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Mechanism of heating of pre-formed plasma electrons in relativistic laser-matter interaction

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The role of the longitudinal ambipolar electric field, present inside a pre-formed plasma, in electron heating and beam generation is investigated by analyzing single electron motion in the presence of one electromagnetic plane wave and "V" shaped potential well (constant electric field) in a one dimensional slab approximation. It is shown that for the electron confined in an infinite potential well, its motion becomes stochastic when the ratio of normalized laser electric field a_0, to normalized longitudinal electric field E_z, exceeds unity, i.e., $a_0/E_z \gtrsim 1$. For a more realistic potential well of finite depth, present inside the pre-formed plasma, the condition for stochastic heating of electrons gets modified to $1 \lesssim a_0/E_z \lesssim \sqrt{L}$, where L is the normalized length of the potential well. The energy of electron beam leaving such a potential well and entering the solid scales $\sim a_0^2/E_z$, which can exceed the laser ponderomotive energy ($\sim a_0$) in the stochastic regime.
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The physics of generation of intense, relativistic electron beams in laser-solid interactions is extensively studied by the plasma physics community due to its potentially interesting and useful applications.[1] The generated electron energy spectrum observed in the experiments shows a distinct departure from single temperature Maxwellian distribution[2,3] indicating that energetic electrons are produced by various laser absorption mechanisms. Recent experiments have shown that the presence of a pre-formed plasma in front of a solid target, which may be produced due to the laser prepulse,[4–10] plays a very important role in establishing the energy spectrum of the beam electrons and has a trend to significantly increase average beam energy. For example, in experiments with planar targets,[5] high-energy fast electrons with energies much greater than the laser ponderomotive potential were observed in the presence of a pre-formed plasma of density scale-length of approximately 10 μm.

In the last decade, considerable progress has been made in understanding the physics of fast electron generation in relativistic laser-solid interactions; however, the underlying physics of effect of a pre-formed plasma on the fast electrons heating has still not been clearly identified. At relativistic intensities, collisionless heating of electrons by laser radiation takes place via several heating mechanisms such as resonance heating,[11,12] vacuum heating (Brunel absorption),[13] anomalous skin effect,[14,15] sheath inverse-bremsstrahlung absorption,[16] relativistic $J \times B$ heating,[17] and stochastic heating by counter propagating electromagnetic (EM) waves.[18–20] Electrons can also get accelerated beyond the laser ponderomotive energy by other mechanisms such as the presence of stochastic fields in the transverse direction[21] and betatron resonance.[22] In addition to these mechanisms, in our recent work,[23] we have shown that the synergistic effects of a large electrostatic potential well formed inside a pre-formed plasma and relativistic laser radiation are responsible for the generation of energetic electrons with energies well beyond the value predicted by the ponderomotive scaling.[3] This longitudinal ambipolar electric field, also reported in recent numerical simulations,[24,25] is formed inside a preformed plasma in order to balance increased electron pressure due to laser heating. However, the fundamental physics of such stochastic heating leading to the electron beam energies greater than the laser ponderomotive energy was not clearly understood.

In this Letter, we give proof-of-principle demonstration of stochastic heating of electrons present inside such a potential well with laser radiation and discuss the energy scaling for electron beam entering into a solid. We show that such heating can occur even with the presence of single electromagnetic wave. This heating resembles the "Fermi acceleration mechanism."[26,27] Here, we have investigated this process of electron heating by analyzing, both analytically and numerically, the dynamics of a single electron in the presence of a longitudinal electrostatic potential well and linearly polarized relativistic laser radiation. The electron motion is first analyzed inside a "V" shaped infinite potential well (Fig. 1(a)) to demonstrate how a longitudinal electric field influences the stochastic heating of an electron. By "infinite potential well," we mean that the potential well is deep enough that the electron inside such potential well can never escape. Since in practice the electric field almost vanishes beyond the relativistic critical density surface, we extend our results to the finite depth potential well (Fig. 1(b)) to make quantitative estimates of kinetic energy for electrons leaving out of such potential well and entering the solid as a relativistic electron beam. We would like to point out that the actual shape of the potential well inside the preformed plasma[23,24] may be different from the "V" shape considered here. But as far as the physics of stochastic heating due to phase-randomization is concerned, such a potential well captures all the important aspects of this heating mechanism while treating the problem analytically.

First, we consider the relativistic electron dynamics in the presence of a plane laser wave with vector potential a(t,z), propagating along the z-direction and longitudinal electric field E_z. The z-momentum and energy equation can be written as (in normalized units)

$$\frac{d(\gamma V_z)}{dt} = \frac{-1}{2\gamma}\frac{\partial a^2}{\partial z} - E_z, \tag{1}$$

$$\frac{d\gamma}{dt} = \frac{1}{2\gamma}\frac{\partial a^2}{\partial t} - E_z V_z, \tag{2}$$

where V_z is the electron velocity component along z direction and the relativistic factor γ is defined as $\gamma = \gamma_A\gamma_z$ with $\gamma_A = \sqrt{1 + a(t,z)^2}$ and $\gamma_z = 1/\sqrt{1 - V_z^2}$.

For one propagating plane wave of the form $a(t,z) = a(t–z)$, from Eqs. (1) and (2), we find

$$\frac{d}{dt}[\gamma_A\gamma_z(1 - V_z)] = E_z(1 - V_z). \tag{3}$$

For constant electric field, Eq. (3) can be integrated and we have

$$\gamma_A\gamma_z(1 - V_z) = \delta_0 + E_z(t - t_0 - z), \tag{4}$$

where t_0 is the time at which the electron crosses the boundary $z = 0$ and $\delta_0 = \gamma_A\gamma_z(1 - V_z)|_{t=t_0}$. Note that for the highly relativistic case, $\delta_0 \simeq \frac{\gamma_A}{2\gamma_z}$. For simplicity, we consider a "V"-shaped normalized electrostatic potential U(z) (see Fig. 1(a)) which is characterized by a constant electric field, E_z for $z > 0$ and "potential wall" at $z = 0$, where the electron is just reflected back preserving its energy.

The trajectory of the electron, $z(t)$ at positive z can be found by introducing a local time $\tau = t - z$ and using V_z from Eq. (4) as (in normalized units)

$$\frac{dz}{d\tau} = \frac{f^2(\tau) - 1}{2}, \tag{5}$$

where $f(\tau) = \gamma_A(t_0 + \tau)/(\delta_0 + E_z\tau)$.

Using the above equation, the dynamics of the electron in the presence of a "V"-shaped electrostatic potential and one plane EM wave can be studied. The consecutive times t_0 and t_1 at which the electron crosses the boundary $z = 0$ are related by the following equation:

$$\int_0^{t_1 - t_0} \frac{[\gamma_A(t_0 + \tau)]^2}{(\delta_0 + E_z\tau)^2} d\tau = t_1 - t_0, \tag{6}$$

and the corresponding parameter $\delta_1 = \gamma_A\gamma_z(1 - V_z)|_{t=t_1}$ can be expressed as following:

$$\delta_1 = \frac{[\gamma_A(t_1)]^2}{\delta_0 + E_z(t_1 - t_0)}. \tag{7}$$

Note that, we have used a perfectly reflecting potential wall at $z = 0$, i.e., $V_z(t_1 + 0) = -V_z(t_1 - 0)$. For the case of a linearly polarized wave, $a(t,z) = a_0\cos(t - z)$ and $\delta_0 \ll 1$ (which corresponds to very large energy) the integral in Eq. (6) can be solved analytically with the required accuracy of $O(\delta_0) \ll 1$. Hence, introducing a parameter $\hat{G}_i = [\gamma_A(t_i)]^2/(E_z\delta_i)$ where $i = 0$, 1, 2., Eqs. (6) and (7) simplify to the following recurrence relations:

$$\hat{G}_i = \hat{G}_{i-1} - \left(\frac{a_0}{E_z}\right)^2 \left\{\frac{\pi}{2}\cos(2t_{i-1}) + \left[\ln\left(\frac{\hat{G}_{i-1}E_z^2}{2(\gamma_A(t_{i-1}))^2}\right) - C\right]\sin(2t_{i-1})\right\}, \tag{8}$$

$$t_i = t_{i-1} + \hat{G}_i, \tag{9}$$

where C is the Euler-Mascheroni constant. Note that in the limit $\delta_0 \ll 1$, $\hat{G}$ is proportional to the non-dimensional electron kinetic energy, $\varepsilon = \gamma_A\gamma_z \gg 1$. The mapping (Eqs. (8) and (9)) is rather similar to the "Chirikov Standard Map."[28] Comparing this mapping with the "standard map," we get a_0/E_z as the parameter governing the degree of stochasticity. In particular, the motion becomes stochastic when $a_0 \gtrsim E_z$. This can be clearly seen by substituting $\gamma_z = 1$ and $V_z = 0$ in Eq. (4) to estimate the phase slip τ_{stop} at which the electron stops and gets reflected back. This gives $\tau_{stop} \simeq \gamma_A/E_z \simeq a_0/E_z$. Thus, for $a_0/E_z \simeq 1$, the electron bounce frequency inside the potential well becomes comparable with the laser frequency thereby resulting in a transition to stochastic motion. Also, note that the maximum energy step-size can be estimated from Eq. (8) as $\sim (a_0^2/E_z)\ln\left(\frac{E_z}{2\delta_0}\right)$. This can be seen by observing that inside the potential well, the change in the electron energy, $\Delta\varepsilon(\tau)$ is given by $\Delta\varepsilon = \gamma_A(\tau)\gamma_z(\tau) + E_z z(\tau) - \gamma_A(0)\gamma_z(0)$ which simplifies into

$$\Delta\varepsilon(\tau_1 - \tau_0) = \frac{1}{2}\int_0^{\tau_1 - \tau_0} \frac{d(\gamma_A(t_0 + \tau))^2/d\tau}{(\delta_0 + E_z\tau)} d\tau \sim \frac{a_0^2}{E_z}. \tag{10}$$

Thus, the above equation shows that, depending upon the initial phase t_0 of the wave at $z = 0$, the electron will gain or lose energy. Also, for a highly relativistic electron ($\delta_0 \ll 1$), most of the energy is gained when $\tau \ll 1$, i.e., within the first cycle of the wave.

The analytical predictions discussed above are verified numerically by solving the electron equation of motion for an ensemble of test electrons randomly placed inside the

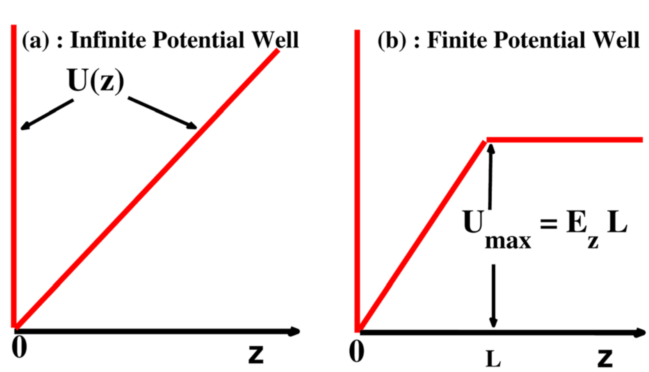

FIG. 1. Electrostatic potential well chosen for the analysis of electron motion. (a) Represents infinite potential well, whereas the finite potential well shown in (b) is used to analyze electron spectrum entering the solid. A perfectly reflecting "potential wall" is assumed at $z = 0$.

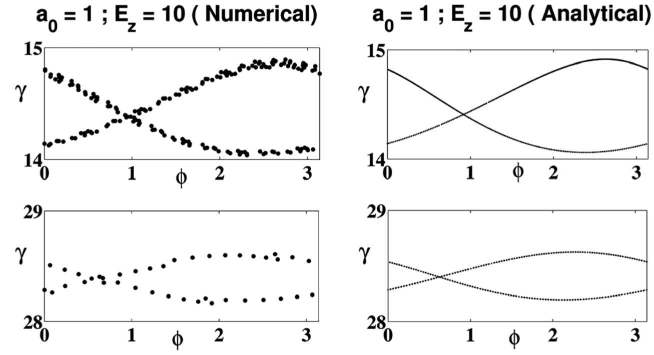

FIG. 2. Comparison of Poincare maps obtained from numerical calculations and analytical results obtained from Eqs. (8) and (9).

potential well. Fig. 2 shows the comparison of the Poincare map in (γ, ϕ) space through $z = 0$ obtained numerically with the analytical mapping given by Eqs. (8) and (9). Here, the phase ϕ is defined as $\phi_i = t_i - \pi[t_i/\pi]$, where [x] is the integer part of x.

This agreement shows that mapping Eqs. (8) and (9) correctly describe the electron dynamics for large energies. The mapping given by Eqs. (8) and (9) (refer Fig. 3) demonstrates the transition from regular (Figs. 3(a) and 3(b)) to stochastic (Fig. 3(d)) motion, with increasing a_0/E_z ratio.

For the highly stochastic regime, i.e., $a_0 \gg E_z$, electron heating can be described by diffusion[28] in energy space ε, $(\gamma \gg 1)$. For this case, the elementary step-size in energy space $\delta\gamma$, and time δt, can be estimated as $\delta\gamma \sim (a_0^2\Lambda)/E_z$ and $\delta t \sim \gamma/E_z$, where $\Lambda \sim$ constant is a slowly varying logarithmic function on the right hand side of Eq. (8). As a result, the energy diffusion coefficient can be estimated as $D_\gamma \sim (\delta\gamma)^2/\delta t \sim (a_0^2\Lambda)^2/(\gamma E_z)$. With such a diffusion coefficient, the asymptotic time evolution of the averaged electron energy $\langle\varepsilon\rangle$, and the electron distribution function, $f(\varepsilon, t)$, is given by the following expressions:

$$\langle\varepsilon\rangle \sim \langle\gamma\rangle \sim (D_\gamma t)^{1/3}, \tag{11}$$

$$f(\varepsilon, t) \sim t^{-1/3}\exp\left(\frac{-\varepsilon^3}{9D_\gamma t}\right). \tag{12}$$

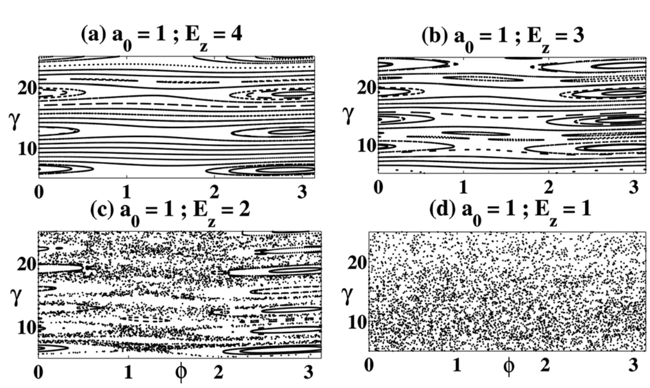

FIG. 3. Poincare map in (γ, ϕ) space for different values of a_0/E_z. Transition from strictly periodic motion ((a) and (b)) to stochastic motion (d) with increasing values of a_0/E_z is clearly demonstrated here.

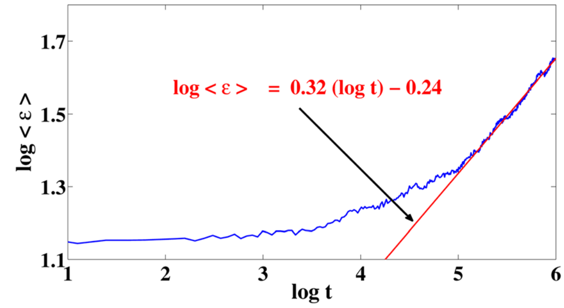

FIG. 4. Average energy of electrons inside the potential well vs time (numerical calculations) confirming $t^{1/3}$ dependance predicted by Eq. (11).

The analytic estimates for this regime which correspond to the case of a large scale-length pre-formed plasma are verified numerically. The temporal dependance of the average energy of the test electrons, placed inside an infinite potential well, is plotted in Fig. 4 which is in agreement with Eq. (11).

To address the issue of the energy of electrons generated in the pre-formed plasma[23] and entering the solid target, we need to consider the potential well of finite depth (see Fig. 1(b)) and analyze the energy distribution of electrons coming out of the well. In this case, when the energy gain $(\delta\gamma \sim a_0^2/E_z)$ exceeds the "depth" of the potential well, $U_{max} = E_z L$, the electron would escape from the potential well and can no longer be stochastically heated inside the potential well. This means the electron will be stochastically heated when $a_0^2/E_z \lesssim E_z L$, i.e., $a_0/E_z \lesssim \sqrt{L}$. Therefore, the condition for the finite potential well, the condition for stochasticity gets modified into $1 \lesssim a_0/E_z \lesssim \sqrt{L}$.

Now, consider an ensemble of electrons at the bottom of finite depth potential well such that the "depth" of the potential well, $U_{max} = E_z L$, is larger than the energy space diffusion step-size, i.e., $U_{max} > \delta\gamma \sim a_0^2/E_z$. The electron inside this potential well will get heated up since it satisfies the stochasticity condition described above. Therefore, such an electron will climb up (in energy space) in the potential well while performing a "random walk" in energy space until it is thrown out of the well. The energy gained by the electron during the last transit from $z = 0$ can be estimated from Eq. (10). Thus, the total energy of the electron beam leaving the potential well can be estimated as $\gamma = 1 + \alpha(a_0^2/E_z)\Lambda$, where $\alpha \sim 1$ is a numerical factor and Λ is the logarithmic function described before. In our model, the escaping electron has both perpendicular $(\gamma_A \sim a_0)$ and parallel $(\gamma_z \sim a_0/E_z)$ energy components. In reality, beyond the relativistic critical density $(z = L)$, the laser field quickly goes to zero which in the 1D case results in conversion of perpendicular energy into parallel energy. This process causes additional ponderomotive acceleration with the energy gain $\simeq \beta a_0$. β is a numerical factor ~ 1. Thus, the electron beam enters the solid with only the parallel energy given by $\gamma_{beam} = 1 + \alpha(a_0^2/E_z)\Lambda + \beta a_0$ which in highly stochastic regime can be much greater than the laser ponderomotive energy $\gamma_{ponder} = \sqrt{1 + a_0^2}$ in agreement with experimental and numerical results.[5,23] Note that beam energy cannot exceed U_{max}. Numerical calculations for a finite potential well confirm these predictions. The distribution of the electrons at the end of the potential well for

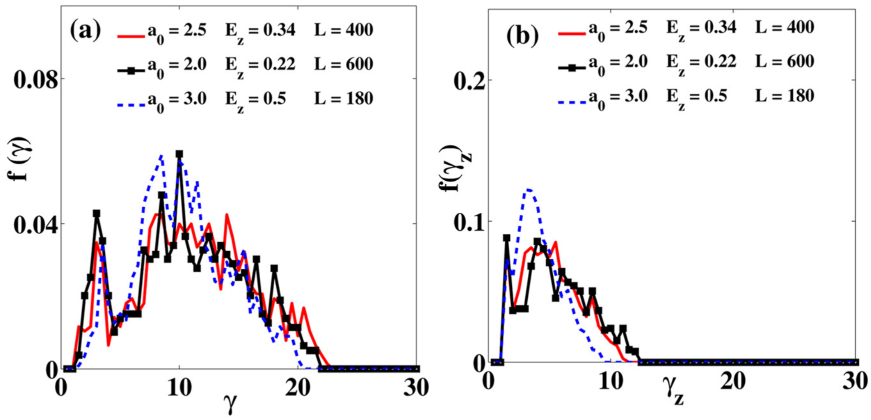

FIG. 5. Energy distribution functions for total energy (a) and parallel energy (b) of electrons escaping the finite depth potential well for different a_0 values such that the elementary energy-step size, a_0^2/E_z is constant.

different values of a_0, E_z, and L, but keeping $\delta\gamma \sim a_0^2/E_z$ constant show that the maximum γ and γ_z at $z = L$ scales proportional to a_0^2/E_z and a_0/E_z, respectively (see Fig. 5).

Thus, the presence of longitudinal potential well inside pre-formed plasma can result in stochastic heating of electrons. But this potential well itself is formed due to the increased pressure by electron heating, i.e., $E_z \sim -\nabla P_e/n_e$. Here P_e and n_e are electron pressure and density, respectively. This results in a positive feedback mechanism where the plasma heating and potential well evolve self-consistently. Therefore, in general E_z is a function of a_0, L, and time, i.e., $E_z \equiv E_z(a_0, L, t)$. For example, for large scale-length pre-formed plasma, E_z should be smaller since $E_z \propto 1/L$ as can be seen from pressure balance equation described above. Therefore, the electrons will enter into the solid with increased temperature ($\sim a_0^2/E_z$) consistent with the numerical simulations and experiments described earlier.

In summary, we analyze the heating mechanism of pre-formed plasma electrons due to relativistic laser radiation and a longitudinal electric field with a "V" shaped potential well. Based on our theoretical results and numerical simulations, we conclude that: (i) For $a_0/E_z \gtrsim 1$, the electron motion in the laser and longitudinal electrostatic field becomes stochastic for a deep potential well. (ii) For finite potential depth, in the highly stochastic regime ($1 \lesssim a_0/E_z \lesssim \sqrt{L}$), the electron undergoes energy space diffusion with the characteristic step-size $\sim a_0^2/E_z$ and the energy of the electron beam entering the solid is estimated as $\gamma_{beam} = 1 + \alpha(a_0^2/E_z)\Lambda + \beta a_0$ which is much larger than the ponderomotive energy.

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