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MEDEA: a DSGE model for the Spanish economy

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Abstract In this paper, we provide a brief introduction to a new macroeconometric model of the Spanish economy named MEDEA (*Modelo de Equilibrio Dinámico de la Economía EspañolA)*. MEDEA is a dynamic stochastic general equilibrium (DSGE) model that aims to describe the main features of the Spanish economy for policy analysis, counterfactual exercises, and forecasting. MEDEA is built in the tradition of New Keynesian models with real and nominal rigidities, but it also incorporates aspects such as a small open economy framework, an outside monetary authority such as the ECB, and population growth, factors that are important in accounting for aggregate fluctuations in Spain. The model is estimated with Bayesian techniques and data from the last two decades. Beyond describing the properties of the model, we perform different exercises to illustrate the potential of MEDEA, including historical decompositions, long-run and short-run simulations, and counterfactual experiments.

Keywords DSGE models · Likelihood estimation · Bayesian methods

JEL Classification C11 · C13 · E30

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1 Introduction

In this paper, we provide a brief introduction to a dynamic equilibrium model of the Spanish economy named MEDEA (*Modelo de Equilibrio Dinámico de la Economía EspañolA*). This model was developed, solved, and estimated with Spanish data while the authors collaborated with the Economic Office of the President of Spain (Oficina Económica del Presidente del Gobierno) from 2007 to 2008.

MEDEA is a dynamic stochastic general equilibrium (DSGE) model that aims to describe the main features of the Spanish economy for policy analysis, counterfactual exercises, and forecasting. MEDEA is built in the tradition of New Keynesian models with real and nominal rigidities (see Christiano et al. 2005; Smets and Wouters 2003, and the book-length descriptions in Woodford 2003 and Galí 2008). New Keynesian models have proven to be a flexible framework that can incorporate many different economic mechanisms of interest, to be rich enough for meaningful policy analysis, and to have a good forecasting track record. At the kernel of MEDEA, we have a neoclassical growth model with optimizing households and firms and long-run growth induced by technological change and population growth. On top of this core, MEDEA has rigidities of prices and wages, habit persistence in consumption, a set of adjustment costs (to investment, to exports, and to imports), a fiscal and monetary authority that determines a short-run nominal interest rate and taxes, and shocks to population growth, technology, preferences, and policy that induce the stochastic dynamics of the economy.

In addition, MEDEA is designed to be a model for a small open economy that belongs to a currency area, in this case, the euro. The open economy aspects are captured by the presence of exporting and importing firms with incomplete pass-through and by the ability of the agents to save or borrow on foreign financial assets. The currency area is modelled through a monetary authority that sets short-term nominal interest rates by following a Taylor rule based on the economic performance of the whole euro area.

MEDEA shares many features with other DSGE models developed at policy-making institutions for use as an input for their activities. Some examples are the Federal Reserve Board (Erceg et al. 2006), the European Central Bank (Christoffel et al. 2008), the Bank of Canada (Murchison and Rennison 2006), the Bank of England (Harrison et al. 2005), the Bank of Finland (Kilponen and Ripatti 2006; Kortelainen 2002), the Bank of Sweden (Adolfson et al. 2005) the Bank of Spain (Andrés et al. 2006), and the Spanish Ministry of Economics and Finance (Ministerio de Economía y Hacienda) (Boscá et al. 2007). As such, MEDEA is a model that it is comparable to its peers and can borrow from many years of experience.

At the same time, MEDEA has many new elements, some that are, in our opinion, interesting advances for DSGE modelling in general, and some that are important to adapt the model to Spain. We would like to highlight four of these:

1. MEDEA has stochastic growth coming from three sources: neutral technological progress, investment-specific technological progress, and population growth. These will allow us to capture two relevant characteristics of Spain in the last decade: the low productivity growth and the large rise in immigration.

By modelling population growth as a random walk in logs with a drift, we can explore both the effects of changing the drift and the consequences of random shocks (such as the unexpected arrival of more immigrants).

2. In comparison with other recent estimated DSGE models, we do not model the foreign world as an equilibrium outcome beyond the behavior of the European Central Bank (ECB) through its Taylor rule. Spain is too small to have a significant impact on the world economy. We prefer to use the extra complexity to enrich other aspects of the model.
3. Fiscal policy. Most of the estimated DSGE models have been developed in central banks. Since monetary policy corresponds to their role, central banks have paid particular attention to issues related to such policy, but the treatment of fiscal policy has been very parsimonious. We pay some detailed attention to the fiscal sector of the economy (although more work is needed), with three tax rates: on capital income, labor income, and consumption. Given the current widespread use of fiscal instruments to fight the 2008–2009 recession, this aspect is particularly interesting.
4. At a more technical level, we design the solution of the model in such a way that we will be able to undertake higher-order approximations in the middle run in a relatively simple way. There is a growing body of literature that emphasizes that there is much to be gained from a non-linear estimation of the model, both in terms of accuracy and in terms of identification (see Fernández-Villaverde and Rubio-Ramírez 2005, 2007; Fernández-Villaverde et al. 2006; An and Schorfheide 2006, among several others). In the current version of the model, for computational reasons, we solve the model by log-linearizing the equilibrium conditions around a transformed stationary steady state. However, our derivations are done with the perspective of performing these higher-order approximations in the future (for example, contrary to common practice, we never substitute variables away to find a Phillips curve, a strategy that works only up to a first-order approximation).

MEDEA is estimated by Bayesian methods. We follow the Bayesian paradigm because it is a powerful, coherent, and flexible perspective for the estimation of dynamic models in economics (see An and Schorfheide 2006; Fernández-Villaverde 2009, for surveys of the literature). First, Bayesian analysis is built on a clear set of axioms and it has a direct link with decision theory. The link is particularly relevant for MEDEA since the model has been designed for applied policy analysis. Many of the relevant policy decisions require an explicit consideration of uncertainty and asymmetric loss assessments. Consequently, the Bayesian approach provides a convenient playground for risk management. Second, the Bayesian approach deals in a transparent way with misspecification and identification problems, which are pervasive in the estimation of DSGE models (Canova and Sala 2006; Iskrev 2008). Third, the Bayesian estimators have desirable small sample and asymptotic properties, even if they are evaluated by classical criteria (Fernández-Villaverde and Rubio-Ramírez 2004). Fourth, priors allow us to introduce presample information and to reduce the dimensionality problem associated with the number of parameters. As Sims (2007) has emphasized, with any model rich enough to fit the data well, the use of priors is essential to do any reasonable inference. Priors will be especially attractive for

MEDEA because the deep changes in the structure of the Spanish economy over the last several decades stop us from using data before the early 1980s, leaving us with a relatively short sample. Fifth, a likelihood-based method, such as our Bayesian estimation, allows us to recover the whole set of parameter values required for policy and welfare analysis. Finally, Bayesian methods have important computational advantages over maximum likelihood in large models like MEDEA. Simulating the posterior distribution of the parameters is a much easier task than maximizing a highly dimensional likelihood.

MEDEA can be employed for three main alternative purposes. First, we can use it to understand the dynamics of aggregate fluctuations. To illustrate this feature, in this paper, we will show a decomposition of the last two business cycles among different sources of variation. Second, we can use MEDEA for policy analysis, including counterfactuals and alternative policy experiments. We will perform several counterfactuals to illustrate the properties of the model. For instance, we will look at the effects of changes in the consumption tax rate and in the wage mark-up or the consequences of alternative scenarios for population growth. Finally, we can use MEDEA for forecasting purposes. Even if DSGE models are not specifically designed with this goal in mind, their forecasting performance has been very satisfactory (see Edge et al. 2009; Christoffel et al. 2007, for the forecasting experience of the DSGE model used by the Federal Reserve System and the ECB, respectively). Because of space considerations, we will leave a thorough analysis of the forecasting properties of MEDEA for the near future.

The structure of this paper is as follows. First, we outline the main structure of the model. Second, we describe MEDEA's theoretical framework in detail. Third, we define the equilibrium of the economy, we transform this equilibrium into a stationary one by appropriately changing the variables, and we solve the model by log-linearizing the equilibrium conditions of the transformed model. Fourth, we build the likelihood of the model. Finally, we estimate the model with data from 1986:1 to 2007:2 and we report the results of a number of exercises undertaken with the model.

2 Outline of the model

MEDEA is a medium-scale model with 82 equations and 10 stochastic shocks. The 82 equations include 16 equations that relate variables in the model to observables (although we will not use all of the observables in all of our exercises), the laws of motion for 37 state variables, and the equations determining 19 endogenous variables that are not states. Given this relatively large number of variables, it is worthwhile to outline the basic structure of the model before we go into further details:

1. A continuum of households consume, save in domestic and foreign assets, hold money, supply labor, and set their own wages subject to a demand curve and Calvo's pricing with partial indexation.
2. The labor of households is aggregated by a perfectly competitive labor packer who sells the aggregated labor to the domestic intermediate good producers.
3. The final domestic good is manufactured by a final domestic good producer, which uses as inputs intermediate domestic goods.

4. The consumption good is packed by a consumption good producer using the final domestic good and the final imported good.
5. Similarly, the investment good is packed by an investment good producer using the final domestic good and the final imported good.
6. Domestic intermediate goods producers rent capital and labor to manufacture their good and are subject to Calvo's pricing with partial indexation.
7. The final imported good is packed by a final imported good producer using intermediate goods produced by monopolistic competitors from a generic import good, with incomplete pass-through specified as Calvo's pricing with partial indexation.
8. The export goods are produced by monopolistic competitors who buy the final domestic good and differentiate it by brand naming. The exporters exhibit local-currency pricing that we specify as Calvo's pricing with partial indexation.
9. There is a monetary authority, the ECB, that implements monetary policy. The ECB's monetary policy fixes the one-period nominal interest rate of the euro area through open market operations, with the euro area inflation as target. The weight of the Spanish economy in this policy target is approximately 10%.
10. There is a government that implements fiscal policy to finance an exogenously given stream of government consumption with taxes on capital and labor income and on consumption.
11. Finally, long-run growth of per capita income is induced by the presence of two unit roots, one in the level of neutral technology and one in the investment-specific technology. Moreover, there is stochastic population growth.

3 The model

We start now by describing the model. We will discuss each of type of agents (households, distribution sector, intermediate good producers, foreign sector, the monetary authority, and the government) and how their actions aggregate.

3.1 Households

There is a continuum of households indexed by $j \in [0, 1]$. Each household is composed of L_t identical workers. The preferences of households are representable by the following lifetime utility function, which is separable into per capita consumption, c_{jt}, per capita real money balances, m_{jt}/p_t (where p_t is the price of the domestic final good), and per capita hours worked, l^s_{jt} (in terms of the proportion of the period spent at work):

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t L_t d_t \left\{ \log \left(c_{jt} - h c_{jt-1} \right) + \upsilon \log \frac{m_{jt}}{p_t} - \varphi_t \psi \frac{\left(l^s_{jt} \right)^{1+\vartheta}}{1+\vartheta} \right\}$$

where $\mathbb{E}_0$ is the conditional expectation operator evaluated at time 0, β is the discount factor, h is the parameter that controls habit persistence, and ϑ is the inverse of Frisch labor supply elasticity. The variable d_t is an intertemporal preference shock, while φ_t is a labor supply shock with laws of motion:

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t} \quad \text{where } \varepsilon_{d,t} \sim \mathcal{N}(0, 1),$$
$$\log \varphi_t = \rho_\varphi \log \varphi_{t-1} + \sigma_\varphi \varepsilon_{\varphi,t} \quad \text{where } \varepsilon_{\varphi,t} \sim \mathcal{N}(0, 1).$$

Note that the preference shifters are common for all households. The preference shock d_t changes the intertemporal first-order conditions, while the preference shock φ_t moves the first-order conditions affecting labor supply and wage determination. We include the shock d_t to capture the changes in valuations between the present and the future that the analysis of intertemporal wedges suggests as key for understanding aggregate fluctuations (Primiceri et al. 2006). We add the shock φ_t to model the changes in labor supply that Hall (1997) and Chari et al. (2007) have pointed out as responsible for a large proportion of the changes in employment over the business cycle. We have selected a utility function where consumption appears in logs. Thus, the marginal relation of substitution between consumption and leisure is linear in consumption and we have a balanced growth path with constant hours (King et al. 1988).

The household's size, L_t, follows a random walk with drift in logs:

$$L_t = L_{t-1} \exp\left(\Lambda_L + z_{L,t}\right) \quad \text{where } z_{L,t} = \sigma_L \varepsilon_{L,t} \quad \text{and} \quad \varepsilon_{L,t} \sim \mathcal{N}(0, 1).$$

Thus, the growth of population is given by

$$\gamma_t^L = \frac{L_t}{L_{t-1}} = \exp\left(\Lambda_L + z_{L,t}\right)$$

This process induces the first unit root in the model. However, this unit root will only affect the absolute levels of the variables and not the per capita terms.

Households hold an amount a_{jt+1} of Arrow securities,[1] an amount b_{jt} of domestic government bonds that pay a nominal gross interest rate of R_t, and an amount in domestic currency $ex_t b_{jt}^W$ of foreign government bonds from the rest of the world, which pay a nominal gross interest rate of $R_t^W \Gamma(\cdot)$. The exchange rate, ex_t, is expressed in terms of the domestic currency per unit of foreign currency. Following Schmitt-Grohé and Uribe (2003), the function $\Gamma(\cdot)$ represents the premium associated with buying foreign bonds and it captures the costs (or benefits) for households of undertaking positions in the international asset market. We assume that $\Gamma(\cdot)$ depends on the per capita holdings of foreign bonds in the entire economy with respect to nominal output

[1] Households can trade on the whole set of possible Arrow securities within the country, indexed both by the household j (since the household faces idiosyncratic wage-adjustment risk that we will describe below) and by time (to capture Spanish aggregate risk). The amount a_{jt+1} indicates the amount of those securities that pay one unit of the domestic final good in event $\omega_{j,t+1,t}$ purchased by household j at time t at a (real) price $q_{jt+1,t}$.

of the final domestic good:

$$\widetilde{b}_t^W = \frac{\int_0^1 b_{jt}^W dj}{p_t y_t^d}$$

Thus, as borrowers, households are charged a premium on the foreign interest rate (that is, if $\widetilde{b}_t^W < 0$, then $\Gamma\left(ex_t \widetilde{b}_t^W, \xi_t^{b^W}\right) < 1$) and get a remuneration when they act as lenders. Moreover, $\Gamma(0) = 1$, $\Gamma(\cdot)' > 0$, and $\Gamma(\cdot)'' < 0$. Domestic households take $\widetilde{b}_t^W$ as given when deciding their optimal holding of foreign bonds. Finally, revenues from the premium are rebated in a lump-sum to the foreign agents.[2] The holding cost of foreign debt is introduced to pin down a well-defined steady state for consumption and assets in the context of international incomplete markets (otherwise, transient dynamics will have permanent effects) and it is motivated empirically by the observation that Spain cannot fully insure against its idiosyncratic shocks in the international capital markets.[3]

Since we do not model the rest of the world, the evolution of the R_t^W is given by an exogenous process:

$$R_t^W = \left(R^W\right)^{(1-\rho_{R^W})} \left(R_{t-1}^W\right)^{\rho_{R^W}} \exp\left(\sigma_{R^W} \varepsilon_{R^W,t}\right)$$

In addition, there is a time-varying shock to the premium $\xi_t^{b^W}$, with the following process:

$$\log \xi_t^{b^W} = \rho_{b^W} \log \xi_{t-1}^{b^W} + \sigma_{b^W} \varepsilon_{b^W,t} \quad \text{where } \varepsilon_{b^W,t} \sim \mathcal{N}(0,1),$$

With all this structure, the j-th household's per capita budget constraint is given by:

$$\begin{aligned}
&(1+\tau_c)\frac{p_t^c}{p_t}c_{jt} + \frac{p_t^i}{p_t}i_{jt} + \frac{m_{jt}}{p_t} + \frac{b_{jt}}{p_t} + \frac{ex_t b_{jt}^W}{p_t} + \int q_{jt+1,t} a_{jt+1} d\omega_{j,t+1,t} \\
&\quad = (1-\tau_w) w_{jt} l_{jt}^s + \left(r_t u_{jt}(1-\tau_k) + \mu_t^{-1}\delta\tau_k - \mu_t^{-1}\Phi\left[u_{jt}\right]\right) k_{jt-1} + \frac{1}{\gamma_t^L}\frac{m_{jt-1}}{p_t} \\
&\qquad + R_{t-1}\frac{1}{\gamma_t^L}\frac{b_{jt-1}}{p_t} + R_{t-1}^W \Gamma\left(ex_t \widetilde{b}_{t-1}^W, \xi_{t-1}^{b^W}\right)\frac{1}{\gamma_t^L}\frac{ex_t b_{jt-1}^W}{p_t} + \frac{1}{\gamma_t^L} a_{jt} + T_t + \digamma_t
\end{aligned}$$

where p_t is the price of the domestic final good, p_t^c is the price level of the consumption final good, p_t^i is the price level of the investment final good, w_{jt} is the real wage in

[2] However, since we do not model the equilibrium behavior of the rest of the world, the way in which this rebate is distributed is irrelevant for our purposes.

[3] Some of the alternatives to this holding cost outlined by Schmitt-Grohé and Uribe (2003) are less attractive for us. Complete international markets are empirically implausible and Uzawa preferences may induce complicated dynamic responses. The final possibility proposed by Schmitt-Grohé and Uribe, a quadratic adjustment cost on the level of the debt, would deliver results that are quantitatively nearly identical to ours.

terms of the domestic final good, r_t the real rental price of capital, also in terms of the domestic final good, $u_{jt} > 0$ the intensity of use of capital, $\mu_t^{-1}\Phi\left[u_{jt}\right]$ is the physical cost of use of capital in resource terms, μ_t is an investment-specific technological shock to be described momentarily, T_t is a lump-sum transfer, F_t are the profits of the firms in the economy, and τ_c, τ_w, and τ_k are the tax rates on consumption, wages, and capital income. Note that the tax on capital income is defined on the net return of capital after depreciation δ and hence we include a tax credit $\mu_t^{-1}\delta\tau_k$, expressed in resource terms. Also, note that we divide the per capita holdings of money and bonds carried into the period by the current population growth to express all quantities in current population per capita terms. Finally, we assume that $\Phi[1] = 0$, Φ' and $\Phi'' > 0$.

Investment i_{jt} induces a law of motion for (per capita) capital held by the j-th household:

$$\gamma_{t+1}^L k_{jt} = (1-\delta)\, k_{jt-1} + \mu_t \left(1 - S\left[\gamma_t^L \frac{i_{jt}}{i_{jt-1}}\right]\right) i_{jt}$$

where $S[\cdot]$ is an adjustment cost function on the level of investment such that $S[\Lambda_i] = 0$, $S'[\Lambda_i] = 0$, and $S''[\cdot] > 0$, and where Λ_i is the growth rate of investment along the balance growth path. Note our capital timing: we index capital at the time its level is decided. Also, the amount of per capita capital in the next period is a random variable because the population next period is also random (obviously, this randomness does not affect total capital at period $t+1$, which is determined at time t).

We include an investment-specific technological shock in our law of motion for capital because we were convinced by Greenwood et al. (1997, 2000) that this mechanism is of key importance to quantitatively account for growth and aggregate fluctuations. The investment-specific technological shock follows an autoregressive process of the form:

$$\mu_t = \mu_{t-1} \exp\left(\Lambda_\mu + z_{\mu,t}\right) \quad \text{where } z_{\mu,t} = \sigma_\mu \varepsilon_{\mu,t} \quad \text{and} \quad \varepsilon_{\mu,t} \sim \mathcal{N}(0,1)$$

This process induces a second unit root in the model.

3.1.1 Household's problem

Given our description of the household's environment, the Lagrangian function associated with it is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \widetilde{\gamma}_t^L \left[\begin{array}{c} d_t \left\{ \log\left(c_{jt} - hc_{jt-1}\right) + \upsilon \log \frac{m_{jt}}{p_t} - \varphi_t \psi \frac{\left(l_{jt}^s\right)^{1+\vartheta}}{1+\vartheta} \right\} \\ -\lambda_{jt} \left\{ \begin{array}{c} (1+\tau_c) \frac{p_t^c}{p_t} c_{jt} + \frac{p_t^i}{p_t} i_{jt} + \frac{m_{jt}}{p_t} + \frac{b_{jt}}{p_t} + \frac{ex_t b_{jt}^W}{p_t} + \int q_{jt+1,t} a_{jt+1} d\omega_{j,t+1,t} \\ -(1-\tau_w) w_{jt} l_{jt}^s - \left(r_t u_{jt} (1-\tau_k) + \mu_t^{-1}\delta\tau_k - \mu_t^{-1}\Phi\left[u_{jt}\right]\right) k_{jt-1} - \frac{1}{\gamma_t^L} \frac{m_{jt-1}}{p_t} \\ -T_t - F_t \\ -R_{t-1} \frac{1}{\gamma_t^L} \frac{b_{jt-1}}{p_t} - R_{t-1}^W \Gamma\left(ex_t \widetilde{b}_{t-1}^W, \xi_{t-1}^{b^W}\right) \frac{1}{\gamma_t^L} \frac{ex_t b_{jt-1}^W}{p_t} - \frac{1}{\gamma_t^L} a_{jt} \end{array} \right\} \\ -Q_{jt} \left\{ \gamma_{t+1}^L k_{jt} - (1-\delta) k_{jt-1} - \mu_t \left(1 - S\left[\gamma_t^L \frac{i_{jt}}{i_{jt-1}}\right]\right) i_{jt} \right\} \end{array}\right]$$

where $\widetilde{\gamma}_t^L = \prod_{i=1}^t \gamma_i^L$ and they maximize over $c_{jt}, b_{jt}, b_{jt}^W, u_{jt}, k_{jt}, i_{jt}, m_{jt}, w_{jt}$, and l_{jt}^s; while λ_{jt} and Q_{jt} are the Lagrangian multipliers associated with the budget constraint and the evolution of installed capital, respectively.

The first-order conditions of this problem (except for labor and wages) are:

$$d_t\left(c_{jt} - hc_{jt-1}\right)^{-1} - h\mathbb{E}_t\beta\gamma_{t+1}^L d_{t+1}\left(c_{jt+1} - hc_{jt}\right)^{-1} = \lambda_{jt}\left(1+\tau_c\right)\frac{p_t^c}{p_t}$$

$$\lambda_{jt} = \mathbb{E}_t\left\{\beta\lambda_{jt+1}\frac{R_t}{\Pi_{t+1}}\right\}$$

$$\lambda_{jt} = \mathbb{E}_t\left\{\beta\lambda_{jt+1}\frac{R_t^W\Gamma\left(ex_t\widetilde{b}_t^W, \xi_t^{b^W}\right)}{\Pi_{t+1}}\frac{ex_{t+1}}{ex_t}\right\}$$

$$r_t = \frac{\mu_t^{-1}\Phi'\left[u_{jt}\right]}{(1-\tau_k)}$$

$$q_{jt}\mathbb{E}_t\gamma_{t+1}^L = \beta\mathbb{E}_t\gamma_{t+1}^L\left\{\frac{\lambda_{jt+1}}{\lambda_{jt}}\left((1-\delta)q_{jt+1} + \left(r_{t+1}u_{jt+1}(1-\tau_k)\right.\right.\right.$$
$$\left.\left.\left. + \mu_{t+1}^{-1}\delta\tau_k - \mu_{t+1}^{-1}\Phi\left[u_{jt+1}\right]\right)\right)\right\}$$

$$\frac{p_t^i}{p_t} = q_{jt}\mu_t\left(1 - S\left[\gamma_t^L\frac{i_{jt}}{i_{jt-1}}\right] - S'\left[\gamma_t^L\frac{i_{jt}}{i_{jt-1}}\right]\gamma_t^L\frac{i_{jt}}{i_{jt-1}}\right)$$
$$+\mathbb{E}_t\beta\gamma_{t+1}^L q_{jt+1}\frac{\lambda_{jt+1}}{\lambda_{jt}}\mu_{t+1}S'\left[\gamma_{t+1}^L\frac{i_{jt+1}}{i_{jt}}\right]\gamma_{t+1}^L\left(\frac{i_{jt+1}}{i_{jt}}\right)^2$$

$$\frac{m_{jt}}{p_t} = d_t\upsilon\left[\beta\mathbb{E}_t\lambda_{jt+1}\frac{R_t-1}{\Pi_{t+1}}\right]^{-1}.$$

where we have defined the (marginal) Tobin's Q as $q_{jt} = \frac{Q_{jt}}{\lambda_{jt}}$ (the ratio of the two Lagrangian multipliers, or more loosely the value of installed capital in terms of its replacement cost), and substituted the Euler equation into the money balances equation.[4] For our analysis, the first order condition with respect to money holdings will not be strictly needed: as we will explain later, the monetary authority will just issue enough money balances such that the optimality condition is satisfied given the allocation, prices, and the nominal interest rate.

The first order condition with respect to investment has a simple interpretation. If $S[\cdot] = 0$ (the case without adjustment costs), we get:

$$q_{jt} = \frac{p_t^i}{p_t}\frac{1}{\mu_t}$$

[4] We do not take first-order conditions with respect to Arrow securities since, in our environment with complete markets and separable utility in labor, their equilibrium price will be such that their demand ensures that the household's consumption does not depend on idiosyncratic shocks (see Erceg et al. 2000).

that is, the marginal Tobin's Q is equal to the replacement cost of capital (the relative price of capital) in terms of the domestic final good. Furthermore, if $\mu_t = 1$ and $p_t^i = p_t$, as in the standard neoclassical growth model, $q_{jt} = 1$.

3.1.2 Labor demand and wage decisions

The first-order conditions with respect to labor and wages are more involved. The labor used by intermediate good producers described below is supplied by a representative competitive firm that hires the labor supplied by each household j, $L_t l_{jt}^s$. The labor supplier aggregates the differentiated labor types with the following production function:

$$L_t^d = L_t \left(\int_0^1 \left(l_{jt}^s \right)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} = L_t l_t^d \tag{1}$$

where $0 \leq \eta < \infty$ is the elasticity of substitution among different types of labor, l_t^d is the per capita labor demand, and L_t^d is the total labor demand.

The labor "packer" maximizes profits subject to the production function (1), taking as given all differentiated labor wages w_{jt} and the aggregate wage w_t:

$$\max_{l_{jt}} w_t L_t l_t^d - \int_0^1 w_{jt} L_t l_{jt}^s dj$$

The first-order conditions of the labor "packer" are:

$$w_t \frac{\eta}{\eta - 1} \left(\int_0^1 \left(l_{jt}^s \right)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}-1} \frac{\eta - 1}{\eta} \left(l_{jt}^s \right)^{\frac{\eta-1}{\eta}-1} - w_{jt} = 0 \quad \forall j$$

Dividing the first-order conditions for two types of labor i and j and integrating over all labor types, we get:

$$\int_0^1 w_{jt} l_{jt}^s dj = w_{it} \left(l_{it}^s \right)^{\frac{1}{\eta}} \int_0^1 \left(l_{jt}^s \right)^{\frac{\eta-1}{\eta}} dj = w_{it} \left(l_{it}^s \right)^{\frac{1}{\eta}} \left(l_{jt}^s \right)^{\frac{\eta-1}{\eta}}$$

Using the zero profits condition implied by perfect competition:

$$w_t l_t^d = \int_0^1 w_{jt} l_{jt}^s dj$$

and solving we obtain the per capita input demand function and aggregate wage:

$$
\begin{aligned}
l_{jt}^{s} &= \left(\frac{w_{jt}}{w_t}\right)^{-\eta} l_t^d \quad \forall j \\
w_t &= \left(\int_0^1 w_{jt}^{1-\eta} dj\right)^{\frac{1}{1-\eta}}
\end{aligned} \tag{2}
$$

Idiosyncratic risk comes about because households set their wages following a Calvo's setting. In each period, a fraction $1-\theta_w$ of households can change their wages. All other households can only partially index their wages to past inflation of the final domestic good. Indexation is controlled by the parameter $\chi_w \in [0, 1]$. This implies that if the household cannot change its wage for τ periods, its normalized wage is $\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} w_{jt}$. When new workers in the household begin to work, they are assigned a wage equal to the wage of the other workers in the household.

Thus, the relevant part of the Lagrangian for the household is:

$$
\max_{w_{jt}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \theta_w^{\tau} \beta^{\tau} \widetilde{\gamma}_{\tau}^{L} \left\{ -d_{t+\tau} \varphi_{t+\tau} \psi \frac{\left(l_{jt+\tau}^{s}\right)^{1+\vartheta}}{1+\vartheta} + \lambda_{jt+\tau} \prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} (1-\tau_w) w_{jt} l_{jt+\tau}^{s} \right\}
$$

$$
\text{s.t. } l_{jt+\tau}^{s} = \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{w_{jt}}{w_{t+\tau}} \right)^{-\eta} l_{t+\tau}^{d}
$$

All households that can optimize their wages in this period set the same wage ($w_t^* = w_{jt}\ \forall j$ that optimizes) because complete markets allow them to hedge the risk of the timing of wage change. Hence, we can drop the jth from the choice of wages and λ_{jt}. Similarly, the ratio of Lagrangians, $\lambda_{t+\tau}/\lambda_t$, will be constant across households and, consequently, the marginal valuation of future income is also constant. The first-order condition of this problem is:

$$
\begin{aligned}
&\frac{\eta-1}{\eta} (1-\tau_w) w_t^* \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^{\tau} \widetilde{\gamma}_{\tau}^{L} \lambda_{t+\tau} \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \right)^{1-\eta} \left(\frac{w_t^*}{w_{t+\tau}} \right)^{-\eta} l_{t+\tau}^{d} \\
&= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^{\tau} \widetilde{\gamma}_{\tau}^{L} \left(d_{t+\tau} \varphi_{t+\tau} \psi \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{w_t^*}{w_{t+\tau}} \right)^{-\eta(1+\vartheta)} \left(l_{t+\tau}^{d} \right)^{1+\vartheta} \right)
\end{aligned}
$$

Note that for those sums to be well defined (and, more generally, for the maximization problem to have a solution), we need to assume that $(\beta\theta_w)^{\tau} \widetilde{\gamma}_{\tau}^{L} \lambda_{t+\tau}$ goes to zero faster than $\left(\prod_{s=1}^{\tau} \Pi_{t+s}^{\chi_w} / \Pi_{t+s-1}\right)^{1-\eta}$ goes to infinity in expectation.

To express this equation recursively, we re-label each part of this equality as f_t^1 and f_t^2:

$$f_t^1 = \frac{\eta-1}{\eta}(1-\tau_w)\, w_t^* \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau\, \widetilde{\gamma}_\tau^L \lambda_{t+\tau} \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \right)^{1-\eta} \left(\frac{w_{t+\tau}}{w_t^*} \right)^{\eta} l_{t+\tau}^d$$

$$f_t^2 = \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau\, \widetilde{\gamma}_\tau^L d_{t+\tau}\varphi_{t+\tau}\psi \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \right)^{-\eta(1+\vartheta)} \left(\frac{w_{t+\tau}}{w_t^*} \right)^{\eta(1+\vartheta)} \left(l_{t+\tau}^d \right)^{1+\vartheta}$$

and add the equality $f_t = f_t^1 = f_t^2$.

Then, in recursive form:

$$f_t = \frac{\eta-1}{\eta}(1-\tau_w)\left(w_t^*\right)^{1-\eta} \lambda_t w_t^\eta l_t^d + \beta\theta_w \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}} \right)^{1-\eta} \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\eta-1} f_{t+1}$$

and

$$f_t = d_t\varphi_t\psi \left(\frac{w_t^*}{w_t} \right)^{-\eta(1+\vartheta)} \left(l_t^d \right)^{1+\vartheta} + \beta\theta_w \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}} \right)^{-\eta(1+\vartheta)} \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\eta(1+\vartheta)} f_{t+1}$$

Later, by solving for the dynamics of f_t, we will be able to compute w_t^*.

Finally, in a symmetric equilibrium and in every period, a fraction $1-\theta_w$ of households set w_t^* as their wage, while the remaining fraction θ_w partially index their price to past inflation. Consequently, the real wage index evolves:

$$w_t^{1-\eta} = \theta_w \left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t} \right)^{1-\eta} w_{t-1}^{1-\eta} + (1-\theta_w)\, w_t^{*1-\eta}.$$

that is, as a geometric average of past real wage and the new optimal wage. This structure is a direct consequence of the memoryless characteristic of Calvo pricing.

3.2 The distribution sector

The distribution sector is composed of two segments. At the end, there is a consumption good producer and an investment good producer, while at the source, a final domestic good producer aggregates all domestic intermediate goods to produce the final domestic good.

3.2.1 Final consumption and investment good producers

At the top of the distribution chain, there is a perfectly competitive consumption good producer and investment good producer who pack domestic consumption and investment $\left(c_t^d, i_t^d\right)$ with imported consumption and investment baskets $\left(c_t^M, i_t^M\right)$ to generate final consumption and investment (c_t, i_t) using a technology described by:

$$c_t = \left[\left(n^c\right)^{\frac{1}{\varepsilon_c}} \left(c_t^d\right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} + \left(1 - n^c\right)^{\frac{1}{\varepsilon_c}} \left(c_t^M \left(1 - \Gamma_t^c\right)\right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} \right]^{\frac{\varepsilon_c}{\varepsilon_c - 1}}$$

$$i_t = \left[\left(n^i\right)^{\frac{1}{\varepsilon_i}} \left(i_t^d\right)^{\frac{\varepsilon_i - 1}{\varepsilon_i}} + \left(1 - n^i\right)^{\frac{1}{\varepsilon_i}} \left(i_t^M \left(1 - \Gamma_t^i\right)\right)^{\frac{\varepsilon_i - 1}{\varepsilon_i}} \right]^{\frac{\varepsilon_i}{\varepsilon_i - 1}}$$

where there is a home bias in the aggregation, measured by n^c and n^i, which determines the steady state degree of openness, and where ε_c (ε_i) represents the elasticity of substitution between imported and domestic consumption (investment) goods. In addition, we assume that it is costly to change the share of imports of consumption and investment in final production. This is modelled by adding a cost term (Γ_t^c and Γ_t^i) to changing the import to consumption (investment) ratio in the production function:

$$\Gamma_t^s = \frac{\Gamma^s}{2} \left(\frac{s_t^M}{s_t} \bigg/ \frac{s_{t-1}^M}{s_{t-1}} - 1 \right)^2 \quad \text{for } s = c, i$$

The producer of the final consumption good maximizes profits subject to the production function, taking as given the price of the final domestic good, of the imported consumption goods (in domestic currency) p_t, p_t^M, and of the final consumption basket p_t^c. Due to adjustment costs, the problem becomes dynamic and the aggregator discounts future income with the pricing kernel $\beta \gamma_{t+1}^L \frac{\lambda_{t+1}}{\lambda_t}$ (below, when talking about discounting by intermediate good producers, we explain this point in more detail). In the case of consumption (and similarly for investment):

$$\max_{c_t^d, c_t^M} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \widetilde{\gamma}_\tau^L \frac{\lambda_{t+\tau}}{\lambda_t} \left[p_t^c c_t - p_t c_t^d - p_t^M c_t^M \right]$$

$$\text{s.t. } c_t = \left[\left(n^c\right)^{\frac{1}{\varepsilon_c}} \left(c_t^d\right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} + \left(1 - n^c\right)^{\frac{1}{\varepsilon_c}} \left(c_t^M \left(1 - \Gamma_t^c\right)\right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} \right]^{\frac{\varepsilon_c}{\varepsilon_c - 1}}$$

Solving (and after some tedious algebra), we get the demand for the domestic and imported consumption good and the price of the final consumption good:

$$c_t^d = n^c \left(\frac{p_t}{p_t^c} \right)^{-\varepsilon_c} c_t$$

$$c_t^M = \mathbb{E}_t \Omega_{t+1}^c \left(1 - n^c\right) \left(\frac{p_t^M}{p_t^c} \right)^{-\varepsilon_c} c_t$$

$$p_t^c = \left[n^c \left(p_t\right)^{1-\varepsilon_c} + \mathbb{E}_t \Omega_{t+1}^c \left(1 - n^c\right) \left(p_t^M\right)^{1-\varepsilon_c} \right]^{\frac{1}{1-\varepsilon_c}}$$

where

$$\mathbb{E}_t \Omega_{t+1}^c = \frac{\left[1-\beta\left(1-n^c\right)^{\frac{1}{\varepsilon_c}} \mathbb{E}_t \frac{\gamma_{t+1}^L \lambda_{t+1}}{\lambda_t} \left(\frac{p_t^c}{p_t^M}\right) \Pi_{t+1}^c \left(\frac{c_{t+1}}{c_{t+1}^M\left(1-\Gamma_{t+1}^c\right)}\right)^{\frac{1}{\varepsilon_c}} \Gamma_{t+1}^{c\,\prime} \frac{\left(\Delta c_{t+1}^M\right)^2}{\Delta c_{t+1}}\right]^{-\varepsilon_c}}{\left(1-\Gamma_t^c\right)\left[1-\Gamma_t^c-\Gamma_t^{c\,\prime}\left(\frac{\Delta c_t^M}{\Delta c_t}\right)\right]^{-\varepsilon_c}}$$

3.2.2 Final domestic good producer

At the start of the distribution chain, we have the final domestic good producer that produces the final domestic good (y_t) by aggregating intermediate domestic goods (y_{it}) with the following production function:

$$y_t = \left(\int_0^1 \left(y_{it}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{3}$$

where ε is the elasticity of substitution across domestic intermediate goods and all the variables are expressed in per capita terms.

The final domestic good producer is perfectly competitive and maximizes profits subject to the production function (3), taking as given all intermediate domestic goods prices p_{it} and the final domestic good price p_t. Following the same steps as for wages, we find the input demand functions and the aggregate price associated with this problem:

$$y_{it} = \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} y_t^d \quad \forall i,$$

$$p_t = \left(\int_0^1 p_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

where y_t^d is the aggregate demand by the final good producer.

3.3 Intermediate good producers

At the bottom of the domestic production process, there is a continuum of intermediate goods producers. Each intermediate good producer i has access to a technology represented by a production function (expressed in per capita terms):

$$y_{it} = A_t k_{it-1}^{\alpha}\left(l_{it}^d\right)^{1-\alpha} - \phi z_t$$

where k_{it-1} is the capital rented by the firm, l_{it}^d is the amount of the “packed” labor input rented by the firm, and where A_t, the neutral technology level, follows the process:

$$A_t = A_{t-1} \exp\left(\Lambda_A + z_{A,t}\right) \quad \text{where } z_{A,t} = \sigma_A \varepsilon_{A,t} \quad \text{and} \quad \varepsilon_{A,t} \sim \mathcal{N}(0,1)$$

which induces a third unit root in the model, the second from technology. The parameter ϕ, which corresponds to the fixed cost of production, and $z_t = A_t^{\frac{1}{1-\alpha}} \mu_t^{\frac{\alpha}{1-\alpha}}$ guarantee that economic profits are roughly equal to zero in the steady state. We rule out the entry and exit of intermediate good producers. Long-run growth of domestic output in per capita terms is determined by $z_t = A_t^{\frac{1}{1-\alpha}} \mu_t^{\frac{\alpha}{1-\alpha}}$, which evolves as:

$$z_t = z_{t-1} \exp\left(\Lambda_z + z_{z,t}\right) \quad \text{where } z_{z,t} = \frac{z_{A,t} + \alpha z_{\mu,t}}{1-\alpha} \quad \text{and} \quad \Lambda_z = \frac{\Lambda_A + \alpha \Lambda_\mu}{1-\alpha}.$$

Intermediate goods producers solve a two-stage problem. In the first stage, taking the input prices w_t and r_t as given, firms rent l_{it}^d and k_{it-1} in perfectly competitive factor markets to minimize real cost:

$$\min_{l_{it}^d, k_{it-1}} \; w_t l_{it}^d + r_t k_{it-1}$$

$$\text{s.t. } y_{it} = \begin{cases} A_t k_{it-1}^{\alpha} \left(l_{it}^d\right)^{1-\alpha} - \phi z_t & \text{if } A_t k_{it-1}^{\alpha} \left(l_{it}^d\right)^{1-\alpha} \geq \phi z_t \\ 0 & \text{otherwise} \end{cases}$$

Assuming an interior solution, the intermediate good producers equate marginal productivity to input prices to get an optimal capital–labor ratio

$$\frac{k_{it-1}}{l_{it}^d} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}$$

and a real marginal cost:

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{w_t^{1-\alpha} r_t^{\alpha}}{A_t}$$

Note that both the optimal capital–labor ratio and the marginal cost do not depend on i: all firms receive the same technology shocks and all firms rent inputs at the same price.

In the second stage, intermediate good producers choose the price that maximizes discounted real profits. We assume they are under the same pricing scheme as households. In each period, a fraction $1 - \theta_p$ of firms can change their prices. All other firms can only index their prices to past inflation of the final domestic good price $\left(\Pi_t = \frac{p_t}{p_{t-1}}\right)$. Indexation is controlled by the parameter $\chi \in [0, 1]$, where $\chi = 0$ is no indexation and $\chi = 1$ is total indexation. The problem of the firms is then:

$$\max_{p_{it}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \left(\beta\theta_p\right)^{\tau} \widetilde{\gamma}_{\tau}^{L} \frac{\lambda_{t+\tau}}{\lambda_t} \left\{ \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \frac{p_{it}}{p_{t+\tau}} - mc_{t+\tau} \right) y_{it+\tau} \right\}$$
$$\text{s.t. } y_{it+\tau} = \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \frac{p_{it}}{p_{t+\tau}} \right)^{-\varepsilon} y_{t+\tau}$$

where the valuation of future profits is done with the common ratio of Lagrangian multipliers $\lambda_{t+\tau}/\lambda_t$ (treated as exogenous by the firm). Since we have complete markets in securities, this ratio is indeed the correct valuation on future profits.[5] The solution p_{it}^* implies the first-order condition:

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \left(\beta\theta_p\right)^{\tau} \widetilde{\gamma}_{\tau}^{L} \lambda_{t+\tau} \left\{ \left((1-\varepsilon) \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi}}{\Pi_{t+s}} \right)^{1-\varepsilon} \frac{p_t^*}{p_t} + \varepsilon \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi}}{\Pi_{t+s}} \right)^{-\varepsilon} mc_{t+\tau} \right) y_{t+\tau} \right\} = 0 \tag{4}$$

where we have dropped irrelevant constants and used the fact that we are in a symmetric equilibrium. This expression nests the usual result in the fully flexible prices case ($\theta_p = 0$):

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} p_t mc_{t+\tau}$$

that is, the price is equal to a mark-up over the nominal marginal cost.

To express Eq. (4) recursively, we define g_t^1 and g_t^2:

$$g_t^1 = \lambda_t mc_t y_t + \beta\theta_p \mathbb{E}_t \gamma_{t+1}^{L} \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}} \right)^{-\varepsilon} g_{t+1}^1$$
$$g_t^2 = \lambda_t \Pi_t^* y_t + \beta\theta_p \mathbb{E}_t \gamma_{t+1}^{L} \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}} \right)^{1-\varepsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*} \right) g_{t+1}^2$$

where $\Pi_t^* = \frac{p_t^*}{p_t}$, and we get that (4) is equivalent to $\varepsilon g_t^1 = (\varepsilon - 1) g_t^2$.

Given Calvo's pricing, the price index evolves:

$$(p_t)^{1-\varepsilon} = \theta_p \left(\Pi_{t-1}^{\chi}\right)^{1-\varepsilon} (p_{t-1})^{1-\varepsilon} + \left(1-\theta_p\right) \left(p_t^*\right)^{1-\varepsilon}$$

[5] As in the household problem, we need $\left(\beta\theta_p\right)^{\tau} \widetilde{\gamma}_{\tau}^{L} \lambda_{t+\tau}/\lambda_t$ to go to zero sufficiently fast in relation to the rate of inflation for the optimization problem to be well defined.

or, dividing by $(p_{t-1})^{1-\varepsilon}$,

$$1 = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t} \right)^{1-\varepsilon} + \left(1 - \theta_p\right) \left(\Pi_t^*\right)^{1-\varepsilon}$$

3.4 Foreign sector

To better describe the foreign sector, we start by explaining the composition of the import and export markets separately and then proceed to show how both types of firms set prices.

Importing firms

The import sector is composed of two segments. At the end, a distributor (or aggregator) mixes differentiated imported goods $\left(y_{it}^M\right)$ to produce the final imported basket $\left(y_t^M\right)$, while at the source, a continuum of importing firms buy the foreign homogeneous final good in the international markets at price p_t^W and turn it into a differentiated import good through a differentiating technology or brand naming.

The distributor (or aggregator) produces the final imported good $\left(y_t^M\right)$ from differentiated imported goods $\left(y_{it}^M\right)$ with the following production function:

$$y_t^M = \left(\int_0^1 \left(y_{it}^M\right)^{\frac{\varepsilon_M - 1}{\varepsilon_M}} di \right)^{\frac{\varepsilon_M}{\varepsilon_M - 1}}.$$

where ε_M is the elasticity of substitution across foreign final goods and all the variables are expressed in per capita terms. Following the same steps as for domestic prices, we find the price of the imported final basket and the import demand functions:

$$y_{it}^M = \left(\frac{p_{it}^M}{p_t^M} \right)^{-\varepsilon_M} y_t^M \quad \forall i,$$

$$p_t^M = \left(\int_0^1 \left(p_{it}^M\right)^{1-\varepsilon_M} di \right)^{\frac{1}{1-\varepsilon_M}}$$

The total per capita amount of imported goods is thus given by:

$$M_t = \int_0^1 y_{it}^M di$$

Exporting firms

The export sector consists of a continuum of firms that buy the final domestic good and differentiate it by brand naming. Then, they sell the continuum of differentiated goods to importers from the rest of the world. Each exporting firm faces the following demand for its products:

$$y_{it}^{x} = \left(\frac{p_{it}^{x}}{p_{t}^{x}}\right)^{-\varepsilon_x} y_t^x \quad \forall i,$$

where both prices are expressed in the foreign currency of the export market. The export deflator is

$$p_t^x = \left(\int_0^1 \left(p_{it}^x\right)^{1-\varepsilon_x} di\right)^{\frac{1}{1-\varepsilon_X}}$$

Therefore, the total amount of exported good is given by:

$$x_t = \int_0^1 y_{it}^x di$$

Finally, appealing to symmetry, we assume that the world demand of our exports is:

$$y_t^x = \left(\frac{p_t^x}{p_t^W}\right)^{-\varepsilon_W} y_t^W.$$

and we have that:

$$y_{it}^x = \left(\frac{p_{it}^x}{p_t^x}\right)^{-\varepsilon_x} \left(\frac{p_t^x}{p_t^W}\right)^{-\varepsilon_W} y_t^W \quad \forall i.$$

The evolution of world demand is exogenously given by:

$$y_t^W = \left(y^W\right)^{\left(1-\rho_{y^W}\right)} \left(y_{t-1}^W\right)^{\rho_{y^W}} e^{\left(\sigma_{y^W}\varepsilon_{y^W,t}\right)}$$

and world inflation by:

$$\Pi_t^W = \left(\Pi^W\right)^{\left(1-\rho_{\Pi^W}\right)} \left(\Pi_{t-1}^W\right)^{\rho_{\Pi^W}} e^{\left(\sigma_{\Pi^W}\varepsilon_{\Pi^W,t}\right)}$$

Price-setting in the foreign sector

To allow for incomplete exchange rate pass-through to import and export prices, we assume that importing and exporting firms in the foreign sector face price stickiness à la Calvo. Since the problem faced by both types of firms is similar, we will describe them together. In particular, in each period, a fraction $1 - \theta_M$ $(1 - \theta_X)$

of importing (exporting) firms can change their prices. All other importing (exporting) firms can only index their prices to past inflation of the final imported (foreign) good $\left(\Pi_t^M = \frac{p_t^M}{p_{t-1}^M}\left(\Pi_t^W = \frac{p_t^W}{p_{t-1}^W}\right)\right)$. Indexation is controlled by the parameter $\chi_M, \chi_X \in [0,1]$, where $\chi_M, \chi_X = 0$ is no indexation and $\chi_M, \chi_X = 1$ is total indexation.

Since importing (exporting) firms buy the homogeneous foreign (domestic) good at price p_t^W (p_t) in the world (domestic) market, their real marginal cost, in domestic (foreign) currency terms, is equal to $mc_t^M = \frac{p_t^W ex_t}{p_t^M}$ $\left(mc_t^x = \frac{p_t}{ex_t p_t^x}\right)$.

The problem of importing (exporting) firm i is then:

$$\max_{p_{it}^{fm}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \left(\beta\theta_{fm}\right)^{\tau} \widetilde{\gamma}_{\tau}^{L} \frac{\lambda_{t+\tau}}{\lambda_t} \left\{\left(\prod_{s=1}^{\tau}\left(\Pi_{t+s-1}^{fm}\right)^{\chi_{fm}} \frac{p_{it}^{fm}}{p_{t+\tau}^{fm}} - mc_{t+\tau}^{fm}\right) y_{it+\tau}^{fm}\right\}$$

$$\text{s.t. } y_{it+\tau}^{fm} = \left(\prod_{s=1}^{\tau}\left(\Pi_{t+s-1}^{fm}\right)^{\chi_{fm}} \frac{p_{it}^{fm}}{p_{t+\tau}^{fm}}\right)^{-\varepsilon_{fm}} y_{t+\tau}^{fm} \quad \text{for } fm = M, x$$

Proceeding as in the case of domestic prices we get:

$$g_t^{M_1} = \begin{bmatrix} \lambda_t \frac{ex_t p_t^W}{p_t^M} y_t^M + \\ \beta\theta_M \mathbb{E}_t \gamma_{t+1}^L \left(\frac{(\Pi_t^M)^{\chi_M}}{\Pi_{t+1}^M}\right)^{-\varepsilon_M} g_{t+1}^{M_1} \end{bmatrix}$$

$$g_t^{M_2} = \left[\lambda_t \Pi_t^{M^*} y_t^M + \beta\theta_M \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\left(\Pi_t^M\right)^{\chi_M}}{\Pi_{t+1}^M}\right)^{1-\varepsilon_M} \left(\frac{\Pi_t^{M^*}}{\Pi_{t+1}^{M^*}}\right) g_{t+1}^{M_2}\right]$$

$$g_t^{x_1} = \begin{bmatrix} \lambda_t \frac{p_t}{ex_t p_t^x} y_t^x + \\ \beta\theta_x \mathbb{E}_t \gamma_{t+1}^L \left(\frac{(\Pi_t^W)^{\chi_x}}{\Pi_{t+1}^x}\right)^{-\varepsilon_x} g_{t+1}^{x_1} \end{bmatrix}$$

$$g_t^{x_2} = \lambda_t \Pi_t^{x^*} y_t^x + \beta\theta_x \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\left(\Pi_t^W\right)^{\chi_x}}{\Pi_{t+1}^x}\right)^{1-\varepsilon_x} \left(\frac{\Pi_t^{x^*}}{\Pi_{t+1}^{x^*}}\right) g_{t+1}^{x_2}$$

$$\varepsilon_M g_t^{M_1} = (\varepsilon_M - 1) g_t^{M_2};\ \varepsilon_x g_t^{X_1} = (\varepsilon_x - 1) g_t^{X_2}$$

where $\Pi_t^{M^*} = \frac{p_t^{M^*}}{p_t^M}$ and $\Pi_t^{x^*} = \frac{p_t^{x^*}}{p_t^x}$.

Given Calvo's pricing, the import and export price indices evolve as:

$$1 = \theta_M \left(\frac{\left(\Pi_{t-1}^M\right)^{\chi_M}}{\Pi_t^M}\right)^{1-\varepsilon_M} + (1-\theta_M)\left(\Pi_t^{M^*}\right)^{1-\varepsilon_M}$$

$$1 = \theta_x \left(\frac{\left(\Pi_{t-1}^W\right)^{\chi_x}}{\Pi_t^x}\right)^{1-\varepsilon_x} + (1-\theta_x)\left(\Pi_t^{x^*}\right)^{1-\varepsilon_x}$$

Evolution of net foreign assets

To close the foreign sector, we have to determine the evolution of net foreign assets. The balance of payments, including the premium on foreign holdings, evolves as follows:

$$\int_0^1 ex_t b_{jt}^W dj = R_{t-1}^W \Gamma\left(ex_t \widetilde{b}_{t-1}^W, \xi_{t-1}^{b^W}\right) ex_t \frac{1}{\gamma_t^L} \int_0^1 b_{jt-1}^W dj + ex_t p_t^x y_t^x - ex_t p_t^W M_t$$

where we have used the fact that:

$$\int_0^1 p_{it}^x y_{it}^x di = \int_0^1 p_{it}^x \left(\frac{p_{it}^x}{p_t^x}\right)^{-\varepsilon_x} y_t^x di = \left(p_t^x\right)^{\varepsilon_x} y_t^x \int_0^1 \left(p_{it}^x\right)^{1-\varepsilon_x} di = p_t^x y_t^x$$

3.5 The monetary authority

Monetary policy is controlled by the ECB, which sets the nominal interest rates for the euro area (R_t) according to the Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\left(\frac{\Pi_t^{EA}}{\Pi^{EA}}\right)^{\gamma_\Pi} \left(\frac{\gamma_t^{L^{EA}} \frac{y_t^{EA}}{y_{t-1}^{EA}}}{\exp\left(\Lambda_{L^{EA}} + \Lambda_{y^{EA}}\right)}\right)^{\gamma_y}\right)^{1-\gamma_R} \exp\left(\xi_t^m\right)$$

through open market operations, where Π^{EA} represents the euro area target level of inflation, R euro area steady state nominal gross return of capital, and Λ_{y^d} the euro area steady state gross growth rate of y_t^{EA}. The term ξ_t^m is a random shock to monetary policy that follows $\xi_t^m = \sigma_m \varepsilon_{mt}$ where $\varepsilon_{mt} \sim \mathcal{N}(0, 1)$. The presence of the previous period interest rate, R_{t-1}, is justified because we want to match the smooth profile of the interest rate over time observed in the data. Note that R is beyond the control of the monetary authority, since it is equal to the steady state real gross return of capital plus the target level of inflation.

The Spanish economy contributes to the euro area inflation and output according to its relative size. Ideally, we would like to account for how shocks to the Spanish economy affect euro area variables and through those, to R_t. Unfortunately, in practice, it is difficult to solve a DSGE model taking Spain's behavior as implied by the model and the rest of the euro data as given. This is because, given the small weight of Spain in the euro area aggregate (10%), the indeterminacy region of such a model is so large that the model becomes nearly useless. One way to solve this problem is to build a model of a two-country small open monetary area, like BEMOD (Andrés et al. 2006). However, we avoid this route because we fear it would make us lose focus.

Instead, we adopt two alternative approaches depending on the objective of the exercise. In the first approach, we assume the domestic economy has an independent local monetary policy that sets the nominal interest rate. This is equivalent to setting a weight of 1 for Spain's aggregates in the Taylor rule above. This would correspond to the time before the euro area was set up and we use it when estimating the model over the whole sample period (1986–2007). We also employ it when doing a historical decomposition of shocks and in some counterfactual exercises. In the second approach, we assume Spain is too small to have a significant influence on the ECB's Taylor rule. Thus, changes in Spanish conditions do not affect R_t, which is determined by the Taylor rule above, evaluated at the observed (or forecasted) values of euro area variables. This is the approach we use when estimating over the most recent period (1997–2007) or when we do policy analysis related to the current situation.

3.6 The government problem

The per capita government budget constraint is:

$$\widetilde{b}_t = \frac{g_t z_t}{y_t^d} + \frac{T_t}{y_t^d} + \frac{\frac{m_{t-1}}{p_{t-1}}}{\gamma_t^L y_t^d \Pi_t} + \frac{R_{t-1}\widetilde{b}_{t-1}}{\gamma_t^L \Pi_t y_t^d}$$
$$- \left(r_t u_{jt} - \mu_t^{-1}\delta\right) \tau_k \frac{k_{t-1}}{y_t^d} - \tau_w w_t \frac{l_t^d}{y_t^d} - \tau_c \frac{p_t^c}{p_t} \frac{c_t}{y_t^d} - \frac{\frac{m_t}{p_t}}{y_t^d}$$

where we have redefined the level of outstanding debt as a proportion of nominal output as $\widetilde{b}_t = \frac{\int_0^1 b_{jt} dj}{p_t y_t^d}$.[6] The level of real government consumption appears multiplied by z_t to keep it a stationary share of output, which is exogenous and determined according to a stochastic process:

$$\log g_t = \left(1 - \rho_g\right) \log g + \rho_g \log g_{t-1} + \sigma_g \varepsilon_{g,t} \quad \text{where } \varepsilon_{g,t} \sim \mathcal{N}(0, 1),$$

Fiscal policy must be designed to prevent the level of debt from exploding. Since all tax rates are assumed to be constant, we assume that lump-sum taxes as a proportion of output in per capita terms ($\frac{T_t}{y_t}$) respond sufficiently to prevent deviations of the level of debt as a proportion of output ($\widetilde{b}_t$) from target $\left(\overline{\widetilde{b}} = \overline{\left(\frac{b}{py^d}\right)}\right)$:

$$\frac{T_t}{y_t^d} = T_0 - T_1 \left(\widetilde{b}_t - \overline{\widetilde{b}}\right)$$

3.7 Aggregation

To close the model, we need aggregation conditions for each of the markets considered: goods, labor, import, and export markets. In the case of the goods market, we

[6] One can show that the government budget constraint is correct by inserting it into the households' budget constraint (evaluated at the aggregate level), which implies that all of the tax terms except g_t cancel out.

start from the expression for per capita aggregate demand of the domestic final good:

$$y_t^d = c_t^d + i_t^d + g_t z_t + \mu_t^{-1} \Phi\left[u_t\right] k_{t-1} + x_t,$$

Then, we use the equation determining the demand for each intermediate producer's good $\left(y_{it} = \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} y_t^d\right)$ and the production function $\left(y_{it} = A_t k_{it-1}^{\alpha} \left(l_{it}^d\right)^{1-\alpha}\right)$, and integrating over all firms, we get the aggregate condition of the goods market:

$$\begin{aligned} & n^c \left(\frac{p_t}{p_t^c}\right)^{-\varepsilon_c} c_t + n^i \left(\frac{p_t}{p_t^i}\right)^{-\varepsilon_i} i_t + g_t z_t + \mu_t^{-1} \Phi\left[u_t\right] k_{t-1} + x_t \\ & \quad = \frac{A_t \left(u_t k_{t-1}\right)^{\alpha} \left(l_t^d\right)^{1-\alpha} - \phi z_t}{v_t^p} \end{aligned}$$

where $v_t^p = \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di$ measures the impact of the price distribution on output. To get this result, we have used the fact that all of the intermediate good producers have the same optimal capital-labor ratio and that by market clearing:

$$l_t^d = \int_0^1 l_{it}^d di$$

$$u_t k_{t-1} = \int_0^1 k_{it-1} di$$

To close the labor market, we start with the demand for household's labor services $\left(l_{jt}^s = \left(\frac{w_{jt}}{w_t}\right)^{-\eta} l_t^d\right)$ and integrate over all households to get the condition equating (per capita) aggregate labor demand (l_t^d) and supply (l_t)

$$l_t^d = \frac{1}{v_t^w} l_t.$$

where $v_t^w = \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\eta} dj$ measures the impact of the wage distribution on employment.

In the case of the imported goods market, using the definition of aggregate imports ($M_t = \int_0^1 y_{it}^M di$) and the fact that its production is distributed to households as imported consumption and investment ($y_t^M = c_t^M + i_t^M$), we get:

$$M_t = v_t^M \left[\mathbb{E}_t \Omega_{t+1}^c \left(1 - n^c\right) \left(\frac{p_t^M}{p_t^c}\right)^{-\varepsilon_c} c_t + \mathbb{E}_t \Omega_{t+1}^i \left(1 - n^i\right) \left(\frac{p_t^M}{p_t^i}\right)^{-\varepsilon_i} i_t\right]$$

where $v_t^M = \int_0^1 \left(\frac{p_{it}^M}{p_t^M}\right)^{-\varepsilon_M} di$ measures the impact of price dispersion in the output of this sector and

$$\mathbb{E}_t \Omega_{t+1}^s = \frac{\left[1-\beta\left(1-n^s\right)^{\frac{1}{\varepsilon_s}} \mathbb{E}_t \frac{\gamma_{t+1}^L \lambda_{t+1}}{\lambda_t} \left(\frac{p_t^s}{p_t^M}\right) \Pi_{t+1}^s \left(\frac{s_{t+1}}{s_{t+1}^M\left(1-\Gamma_{t+1}^s\right)}\right)^{\frac{1}{\varepsilon_s}} \Gamma_{t+1}^{s\,\prime} \frac{\left(\Delta s_{t+1}^M\right)^2}{\Delta s_{t+1}}\right]^{-\varepsilon_s}}{\left(1-\Gamma_t^s\right)\left[1-\Gamma_t^s-\Gamma_t^{s\,\prime}\left(\frac{\Delta s_t^M}{\Delta s_t}\right)\right]^{-\varepsilon_s}}$$

for $s = c, i$.

In the exported goods market, using the definition of exports ($x_t = \int_0^1 y_{it}^x di$) and the demand functions for their output

$$y_{it}^x = \left(\frac{p_{it}^x}{p_t^x}\right)^{-\varepsilon_x} \left(\frac{p_t^x}{p_t^W}\right)^{-\varepsilon_W} y_t^W$$

we have that aggregate exports are equal to

$$x_t = v_t^x y_t^x = v_t^x \left(\frac{p_t^x}{p_t^W}\right)^{-\varepsilon_W} y_t^W$$

where $v_t^x = \int_0^1 \left(\frac{p_{it}^x}{p_t^x}\right)^{-\varepsilon_x} di$.

Finally, by the properties of price indices under Calvo's pricing mechanism, price dispersions evolve according to:

$$\begin{aligned}
v_t^p &= \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon} v_{t-1}^p + \left(1-\theta_p\right)\left(\Pi_t^*\right)^{-\varepsilon} \\
v_t^w &= \theta_w \left(\frac{w_{t-1}}{w_t}\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{-\eta} v_{t-1}^w + (1-\theta_w)\left(\Pi_t^{*w}\right)^{-\eta} \\
v_t^M &= \theta_M \left(\frac{\left(\Pi_{t-1}^M\right)^{\chi_M}}{\Pi_t^M}\right)^{-\varepsilon_M} v_{t-1}^M + (1-\theta_M)\left(\Pi_t^{M*}\right)^{-\varepsilon_M} \\
v_t^x &= \theta_x \left(\frac{\left(\Pi_{t-1}^W\right)^{\chi_x}}{\Pi_t^x}\right)^{-\varepsilon_x} v_{t-1}^x + (1-\theta_x)\left(\Pi_t^{x*}\right)^{-\varepsilon_x}.
\end{aligned}$$

4 Equilibrium and model solution

The definition of equilibrium in this economy is standard and we omit it in the interest of space. Since there is growth in the model induced by technological change, most of the variables are growing on average along the equilibrium path.[7] Before we can solve the model, we need to rescale all the relevant variables to obtain a system of stationary variables. Hence, we define $\widetilde{c}_t = \frac{c_t}{z_t}$, $\widetilde{\lambda}_t = \lambda_t z_t$, $\widetilde{r}_t = r_t \mu_t$, $\widetilde{q}_t = q_t \mu_t$, $\widetilde{x}_t = \frac{x_t}{z_t}$, $\widetilde{w}_t = \frac{w_t}{z_t}$, $\widetilde{w}_t^* = \frac{w_t^*}{z_t}$, $\widetilde{k}_t = \frac{k_t}{z_t \mu_t}$, $\widetilde{m}_t = \frac{m_t}{z_t}$, $\widetilde{y}_t^d = \frac{y_t^d}{z_t}$, and the growth rates $\widetilde{z}_t = \frac{z_t}{z_{t-1}}$, $\widetilde{A}_t = \frac{A_t}{A_{t-1}}$, $\widetilde{\mu}_t = \frac{\mu_t}{\mu_{t-1}}$, $\widetilde{L}_t = \frac{L_t}{L_{t-1}}$. In addition, we express all the relative prices in terms of the vector $\left(\frac{p_t^c}{p_t}, \frac{p_t^i}{p_t}, \frac{p_t^M}{p_t}, \frac{ex_t p_t^x}{p_t} \text{ and } \frac{ex_t p_t^W}{p_t}\right)$. To solve the model, we find the steady state and log-linearize the equilibrium conditions around it. For completeness, the full set of non-linear and log-linearized equilibrium conditions is included in the appendix.

4.1 The steady state

Now, we find the deterministic steady state of the model. We know several of its properties. First, the law of one price must hold, $\left(\frac{ex\ p^W}{p}\right) = 1$. Second, the exchange rate is assumed to be constant ($\Delta ex = 1$), which means that the domestic nominal interest rate is equal to the world nominal interest rate $\left(R = R^W = \frac{\Pi \widetilde{z}}{\beta}\right)$, and the net foreign asset position is assumed to be equal to zero (expressed in domestic currency), so that nominal exports equal nominal imports $\left(\widetilde{x}\left(\frac{ex\ p^x}{p}\right) = \upsilon^x \left(\frac{ex\ p^W}{p}\right) \widetilde{M}\right)$. Third, let $\widetilde{z} = \exp(\Lambda_z)$, $\widetilde{\mu} = \exp(\Lambda_\mu)$, $\widetilde{A} = \exp(\Lambda_A)$, and $\gamma^L = \exp(\Lambda_L)$. Also, given the definition of $\widetilde{c}, \widetilde{x}_t, \widetilde{w}_t, \widetilde{w}_t^*$, and $\widetilde{y}_t^d$, we have that $\Lambda_c = \Lambda_i = \Lambda_w = \Lambda_{w^*} = \Lambda_{y^d} = \Lambda_z$, and $u = d = \varphi = 1$ and $g = g$.

In addition, we need to choose functional forms for all of the adjustment cost functions in the model: $\Phi[\cdot]$, $S[\cdot]$, $\Gamma^s[\cdot]$ and $\Gamma\left(\widetilde{b}^*\right)$. For $\Phi[u]$, we pick: $\Phi[u_t] = \Phi_1(u_t - 1) + \frac{\Phi_2}{2}(u_t - 1)^2$. We select Φ_1 as a function of the other parameters of the model to normalize $u = 1$ in steady state. Therefore, $\Phi[1] = 0$ and $\Phi'[1] = \Phi_1$. The investment adjustment cost function is expressed in terms of quadratic deviations with respect to the average growth of investment:

$$S\left[\gamma_t^L \frac{\widetilde{i}_t}{\widetilde{i}_{t-1}} \widetilde{z}_t\right] = S\left[\frac{i_t}{i_{t-1}}\right] = \frac{\kappa}{2}\left(\gamma_t^L \frac{\widetilde{i}_t}{\widetilde{i}_{t-1}} \widetilde{z}_t - \Lambda_i\right)^2.$$

Then, along the balanced growth path, $S\left[\gamma^L \widetilde{z}\right] = S[\Lambda_i] = S'[\Lambda_i] = 0$. Finally, the imports adjustment cost function along the balanced growth path is

$$\Gamma_t^s = \frac{\Gamma^s}{2}\left(\left(\frac{s_t^M}{s_t}\right) \Big/ \left(\frac{s_{t-1}^M}{s_{t-1}}\right) - 1\right)^2,$$

[7] Population growth does not appear explicitly in the equilibrium since the variables are already expressed in per capita terms.

thus $\Gamma^s = \Gamma^{s\,\prime} = 0$. With respect to the adjustment cost of the premium for holding foreign assets, we assume:

$$\Gamma\left(ex_t\widetilde{b}_t^*, \xi_t^{b^*}\right) = e^{\left(-\Gamma^{b^*}\left(ex_t\widetilde{b}_t^* - ex\widetilde{b}^*\right) + \xi_t^{b^*}\right)}.$$

Since in the steady state the domestic and world interest rates are the same, $R = R^*\Gamma\left(ex\widetilde{b}^*, 0\right)$, we have $\Gamma\left(ex\widetilde{b}^*, 0\right) = e^0 = 1$ and $\Gamma'\left(ex\widetilde{b}^*\right) = -\Gamma^{b^*}e^0 = -\Gamma^{b^*}$.

Therefore, using these results and the equilibrium conditions we can simplify the steady state to the following set of equations determining l^d, while all the rest of the variables are recursive to these:

$$\widetilde{k} = \left(\frac{\alpha}{1-\alpha}\frac{\widetilde{w}}{\widetilde{r}}\widetilde{z}\widetilde{\mu}\right) l^d = \Omega l^d.$$

$$\widetilde{y}^d = \frac{\frac{\widetilde{A}}{\widetilde{z}}\widetilde{k}^\alpha \left(l^d\right)^{1-\alpha} - \phi}{v^p}$$

$$\widetilde{i} = \left(\frac{\gamma^L - (1-\delta)}{\widetilde{\mu}\widetilde{z}}\right)\widetilde{k}$$

$$\widetilde{c} = \frac{1}{n^c}\left(\frac{p^c}{p}\right)^{-\varepsilon_c}\left[\widetilde{y}^d - n^i\left(\frac{p^i}{p}\right)^{\varepsilon_i}\widetilde{i} - v^x\frac{\left(\frac{ex\ p^W}{p}\right)}{\left(\frac{ex\ p^x}{p}\right)}\widetilde{M} - g\right]$$

$$\widetilde{M} = v^M\left[\left(1-n^c\right)\left[\frac{\left(\frac{p^M}{p}\right)}{\left(\frac{p^c}{p}\right)}\right]^{-\varepsilon_c}\widetilde{c} + \left(1-n^i\right)\left[\frac{\left(\frac{p^M}{p}\right)}{\left(\frac{p^i}{p}\right)}\right]^{-\varepsilon_i}\widetilde{i}\right]$$

$$\widetilde{\lambda} = \left(\frac{\widetilde{z} - h\beta\gamma^L}{\widetilde{z} - h}\right)\frac{1}{\widetilde{c}\left(1+\tau_c\right)\frac{p^c}{p}}$$

$$\frac{1 - \beta\theta_w\widetilde{z}^{\,\eta(1+\vartheta)}\Pi^{\eta(1-\chi_w)(1+\vartheta)}\gamma^L}{1 - \beta\theta_w\widetilde{z}^{\,\eta-1}\Pi^{-(1-\chi_w)(1-\eta)}\gamma^L} = \frac{\psi\left(\frac{\widetilde{w}^*}{\widetilde{w}}\right)^{-\eta\vartheta}\left(l^d\right)^\vartheta}{\frac{\eta-1}{\eta}\left(1-\tau_w\right)\widetilde{w}^*\widetilde{\lambda}}$$

Or alternatively, after some algebra, we have the following equation on l^d:

$$\frac{1 - \beta\theta_w\widetilde{z}^{\,\eta(1+\vartheta)}\Pi^{\eta(1-\chi_w)(1+\vartheta)}\gamma^L}{1 - \beta\theta_w\widetilde{z}^{\,\eta-1}\Pi^{-(1-\chi_w)(1-\eta)}\gamma^L} = \frac{\psi\left(\frac{\widetilde{w}^*}{\widetilde{w}}\right)^{-\eta\vartheta}\left(l^d\right)^\vartheta}{\frac{\eta-1}{\eta}\left(1-\tau_w\right)\widetilde{w}^*}\left(\frac{\widetilde{z} - h}{\widetilde{z} - h\beta\gamma^L}\right)$$
$$* \left\{\frac{\left(1+\tau_c\right)}{\Lambda^c}\left(\frac{p^c}{p}\right)^{1-\varepsilon_c}\left[\begin{matrix}\left[\frac{\widetilde{A}}{\widetilde{z}}\Omega^\alpha\left(v^p\right)^{-1} - \Lambda^i\left(\frac{p^i}{p}\right)^{\varepsilon_i}\left(\frac{\gamma^L-(1-\delta)}{\widetilde{\mu}\widetilde{z}}\right)\Omega\right] l^d \\ -\phi\left(v^p\right)^{-1} - g\end{matrix}\right]\right\}$$

This is a non-linear equation that we solve for l^d with a root finder. Note that $\widetilde{w}^*$, $\widetilde{w}$, $\left(\frac{p^c}{p}\right)$, $\left(\frac{p^i}{p}\right)$, and v^p are functions of parameters of the model, Π and Π^M are parameters to be estimated, and Π^W and $\widetilde{y}^W$ are exogenously given.

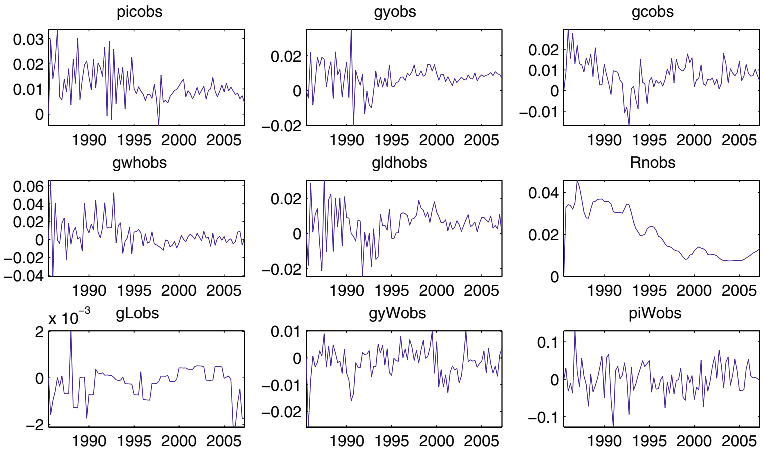

Fig. 1 Data series used in the estimation

5 Estimating the model

As motivated in the introduction, we will confront our model with the data using Bayesian methods. Formally, we stack all the parameters in the model in the vector $\Psi \in \Theta$ and we elicit a prior distribution for them, $p(\Psi)$. The model implies a likelihood $p(Y^T|\Psi)$ given some observed data, $Y^T = \{y^1, \ldots, y^T\}$. Then, we have the posterior distribution of Ψ:

$$p\left(\Psi|Y^T\right) \propto p\left(Y^T|\Psi\right) p(\Psi),$$

where "$\propto$" indicates proportionality. The posterior summarizes the uncertainty regarding the parameter values and it can be used for point estimation once we have specified a loss function. For example, under a quadratic loss, our point estimates will be the mean of the posterior. Since the posterior is also difficult to characterize, we generate draws from it using a Metropolis-Hastings algorithm. We use the resulting empirical distribution to obtain point estimates, standard deviations, etc.

5.1 Data

We use time series for 9 variables to estimate MEDEA (see Fig. 1): real GDP growth (gyobs), real private consumption growth (gcobs), total employment in hours growth (gldhobs), real compensation per hour growth (total compensation/total hours/GDP deflator) (gwhobs), consumption deflator inflation (picobs), total population over 16 years of age growth (gLobs), euro area nominal interest rate (Rnobs), inflation of Spanish competitors prices (piWobs), and Spain's foreign demand growth (gyWobs). All the time series are taken from national accounts published by INE, except for the foreign-sector variables and the nominal interest rate, which come from the database developed for the REMS model (BDREMS).[8] We have excluded real investment (or the investment deflator) from the baseline estimation since it has grown in the last decade at an unprecedented pace, mainly due to the construction sector, which we believe would be difficult to explain using a model without housing and financial frictions (see Andrés and Arce 2008, for a theoretical model tackling this issue). Nevertheless, we check the robustness of this baseline estimation by adding real investment, investment deflator inflation, or public consumption.

The choice of the sample period over which to estimate the parameters of the model is controversial. There have been significant changes in the Spanish economy since the mid-nineties, mainly related to the set-up of the euro area but also to the increase in labor force participation and the large immigration flows. Some papers in the literature have thus decided to use only the period since the euro area was conceived, that is, from 1997 onward. In this way, these papers avoid having to deal with structural breaks in the sample and with the change in the implementation of monetary policy. However, since it is likely that the impact of the creation of the monetary union lasted for several years after 1997, it is not certain that the structural break problem will disappear. Moreover, this will not avoid the other structural changes such as immigration. The main drawback of this approach is, however, that the sample becomes fairly short, probably requiring tighter priors in the estimation.

Instead, in this paper we have decided to proceed in three stages. First, we use data for the full sample 1986–2007, as if Spain had an independent monetary policy during this period. This allows us to set fairly loose priors and let the data speak up as much as possible. We had to drop the data before 1986 because the changes in the structure of the Spanish economy in the early eighties were too substantial. Second, we check the stability of the point estimates by estimating the model separately for two subsamples but maintaining the assumption of an independent monetary authority: one for the period before the euro area was set up (1986–1996) and the other from 1997 onward. Third, the model is re-estimated over the most recent subsample assuming Spain has no independent monetary policy; that is, the interest rate is exogenous and the exchange rate is constant.

The model incorporates economic growth. Therefore, to take the model to the data, it is not necessary to transform our observables. Instead, we add transition equations to our state space representation relating model and empirical variables. These transition equations account for the following differences. First, in the log-linearized version of the model, all variables are expressed as log deviations with respect to their steady state value. Second, all variables in the model are expressed in per capita terms dividing by the population (L_t). Third, some real variables in the model are made stationary by dividing by z_t. Therefore, in the case of real per capita variables, like real GDP per capita, the growth rate of the empirical variable $\left(y_t^{d,O}\right)$ is equal to the growth rate of

[8] The BDREMS database is described in Boscá et al. (2008) and is available for open download at: http://www.sgpg.pap.meh.es/SGPG/Cln_Principal/Presupuestos/Documentacion/BasedatosmodeloREMS.htm.

the model variable (in per capita terms) $\left(y_t^d\right)$ plus population growth (γ_t^L):

$$\triangle \log y_t^{d,O} = \Delta \log y_t + \gamma_t^L$$

But the variable included in the stationary log-linearized version of the model is $\widehat{\widetilde{y}}_t$, which has been made stationary by dividing by technology and expressed as a deviation with respect to the steady state ($\widehat{\widetilde{y}}_t = \log \frac{y_t}{z_t} - \log \frac{y}{z}$). The same point applies to the growth rate of technology ($\widehat{\widetilde{z}}_t = \log \frac{z_t}{z_{t-1}} - \log \widetilde{z}$). Considering all of this, the transition equation for real per capita variables, such as output, is

$$\triangle \log y_t^{d,O} = \Delta \log y_t + \gamma_t^L = \Delta\widehat{\widetilde{y}}_t + \widehat{\widetilde{z}}_t + \widehat{\gamma}_t^L + \log \widetilde{z} + \gamma^L$$

An exception is employment in hours, which is stationary in per capita terms in the model ($\widehat{l}_t^d = \log l_t^d - \log l$), so we only have to add population growth:

$$\triangle \log l_t^{d,O} = \Delta \log l_t^d + \gamma_t^L = \Delta\widehat{l}_t^d + \widehat{\gamma}_t^L + \gamma^L$$

In the case of nominal variables, such as inflation and interest rates, model and empirical variables are the same, so we express them as deviation with respect to the steady state:

$$\log \Pi_t^{c,O} = \widehat{\Pi}_t^c + \log \Pi^c$$
$$\log R_t^O = \widehat{R}_t + \log R$$

5.2 Calibration and prior distributions

The model has 59 parameters, 12 of which are calibrated and the remaining 47 estimated. The calibration is shown in Table 1. Several theoretical and empirical reasons explain why one may not want to estimate all the parameters of the model. First, some parameters are difficult to identify with the model structure, such as the discount factor β. This parameter is set to be consistent with an annualized nominal interest rate of 2.5% and an inflation objective of 2%, so that the steady state annual nominal interest rate $\left(R = R^W = \frac{\Pi\widetilde{z}}{\beta}\right)$ is 4.5%. Second, other parameters such as the depreciation rate δ or the labor share α, are better estimated using micro data, while others would require adding more data series to the estimation, such as the three tax rates (τ_c, τ_w, τ_k). Third, there are parameters that are irrelevant for the model solution, such as the coefficient of money demand in the utility function, υ. Finally, the parameters of the Taylor rule are set equal to the standard estimation results for the euro area (Christoffel et al. 2008). The two fiscal parameters have not yet been included in the estimation and thus are set to their empirical values.

The first vertical panel of Table 2 summarizes our assumptions regarding prior distributions for the estimated parameters. Our approach has been to set priors as loose as possible. Therefore, for most parameters, we have chosen as our priors uniform

Table 1 Calibrated parameters

	Value	Reason		Value	Reason
β	0.99	Difficult to identify	γ_y	0.125	Taylor rule U.E.
υ	0.1	Irrelevant	τ_c	0.113	Data on taxes
δ	0.0175	Micro data	τ_w	0.341	Data on taxes
α	0.3621	Micro data	τ_k	0.219	Data on taxes
γ_R	0.8	Taylor rule U.E.	g	0.17	–
γ_Π	1.7	Taylor rule U.E.	b	0.40	–

distributions with a range covering all the theoretically feasible values. In particular, we have set a range of (0, 1) , for the labor supply coefficient, price and wage Calvo and indexation parameters, adjustment cost parameters, autoregressive coefficients (except the labor supply shock for which we have chosen (0, 0.9)), and the standard deviations of shocks. In the case of the elasticity of substitution parameters, we have set a range of (6, 10). In the case of the habits coefficient and the home bias coefficients, we have imposed stronger priors by assuming beta distributions, because the data moved them toward unrealistic parameter values. Finally, we also set beta distributions for the parameters determining growth in the model, to help identify them.[9]

5.3 Estimation results

The right-hand panel of Table 2 presents the estimation results for the full sample (1986Q1–2007Q4). Table 3 presents the results for the two subsamples considered (1986Q1–1996Q4 and 1997Q1–2007Q4) as well as the model without independent monetary policy. The columns of both tables report the mean, mode, median, standard deviation, and the 5th and 95th percentiles of the posterior distribution of the parameters. All of them are computed using a Metropolis-Hastings algorithm in `Dynare`, based on a Markov chain with 5 million draws, with the first 2.5 million draws being discarded as burn-in draws, and the appropriate acceptance ratio (Roberts et al. 1997).

We start by studying the goodness of the estimation. We have implemented standard convergence diagnostic tests, which show that the draws of the posterior sampling converged for all of the estimated parameters (see Mengersen et al. 1999). Details are available upon request. Moreover, the smoothed estimates of the innovation component of structural shocks (see Fig. 2) are clearly stationary. Note that the variance of the shocks seems to have fallen in the second part of the sample, with the exception of the population growth shock, which has increased, in line with the rise in population growth in Spain over the last decade.

Another way to check the quality of the estimation is by comparing the prior and posterior distributions of each parameter, as shown by Figs. 3, 4, 5, 6, 7. In general,

[9] Unfortunately, for some parameters (in particular, those related to the open sector of the economy), uniform priors seem to bring insufficient information for some empirical exercises. We need a more thorough assessment of the robustness of the empirical estimates with respect to priors.

Table 2 Prior and posterior distribution of structural parameters [full sample (86q1–07q4)]

Parameter		Prior distribution			Posterior distribution					
		Type	Mean	s.d.	Mean	Mode	s.d.	Median	5%	95%
Preferences										
Habits	h	Beta	0.70	0.10	0.847	0.795	0.010	0.847	0.831	0.864
Labour supply coef.	ψ	Uniform	0.50	0.29	6.772	6.744	0.059	6.792	6.673	6.847
Frisch elasticity	ϑ	Uniform	1.55	0.84	1.835	1.970	0.110	1.840	1.648	2.003
Adjustment costs										
Investment	κ	Uniform	25.0	14	28.954	28.995	0.042	28.960	28.887	29.016
Fixed cost of production	Φ	Uniform	0.50	0.29	0.127	0.051	0.058	0.127	0.034	0.223
Capital utilization	$\Phi 2$	Uniform	1.00	0.58	0.248	0.461	0.017	0.246	0.219	0.273
Risk premium	ΓbW	Uniform	0.50	0.29	0.832	0.859	0.064	0.827	0.742	0.959
Import consumption	Γc	Uniform	0.50	0.29	0.449	0.259	0.153	0.440	0.211	0.692
Import investment	Γi	Uniform	0.50	0.29	0.618	0.647	0.046	0.629	0.538	0.691
Elasticities of substitution										
Domestic goods	ε	Uniform	8.00	1.15	8.577	8.480	0.132	8.575	8.396	8.800
Import goods	εM	Uniform	8.00	1.15	8.787	8.727	0.064	8.778	8.690	8.892
Export goods	εX	Uniform	8.00	1.15	9.491	9.437	0.101	9.498	9.321	9.622
World goods	εW	Uniform	8.00	1.15	6.791	7.300	0.064	6.785	6.689	6.900
Consumption goods	εc	Uniform	8.00	1.15	7.512	7.671	0.044	7.517	7.441	7.585
Investment goods	εi	Uniform	8.00	1.15	7.851	8.056	0.207	7.867	7.595	8.108
Labour types	η	Uniform	8.00	1.15	7.758	7.706	0.057	7.754	7.670	7.865
Price and wage and price setting										
Calvo dom. prices	θp	Uniform	0.50	0.29	0.898	0.904	0.001	0.898	0.897	0.900
Calvo exp. Prices	θX	Uniform	0.50	0.29	0.330	0.327	0.032	0.335	0.272	0.378
Calvo imp.	θM	Uniform	0.50	0.29	0.064	0.050	0.053	0.046	0.000	0.147
Calvo wages	θw	Uniform	0.50	0.29	0.457	0.235	0.025	0.457	0.417	0.501
Indexation dom. prices	χp	Uniform	0.50	0.29	0.004	0.003	0.004	0.003	0.000	0.008
Indexation imp. Prices	χM	Uniform	0.50	0.29	0.064	0.287	0.047	0.055	0.000	0.148
Indexation exp. Prices	χX	Uniform	0.50	0.29	0.027	0.013	0.022	0.020	0.000	0.062
Indexation wages	χw	Uniform	0.50	0.29	0.961	0.967	0.031	0.969	0.919	1.000
Fiscal policy										
Fiscal rule coeff.	T1	Uniform	0.05	0.03	0.051	0.051	0.028	0.047	0.015	0.100
Home bias										
In consumption	nc	Beta	0.70	0.10	0.962	0.813	0.0127	0.962	0.943	0.982
In investment	ni	Beta	0.50	0.20	0.072	0.100	0.0129	0.071	0.053	0.094
Growth rates										
Inv. especific tech.	$\Lambda\mu$	Beta	0.004	0.001	0.004	0.003	0.001	0.004	0.002	0.006
General technology	ΛA	Beta	0.004	0.001	0.004	0.002	0.001	0.004	0.002	0.006
Population	γL	Beta	0.004	0.001	0.003	0.003	0.001	0.003	0.002	0.004
Autorregressive coefficients of shocks										
Intertemp. preferences	ρd	Uniform	0.50	0.29	0.978	0.900	0.006	0.978	0.969	0.989
Hours preferences	$\rho\psi$	Uniform	0.45	0.26	0.895	0.800	0.005	0.897	0.889	0.900
Public consumption	ρg	Uniform	0.50	0.29	0.979	0.978	0.011	0.982	0.967	0.990
Foreign prices	$\rho\pi$w	Uniform	0.50	0.29	0.361	0.366	0.040	0.363	0.285	0.422
Foreign demand	ρyW	Uniform	0.50	0.29	0.033	0.459	0.029	0.025	0.000	0.070
World interest rate	ρRW	Uniform	0.50	0.29	0.876	0.963	0.042	0.869	0.802	0.957

Table 2 continued

Parameter		Prior distribution			Posterior distribution					
		Type	Mean	s.d.	Mean	Mode	s.d.	Median	5%	95%
Standard deviations of shocks										
Inv. especific tech.	$\sigma\mu$	Uniform	0.50	0.29	0.403	0.300	0.037	0.400	0.347	0.463
General technology	σA	Uniform	0.50	0.29	0.009	0.012	0.001	0.009	0.008	0.010
Population	σL	Uniform	0.50	0.29	0.001	0.001	0.0001	0.001	0.001	0.001
Intertemp. preferences	σd	Uniform	0.50	0.29	0.174	0.933	0.046	0.161	0.109	0.250
Hours preferences	$\sigma\psi$	Uniform	0.50	0.29	0.266	0.137	0.028	0.262	0.223	0.313
Public consumption	σg	Uniform	0.50	0.29	0.062	0.076	0.009	0.062	0.047	0.076
Interest rate	σR	Uniform	0.50	0.29	0.003	0.003	0.0002	0.003	0.003	0.004
Foreign prices	$\sigma\pi$w	Uniform	0.50	0.29	0.044	0.044	0.004	0.044	0.038	0.049
Foreign demand	σyW	Uniform	0.50	0.29	0.145	0.150	0.013	0.144	0.124	0.165
World interest rate	σRW	Uniform	0.50	0.29	0.005	0.001	0.003	0.005	0.000	0.009

Table 3 Prior and posterior distribution of structural parameters (subsamples)

Parameter		1986Q1–1996Q4			1997Q1–2007Q4			1997Q1–2007Q4 (ex MP)		
		Mean	5%	95%	Mean	5%	95%	Mean	5%	95%
Preferences										
Habits	h	0.861	0.836	0.886	0.827	0.796	0.863	0.886	0.851	0.911
Labour supply coef.	ψ	7.005	6.653	7.336	6.982	6.815	7.168	6.642	6.541	6.736
Frisch elasticity	ϑ	1.909	1.675	2.093	1.775	1.564	1.940	2.274	2.072	2.437
Adjustment costs										
Investment	κ	29.722	29.477	29.997	29.052	28.946	29.138	28.964	28.873	29.067
Fixed cost of production	Φ	0.760	0.626	0.894	0.209	0.097	0.338	0.024	0.000	0.058
Capital utilization	Φ2	0.244	0.208	0.278	0.394	0.335	0.454	0.687	0.637	0.749
Risk premium	ΓbW	0.827	0.689	0.987	0.865	0.692	1.000	0.000	0.000	0.001
Import consumption	Γc	0.036	0.000	0.081	0.501	0.172	0.860	0.436	0.249	0.533
Import investment	Γi	0.503	0.293	0.635	0.758	0.644	0.874	0.889	0.828	0.972
Elasticities of substitution										
Domestic goods	ε	7.494	6.904	7.901	8.618	8.353	8.809	8.947	8.771	9.088
Import goods	εM	8.690	8.446	8.946	8.741	8.618	8.857	8.356	8.291	8.426
Export goods	εX	9.553	9.374	9.727	9.591	9.440	9.774	9.177	9.126	9.216
World goods	εW	6.676	6.354	6.950	7.067	6.886	7.183	6.649	6.597	6.691
Consumption goods	εc	8.736	8.425	8.945	7.874	7.765	8.046	7.958	7.871	8.038
Investment goods	εi	7.266	7.130	7.435	8.371	8.199	8.499	8.282	8.210	8.327
labour types	η	7.784	7.565	8.101	7.860	7.768	7.960	8.032	7.982	8.097
Price and wage and price setting										
Calvo dom. prices	θp	0.901	0.898	0.905	0.895	0.893	0.897	0.529	0.490	0.573
Calvo exp. Prices	θX	0.241	0.130	0.365	0.148	0.053	0.259	0.592	0.486	0.673
Calvo imp. Prices	θM	0.077	0.000	0.150	0.297	0.125	0.455	0.285	0.254	0.339

Table 3 continued

Parameter		1986Q1–1996Q4			1997Q1–2007Q4			1997Q1–2007Q4 (ex MP)		
		Mean	5%	95%	Mean	5%	95%	Mean	5%	95%
Calvo wages	θw	0.418	0.328	0.538	0.527	0.486	0.573	0.030	0.000	0.075
Indexation dom. prices	χp	0.006	0.000	0.014	0.007	0.000	0.016	0.031	0.000	0.081
Indexation imp. Prices	χM	0.344	0.232	0.433	0.201	0.089	0.336	0.028	0.000	0.052
Indexation exp. Prices	χX	0.096	0.000	0.196	0.058	0.000	0.126	0.180	0.095	0.250
Indexation wages	χw	0.767	0.592	0.953	0.891	0.741	1.000	0.954	0.921	1.000
Fiscal policy										
Fiscal rule coeff.	T1	0.043	0.000	0.084	0.052	0.009	0.100	0.051	0.017	0.091
Home bias										
In consumption	nc	0.940	0.905	0.972	0.884	0.842	0.919	0.860	0.809	0.903
In investment	ni	0.109	0.073	0.145	0.148	0.112	0.197	0.088	0.032	0.140
Growth rates										
Inv. especific tech.	$\Lambda\mu$	0.004	0.002	0.006	0.004	0.002	0.006	0.004	0.002	0.006
General technology	ΛA	0.004	0.002	0.006	0.004	0.002	0.006	0.004	0.002	0.006
Population	γL	0.003	0.002	0.004	0.003	0.002	0.004	0.003	0.002	0.004
Autorregressive coefficients of shocks										
Intertemp. preferences	ρd	0.954	0.934	0.973	0.973	0.958	0.990	0.883	0.816	0.990
Hours preferences	$\rho\psi$	0.875	0.847	0.900	0.888	0.873	0.900	0.861	0.831	0.891
Public consumption	ρg	0.623	0.523	0.707	0.899	0.817	0.990	0.974	0.961	0.990
Foreign prices	$\rho\pi$w	0.278	0.175	0.398	0.170	0.064	0.306	0.030	0.000	0.051
Foreign demand	ρyW	0.129	0.015	0.239	0.059	0.000	0.125	0.025	0.000	0.059
World interest rate	ρRW	0.900	0.823	0.990	0.739	0.548	0.990	0.954	0.925	0.990
Standard deviations of shocks										
Inv. especific tech.	$\sigma\mu$	0.662	0.575	0.764	0.185	0.147	0.222	0.179	0.100	0.268
General technology	σA	0.011	0.009	0.013	0.003	0.003	0.004	0.012	0.009	0.014
Population	σL	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.002
Intertemp. preferences	σd	0.109	0.079	0.138	0.093	0.052	0.137	0.101	0.054	0.222
Hours preferences	$\sigma\psi$	0.326	0.208	0.505	0.131	0.097	0.166	0.072	0.054	0.088
Public consumption	σg	0.057	0.006	0.092	0.037	0.029	0.044	0.598	0.558	0.636
Interest rate	σR	0.004	0.003	0.005	0.002	0.002	0.002	0.143	0.074	0.240
Foreign prices	$\sigma\pi$w	0.049	0.040	0.057	0.037	0.031	0.044	0.037	0.031	0.044
Foreign demand	σyW	0.221	0.176	0.259	0.081	0.065	0.097	0.160	0.116	0.196
World interest rate	σRW	0.010	0.000	0.016	0.001	0.000	0.002	0.001	0.001	0.001

the results show that the data are very informative about the posterior distribution of the parameters. An exception is the elasticity of substitution of investment goods (ε_i), with a twin-peaked posterior distribution, although both peaks imply fairly similar estimates. More relevant is the fact that the data contain little information on the posterior distribution of the steady state growth rate of technology and population (the posterior lies on top of the beta prior), the coefficient of the fiscal rule (T_1), and the adjustment cost parameter of imported consumption (Γ^c). In the case of the growth rates, we have set fairly tight priors centered on the sample means of observed growth

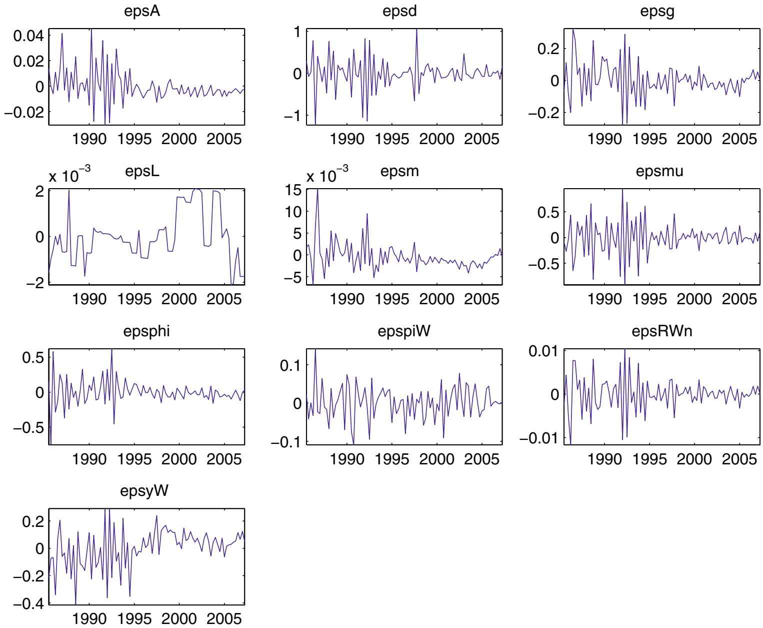

Fig. 2 Smoothed estimates of innovations

rates of population and output per capita. Given this result, one should be cautious when making inferences about the relative importance in the observed data of the drifts in technological growth (neutral and investment specific) included in the model.

Moving to the point estimates, a number of findings are worth noting. First, the estimates of the utility parameters are quite standard. The data strongly support a high estimate of habit persistence, which is not surprising given the persistence of observed consumption, while fixed costs of production are very close to zero. The Frisch elasticity posterior mean of 1.83 is in line with most estimations for other countries and rather plausible once we think about both the intensive and the extensive margin of labor supply.

Second, the estimated elasticities of substitution between different types of intermediate goods produced are relatively similar, implying a mark-up between 13.5% in the case of domestic goods and 12% in the case of export goods, while the wage mark-up is somewhat higher, at around 15%. This is not surprising given the rigidities of the Spanish labor market, where wages are mainly set by insiders with long-term contracts and thus high bargaining power. The estimates are also similar for the demand elasticity of substitution between imported and domestically produced goods. In contrast, the adjustment cost parameter associated with changing the import content varies substantially across both types of goods. We estimate that the adjustment cost is much higher for investment. Moreover, the data are very informative about this. This is evidence of the technological constraints that the Spanish economy still faces in areas such as advanced capital goods or IT, which require a large import content.

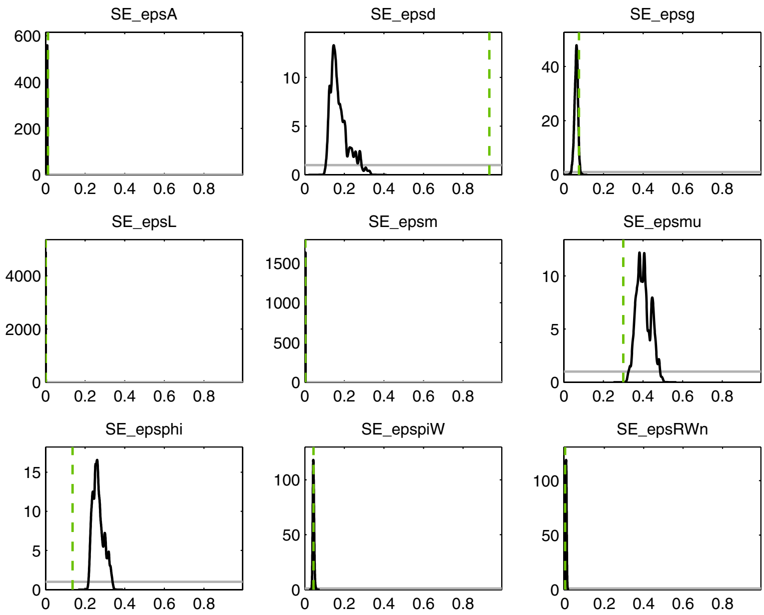

Fig. 3 Prior and posterior distributions of the structural parameters

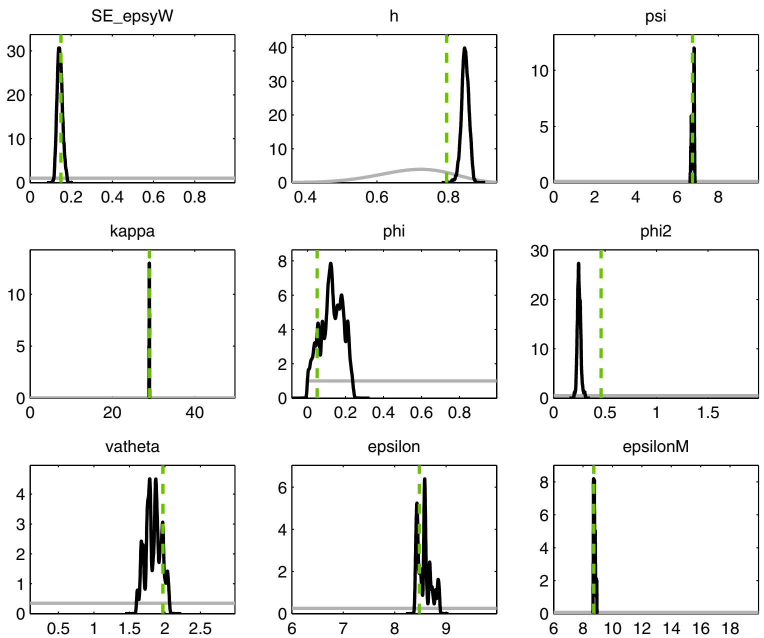

Fig. 4 Prior and posterior distributions of the structural parameters (continued)

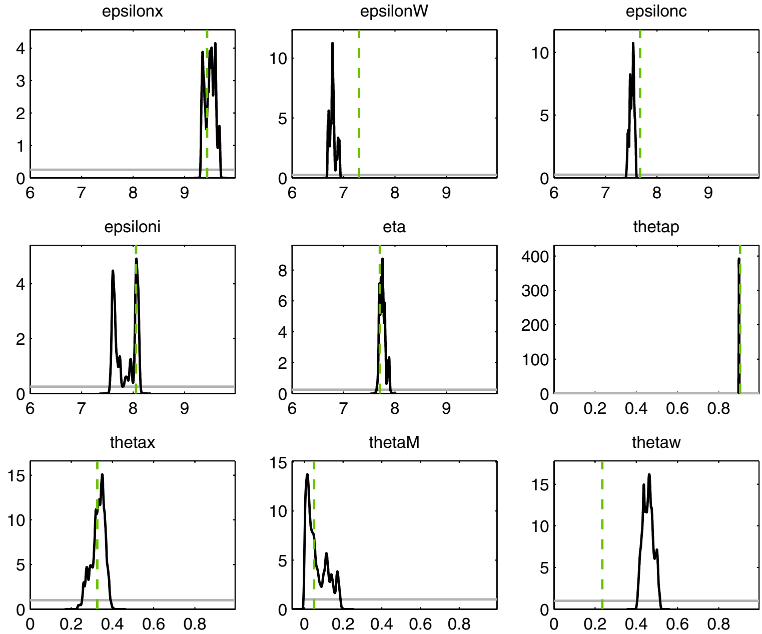

Fig. 5 Prior and posterior distributions of the structural parameters (continued)

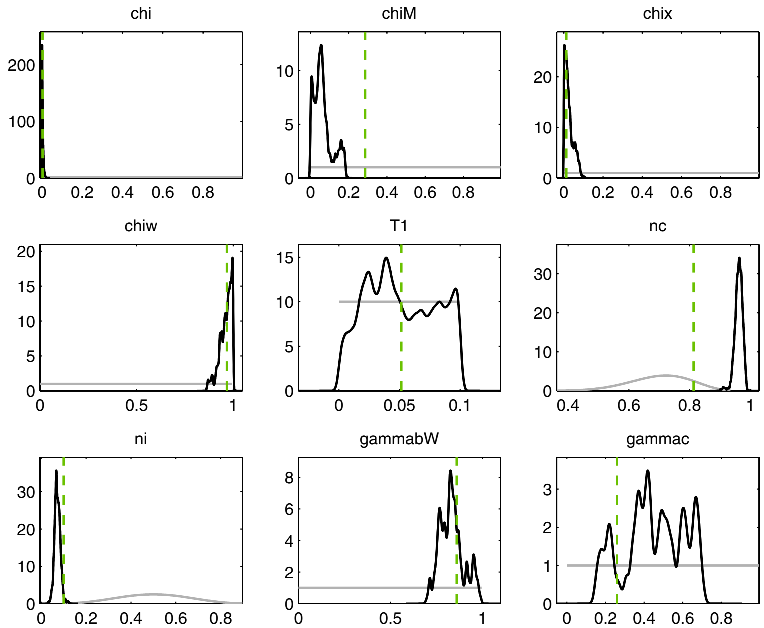

Fig. 6 Prior and posterior distributions of the structural parameters (continued)

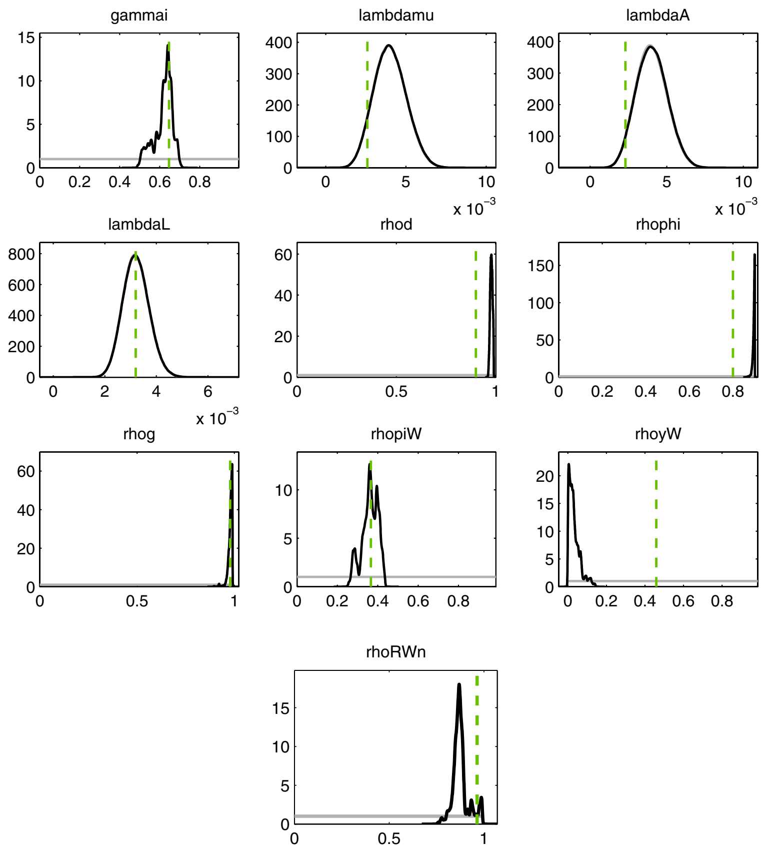

Fig. 7 Prior and posterior distributions of the structural parameters (continued)

Third, on the nominal side, we find important differences across sectors of the economy. The estimate of the Calvo parameter is very high for intermediate domestic goods, although not different from values generally obtained for the euro area (Smets and Wouters 2003), but quite low for the import and export intermediate goods. The same is true for wages. In contrast, indexation is very close to one for wages,which seems the direct consequence of the (ex-ante) indexation mechanism inherent in Spanish wage agreements. However, indexation is non-existent in prices of domestically produced goods, while indexation is a bit higher in the import and export sectors. These differences across sectors of production and the labor market are strongly supported in the data.

Fourth, the estimation confirms the evidence in input-output tables that in Spain there is a much stronger home bias in consumption than investment (remember our earlier comment about the technological constraints faced by our economy). However, the point estimates are too large given the micro evidence. Nevertheless, the data are very informative about this result, since imposing tighter priors does not change the results, even when data on real investment or on the investment deflator are used (see Table 5).

Finally, the point estimates for the autoregressive parameters of shock processes show that domestic shocks are very persistent, especially those related to demand, public consumption, and preferences. This may suggest that the model has some difficulty endogenously generating the level of persistence present in the data and, thus, it opts for these exogenous shocks to be highly persistent. Alternatively, one could argue that this is the consequence of a structural break in the data, but this hypothesis is rejected when the estimation is performed recursively over the final sample (see Table 4). In comparison, the foreign demand and inflation shocks have much lower persistence.

5.4 Subsample analysis and robustness

Comparing the results across the two subsamples (see Table 3), we observe that the point estimates are rather similar for most parameters. This suggests that our fears about pervasive structural breaks over the most recent years may have been exaggerated. The recursive estimation in Table 4 confirms this impression. The exception is the standard deviation of shocks, which have all fallen markedly, except for the population shock, which has increased. This was already noticeable in the graph of the innovations for the whole sample period. This seems to be another manifestation of the "great moderation" that the western economies experienced from 1984 to 2007 (McConnell and Pérez-Quirós 2000; Stock and Watson 2003).

In addition, the estimate of the adjustment cost of the import content of consumption and investment goods is larger for the most recent sample. Finally, the estimated elasticities of substitution suggest a more competitive economy since 1997, with slightly lower steady-state mark-ups, especially for domestic goods (13 vs. 15%), and more flexible prices, while wages have become stickier and more persistent.

When the model is estimated assuming an exogenous monetary policy (see third panel of Table 3), the estimates of open economy and monetary policy parameters change markedly. In particular, the mark-up on imports, exports, and world goods rises and the premium on foreign interest rate falls, while domestic prices become more flexible and competitive. Moreover, since the interest rate does not react to Spanish economic conditions, the standard deviation of the Taylor rule shock rises greatly. This suggests that more informative priors for these parameters may help the model to deliver a more consistent performance.

Finally, several robustness checks have been performed. First, a recursive estimation (adding two years every time) over the most recent period (see Table 4) confirms that there are very few signs of structural instability in our sample, since most parameters change little and gradually, with the exception of the fixed cost of production and the indexation of import prices. Second, adding real investment to the estimation has a significant impact on the point estimates (see the second panel of Table 5), reducing most steady-state mark-ups (except for domestic prices, which increase) and increasing price indexation parameters. This is not surprising given that investment has grown very quickly during the last economic cycle, especially due to housing investment.

Table 4 Prior and posterior distribution of structural parameters (recursive estimation)

Parameter		Prior distribution			Mean of posterior distribution						
		Type	Mean	s.d.	86–96	86–98	86–00	86–02	86–04	86–06	86–07
Preferences											
Habits	h	Beta	0.70	0.10	0.861	0.848	0.850	0.854	0.845	0.847	0.847
Labour supply coef.	ψ	Uniform	0.50	0.29	7.005	6.684	6.671	6.603	6.827	6.859	6.772
Frisch elasticity	ϑ	Uniform	1.55	0.84	1.909	2.040	1.927	2.017	1.826	1.857	1.835
Adjustment costs											
Investment	K	Uniform	25.0	14	29.722	28.949	28.984	28.950	28.857	28.911	28.954
Fixed cost of production	Φ	Uniform	0.50	0.29	0.760	0.039	0.039	0.048	0.120	0.096	0.127
Capital utilization	$\Phi 2$	Uniform	1.00	0.58	0.244	0.238	0.243	0.243	0.244	0.290	0.248
Risk premium	ΓbW	Uniform	0.50	0.29	0.827	0.871	0.962	0.894	0.974	0.825	0.832
Import consumption	Γc	Uniform	0.50	0.29	0.036	0.415	0.330	0.517	0.133	0.259	0.449
Import investment	Γi	Uniform	0.50	0.29	0.503	0.618	0.794	0.867	0.654	0.621	0.618
Elasticities of substitution											
Domestic goods	ε	Uniform	8.00	1.15	7.494	8.302	8.527	8.627	8.435	8.505	8.577
Import goods	εM	Uniform	8.00	1.15	8.690	8.696	8.674	8.751	8.518	8.592	8.787
Export goods	εX	Uniform	8.00	1.15	9.553	9.339	9.257	9.293	9.386	9.394	9.491
World goods	εW	Uniform	8.00	1.15	6.676	7.107	7.390	7.318	7.026	7.094	6.791
Consumption goods	εc	Uniform	8.00	1.15	8.736	7.668	7.818	7.660	7.722	7.782	7.512
Investment goods	εi	Uniform	8.00	1.15	7.266	7.799	8.001	7.847	8.215	8.254	7.851
Labour types	η	Uniform	8.00	1.15	7.784	7.800	7.820	7.767	7.688	7.563	7.758
Price and wage and price setting											
Calvo dom. prices	θp	Uniform	0.50	0.29	0.901	0.900	0.900	0.899	0.900	0.900	0.898
Calvo exp. Prices	θX	Uniform	0.50	0.29	0.241	0.305	0.330	0.289	0.367	0.320	0.330
Calvo imp. Prices	θM	Uniform	0.50	0.29	0.077	0.040	0.186	0.135	0.156	0.129	0.064
Calvo wages	θw	Uniform	0.50	0.29	0.418	0.404	0.365	0.415	0.431	0.410	0.457
Indexation dom. prices	**X**p	Uniform	0.50	0.29	0.006	0.004	0.005	0.005	0.003	0.003	0.004
Indexation imp. Prices	**X**M	Uniform	0.50	0.29	0.344	0.278	0.063	0.167	0.348	0.187	0.064
Indexation exp. Prices	**X**X	Uniform	0.50	0.29	0.096	0.036	0.039	0.020	0.035	0.041	0.027
Indexation wages	**X**w	Uniform	0.50	0.29	0.767	0.927	0.905	0.972	0.932	0.965	0.961
Fiscal policy											
Fiscal rule coeff.	T1	Uniform	0.05	0.03	0.043	0.048	0.060	0.050	0.046	0.056	0.051
Home bias											
In consumption	nc	Beta	0.70	0.10	0.940	0.975	0.975	0.965	0.971	0.970	0.962
In investment	ni	Beta	0.50	0.20	0.109	0.065	0.054	0.069	0.066	0.053	0.072
Growth rates											
Inv. especific tech.	$\Lambda\mu$	Beta	0.004	0.001	0.004	0.004	0.004	0.004	0.004	0.004	0.004
General technology	ΛA	Beta	0.004	0.001	0.004	0.004	0.004	0.004	0.004	0.004	0.004
Population	yL	Beta	0.004	0.001	0.003	0.003	0.003	0.003	0.003	0.003	0.003
Autorregressive coefficients of shocks											
Intertemp. preferences	ρd	Uniform	0.50	0.29	0.954	0.989	0.989	0.987	0.989	0.986	0.978
Hours preferences	$\rho\psi$	Uniform	0.45	0.26	0.875	0.896	0.895	0.893	0.896	0.897	0.895

Table 4 continued

Parameter		Prior distribution			Mean of posterior distribution						
		Type	Mean	s.d.	86–96	86–98	86–00	86–02	86–04	86–06	86–07
Public consumption	ρg	Uniform	0.50	0.29	0.623	0.920	0.973	0.964	0.938	0.976	0.979
Foreign prices	ρTTw	Uniform	0.50	0.29	0.278	0.341	0.380	0.324	0.422	0.366	0.361
Foreign demand	ρyW	Uniform	0.50	0.29	0.129	0.111	0.035	0.038	0.094	0.043	0.033
World interest rate	ρRW	Uniform	0.50	0.29	0.900	0.869	0.963	0.954	0.864	0.925	0.876
Standard deviations of shocks											
Inv. especific tech.	$\sigma\mu$	Uniform	0.50	0.29	0.662	0.415	0.428	0.446	0.398	0.376	0.403
General technology	σA	Uniform	0.50	0.29	0.011	0.009	0.009	0.010	0.009	0.009	0.009
Population	σL	Uniform	0.50	0.29	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Intertemp. preferences	σd	Uniform	0.50	0.29	0.109	0.433	0.503	0.376	0.355	0.343	0.174
Hours preferences	$\sigma\psi$	Uniform	0.50	0.29	0.326	0.246	0.200	0.285	0.239	0.206	0.266
Public consumption	σg	Uniform	0.50	0.29	0.057	0.064	0.067	0.069	0.061	0.063	0.062
Interest rate	σR	Uniform	0.50	0.29	0.004	0.003	0.003	0.003	0.003	0.003	0.003
Foreign prices	σTTw	Uniform	0.50	0.29	0.049	0.046	0.047	0.046	0.048	0.044	0.044
Foreign demand	σyW	Uniform	0.50	0.29	0.221	0.164	0.160	0.164	0.168	0.144	0.145
World interest rate	σRW	Uniform	0.50	0.29	0.010	0.006	0.006	0.007	0.006	0.005	0.005

However, our model is not able (and it was not designed) to account for the boom in housing investment. Third, adding the investment deflator or public consumption affects the estimation results only marginally (see the last two panels of Table 5).

6 Applications

In this section, we consider a number of properties and applications of our model to illustrate the contributions that MEDEA can make to policy analysis. First, we briefly describe the basic properties of the model. In the interest of space, we offer only some concise information. Second, we show how the model can help in understanding the evolution of the Spanish economy over the last several decades by interpreting historical developments through the lens of equilibrium theory. Many of the answers that MEDEA will give us are not surprising and either have been suggested before by the literature or fit into our economic intuition (although it is comforting to have a quantitative corroboration), but others will be relatively new. Third, we evaluate the impact and dynamics after a change in some relevant steady state parameters. Finally, we illustrate how MEDEA can be used to conduct alternative scenarios for observed variables. This exercise is particularly important for the assessment of policy interventions by the government and the monetary authority.

6.1 Model properties

Table 6 reports the steady state ratios implied by our point estimates. The ratios are comparable to the ones observed in the data, and in the case of ratios for which the

Table 5 Prior and posterior distribution of structural parameters (adding observables)

Parameter		Baseline			Add real investment			Add investment deflator			Add public consumption		
		Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%
Preferences													
Habits	h	0.847	0.831	0.864	0.926	0.912	0.940	0.857	0.826	0.891	0.820	0.806	0.831
Labour supply coef.	ψ	6.772	6.673	6.847	6.747	6.561	6.876	6.753	6.577	6.856	6.724	6.695	6.751
Frisch elasticity	ϑ	1.835	1.648	2.003	2.571	2.294	2.755	1.799	1.663	1.958	1.928	1.905	1.950
Adjustment costs													
Investment	K	28.954	28.887	29.016	28.780	28.565	28.989	28.817	28.762	28.893	28.992	28.977	29.006
Fixed cost of production	Φ	0.127	0.034	0.223	0.220	0.026	0.346	0.088	0.009	0.161	0.016	0.000	0.036
Capital utilization	$\Phi 2$	0.248	0.219	0.273	0.344	0.285	0.406	0.233	0.207	0.260	0.471	0.460	0.485
Risk premium	ΓbW	0.832	0.742	0.959	0.763	0.572	1.000	0.249	0.010	0.489	0.875	0.867	0.884
Import consumption	Γc	0.449	0.211	0.692	0.482	0.321	0.638	0.012	0.000	0.026	0.257	0.245	0.264
Import investment	Γi	0.618	0.538	0.691	0.518	0.342	0.726	0.782	0.625	1.000	0.695	0.676	0.715
Elasticities of substitution													
Domestic goods	ε	8.577	8.396	8.800	7.771	7.668	7.888	8.686	8.538	8.802	8.488	8.479	8.497
Import goods	εM	8.787	8.690	8.892	9.207	9.134	9.315	8.547	8.463	8.639	8.774	8.763	8.784
Export goods	εX	9.491	9.321	9.622	9.716	9.404	10.000	9.466	9.291	9.633	9.413	9.395	9.437
World goods	εW	6.791	6.689	6.900	7.572	7.469	7.669	6.174	6.000	6.422	7.264	7.253	7.278
Consumption goods	εc	7.512	7.441	7.585	7.776	7.658	7.891	7.760	7.709	7.818	7.679	7.668	7.691
Investment goods	εi	7.851	7.595	8.108	8.740	8.504	9.065	7.795	7.526	7.975	8.057	8.047	8.065
Labour types	η	7.758	7.670	7.865	7.517	7.415	7.615	7.673	7.419	7.883	7.692	7.681	7.703
Price and wage and price setting													
Calvo dom. prices	θp	0.898	0.897	0.900	0.878	0.869	0.887	0.899	0.897	0.900	0.902	0.901	0.903
Calvo exp. Prices	θX	0.330	0.272	0.378	0.081	0.000	0.147	0.552	0.477	0.638	0.342	0.328	0.355
Calvo imp. Prices	θM	0.064	0.000	0.147	0.543	0.203	0.806	0.042	0.000	0.083	0.083	0.048	0.102

Table 5 continued

Parameter		Baseline			Add real investment			Add investment deflator			Add public consumption		
		Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%
Calvo wages	θw	0.457	0.417	0.501	0.355	0.271	0.445	0.441	0.375	0.516	0.304	0.296	0.313
Indexation dom. prices	χp	0.004	0.000	0.008	0.005	0.000	0.011	0.004	0.000	0.010	0.002	0.000	0.004
Indexation imp. Prices	χM	0.064	0.000	0.148	0.584	0.445	0.683	0.135	0.000	0.213	0.325	0.314	0.336
Indexation exp. Prices	χX	0.027	0.000	0.062	0.295	0.189	0.373	0.061	0.000	0.157	0.006	0.000	0.012
Indexation wages	χw	0.961	0.919	1.000	0.973	0.941	1.000	0.944	0.884	1.000	0.988	0.980	0.998
Fiscal policy													
Fiscal rule coeff.	T1	0.051	0.015	0.100	0.053	0.007	0.095	0.057	0.014	0.100	0.009	0.000	0.019
Home bias													
In consumption	nc	0.962	0.943	0.982	0.928	0.877	0.975	0.875	0.827	0.915	0.847	0.832	0.859
In investment	ni	0.072	0.053	0.094	0.036	0.011	0.057	0.017	0.006	0.029	0.010	0.004	0.020
Growth rates													
Inv. especific tech.	$\Lambda\mu$	0.004	0.002	0.006	0.004	0.002	0.006	0.004	0.002	0.006	0.003	0.001	0.004
General technology	ΛA	0.004	0.002	0.006	0.004	0.002	0.006	0.004	0.002	0.006	0.002	0.001	0.004
Population	YL	0.003	0.002	0.004	0.003	0.002	0.004	0.003	0.002	0.004	0.003	0.002	0.004
Autorregressive coefficients of shocks													
Intertemp. preferences	ρd	0.978	0.969	0.989	0.984	0.977	0.990	0.972	0.953	0.989	0.990	0.989	0.990
Hours preferences	$\rho\psi$	0.895	0.889	0.900	0.810	0.722	0.900	0.894	0.885	0.900	0.899	0.898	0.900
Public consumption	ρg	0.979	0.967	0.990	0.979	0.970	0.987	0.736	0.692	0.783	0.849	0.813	0.873
Foreign prices	ρTTw	0.361	0.285	0.422	0.072	0.000	0.141	0.290	0.146	0.453	0.349	0.339	0.361
Foreign demand	ρyW	0.033	0.000	0.070	0.288	0.169	0.388	0.047	0.000	0.080	0.440	0.428	0.450
World interest rate	ρRW	0.876	0.802	0.957	0.918	0.869	0.967	0.988	0.986	0.990	0.987	0.984	0.990

Table 5 continued

Parameter		Baseline			Add real investment			Add investment deflator			Add public consumption		
		Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%
Standard deviations of shocks													
Inv. especific tech.	$\sigma\mu$	0.403	0.347	0.463	0.322	0.278	0.361	0.302	0.266	0.349	0.304	0.297	0.312
General technology	σA	0.009	0.008	0.010	0.008	0.007	0.010	0.008	0.007	0.009	0.011	0.009	0.012
Population	σL	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Intertemp. preferences	σd	0.174	0.109	0.250	0.230	0.151	0.304	0.161	0.084	0.246	0.899	0.862	0.922
Hours preferences	$\sigma\psi$	0.266	0.223	0.313	0.251	0.164	0.361	0.248	0.174	0.340	0.159	0.152	0.169
Public consumption	σg	0.062	0.047	0.076	0.817	0.737	0.892	0.454	0.382	0.514	0.180	0.165	0.192
Interest rate	σR	0.003	0.003	0.004	0.003	0.003	0.004	0.003	0.003	0.003	0.003	0.003	0.004
Foreign prices	σTTw	0.044	0.038	0.049	0.042	0.036	0.047	0.043	0.038	0.049	0.043	0.037	0.049
Foreign demand	σyW	0.145	0.124	0.165	0.148	0.129	0.168	0.125	0.107	0.143	0.159	0.146	0.170
World interest rate	σRW	0.005	0.000	0.009	0.100	0.075	0.130	0.053	0.007	0.101	0.016	0.014	0.018

Table 6 Steady state ratios

	Model	Data		Model	Data
k/y	10.1	–	T/y	0.24	–
c/y	0.62	0.59	R	1.01	–
i/y	0.29	0.25	r	0.03	–
m/y	0.33	0.30	q	1.12	–
x/y	0.29	0.24			

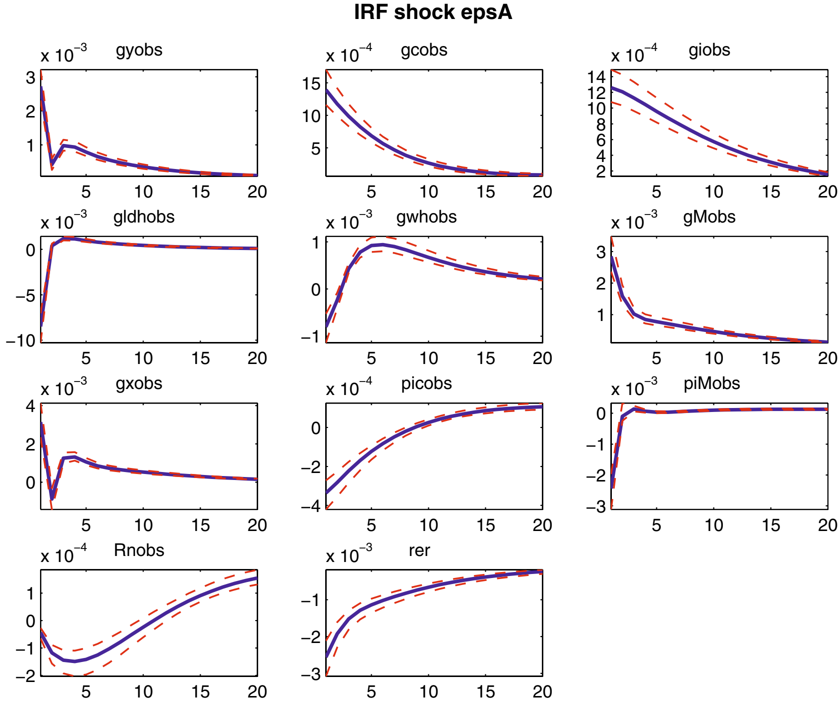

Fig. 8 Impulse response function to a neutral technology shock

data are less precise (such as the capital/output ratio), we have values that are comparable to the numbers usually employed in the literature. In addition, in the class of DSGE models to which MEDEA belongs, small changes in steady state ratios have only second-order effects on the dynamics of aggregate variables.

Figures 8, 9, 10, 11, 12, 13 show the impulse response functions (IRFs) to some of the stochastic shocks of the model, as well as the 5 and 95% confidence bands implied by the posterior distribution of parameters. Since there is growth in the model due to technological progress and the increase in population, the real variables in our solution are expressed in per capita efficiency units. In some cases, mainly supply side shocks, the behavior of variables defined in this way may seem confusing. Thus, we show instead in the simulations the growth rates of real variables expressed in the

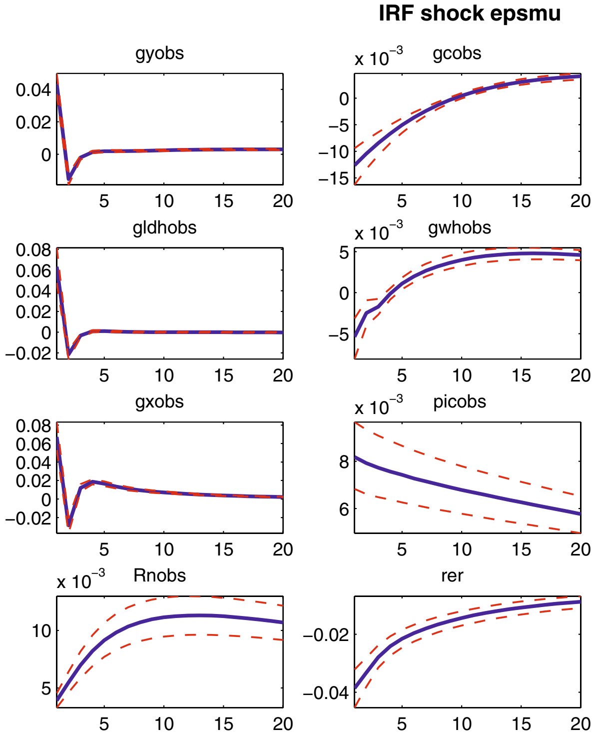

Fig. 9 Impulse response function to an investment-specific technology shock

same units as in the data. That is, for example *gyobs* is equal to total real GDP growth. In all cases, we show a one-standard-deviation shock to the corresponding innovation. In all Figs. 8, 9, 10, 11, 12, 13, the order of variables (from left to right and from top to bottom) is output growth, consumption growth, investment growth, hours growth, wage per hour growth, imports growth, exports growth, consumption deflator, imports deflator, nominal interest rate, and real interest rate.

Figure 8 reports the IRFs for a neutral technology shock. In a rather standard way, output, consumption, and investment respond positively to the shock. Hours fall at impact (with sticky prices and wages, the demand for total labor services is rigid in the short run and since, thanks to the technological shocks, we need less labor to produce the same output, firms hire fewer workers), but they recover in the second quarter and become positive. Prices and the nominal interest rate go down because of higher productivity (marginal cost falls and the monetary authority is less worried about inflation).

Figure 9 plots the IRFs for an investment-specific technological shock. Here, investment goes up rapidly, but since the economy is not more productive in the short run, it has to do so at the expense of lower consumption and longer hours. Note that the impact on hours is the opposite of that in the previous case: now it goes up and then falls. Our model, thus, reproduces the insights of Fisher (2006), who emphasizes that the response of hours to technological shocks depends on the specifics of the techno-

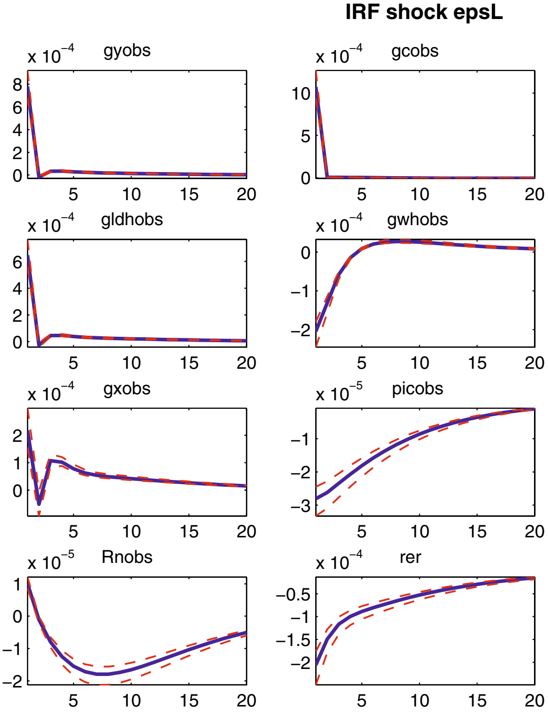

Fig. 10 Impulse response function to a population growth shock

logical process assumed by the model. Imports rise because we want to invest more and exports increase at impact (to later fall) because the investment-specific technological shock makes the national investment product relatively cheap in the world market. Consumption prices increase because fewer resources are concentrated in its production.

The IRFs to a population growth shock are drawn in Fig. 10. Output, investment, and imports grow (we have more workers and we need more capital for them). Interestingly, the consumption deflator goes down because the arrival of new workers lowers wages at impact. Figure 11 displays the IRFs of a labor supply shock. Figure 11 is nearly the opposite view of Fig. 10: a labor supply shock reduces hours worked for all levels of wages, and therefore, it works in nearly the same way as an increase in population. Figs. 10 and 11 suggest that part of the reason why Spain has had such high levels of investment and imports over the last decade is that there have been large immigration flows.

Figures 12 and 13 show the responses of the economy to two important policy shocks: a shock to monetary policy and a shock to government consumption. Two aspects are relevant. One, both shocks have an expansionary effect (as conventionally done, Fig. 12 is expressed in terms of a rise in the interest rate, so to think about a fall in the nominal rate, the reader only needs to flip the lines). Second, the expansionary effect is, however, rather small. For instance, the multiplier of a shock to government

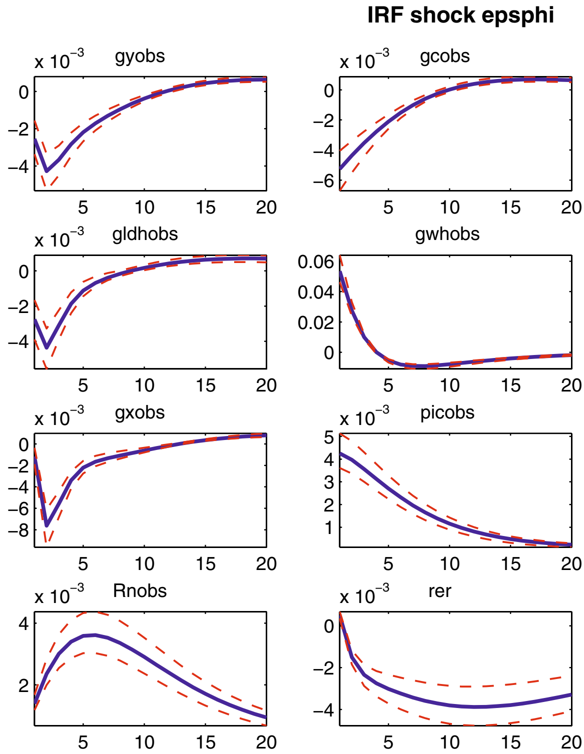

Fig. 11 Impulse response function to a labour supply shock

consumption is slightly less than 0.8 and it rapidly falls to zero. Moreover, shocks to government consumption are associated with falls in consumption and investment (given the low impact multiplier, this is nearly an accounting truism) and increases in prices. Thus, MEDEA does not support the view that increases in government consumption are effective tools for stabilizing output. On the positive side, Fig. 12 tells us that monetary shocks seem effective in controlling inflation in a relatively fast way.

6.2 Historical decompositions

MEDEA can be used to investigate what the driving forces have been behind Spanish economic growth during the last three decades by decomposing the observed GDP growth into the contributions of the structural shocks. The summary results are reported in Fig. 14 and in Table 7. To facilitate the presentation, we group the shocks into five categories: technology shocks, labor shocks, demand shocks, fiscal and monetary policy shocks, and foreign shocks. Then, Figs. 15 and 16 and Table 8 decompose the contribution of labor supply shocks into labor participation (preference between work and leisure) and population growth (creation of new households in

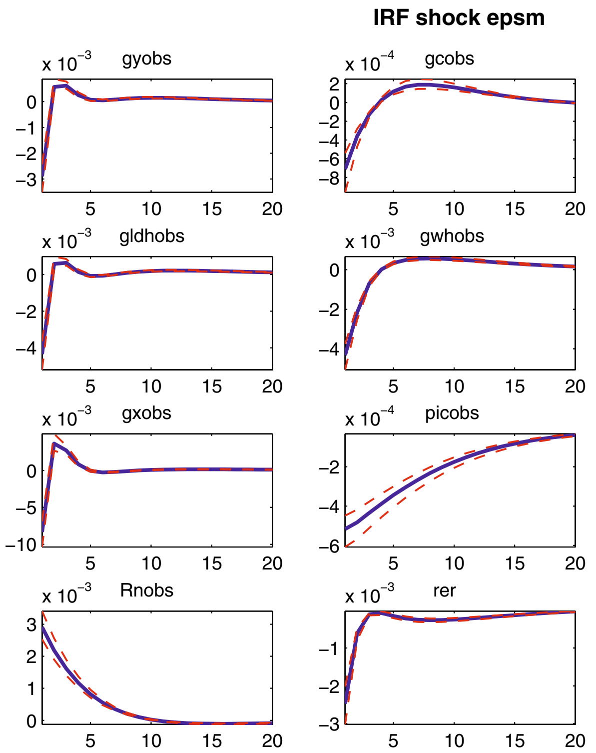

Fig. 12 Impulse response function to a monetary policy shock

the economy), and the contribution of technology into neutral and investment-specific components.

Looking at the period as a whole, the main contributors to growth have been the labor supply, mainly population, and demand shocks. Each of them accounts for around 40% (1.3 percentage points, p.p. hereafter) of real GDP growth. Productivity is the third factor in importance explaining over 15% (0.5 p.p.). The remaining shocks explain little over the long run, something that we could have expected from a neoclassical growth model (such as the one at the core of our paper). This main picture presents a scenario of a Spanish economy that has enjoyed many years of good shocks (immigration, incorporation of women and younger workers into the labor market, low real interest rates, positive world demand, and moderate energy prices), but that has not broken free from the historical constraints of low productivity and poor innovation.

Nevertheless, the contributions have been different over time. We will divide the analysis into three relevant periods: boom in the late 1980s, the crisis of 1993–95, and the expansion since then until 2007. The boom of the late eighties was characterized by large productivity growth but also by a rise in labor supply, mainly population as the large cohorts of the 1960s joined the labor market and women started to search for jobs in the market, but also by higher participation, and positive demand shocks

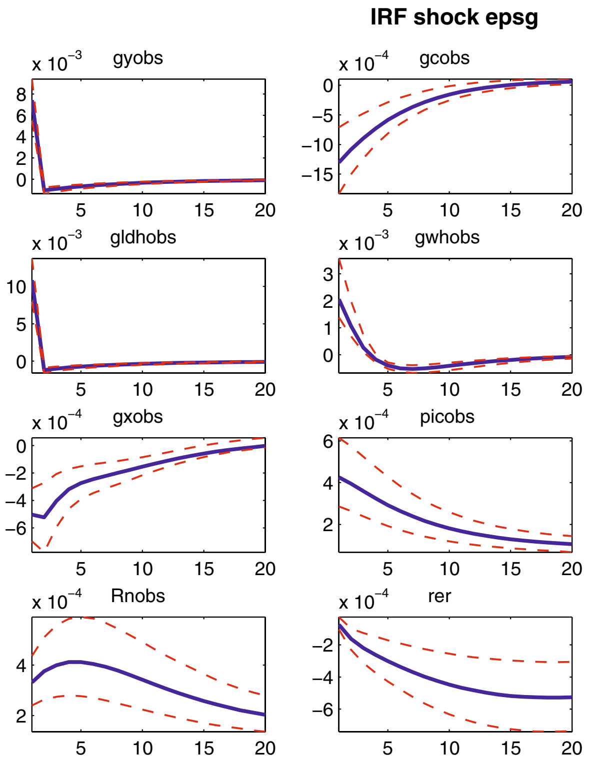

Fig. 13 Impulse response function to a government consumption shock

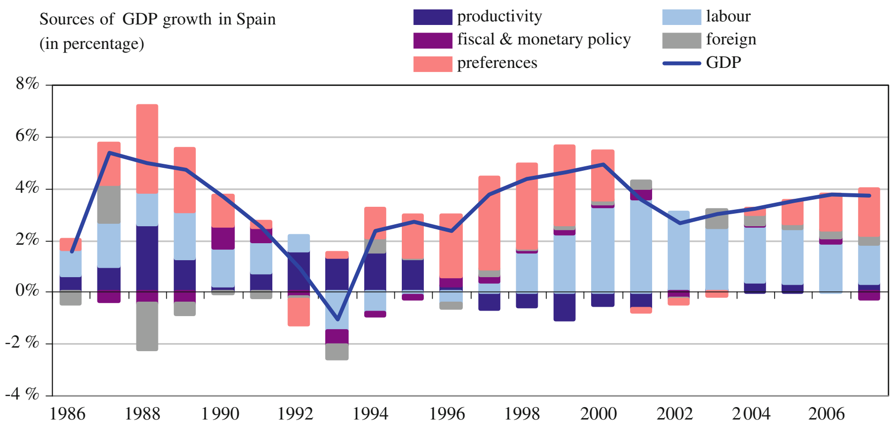

Fig. 14 Sources of GDP growth in Spain

(probably related to the reduction in uncertainty after the large crisis of the transition to democracy and the fall in oil prices). Each of these elements explains about one-third of the increase in real GDP, while fiscal policy accounts for only around 5%. Monetary

Table 7 Sources of GDP growth in Spain

Period	GDP growth (%)	Average contribution to GDP growth				
		Productivity (%)	Labour (%)	Preferences (%)	Policies (%)	Foreign (%)
1986–91	3.82	**1.11**	**1.41**	**1.49**	0.03	−0.22
1992–94	0.75	**1.53**	−0.59	0.10	−0.28	−0.01
1995–07	3.57	**−0.03**	**1.83**	**1.50**	0.08	0.20
1986–07	3.26	0.49	1.38	1.30	0.02	0.06

policy and foreign shocks contributed negatively, limiting GDP growth by around 0.15 and 0.2 p.p. on average, respectively. Those two are likely explained by the tough stand that the Bank of Spain took against inflation with a policy of competitive disinflation that brought high real interest rates and high value of the peseta.

The crisis of the early nineties was characterized by a very strong negative labor supply shock, mainly due to the large increase in unemployment, which the model interprets as a reduction in labor participation. This mechanism limited growth by almost 1.9 p.p. over this period. Labor shocks did not become positive again until 1998. At the same time, the Spanish economy suffered a fairly large negative demand shock, lasting from the end of 1991 until mid 1993, that reduced GDP growth by around 1 p.p.

In contrast, the long period of continuous real GDP growth experienced since the mid-nineties was mainly explained by favorable labor supply and demand shocks, probably a manifestation of immigration, changes in the age composition of the population, and the adoption of the euro and the associated historically low real interest rates. Technology shocks limited growth until 2001, a moment after which its contribution became positive, although rather small.[10] In addition, monetary policy shocks and foreign shocks have had a positive but much smaller contribution. Figure 15 suggests that both types of labor supply shocks have been very important, contributing on average 1.9 p.p., which represents over 50% of GDP growth since 1995, reaching 3.5 p.p. in the early 2000s, with population growth accounting for over 40% of growth and labor participation around 10%.

6.3 Permanent shocks

Another application of DSGE models is to trace the consequences of permanent changes in some variables or parameters.[11] For instance, we can evaluate the effects

[10] There is the caveat, however. Since the new workers joining the labor in the last decade were likely to have lower human capital than the existing workers, MEDEA might be picking up a composition effect that biases downward the contribution of technological shocks.

[11] The parameters of MEDEA are behavioral, in the sense that they have a clear interpretation rooted in economic theory but they are not necessarily structural in the sense of being invariant to the class of interventions we might be interested in. See Fernandez-Villaverde and Rubio-Ramirez (2008) for a more detail discussion.

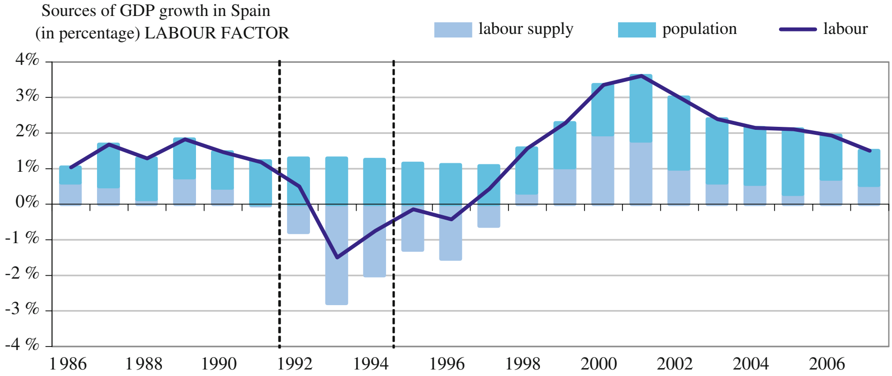

Fig. 15 Sources of GDP growth in Spain. Labour factor

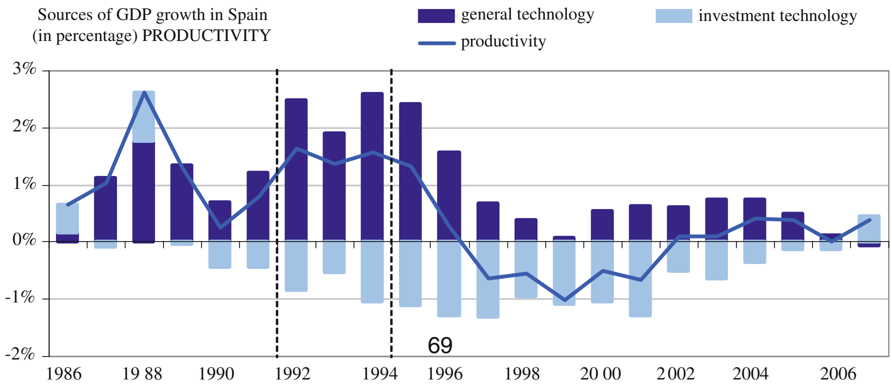

Fig. 16 Sources of GDP growth in Spain. Technology

Table 8 Sources of GDP growth in Spain (labour and productivity)

Period	Average contribution to GDP growth					
	Labour (%)	Population (%)	Labour supply (%)	Productivity (%)	General technology (%)	Investment technology (%)
1986–91	**1.41**	1.00	0.41	**1.11**	**1.06**	0.06
1992–94	**0.59**	1.27	**−1.86**	**1.53**	**2.33**	**−0.80**
1995–07	**−1.83**	**1.41**	0.42	**−0.03**	0.69	**−0.72**
1986–07	1.38	1.28	0.10	0.49	1.01	−0.52

of a reduction in distortionary taxation and the impact of an increase in competition in the labor or goods market. These are but two out of many other exercises of the kind we can select. However, these two are particularly illustrative given our current downturn.

There are three types of distortionary taxes in MEDEA: a tax on capital income, on labor income, and on consumption. The first panel of Table 9 reports the long-run

Table 9 Long run effects of a permanent reduction in tax rates, prices and wage mark-ups

Variable	Baseline steady state	% Change of steady state values						
	Level change	Capital tax	Labour income tax	VAT	Wage mark-up	Price markup domestic goods	Price markup imported goods	Price markup exported goods
		0.22–1.00	0.34–1.00	0.11–1.00	15–0.50	13–0.50	13–0.50	12–0.50
Output	1.407	0.36	0.46	0.28	0.13	0.01	0.18	0.18
Consumption	0.861	0.16	0.55	0.33	0.15	−0.34	0.11	0.11
Investment	0.408	0.82	0.40	0.24	0.11	0.73	0.43	0.44
Exports	0.348	0.57	0.35	0.21	0.10	0.43	1.27	1.28
Imports	0.390	0.49	0.30	0.18	0.08	0.38	1.09	1.10
Employment	0.410	0.06	0.47	0.28	0.13	0.38	0.03	0.03
Wage	1.996	0.29	−0.02	−0.01	−0.01	0.41	0.14	0.14
Wage*hours	0.818	0.35	0.45	0.27	0.13	0.79	0.17	0.17
Rental price of capital	0.032	−0.51	0.04	0.03	0.01	0.05	−0.25	−0.25
Marginal cost	0.863	0.00	0.00	0.00	0.00	0.28	0.00	0.00
Tobin’s q	1.109	0.06	0.04	0.02	0.01	0.05	−0.21	−0.22
Transfers	0.373	−0.22	−1.44	−2.16	0.20	0.49	0.16	0.16
Terms of trade	0.990	−0.08	−0.05	−0.03	−0.01	−0.06	0.27	−0.17

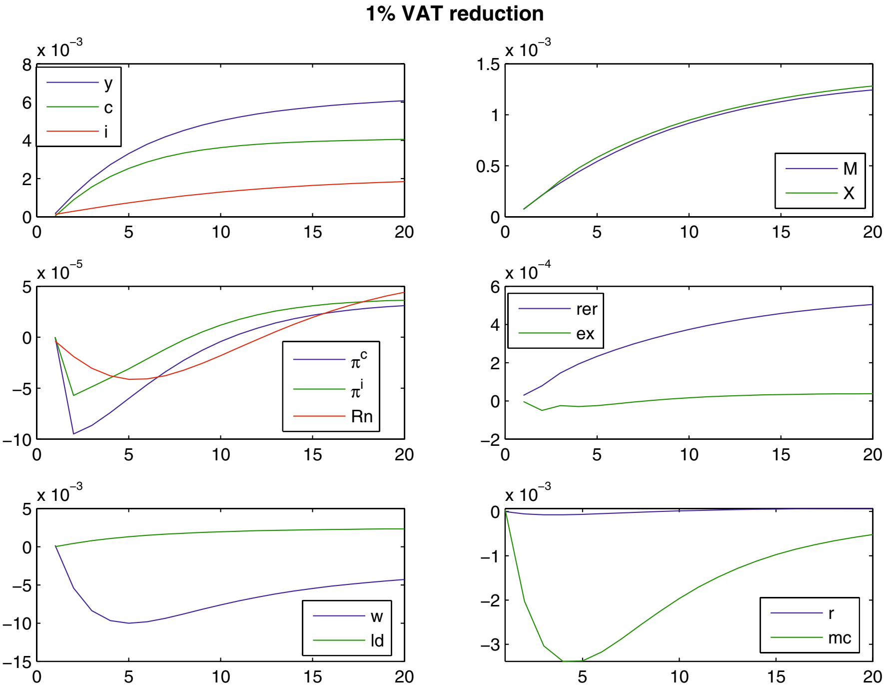

Fig. 17 Transitional dynamics after a permanent reduction of 1% in the consumption tax rate

impact of unexpectedly reducing each of these taxes by 0.5 p.p. To save on space, and since changes in the VAT have been proposed by many economists (and implemented in the UK) as a fiscal policy tool, we will concentrate on describing the effects of cutting the consumption tax. An unexpected reduction in the VAT by 1 p.p. has a long-run positive effect on the Spanish economy, by increasing output per capita in efficiency units and hours worked. Higher labor input pushes up the marginal productivity of capital and increases investment. On the demand side, the increase in the payments to capital and the rise in real compensation per worker (the rise in hours compensates for the fall in real wages) increase households' income and consumption. In order to equilibrate demand and supply, the terms of trade (p^x/p^M) fall to improve the external position. Figure 17 draws the transitional dynamics after the shock (to make the comparison easier, in the charts, all variables are expressed as differences with respect to the initial steady state). Most variables move smoothly to the new steady state. The exception is the real wage, which falls initially below its long-run level and then rises back toward the new equilibrium.

The degree of competition in the goods and labor markets is determined by the mark-up over prices or wages in each case. The second panel of Table 9 reports the long-run impact of unexpectedly reducing each of these mark-ups by 0.5 p.p. Increasing competition in the labor market reduces the wage per hour and increases the number of hours worked, expanding output per capita in efficiency units. Higher labor input pushes up the marginal productivity of capital and increases investment. The

Table 10 Long run effects of a permanent change in several parameters

Variable	Baseline steady state	% Change of steady state values									
	Level change	Habits 0.80–0.10	Labour supply coefficient 6.74–10.00	Elasticity of labour subst. 1.97–1.00	Elasticity of subst. consumption goods 7.67–3.00	Elasticity of subst. investment goods 8.06–3.20	Consumption home bias 0.81–10.00	Investment home bias 0.10–10.00	Neutral tech. growth rate 0.0023–1.00	Investment specific tech. growth rate 0.0026–1.00	Population growth rate 0.0032–1.00
Output	1.407	0.11	−1.49	0.54	0.04	0.06	−0.67	−0.43	−0.51	−0.55	−0.69
Consumption	0.861	0.12	−1.76	0.64	0.02	0.02	−0.91	−0.77	−0.74	−0.77	−0.87
Investment	0.408	0.09	−1.29	0.47	0.08	0.13	−1.45	−0.86	−1.27	−1.30	−1.57
Exports	0.348	0.08	−1.12	0.40	−0.33	−0.36	6.41	5.21	6.56	6.54	6.35
Imports	0.390	0.07	−0.97	0.35	−0.28	−0.31	5.51	4.48	5.64	5.62	5.45
Employment	0.410	0.11	−1.51	0.55	0.01	0.01	−0.21	−0.17	−0.21	−0.22	−0.22
Wage	1.996	−0.01	0.08	−0.03	0.02	0.04	−0.44	−0.24	−0.27	−0.31	−0.44
Wage*hours	0.818	0.10	−1.44	0.52	0.03	0.05	−0.65	−0.41	−0.49	−0.52	−0.66
Rental price of capital	0.032	0.01	−0.14	0.05	−0.04	−0.08	0.77	0.42	0.63	0.61	0.77
Marginal cost	0.863	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.02	0.00
Tobin's q	1.109	0.01	−0.12	0.04	−0.04	−0.07	0.67	0.37	0.69	0.69	0.67
Transfers	0.373	0.06	−2.29	0.83	0.02	0.05	−0.46	−0.24	−0.21	−0.29	−0.54
Terms of trade	0.990	−0.01	0.16	−0.06	0.04	0.05	−0.85	−0.69	−0.87	−0.86	−0.84

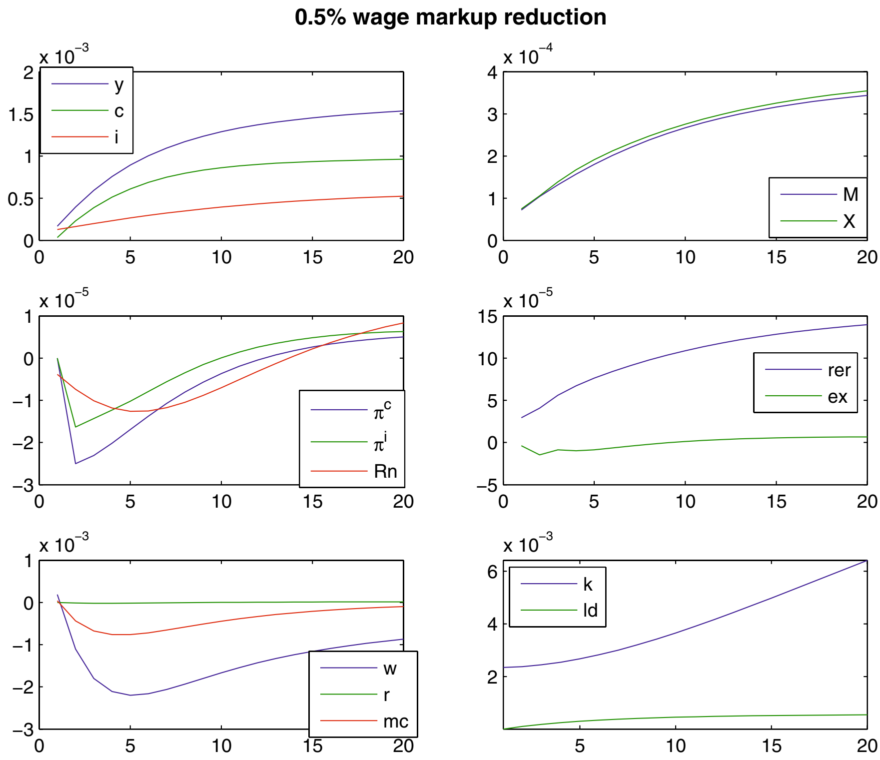

Fig. 18 Transitional dynamics after a permanent reduction of 0.5 p.p. in the wage mark-up

demand side is very similar to the case of a reduction of consumption taxes. During the transition to the new steady state (see Fig. 18), the real wage initially undershoots its long-run level and then recovers.

We complete this subsection with Table 10, which reports the long-run effects of changes in several parameters of the model.

6.4 Alternative scenarios

The historical decomposition of GDP growth showed that population growth created by immigration has been one of the important determinants of economic growth during the recent expansion, explaining around 1.4 p.p. of GDP growth (see Table 11). However, this contribution has not been constant over time. Instead, Fig. 19 shows that there has been an important change in the long-run population growth rate during the sample, increasing from 0.28% on average over the period 1986–96 to 0.35% since then. Therefore, an interesting question is what would GDP growth have been had this rise in population growth not taken place (for example, if the conservative and socialist governments had followed a more restrictive immigration policy).

Table 11 Alternative scenario for population growth

Period	Baseline			Change in alternative scenario		
	GDP growth (%)	Population growth (%)	Population contribution (%)	GDP growth (%)	Population growth (%)	Population contribution (%)
1986–96	2.75	0.284	1.11	−0.24	0.000	−0.26
1997–07	3.76	0.353	1.42	**−0.79**	**−0.067**	**−0.67**
1986–07	3.26	0.318	1.28	−0.48	−0.033	−0.47

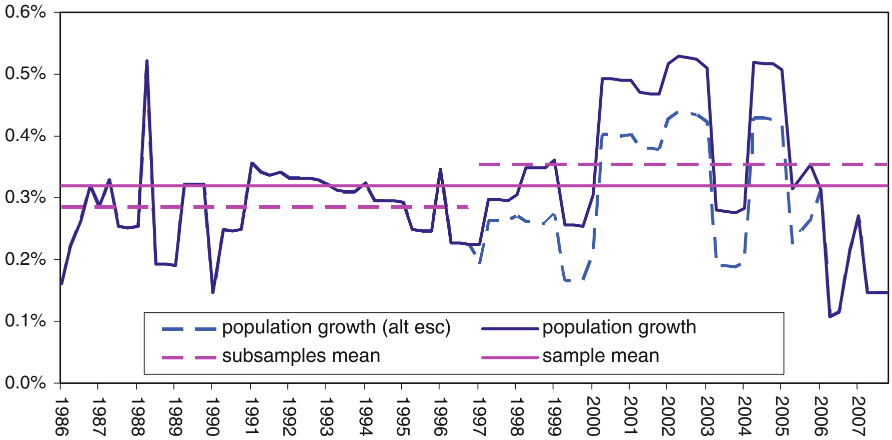

Fig. 19 Alternative scenario for population growth

This important question can be easily answered with MEDEA.[12] In particular, we would like to see what would have happened if population growth followed the alternative scenario depicted in Fig. 19 (discontinuous line). In that figure, we assume that population growth followed a path similar to the historical one, but shifted downward, so that the long-run mean is equal to the one in the first part of the sample. Figure 20 and Table 11 show the impact of this alternative scenario. We find that had population grown at this alternative slower pace, GDP growth would have been 0.5 p.p. lower. That is, the population growth shock experienced in Spain in the current decade added half a percentage point to annual output growth.

12 We want to be careful, though, since this exercise assumes that our parameters and the shocks recovered by the estimation are structural in the sense of being invariant to the change in population growth. This assumption may be problematic, although, unfortunately, difficult to test. We thank Fabio Canova for emphasizing this caveat to us.

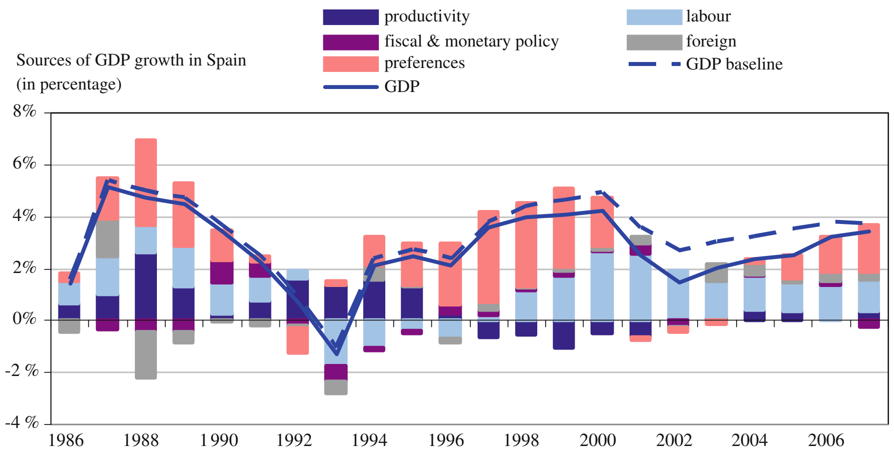

Fig. 20 Alternative scenario for population growth. Sources of growth

7 Conclusions

In this paper, we have introduced MEDEA, a DSGE model of the Spanish economy, and estimated it with data from the last several decades. To illustrate the potentialities of MEDEA, we have applied the model to policy analysis and counterfactual evaluations. We think that our enterprise has been a success. We now have an operational model of the Spanish economy, rich enough for detailed study and yet sufficiently concise to be solved and estimated with off-the-shelf software and a regular workstation. The estimates are reasonable and they tell us important lessons about how our economy works. Some we suspected, such as the differences in behavior of investment good imports versus consumption imports, some we did not, such as the small punch of fiscal policy. The model's performance as a forecasting tool (not a primary design consideration, but a relevant aspect nevertheless) still needs more time before we can establish it.

There are, however, many dimensions along which we would like to improve our work and make DSGE models an important element in the toolbox of policy makers. In particular, we will like to:

1. Incorporate a richer specification of fiscal policy, including tax and transfer shocks, a distinction between public consumption and public investment, and public capital in the production function. The recent active use of fiscal policy as an instrument to stabilize the economy suggests that we need a more detailed understanding of the propagation effects of fiscal policy in Spain.
2. Specify a social security system through the device of stochastic aging of households. As the Spanish population ages over the next decades and the social security system is strained to its limits, we need to know how the steady state and aggregate fluctuations will be affected by this aging and by possible re-adjustments in the system.

3. Model energy consumption more explicitly. Given the large exposure of the Spanish economy to oil shocks, this seems to be an important mechanism for understanding aggregate fluctuations.
4. Pay more attention to the behavior of the labor market. The Spanish economy's biggest open problem has been, for over three decades, its schizophrenic labor market, a heritage of darker eras of our economic policy that no government has dared to tackle. Beyond bitterly complaining about it, our task as macroeconomists is to add to our models a more realistic description of our outmoded set of labor market institutions.
5. Incorporate a financial sector. The recent financial crisis highlights how we want to trace the effects of different financial shocks on the economy and how to design macroeconomic policies that help to correct the problems caused by these financial shocks.
6. Estimate the model non-linearly and allow for stochastic volatility of the shocks and possible parameter drifting.

Hopefully, the support of research institutions and of the profession in general will allow us to see MEDEA or one of her descendants grow over the next years.

Appendix A: Equilibrium conditions

We present now the full set of equilibrium conditions.

- The first-order conditions of the household:

$$d_t \left(\widetilde{c}_t - h\frac{\widetilde{c}_{t-1}}{\widetilde{z}_t}\right)^{-1} - h\mathbb{E}_t\beta\gamma_{t+1}^L d_{t+1}\left(\widetilde{c}_{t+1}\widetilde{z}_{t+1} - h\widetilde{c}_t\right)^{-1} = \widetilde{\lambda}_t\left(1+\tau_c\right)\left(\frac{p_t^c}{p_t}\right) \tag{5}$$

$$\widetilde{\lambda}_t = \mathbb{E}_t\left\{\beta\frac{\widetilde{\lambda}_{t+1}}{\widetilde{z}_{t+1}}\frac{R_t}{\Pi_{t+1}}\right\} \tag{6}$$

$$\widetilde{\lambda}_t = \mathbb{E}_t\left\{\beta\frac{\widetilde{\lambda}_{t+1}}{\widetilde{z}_{t+1}}\frac{R_t^W\Gamma\left(ex_t\widetilde{b}_t^W, \xi_t^{b^W}\right)}{\Pi_{t+1}}\frac{ex_{t+1}}{ex_t}\right\} \tag{7}$$

$$\widetilde{r}_t = \frac{\Phi'\left[u_t\right]}{\left(1-\tau_k\right)} \tag{8}$$

$$\widetilde{q}_t\mathbb{E}_t\gamma_{t+1}^L = \beta\mathbb{E}_t\gamma_{t+1}^L\left\{\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t\widetilde{z}_{t+1}\widetilde{\mu}_{t+1}}\begin{pmatrix}\left(1-\delta\right)\widetilde{q}_{t+1} + \widetilde{r}_{t+1}u_{t+1}\left(1-\tau_k\right)\\ +\delta\tau_k - \Phi\left[u_{t+1}\right]\end{pmatrix}\right\} \tag{9}$$

$$\begin{aligned}\left(\frac{p_t^i}{p_t}\right) = \widetilde{q}_t &\left(1 - S\left[\gamma_t^L\frac{\widetilde{i}_t}{\widetilde{i}_{t-1}}\widetilde{z}_t\right] - S'\left[\gamma_t^L\frac{\widetilde{i}_t}{\widetilde{i}_{t-1}}\widetilde{z}_t\right]\gamma_t^L\frac{\widetilde{i}_t}{\widetilde{i}_{t-1}}\widetilde{z}_t\right)\\ &+\mathbb{E}_t\beta\widetilde{q}_{t+1}\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t\widetilde{z}_{t+1}}S'\left[\gamma_{t+1}^L\frac{\widetilde{i}_{t+1}}{\widetilde{i}_t}\widetilde{z}_{t+1}\right]\left(\gamma_{t+1}^L\frac{\widetilde{i}_{t+1}}{\widetilde{i}_t}\widetilde{z}_{t+1}\right)^2\end{aligned} \tag{10}$$

$$\left(\frac{\widetilde{m}_t}{p_t}\right) = d_t \upsilon \left[\beta \mathbb{E}_t \frac{\widetilde{\lambda}_{t+1}}{\widetilde{z}_{t+1}} \frac{R_t - 1}{\Pi_{t+1}}\right]^{-1} \tag{11}$$

$$\begin{aligned} f_t = {} & \frac{\eta - 1}{\eta} (1 - \tau_w) \left(\widetilde{w}_t^*\right)^{1-\eta} \widetilde{\lambda}_t \left(\widetilde{w}_t\right)^{\eta} l_t^d \\ & + \beta \theta_w \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{1-\eta} \left(\frac{\widetilde{w}_{t+1}^*}{\widetilde{w}_t^*} \widetilde{z}_{t+1}\right)^{\eta-1} f_{t+1} \end{aligned} \tag{12}$$

$$\begin{aligned} f_t = {} & d_t \varphi_t \psi \left(\frac{\widetilde{w}_t^*}{\widetilde{w}_t}\right)^{-\eta(1+\vartheta)} \left(l_t^d\right)^{1+\vartheta} \\ & + \beta \theta_w \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{-\eta(1+\vartheta)} \left(\frac{\widetilde{w}_{t+1}^*}{\widetilde{w}_t^*} \widetilde{z}_{t+1}\right)^{\eta(1+\vartheta)} f_{t+1} \end{aligned} \tag{13}$$

- The intermediate domestic firms that can change prices set them to satisfy:

$$g_t^1 = \widetilde{\lambda}_t mc_t \widetilde{y}_t^{\,d} + \beta \theta_p \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\varepsilon} g_{t+1}^1 \tag{14}$$

$$g_t^2 = \widetilde{\lambda}_t \Pi_t^* \widetilde{y}_t^d + \beta \theta_p \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\varepsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{t+1}^2 \tag{15}$$

$$\varepsilon g_t^1 = (\varepsilon - 1) g_t^2 \tag{16}$$

where they rent inputs to satisfy their static minimization problem:

$$\frac{u_t}{l_t^d} \widetilde{k}_{t-1} = \frac{\alpha}{1-\alpha} \frac{\widetilde{w}_t}{\widetilde{r}_t} \widetilde{z}_t \widetilde{\mu}_t \tag{17}$$

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \widetilde{w}_t^{1-\alpha} \widetilde{r}_t^{\alpha} \tag{18}$$

- The importing and exporting domestic firms that can change prices set them to satisfy:

$$g_t^{M_1} = \widetilde{\lambda}_t \left[\frac{\left(\frac{ex_t p_t^W}{p_t}\right)}{\left(\frac{p_t^M}{p_t}\right)}\right] \widetilde{y}_t^M + \beta \theta_M \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\left(\Pi_t^M\right)^{\chi_M}}{\Pi_{t+1}^M}\right)^{-\varepsilon_M} g_{t+1}^{M_1} \tag{19}$$

$$g_t^{x_1} = \widetilde{\lambda}_t \frac{\widetilde{y}_t^x}{\left(\frac{ex_t p_t^x}{p_t}\right)} + \beta \theta_x \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\left(\Pi_t^W\right)^{\chi_x}}{\Pi_{t+1}^x}\right)^{-\varepsilon_x} g_{t+1}^{x_1} \tag{20}$$

$$g_t^{M_2} = \widetilde{\lambda}_t \Pi_t^{M^*} \widetilde{y}_t^M + \beta \theta_M \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\left(\Pi_t^M\right)^{\chi_M}}{\Pi_{t+1}^M}\right)^{1-\varepsilon_M} \left(\frac{\Pi_t^{M^*}}{\Pi_{t+1}^{M^*}}\right) g_{t+1}^{M_2} \tag{21}$$

$$g_t^{x_2} = \widetilde{\lambda}_t \Pi_t^{x^*} \widetilde{y}_t^x + \beta \theta_x \mathbb{E}_t \gamma_{t+1}^L \left(\frac{\left(\Pi_t^W\right)^{\chi_x}}{\Pi_{t+1}^x}\right)^{1-\varepsilon_x} \left(\frac{\Pi_t^{x^*}}{\Pi_{t+1}^{x^*}}\right) g_{t+1}^{x_2} \tag{22}$$

$$\varepsilon_M g_t^{M_1} = (\varepsilon_M - 1) g_t^{M_2}; \quad \varepsilon_x g_t^{x_1} = (\varepsilon_x - 1) g_t^{x_2} \tag{23}$$

- Wages and prices evolve as:

$$1 = \theta_w \left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t} \right)^{1-\eta} \left(\frac{\widetilde{w}_{t-1}}{\widetilde{w}_t \widetilde{z}_t} \right)^{1-\eta} + (1-\theta_w) \left(\frac{\widetilde{w}_t^*}{\widetilde{w}_t} \right)^{1-\eta}, \tag{24}$$

$$1 = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t} \right)^{1-\varepsilon} + \left(1-\theta_p\right) \Pi_t^{*1-\varepsilon} \tag{25}$$

$$1 = \theta_M \left(\frac{\left(\Pi_{t-1}^M\right)^{\chi_M}}{\Pi_t^M} \right)^{1-\varepsilon_M} + (1-\theta_M) \left(\Pi_t^{M^*} \right)^{1-\varepsilon_M} \tag{26}$$

$$1 = \theta_x \left(\frac{\left(\Pi_{t-1}^W\right)^{\chi_x}}{\Pi_t^x} \right)^{1-\varepsilon_x} + (1-\theta_x) \left(\Pi_t^{x^*} \right)^{1-\varepsilon_x} \tag{27}$$

- Monetary authority follows its Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi} \right)^{\gamma_\Pi} \left(\frac{\gamma_t^L \frac{\widetilde{y}_t^d}{\widetilde{y}_{t-1}^d} \widetilde{z}_t}{\exp\left(\Lambda_L + \Lambda_{y^d}\right)} \right)^{\gamma_y} \right)^{1-\gamma_R} \exp\left(\xi_t^m\right) \tag{28}$$

While the government's policy comprises transfers to households and the level of debt, which are determined by the government's budget constraint:

$$\begin{aligned} \widetilde{b}_t &= \frac{g_t}{\widetilde{y}_t^d} + \frac{\widetilde{T}_t}{\widetilde{y}_t^d} + \frac{\widetilde{m}_{t-1}}{p_{t-1}} \frac{1}{\gamma_t^L \widetilde{y}_t^d \widetilde{z}_t \Pi_t} + R_{t-1} \widetilde{b}_{t-1} \frac{\widetilde{y}_{t-1}^d}{\gamma_t^L \Pi_t \widetilde{y}_t^d \widetilde{z}_t} \\ &\quad - (\widetilde{r}_t u_t - \delta) \tau_k \frac{\widetilde{k}_{t-1}}{\widetilde{y}_t^d \widetilde{z}_t \widetilde{\mu}_t} - \tau_w \frac{\widetilde{w}_t l_t^d}{\widetilde{y}_t^d} - \tau_c \left(\frac{p_t^c}{p_t} \right) \frac{\widetilde{c}_t}{\widetilde{y}_t^d} - \frac{\widetilde{m}_t}{p_t} \frac{1}{\widetilde{y}_t^d} \end{aligned} \tag{29}$$

and the fiscal policy rule:

$$\frac{\widetilde{T}_t}{\widetilde{y}_t^d} = T_0 - T_1 \left(\widetilde{b}_t - \overline{\widetilde{b}} \right) \tag{30}$$

- Net foreign assets evolve as:

$$\begin{aligned} ex_t \widetilde{b}_t^W &= R_{t-1}^W \Gamma \left(\Delta ex_t ex_{t-1} \widetilde{b}_{t-1}^W, \xi_{t-1}^{b^W} \right) \Delta ex_t \frac{\widetilde{y}_{t-1}^d}{\gamma_t^L \widetilde{z}_t \Pi_t \widetilde{y}_t^d} ex_{t-1} \widetilde{b}_{t-1}^W \\ &\quad + \left(\frac{ex_t p_t^W}{p_t} \right)^{\varepsilon_W} \left(\frac{ex_t p_t^x}{p_t} \right)^{1-\varepsilon_W} \left(\frac{\widetilde{y}_t^W}{\widetilde{y}_t^d} \right) - \left(\frac{ex_t p_t^W}{p_t} \right) \left(\frac{\widetilde{M}_t}{\widetilde{y}_t^d} \right) \end{aligned} \tag{31}$$

- Aggregate imports and exports evolve as:

$$\widetilde{M}_t = \upsilon_t^M \left[\mathbb{E}_t \Omega_{t+1}^c \left(1 - n^c\right) \left[\frac{\left(\frac{p_t^M}{p_t}\right)}{\left(\frac{p_t^c}{p_t}\right)} \right]^{-\varepsilon_c} \widetilde{c}_t + \mathbb{E}_t \Omega_{t+1}^i \left(1 - n^i\right) \left[\frac{\left(\frac{p_t^M}{p_t}\right)}{\left(\frac{p_t^i}{p_t}\right)} \right]^{-\varepsilon_i} \widetilde{i}_t \right] \tag{32}$$

$$\widetilde{x}_t = \upsilon_t^x \left[\frac{\left(\frac{ex_t p_t^x}{p_t}\right)}{\left(\frac{ex_t p_t^W}{p_t}\right)} \right]^{-\varepsilon_W} \widetilde{y}_t^W \tag{33}$$

where

$$\mathbb{E}_t \Omega_{t+1}^s = \frac{\left[1 - \beta \left(1 - n^s\right)^{\frac{1}{\varepsilon_s}} \mathbb{E}_t \frac{\gamma_{t+1}^L \widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t \widetilde{z}_{t+1}} \left[\frac{\left(\frac{p_t^s}{p_t}\right)}{\left(\frac{p_t^M}{p_t}\right)} \right] \Pi_{t+1}^s \left(\frac{\widetilde{s}_{t+1}}{\widetilde{s}_{t+1}^M \left(1 - \Gamma_{t+1}^s\right)} \right)^{\frac{1}{\varepsilon_s}} \Gamma_{t+1}^{s\,\prime} \frac{\left(\Delta \widetilde{s}_{t+1}^M\right)^2}{\Delta \widetilde{s}_{t+1}} \right]^{-\varepsilon_s}}{\left(1 - \Gamma_t^s\right) \left[1 - \Gamma_t^s - \Gamma_t^{s\,\prime} \left(\frac{\Delta \widetilde{s}_t^M}{\Delta \widetilde{s}_t} \right) \right]^{-\varepsilon_s}}$$

for $s = c, i$ and the distribution of relative prices of imports and exports:

$$\upsilon_t^M = \theta_M \left(\frac{\left(\Pi_{t-1}^M\right)^{\chi_M}}{\Pi_t^M} \right)^{-\varepsilon_M} \upsilon_{t-1}^M + (1 - \theta_M) \left(\Pi_t^{M^*}\right)^{-\varepsilon_M} \tag{34}$$

$$\upsilon_t^x = \theta_x \left(\frac{\left(\Pi_{t-1}^W\right)^{\chi_x}}{\Pi_t^x} \right)^{-\varepsilon_x} \upsilon_{t-1}^x + (1 - \theta_x) \left(\Pi_t^{x^*}\right)^{-\varepsilon_x}. \tag{35}$$

The production of importing and exporting firms is:

$$\widetilde{y}_t^M = \widetilde{c}_t^M + \widetilde{i}_t^M \tag{36}$$

$$\widetilde{y}_t^x = \left[\frac{\left(\frac{ex_t p_t^x}{p_t}\right)}{\left(\frac{ex_t p_t^W}{p_t}\right)} \right]^{-\varepsilon_W} \widetilde{y}_t^W \tag{37}$$

while the demands for consumption and investment imports relative to the corresponding domestic components are:

$$\frac{\widetilde{c}_t^M}{\widetilde{c}_t^d} = \frac{\mathbb{E}_t \Omega_{t+1}^c \left(1 - n^c\right)}{n^c} \left(\frac{p_t^M}{p_t}\right)^{-\varepsilon_c} \tag{38}$$

$$\frac{\widetilde{i}_t^M}{\widetilde{i}_t^d} = \frac{\mathbb{E}_t \Omega_{t+1}^i \left(1 - n^i\right)}{n^i} \left(\frac{p_t^M}{p_t}\right)^{-\varepsilon_i} \tag{39}$$

- Markets clear:

$$n^c\left(\frac{p_t^c}{p_t}\right)^{\varepsilon_c}\widetilde{c}_t+n^i\left(\frac{p_t^i}{p_t}\right)^{\varepsilon_i}\widetilde{i}_t+g_t+\Phi\left[u_t\right]\frac{\widetilde{k}_{t-1}}{\widetilde{\mu}_t\widetilde{z}_t}+\widetilde{x}_t$$

$$=\frac{\frac{\widetilde{A}_t}{\widetilde{z}_t}\left(u_t\widetilde{k}_{t-1}\right)^{\alpha}\left(l_t^d\right)^{1-\alpha}-\phi}{v_t^p} \tag{40}$$

where

$$l_t=v_t^w l_t^d \tag{41}$$

$$v_t^p=\theta_p\left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon}v_{t-1}^p+\left(1-\theta_p\right)\Pi_t^{*-\varepsilon} \tag{42}$$

$$v_t^w=\theta_w\left(\frac{\widetilde{w}_{t-1}}{\widetilde{w}_t\widetilde{z}_t}\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{-\eta}v_{t-1}^w+(1-\theta_w)\left(\frac{\widetilde{w}_t^*}{\widetilde{w}_t}\right)^{-\eta} \tag{43}$$

and

$$\mathbb{E}_t\gamma_{t+1}^L\widetilde{k}_t-(1-\delta)\frac{\widetilde{k}_{t-1}}{\widetilde{z}_t\widetilde{\mu}_t}-\left(1-S\left[\gamma_t^L\frac{\widetilde{i}_t}{\widetilde{i}_{t-1}}\widetilde{z}_t\right]\right)\widetilde{i}_t=0 \tag{44}$$

Finally, aggregate consumption and investment evolve as:

$$\widetilde{c}_t=\left[\left(n^c\right)^{\frac{1}{\varepsilon_c}}\left(\widetilde{c}_t^d\right)^{\frac{\varepsilon_c-1}{\varepsilon_c}}+\left(1-n^c\right)^{\frac{1}{\varepsilon_c}}\left(\widetilde{c}_t^M\left(1-\Gamma_t^c\right)\right)^{\frac{\varepsilon_c-1}{\varepsilon_c}}\right]^{\frac{\varepsilon_c}{\varepsilon_c-1}} \tag{45}$$

$$\widetilde{i}_t=\left[\left(n^i\right)^{\frac{1}{\varepsilon_i}}\left(\widetilde{i}_t^d\right)^{\frac{\varepsilon_i-1}{\varepsilon_i}}+\left(1-n^i\right)^{\frac{1}{\varepsilon_i}}\left(\widetilde{i}_t^M\left(1-\Gamma_t^i\right)\right)^{\frac{\varepsilon_i-1}{\varepsilon_i}}\right]^{\frac{\varepsilon_i}{\varepsilon_i-1}} \tag{46}$$

- Relative consumption and investment prices evolve as:

$$\frac{p_t^c}{p_t}=\left[n^c+\Omega_t^{c^M}\left(1-n^c\right)\left(\frac{p_t^M}{p_t}\right)^{1-\varepsilon_c}\right]^{\frac{1}{1-\varepsilon_c}} \tag{47}$$

$$\frac{p_t^i}{p_t}=\left[n^i+\Omega_t^{i^M}\left(1-n^i\right)\left(\frac{p_t^M}{p_t}\right)^{1-\varepsilon_i}\right]^{\frac{1}{1-\varepsilon_i}} \tag{48}$$

- The definitions of inflation rates are the following:

$$\Pi_t^c = \frac{\left(\frac{p_t^c}{p_t}\right)}{\left(\frac{p_{t-1}^c}{p_{t-1}}\right)}\Pi_t; \quad \Pi_t^i = \frac{\left(\frac{p_t^i}{p_t}\right)}{\left(\frac{p_{t-1}^i}{p_{t-1}}\right)}\Pi_t$$

$$\Pi_t^M = \frac{\left(\frac{p_t^M}{p_t}\right)}{\left(\frac{p_{t-1}^M}{p_{t-1}}\right)}\Pi_t \quad \Pi_t^x = \frac{\left(\frac{ex_t p_t^x}{p_t}\right)}{\left(\frac{ex_{t-1} p_{t-1}^x}{p_{t-1}}\right)}\frac{\Pi_t}{\Delta ex_t} \tag{49}$$

$$\Pi_t^W = \frac{\left(\frac{ex_t p_t^W}{p_t}\right)}{\left(\frac{ex_{t-1} p_{t-1}^W}{p_{t-1}}\right)}\frac{\Pi_t}{\Delta ex_t}$$

- The growth rate of technology:

$$\widetilde{z}_t = \widetilde{A}_t^{\frac{1}{1-\alpha}} \widetilde{\mu}_t^{\frac{\alpha}{1-\alpha}} \tag{50}$$

Appendix B: Log-linearized equilibrum conditions

The full set of log-linearized equilibrum conditions is:

$$\begin{aligned}
&\left(1 - \frac{h\beta\gamma^L}{\widetilde{z}}\right)\left(\widehat{\widetilde{\lambda}}_t + \left(\widehat{p}_t^c - \widehat{p}_t\right)\right) \\
&= \widehat{d}_t - \frac{h\beta\gamma^L}{\widetilde{z}}\mathbb{E}_t\widehat{d}_{t+1} - \frac{1 + \frac{h^2\beta\gamma^L}{\widetilde{z}^2}}{\left(1 - \frac{h}{\widetilde{z}}\right)}\widehat{\widetilde{c}}_t + \frac{h}{\widetilde{z}\left(1 - \frac{h}{\widetilde{z}}\right)}\widehat{\widetilde{c}}_{t-1} \\
&\quad + \frac{\beta h\gamma^L}{\widetilde{z}\left(1 - \frac{h}{\widetilde{z}}\right)}\mathbb{E}_t\widehat{\widetilde{c}}_{t+1} - \frac{h}{\widetilde{z}\left(1 - \frac{h}{\widetilde{z}}\right)}\widehat{\widetilde{z}}_t
\end{aligned} \tag{51}$$

$$\widehat{\widetilde{\lambda}}_t = \mathbb{E}_t\left\{\widehat{\widetilde{\lambda}}_{t+1} + \widehat{R}_t - \widehat{\Pi}_{t+1}\right\}. \tag{52}$$

$$\mathbb{E}_t\widehat{\widetilde{\lambda}}_{t+1} + \widehat{R}_t^* - \mathbb{E}_t\widehat{\Pi}_{t+1} - \Gamma^{b^*}\left(ex_t + \widetilde{b}_t^*\right) - \Gamma^{b^*}\xi^{b^*}\xi_t^{b^*} + \mathbb{E}_t\Delta\widehat{ex}_{t+1} - \widehat{\widetilde{\lambda}}_t = 0 \tag{53}$$

$$\widehat{\widetilde{r}}_t = \frac{\Phi_2}{\Phi_1}\hat{u}_t. \tag{54}$$

$$\widehat{\widetilde{q}}_t = \mathbb{E}_t\Delta\widehat{\widetilde{\lambda}}_{t+1} + \frac{\beta(1-\delta)}{\widetilde{z}\widetilde{\mu}}\mathbb{E}_t\widehat{\widetilde{q}}_{t+1} + \frac{\beta\Phi_1}{\widetilde{z}\widetilde{\mu}\left(\frac{p^i}{p}\right)}\mathbb{E}_t\widehat{\widetilde{r}}_{t+1}. \tag{55}$$

$$\left(\widehat{p}_t^i - \widehat{p}_t\right) + \kappa\left(\gamma^L\widetilde{z}\right)^2\left(\Delta\widehat{\widetilde{i}}_t + \widehat{\widetilde{z}}_t + \widehat{\gamma}_t^L\right) = \widehat{\widetilde{q}}_t + \beta\kappa\left(\gamma^L\right)^3\widetilde{z}^2\mathbb{E}_t\Delta\widehat{\widetilde{i}}_{t+1} \tag{56}$$

$$\left(\widehat{\widetilde{m}}_t - \widehat{p}_t\right) = \widehat{d}_t - \left(\frac{R}{R-1}\right)\widehat{R}_t - \mathbb{E}_t\left(\widehat{\widetilde{\lambda}}_{t+1} - \widehat{\Pi}_{t+1}\right) \tag{57}$$

$$\begin{aligned}\widehat{f}_t &= \left(1-\beta\theta_w\gamma^L\widetilde{z}^{\eta-1}\Pi^{-(1-\eta)(1-\chi_w)}\right)\left((1-\eta)\,\widehat{\widetilde{w}}_t^* + \widehat{\widetilde{\lambda}}_t + \eta\widehat{\widetilde{w}}_t + \widehat{l}_t^d\right)\\ &\quad+\beta\theta_w\gamma^L\Pi^{-(1-\eta)(1-\chi_w)}\widetilde{z}^{\eta-1}\mathbb{E}_t\left(\widehat{f}_{t+1} - (1-\eta)\left(\widehat{\Pi}_{t+1} - \chi_w\widehat{\Pi}_t + \Delta\widehat{\widetilde{w}}_{t+1}^*\right)\right)\end{aligned} \tag{58}$$

$$\begin{aligned}\widehat{f}_t &= \left(1-\beta\theta_w\gamma^L\widetilde{z}^{\eta(1+\vartheta)}\Pi^{\eta(1+\vartheta)(1-\chi_w)}\right)\\ &\quad\times\left(\widehat{d}_t + \widehat{\varphi}_t + \eta\,(1+\vartheta)\left(\widehat{\widetilde{w}}_t - \widehat{\widetilde{w}}_t^*\right) + (1+\vartheta)\widehat{l}_t^d\right)\\ &\quad+\beta\theta_w\gamma^L\widetilde{z}^{\eta(1+\vartheta)}\Pi^{\eta(1+\vartheta)(1-\chi_w)}\mathbb{E}_t\left(\widehat{f}_{t+1} + \eta\,(1+\vartheta)\right.\\ &\quad\times\left.\left(\widehat{\Pi}_{t+1} - \chi_w\widehat{\Pi}_t + \Delta\widehat{\widetilde{w}}_{t+1}^*\right)\right)\end{aligned} \tag{59}$$

$$\begin{aligned}\widehat{g}_t^1 &= \left(1-\beta\theta_p\gamma^L\Pi^{\varepsilon(1-\chi)}\right)\left(\widehat{\widetilde{\lambda}}_t + \widehat{mc}_t + \widehat{\widetilde{y}}_t^d\right)\\ &\quad+\beta\theta_p\gamma^L\Pi_t^{\varepsilon(1-\chi)}\mathbb{E}_t\left(\widehat{g}_{t+1}^1 + \varepsilon(\widehat{\Pi}_{t+1} - \chi\widehat{\Pi}_t)\right).\end{aligned} \tag{60}$$

$$\begin{aligned}\widehat{g}_t^2 &= \left(1-\beta\theta_p\gamma^L\Pi^{-(1-\varepsilon)(1-\chi)}\right)\left(\widehat{\widetilde{\lambda}}_t + \widehat{\Pi}_t^* + \widehat{\widetilde{y}}_t^d\right)\\ &\quad+\beta\theta_p\gamma^L\Pi^{-(1-\varepsilon)(1-\chi)}\mathbb{E}_t\left(\widehat{g}_{t+1}^2 - (1-\varepsilon)\left(\widehat{\Pi}_{t+1} - \chi\widehat{\Pi}_t\right) - \left(\widehat{\Pi}_{t+1}^* - \widehat{\Pi}_t^*\right)\right).\end{aligned} \tag{61}$$

$$\widehat{g}_t^1 = \widehat{g}_t^2. \tag{62}$$

$$\widehat{u}_t + \widehat{\widetilde{k}}_{t-1} - \widehat{l}_t^d = \widehat{\widetilde{w}}_t - \widehat{\widetilde{r}}_t + \widehat{\widetilde{z}}_t + \widehat{\widetilde{\mu}}_t. \tag{63}$$

$$\widehat{mc}_t = (1-\alpha)\widehat{\widetilde{w}}_t + \alpha\widehat{\widetilde{r}}_t. \tag{64}$$

$$\begin{aligned}\widehat{g}_t^{M_1} &= \left(1-\beta\theta_M\gamma^L\left(\Pi^M\right)^{\varepsilon_M(1-\chi_M)}\right)\\ &\quad\times\left(\widehat{\widetilde{\lambda}}_t + \left(\widehat{ex}_t + \widehat{p}_t^W - \widehat{p}_t\right) - \left(\widehat{p}_t^M - \widehat{p}_t\right) + \widehat{\widetilde{y}}_t^M\right)\\ &\quad+\beta\theta_M\gamma^L\left(\Pi^M\right)_t^{\varepsilon_M(1-\chi_M)}\mathbb{E}_t\left(\widehat{g}_{t+1}^{M_1} + \varepsilon_M\left(\widehat{\Pi}_{t+1}^M - \chi_M\widehat{\Pi}_t^M\right)\right).\end{aligned} \tag{65}$$

$$\begin{aligned}\widehat{g}_t^{M_2} &= \left(1-\beta\theta_M\gamma^L\left(\Pi^M\right)^{-(1-\varepsilon_M)(1-\chi_M)}\right)\left(\widehat{\widetilde{\lambda}}_t + \widehat{\Pi}_t^{M*} + \widehat{\widetilde{y}}_t^M\right)\\ &\quad+\beta\theta_M\gamma^L\left(\Pi^M\right)^{-(1-\varepsilon_M)(1-\chi_M)}\mathbb{E}_t\begin{pmatrix}\widehat{g}_{t+1}^{M_2} - (1-\varepsilon_M)\left(\widehat{\Pi}_{t+1}^M - \chi_M\widehat{\Pi}_t^M\right)\\ -\left(\widehat{\Pi}_{t+1}^{M*} - \widehat{\Pi}_t^{M*}\right)\end{pmatrix}.\end{aligned} \tag{66}$$

$$\widehat{g}_t^{M_1} = \widehat{g}_t^{M_2}. \tag{67}$$

$$\widehat{g}_t^{x_1} = \left(1 - \beta\theta_x\gamma^L \left(\Pi^W\right)^{-\varepsilon_x\chi_x} \left(\Pi^x\right)^{\varepsilon_x}\right)\left(\widehat{\widetilde{\lambda}}_t + \widehat{\widetilde{y}}_t^x - \left(\widehat{ex}_t + \widehat{p}_t^x - \widehat{p}_t\right)\right)$$
$$+\beta\theta_x\gamma^L \left(\Pi^W\right)^{-\varepsilon_x\chi_x} \left(\Pi^x\right)^{\varepsilon_x} \mathbb{E}_t\left(\widehat{g}_{t+1}^{x_1} + \varepsilon_x\left(\widehat{\Pi}_{t+1}^x - \chi_x\widehat{\Pi}_t^W\right)\right). \tag{68}$$

$$\widehat{g}_t^{x_2} = \left(1 - \beta\theta_x\gamma^L \left(\Pi^W\right)^{(1-\varepsilon_x)\chi_x} \left(\Pi^x\right)^{-(1-\varepsilon_x)}\right)\left(\widehat{\widetilde{\lambda}}_t + \widehat{\Pi}_t^{x*} + \widehat{\widetilde{y}}_t^x\right)$$
$$+\beta\theta_x\gamma^L \left(\Pi^W\right)^{(1-\varepsilon_x)\chi_x} \left(\Pi^x\right)^{-(1-\varepsilon_x)} \mathbb{E}_t\left(\begin{array}{c}\widehat{g}_{t+1}^{x_2} - (1-\varepsilon_x)\left(\widehat{\Pi}_{t+1}^x - \chi_x\widehat{\Pi}_t^W\right)\\ -\left(\widehat{\Pi}_{t+1}^{x*} - \widehat{\Pi}_t^{x*}\right)\end{array}\right). \tag{69}$$

$$\widehat{g}_t^{x_1} = \widehat{g}_t^{x_2}. \tag{70}$$

$$\frac{\theta_w \Pi^{-(1-\chi_w)(1-\eta)}\widetilde{z}^{-(1-\eta)}}{(1-\theta_w)\left(\Pi^{w*}\right)^{1-\eta}}\left(\widehat{\Pi}_t - \chi_w\widehat{\Pi}_{t-1} + \widehat{\Pi}_t^w + \widetilde{z}_t\right) = \widehat{\widetilde{w}}_t^* - \widehat{\widetilde{w}}_t. \tag{71}$$

$$\frac{\theta_p \Pi^{-(1-\varepsilon)(1-\chi)}}{\left(1-\theta_p\right)\left(\Pi^*\right)^{(1-\varepsilon)}}\left(\widehat{\Pi}_t - \chi\widehat{\Pi}_{t-1}\right) = \widehat{\Pi}_t^*. \tag{72}$$

$$\frac{\theta_M \left(\Pi^M\right)^{-(1-\varepsilon_M)(1-\chi_M)}}{(1-\theta_M)\left(\Pi^{M*}\right)^{(1-\varepsilon_M)}}\left(\widehat{\Pi}_t^M - \chi_M\widehat{\Pi}_{t-1}^M\right) = \widehat{\Pi}_t^{M*}. \tag{73}$$

$$\frac{\theta_x \left(\Pi^W\right)^{(1-\varepsilon_x)\chi_x}\left(\Pi^x\right)^{-(1-\varepsilon_x)}}{(1-\theta_x)\left(\Pi^{x*}\right)^{(1-\varepsilon_x)}}\left(\widehat{\Pi}_t^x - \chi_x\widehat{\Pi}_{t-1}^W\right) = \widehat{\Pi}_t^{x*}. \tag{74}$$

$$\widehat{R}_t = \gamma_R\widehat{R}_{t-1} + (1-\gamma_R)\left(\gamma_\Pi\widehat{\Pi}_t + \gamma_y(\triangle\widehat{\widetilde{y}}_t^d + \widehat{\widetilde{z}}_t + \widehat{\gamma}_t^L)\right) + \widehat{\xi}_t^m. \tag{75}$$

$$\widehat{\widetilde{b}}_t + \widehat{\widetilde{y}}_t^d = \left(\frac{\widetilde{m}}{p}\right)\frac{1}{\widetilde{b}\widetilde{y}^d\Pi\gamma^L\widetilde{z}}\left((\widehat{\widetilde{m}}_{t-1} - \widehat{p}_{t-1}) - \widehat{\Pi}_t - \widehat{\widetilde{z}}_t - \widehat{\widetilde{\gamma}}_t^L\right)$$
$$-\left(\frac{\widetilde{m}}{p}\right)\frac{1}{\widetilde{b}\widetilde{y}^d}\left(\widehat{\widetilde{m}}_t - \widehat{p}_t\right)$$
$$+\frac{1}{\beta\gamma^L}\left(\widehat{R}_{t-1} + \widehat{\widetilde{b}}_{t-1} + \widehat{\widetilde{y}}_{t-1}^d - \widehat{\Pi}_t - \widehat{\widetilde{z}}_t - \widehat{\widetilde{\gamma}}_t^L\right) - \tau_w\frac{\widetilde{w}l^d}{\widetilde{b}\widetilde{y}^d}\left(\widehat{\widetilde{w}}_t + \widehat{l}_t^d\right)$$
$$-\frac{\tau_k\left(\widetilde{r} - \delta\right)}{\widetilde{z}\widetilde{\mu}}\left(\frac{\widetilde{k}}{\widetilde{b}\widetilde{y}^d}\right)\left(\widehat{\widetilde{k}}_{t-1} - \widehat{\widetilde{z}}_t - \widehat{\widetilde{\mu}}_t\right) - \frac{\tau_k\widetilde{r}}{\widetilde{z}\widetilde{\mu}}\left(\frac{\widetilde{k}}{\widetilde{b}\widetilde{y}^d}\right)\left(\widehat{\widetilde{r}}_t + \widehat{u}_t\right)$$
$$-\tau_c\left(\frac{p^c}{p}\right)\left(\frac{\widetilde{c}}{\widetilde{b}\widetilde{y}^d}\right)\left(\left(\widehat{p}_t^c - \widehat{p}_t\right) + \widehat{\widetilde{c}}_t\right) + \frac{g}{\widetilde{b}\widetilde{y}^d}\widehat{g}_t + \frac{\widetilde{T}_0}{\widetilde{b}}\widehat{\widetilde{T}}_t \tag{76}$$

$$\frac{\widetilde{T}}{\widetilde{y}^d}\left(\widehat{\widetilde{T}}_t-\widehat{\widetilde{y}}_t^d\right)=-\left(\frac{T_1\widetilde{b}}{T_0}\right)\widehat{\widetilde{b}}_t \tag{77}$$

$$\begin{aligned}\left(ex_t+\widetilde{b}_t^*\right)&=\frac{1}{\gamma^L\beta}\left(ex_{t-1}+\widetilde{b}_{t-1}^*\right)\\&\quad+\left(\frac{ex\ p^W}{p}\right)\left(\frac{\widetilde{M}}{\widetilde{y}^d}\right)\left((1-\varepsilon_W)\begin{bmatrix}\left(\widehat{ex}_t+\widehat{p}_t^x-\widehat{p}_t\right)\\-\left(\widehat{ex}_t+\widehat{p}_t^W-\widehat{p}_t\right)\end{bmatrix}-\widehat{\widetilde{M}}_t+\widehat{y}_t^W\right)\end{aligned} \tag{78}$$

$$\widehat{\widetilde{M}}_t=\widehat{v}_t^M+\widehat{\widetilde{y}}_t^M \tag{79}$$

$$\widehat{\widetilde{x}}_t=\widehat{v}_t^x-\varepsilon_W\left[\left(\widehat{ex}_t+\widehat{p}_t^x-\widehat{p}_t\right)-\left(\widehat{ex}_t+\widehat{p}_t^W-\widehat{p}_t\right)\right]+\widehat{\widetilde{y}}_t^W \tag{80}$$

$$\begin{aligned}\widehat{v}_t^M&=\theta_M\left(\Pi^M\right)^{\varepsilon_M(1-\chi_M)}\left(\varepsilon_M(\widehat{\Pi}_t^M-\chi_M\widehat{\Pi}_{t-1}^M)+\widehat{v}_{t-1}^M\right)\\&\quad-\left(1-\theta_M\left(\Pi^M\right)^{\varepsilon_M(1-\chi_M)}\right)\varepsilon_M\widehat{\Pi}_t^{M*}\end{aligned} \tag{81}$$

$$\begin{aligned}\widehat{v}_t^x&=\theta_x\left(\Pi^W\right)^{-\varepsilon_x\chi_x}\left(\Pi^x\right)^{\varepsilon_x}\left(\varepsilon_x(\widehat{\Pi}_t^x-\chi_x\widehat{\Pi}_{t-1}^W)+\widehat{v}_{t-1}^x\right)\\&\quad-\left(1-\theta_x\left(\Pi^W\right)^{-\varepsilon_x\chi_x}\left(\Pi^x\right)^{\varepsilon_x}\right)\varepsilon_x\widehat{\Pi}_t^{x*}\end{aligned} \tag{82}$$

$$\widehat{\widetilde{y}}_t^M=\left(\frac{\widetilde{c}^M}{\widetilde{c}^M+\widetilde{i}^M}\right)\widehat{\widetilde{c}}_t^M+\left(\frac{\widetilde{i}^M}{\widetilde{c}^M+\widetilde{i}^M}\right)\widehat{\widetilde{i}}_t^M \tag{83}$$

$$\widehat{\widetilde{y}}_t^x=-\varepsilon_W\left[\left(\widehat{ex}_t+\widehat{p}_t^x-\widehat{p}_t\right)-\left(\widehat{ex}_t+\widehat{p}_t^W-\widehat{p}_t\right)\right]+\widehat{\widetilde{y}}_t^W \tag{84}$$

$$\widehat{\widetilde{c}}_t^M-\widehat{\widetilde{c}}_t^d=-a_1^c\mathbb{E}_t\left(\triangle\widetilde{c}_t^M-\triangle\widetilde{c}_t\right)+a_2^c\mathbb{E}_t\left(\triangle\widetilde{c}_{t+1}^M-\triangle\widetilde{c}_{t+1}\right)-\varepsilon_c\left(\widehat{p}_t^M-\widehat{p}_t\right) \tag{85}$$

$$\widehat{\widetilde{i}}_t^M-\widehat{\widetilde{i}}_t^d=-a_1^i\mathbb{E}_t\left(\triangle\widetilde{i}_t^M-\triangle\widetilde{i}_t\right)+a_2^i\mathbb{E}_t\left(\triangle\widetilde{i}_{t+1}^M-\triangle\widetilde{i}_{t+1}\right)-\varepsilon_i\left(\widehat{p}_t^M-\widehat{p}_t\right) \tag{86}$$

$$\left(\widetilde{y}^d v^p\right)\left(\widehat{v}_t^p+\widehat{\widetilde{y}}_t^d\right)=\frac{\widetilde{A}}{\widetilde{z}}\widetilde{k}^\alpha\left(\widetilde{l}^d\right)^{1-\alpha}\left(\widehat{\widetilde{A}}_t-\widehat{\widetilde{z}}_t+\alpha\left(\widehat{u}_t+\widehat{\widetilde{k}}_{t-1}\right)+(1-\alpha)\widehat{\widetilde{l}}_t^d\right). \tag{87}$$

$$\begin{aligned}\widetilde{y}^d\,\widehat{\widetilde{y}}_t^d&=n^c\left(\frac{p^c}{p}\right)^{\varepsilon_c}\widetilde{c}\left(\widehat{\widetilde{c}}_t+\varepsilon_c\left(\widehat{p}_t^c-\widehat{p}_t\right)\right)+n^i\left(\frac{p^i}{p}\right)^{\varepsilon_i}\widetilde{i}\left(\widehat{\widetilde{i}}_t+\varepsilon_i\left(\widehat{p}_t^i-\widehat{p}_t\right)\right)\\&\quad+g\widehat{g}_t+\frac{\Phi_1\widetilde{k}}{\widetilde{\mu}\widetilde{z}}\widehat{u}_t+\widetilde{x}\widehat{\widetilde{x}}_t,\end{aligned} \tag{88}$$

$$\widehat{l}_t = \widehat{v}_t^w + \widehat{l}_t^d, \tag{89}$$

$$\begin{aligned}\widehat{v}_t^p &= \theta_p \Pi^{\varepsilon(1-\chi)} \left(\varepsilon(\widehat{\Pi}_t - \chi \widehat{\Pi}_{t-1}) + \widehat{v}_{t-1}^p\right) \\ &\quad - \left(1 - \theta_p \Pi^{\varepsilon(1-\chi)}\right) \varepsilon \widehat{\Pi}_t^*.\end{aligned} \tag{90}$$

$$\begin{aligned}\widehat{v}_t^w &= \theta_w \Pi^{\eta(1-\chi_w)} \widetilde{z}^\eta \left(\eta\left(\widehat{\Pi}_t - \chi_w \widehat{\Pi}_{t-1} + \widehat{\widetilde{w}}_t - \widehat{\widetilde{w}}_{t-1} + \widehat{\widetilde{z}}_t\right) + \widehat{v}_{t-1}^w\right) \\ &\quad - \left(1 - \theta_w \Pi^{\eta(1-\chi_w)} \widetilde{z}^\eta\right) \eta(\widehat{\widetilde{w}}_t^* - \widehat{\widetilde{w}}_t).\end{aligned} \tag{91}$$

$$\widehat{\widetilde{k}}_t = \frac{(1-\delta)}{\widetilde{z}\widetilde{\mu}\gamma^L} \widehat{\widetilde{k}}_{t-1} + \left(1 - \frac{1-\delta}{\widetilde{z}\widetilde{\mu}\gamma^L}\right) \widehat{\widetilde{i}}_t - \frac{(1-\delta)}{\widetilde{z}\widetilde{\mu}\gamma^L} \left(\widehat{\widetilde{z}}_t + \widehat{\widetilde{\mu}}_t\right). \tag{92}$$

$$\widehat{\widetilde{c}}_t = \left(n^c\right)^{\frac{1}{\varepsilon_c}} \left(\frac{\widetilde{c}^d}{\widetilde{c}}\right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} \widehat{\widetilde{c}}_t^d + \left[1 - \left(n^c\right)^{\frac{1}{\varepsilon_c}} \left(\frac{\widetilde{c}^d}{\widetilde{c}}\right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}}\right]^{\frac{1}{\varepsilon_c}} \widehat{\widetilde{c}}_t^M \tag{93}$$

$$\widehat{\widetilde{i}}_t = \left(n^i\right)^{\frac{1}{\varepsilon_i}} \left(\frac{\widetilde{i}^d}{\widetilde{i}}\right)^{\frac{\varepsilon_i - 1}{\varepsilon_i}} \widehat{\widetilde{i}}_t^d + \left[1 - \left(n^i\right)^{\frac{1}{\varepsilon_i}} \left(\frac{\widetilde{i}^d}{\widetilde{i}}\right)^{\frac{\varepsilon_i - 1}{\varepsilon_i}}\right]^{\frac{1}{\varepsilon_i}} \widehat{\widetilde{i}}_t^M \tag{94}$$

$$\begin{aligned}\left(\widehat{p}_t^c - \widehat{p}_t\right) &= \left(\frac{1-n^c}{1-\varepsilon_c}\right) \left(\frac{\left(\frac{p^M}{p}\right)}{\left(\frac{p^c}{p}\right)}\right)^{1-\varepsilon_c} \\ &\quad \times \left[\begin{array}{c}(1-\varepsilon_c)\left(\widehat{p}_t^M - \widehat{p}_t\right) \\ -a_1^c \mathbb{E}_t\left(\Delta\widetilde{c}_t^M - \Delta\widetilde{c}_t\right) + a_2^c \mathbb{E}_t\left(\Delta\widetilde{c}_{t+1}^M - \Delta\widetilde{c}_{t+1}\right)\end{array}\right]\end{aligned} \tag{95}$$

$$\begin{aligned}\left(\widehat{p}_t^i - \widehat{p}_t\right) &= \left(\frac{1-n^i}{1-\varepsilon_i}\right) \left(\frac{\left(\frac{p^M}{p}\right)}{\left(\frac{p^i}{p}\right)}\right)^{1-\varepsilon_i} \\ &\quad \times \left[\begin{array}{c}(1-\varepsilon_i)\left(\widehat{p}_t^M - \widehat{p}_t\right) \\ -a_1^i \mathbb{E}_t\left(\Delta\widetilde{i}_t^M - \Delta\widetilde{i}_t\right) + a_2^i \mathbb{E}_t\left(\Delta\widetilde{i}_{t+1}^M - \Delta\widetilde{i}_{t+1}\right)\end{array}\right]\end{aligned} \tag{96}$$

$$\widehat{\Pi}_t^c = \left(\widehat{p}_t^c - \widehat{p}_t\right) - \left(\widehat{p}_{t-1}^c - \widehat{p}_{t-1}\right) + \Pi_t \tag{97}$$

$$\widehat{\Pi}_t^i = \left(\widehat{p}_t^i - \widehat{p}_t\right) - \left(\widehat{p}_{t-1}^i - \widehat{p}_{t-1}\right) + \Pi_t \tag{98}$$

$$\widehat{\Pi}_t^M = \left(\widehat{p}_t^M - \widehat{p}_t\right) - \left(\widehat{p}_{t-1}^M - \widehat{p}_{t-1}\right) + \Pi_t \tag{99}$$

$$\widehat{\Pi}_t^x = \left(ex_t + \widehat{p}_t^x - \widehat{p}_t\right) - \left(ex_{t-1} + \widehat{p}_{t-1}^x - \widehat{p}_{t-1}\right) + \Pi_t - \Delta\widehat{ex}_t \tag{100}$$

$$\widehat{\Pi}_t^W = \left(\widehat{p}_t^W + ex_t - \widehat{p}_t\right) - \left(\widehat{p}_{t-1}^W + ex_{t-1} - \widehat{p}_{t-1}\right) + \Pi_t - \Delta\widehat{ex}_t \tag{101}$$

$$\widehat{\widetilde{z}}_t = \frac{\widehat{\widetilde{A}}_t + \alpha\widehat{\widetilde{\mu}}_t}{1-\alpha} \tag{102}$$

$$\widehat{\Pi}_t^W = \widehat{\xi}_t^{\Pi^W} \tag{103}$$

$$\widehat{R}_t^* = \widehat{\xi}_t^{R^*} \tag{104}$$

$$\widehat{\widetilde{y}}_t^W - \widehat{\xi}_t^{y^W} = 0 \tag{105}$$

$$\widehat{d}_t = \rho_d \widehat{d}_{t-1} + \sigma_d \varepsilon_{d,t} \tag{106}$$

$$\widehat{\varphi}_t = \rho_\varphi \widehat{\varphi}_{t-1} + \sigma_\varphi \varepsilon_{\varphi,t} \tag{107}$$

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \sigma_g \varepsilon_{g,t} \tag{108}$$

$$\widehat{\xi}_t^m = \sigma_m \varepsilon_{m,t} \tag{109}$$

$$\widehat{\gamma}_t^L = \sigma_L \varepsilon_{L,t} \tag{110}$$

$$\widehat{\widetilde{A}}_t = \sigma_A \varepsilon_{A,t} \tag{111}$$

$$\widehat{\widetilde{\mu}}_t = \sigma_\mu \varepsilon_{\mu,t} \tag{112}$$

$$\widehat{\xi}_t^{\Pi^W} = \rho_{\Pi^W} \widehat{\xi}_{t-1}^{\Pi^W} + \sigma_{\Pi^W} \varepsilon_{\Pi^W,t} \tag{113}$$

$$\widehat{\xi}_t^{y^W} = \rho_{y^W} \widehat{\xi}_{t-1}^{y^W} + \sigma_{y^W} \varepsilon_{y^W,t} \tag{114}$$

$$\widehat{\xi}_t^{R^*} = \rho_{R^*} \widehat{\xi}_{t-1}^{R^*} + \sigma_{R^*} \varepsilon_{R^*,t} \tag{115}$$

$$\widehat{\xi}_t^{b^*} = \rho_{b^*} \widehat{\xi}_{t-1}^{b^*} + \sigma_{b^*} \varepsilon_{b^*,t} \tag{116}$$

$$\Pi_t^O - \log \Pi = \widehat{\Pi}_t \tag{117}$$

$$w_t^O - w_{t-1}^O - \log \widetilde{z} = \left(\widehat{\widetilde{w}}_t - \widehat{\widetilde{w}}_{t-1}\right) + \widehat{\widetilde{z}}_t \tag{118}$$

$$R_t^O - \log R = \widehat{R}_t \tag{119}$$

$$y_t^{d,O} - y_{t-1}^{d,O} - \log \widetilde{z} - \log \widetilde{\gamma}^L = \left(\widehat{\widetilde{y}}_t^d - \widehat{\widetilde{y}}_{t-1}^d\right) + \widehat{\widetilde{z}}_t + \widehat{\widetilde{\gamma}}_t^L \tag{120}$$

$$c_t^O - c_{t-1}^O - \log \widetilde{z} - \log \widetilde{\gamma}^L = \left(\widehat{\widetilde{c}}_t - \widehat{\widetilde{c}}_{t-1}\right) + \widehat{\widetilde{z}}_t + \widehat{\widetilde{\gamma}}_t^L \tag{121}$$

$$i_t^O - i_{t-1}^O - \log \widetilde{z} - \log \widetilde{\gamma}^L = \left(\widehat{\widetilde{i}}_t - \widehat{\widetilde{i}}_{t-1}\right) + \widehat{\widetilde{z}}_t + \widehat{\widetilde{\gamma}}_t^L \tag{122}$$

$$\Pi_t^{c,O} - \log \Pi^c = \widehat{\Pi}_t^c \tag{123}$$

$$\Pi_t^{i,O} - \log \Pi^i = \widehat{\Pi}_t^i \tag{124}$$

$$l_t^{d,O} - l_{t-1}^{d,O} - \log \widetilde{\gamma}^L = \left(\widehat{l}_t^d - \widehat{l}_{t-1}^d\right) = \widehat{\widetilde{\gamma}}_t^L \tag{125}$$

$$M_t^O - M_{t-1}^O - \log \widetilde{z} - \log \widetilde{\gamma}^L = \left(\widehat{\widetilde{M}}_t - \widehat{\widetilde{M}}_{t-1}\right) + \widehat{\widetilde{z}}_t + \widehat{\widetilde{\gamma}}_t^L \tag{126}$$

$$x_t^O - x_{t-1}^O - \log \widetilde{z} - \log \widetilde{\gamma}^L = \left(\widehat{\widetilde{x}}_t - \widehat{\widetilde{x}}_{t-1}\right) + \widehat{\widetilde{z}}_t + \widehat{\widetilde{\gamma}}_t^L \tag{127}$$

$$\Pi_t^{M,O} - \log \Pi^M = \widehat{\Pi}_t^M \tag{128}$$

$$\Pi_t^{x,O} - \log \Pi^x = \widehat{\Pi}_t^x \tag{129}$$

$$\Pi_t^{W,O} - \log \Pi^W = \widehat{\Pi}_t^W \tag{130}$$

$$R_t^{*,O} - \log R^* = \widehat{R}_t^* \tag{131}$$

$$y_t^{W,O} - y_{t-1}^{W,O} - \log \widetilde{z} - \log \widetilde{\gamma}^L = \left(\widehat{\widetilde{y}}_t^W - \widehat{\widetilde{y}}_{t-1}^W\right) + \widehat{\widetilde{z}}_t + \widehat{\widetilde{\gamma}}_t^L \tag{132}$$

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