant, because they are determined by amplitude factors of α_N/λ_N.

The contribution of the single waves of higher order to the contact reactions are basically the same as those of the fundamental wave, so that the contact-stresses theoretically seam to become infinitely large.

This result shows the limits of the thesis of classical mechanics. For the expansion of the waves of higher order the shear-deformation as well as the rotatory inertia are apparently of fundamental importance, so that neglecting those leads to the above mentioned result.

3.2) Energyconsideration

For the calculation of dynamical charges the mechanical energy taken up by the elastic body is often used. An application of this method appears to be possible for our problem too. For the present case we assume that in the ringplate we consider, at the end of the very short impact period, a velocity distribution is established which suffices the end conditions and that the kinetic energy before and after the stroke is the same. With equation (23) this leads to

$$\frac{v_0^2(1-\beta^2)}{2} = A_0^2\,\omega^2 \int \mathcal{E}^2(\rho)\cdot\rho\cdot d\rho \qquad (38)$$

from which we get, taking into account that $\mathcal{E}(\rho)$ represents a normalised normal mode, for the amplitude:

$$A_0 = \frac{v_0}{\omega}\sqrt{\frac{1-\beta^2}{2}} \qquad (39)$$

The equation for the deformation of the plate can be written as, with the aid of equations (14),(27) and (39):

$$W(\rho,t) = W_0\sqrt{\frac{1-\beta^2}{2}}\cdot\frac{1}{\lambda^2}\cdot\mathcal{E}(\rho)\cdot\sin\omega t \qquad (40)$$

The distribution of the moments of bending respectively the contactreactions fallow from (40),(16),(29) to (33):

$$m_r(\rho,t) = \frac{-W_0 D}{R_A^2}\sqrt{\frac{1-\beta^2}{2}}\cdot\mathcal{R}(\rho)\cdot\sin\omega t \qquad (41)$$

$$m_t(\rho,t) = \frac{-W_0 D}{R_A^2}\sqrt{\frac{1-\beta^2}{2}}\cdot\mathcal{T}(\rho)\cdot\sin\omega t \qquad (42)$$

$$q_r(\rho,t) = \frac{-W_0 D}{R_A^3}\sqrt{\frac{1-\beta^2}{2}}\cdot\lambda\cdot\mathcal{Q}(\rho)\cdot\sin\omega t \qquad (43)$$

From equations (41) to (43) the (for $t=\pi/2\omega$) maximum to be expected moments of bending and the contactreactions can be calculated. A representation of these quantities is shown in fugures D5 to D7.

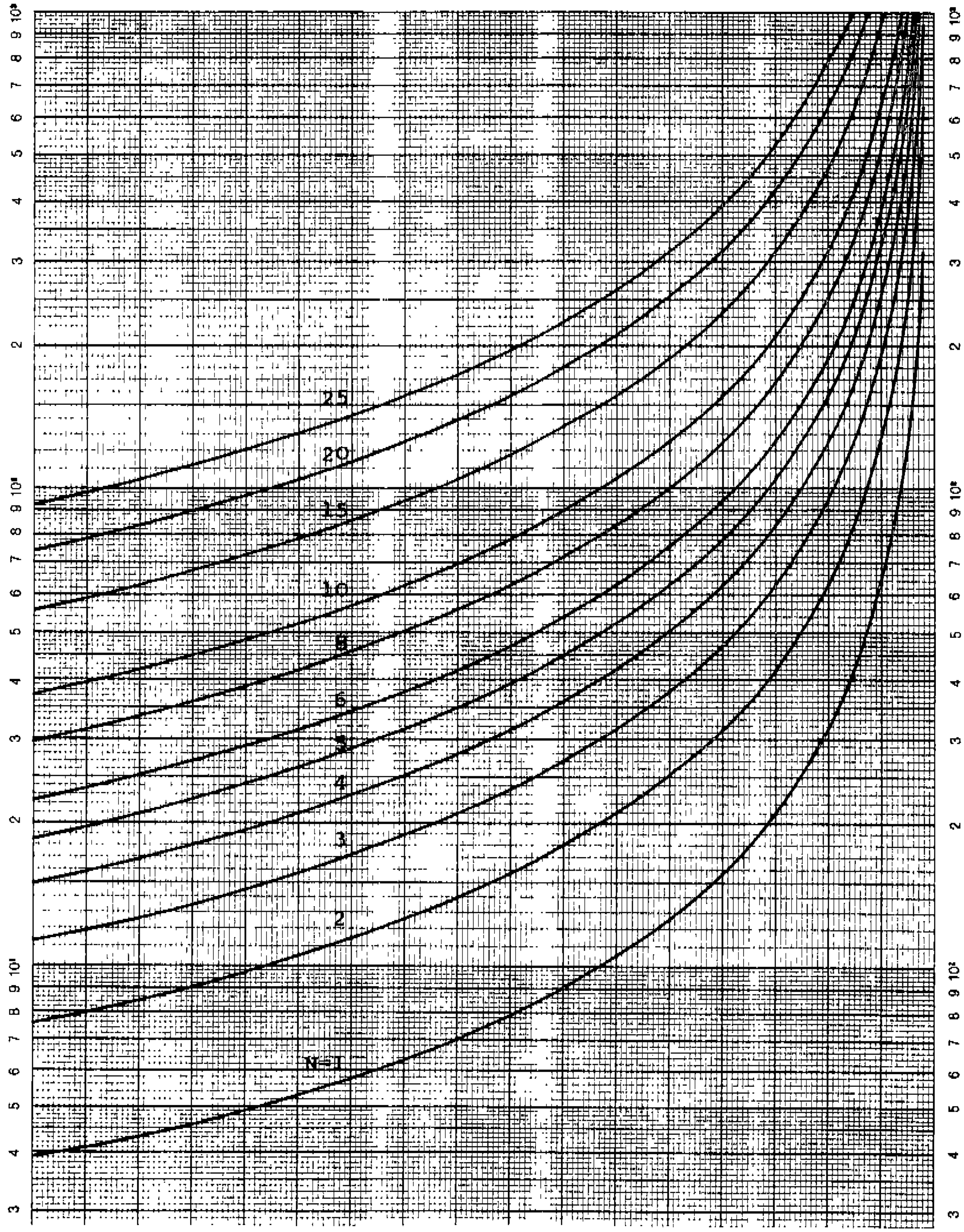


Figure D1: Eigenvalues λ in function of the ratio of the ring radii β

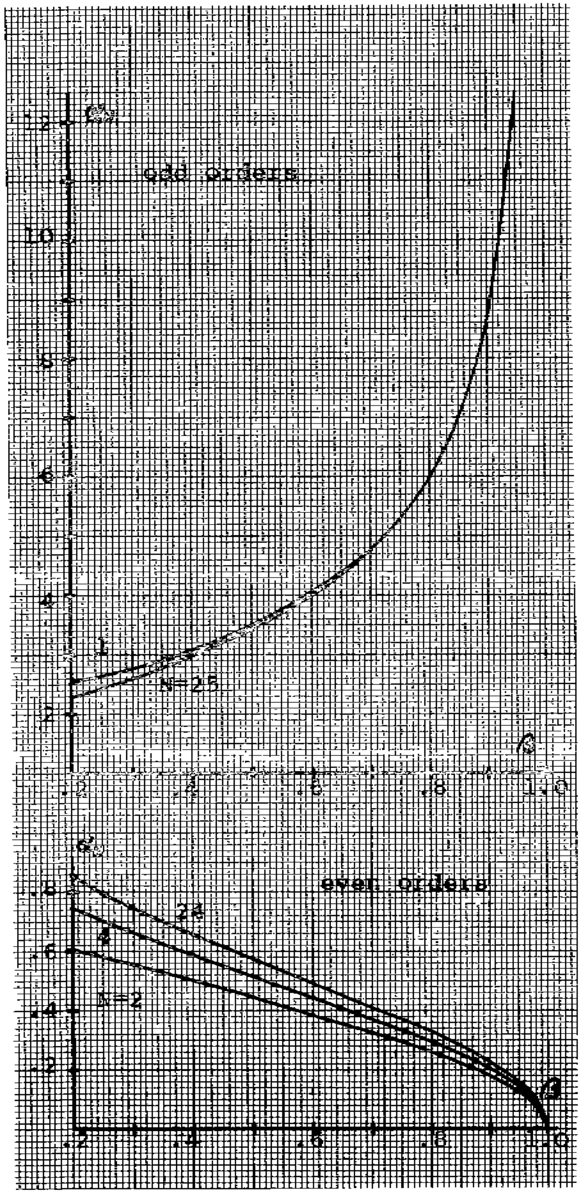


Figure D2: Normal mode integrals α_N

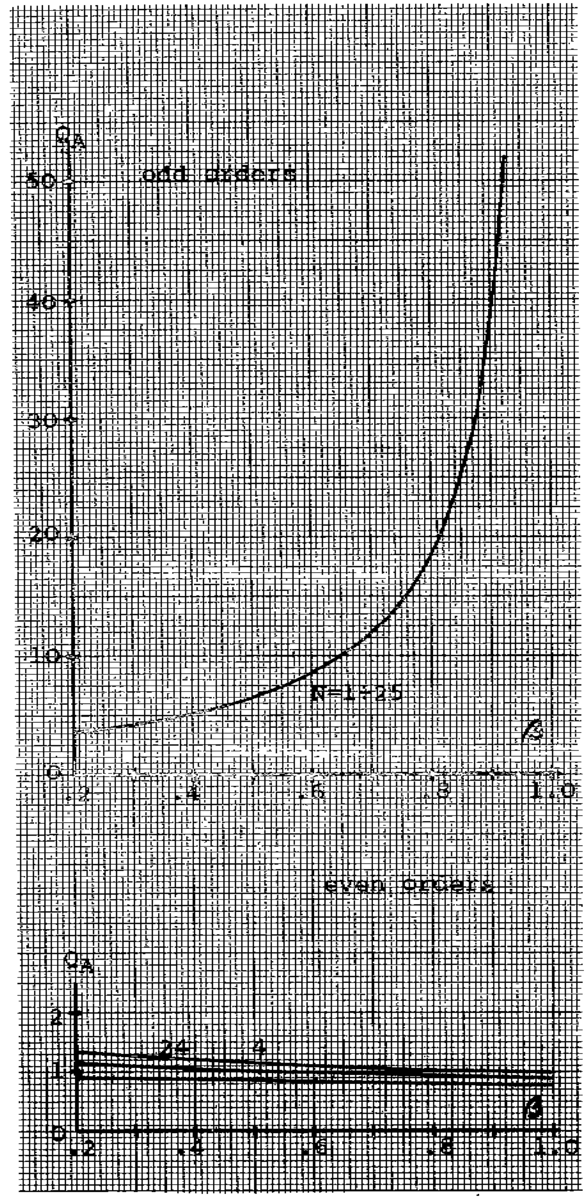


Figure D3: Support functions Q_A

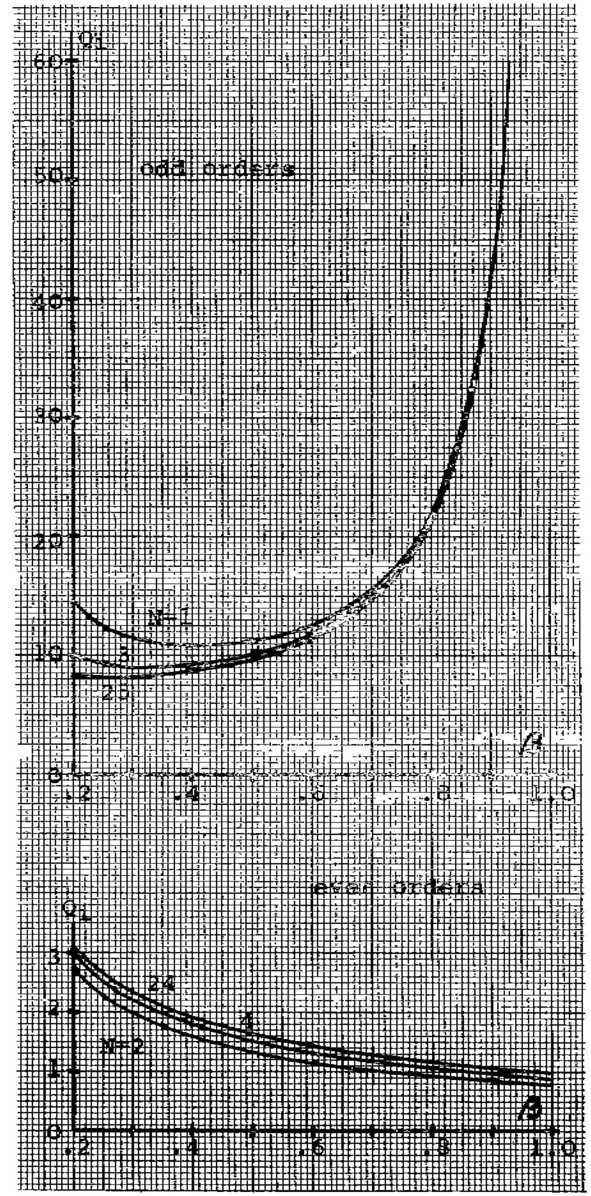


Figure D4: Support functions Q_i

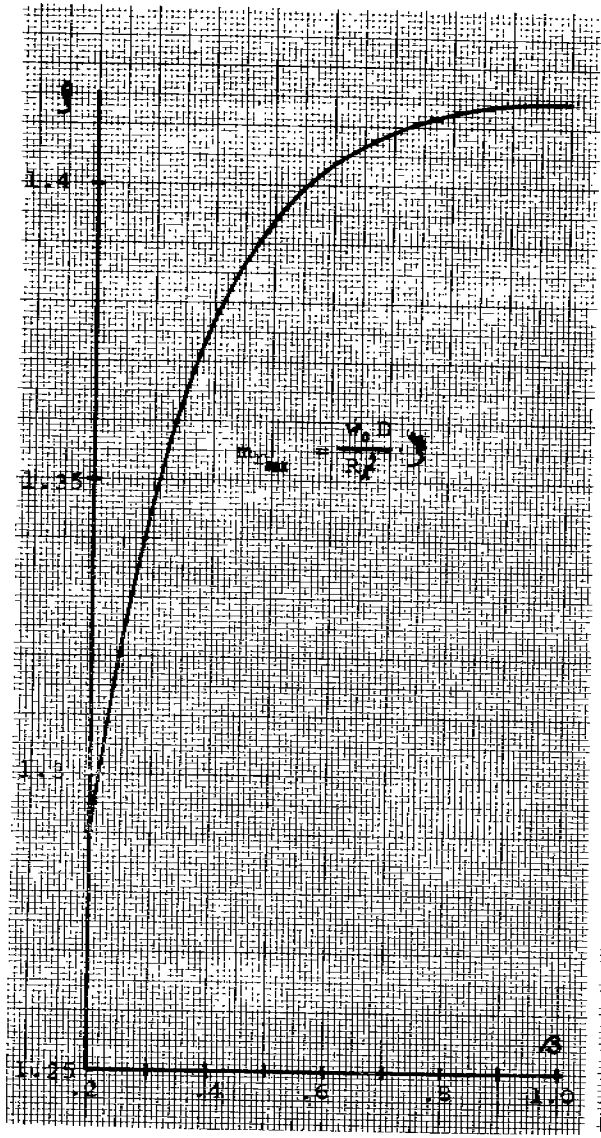


Figure D5: Maximum values for the radial moment of bending

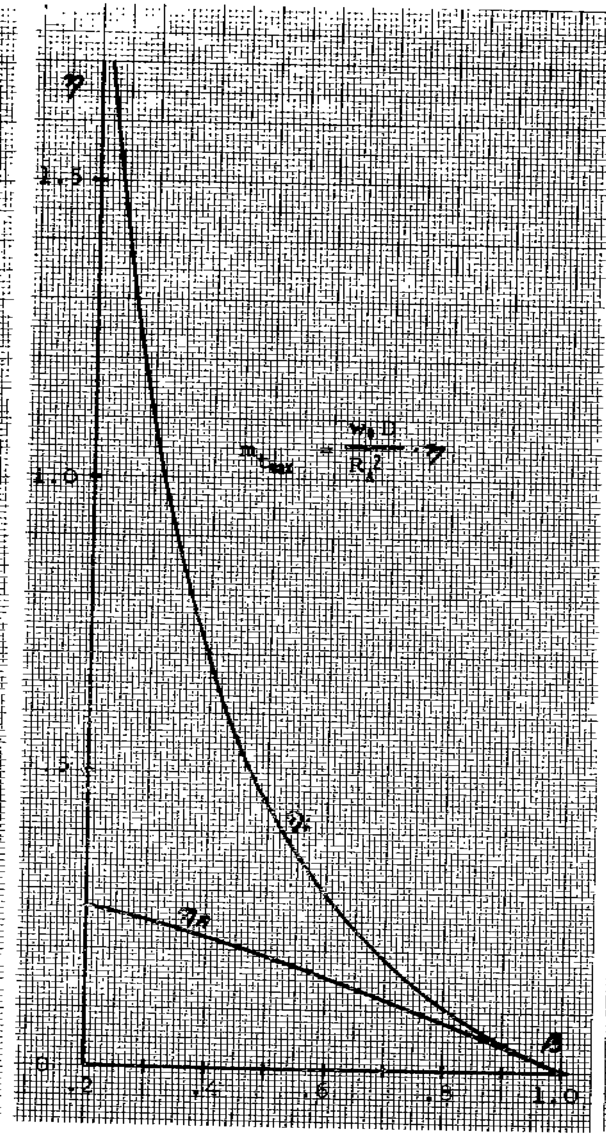


Figure D6: Maximum values for the tangential moment of bending

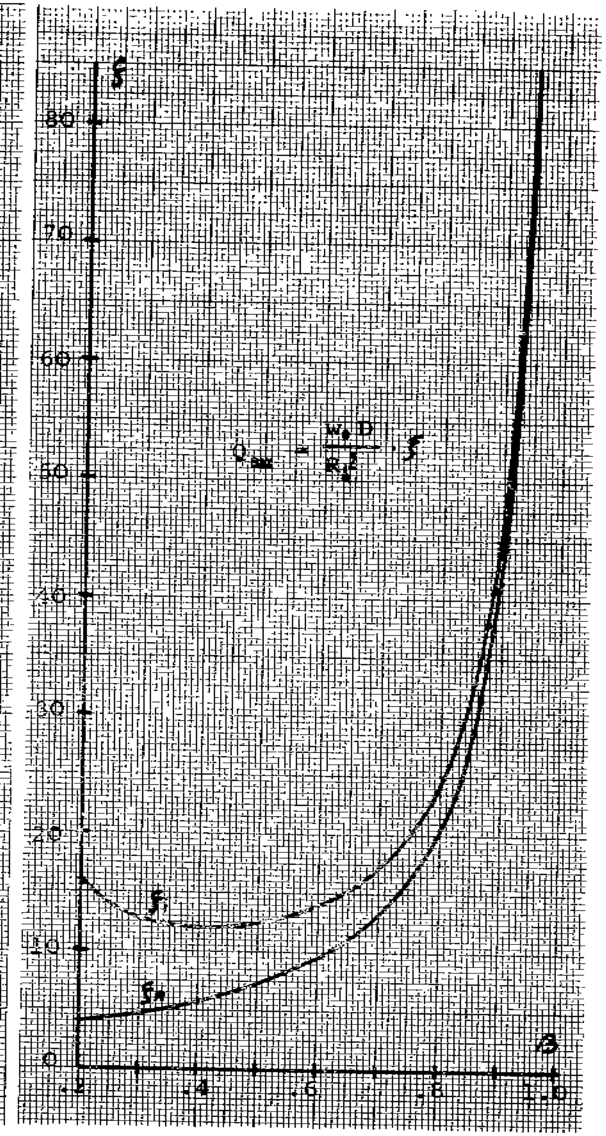


Figure D7: Maximum values for the support reactions