

MEDIEVAL EUROPE'S SATANIC CIPHERS: ON THE GENESIS OF A MODERN MYTH

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Abstract

The purpose of this article is to investigate the genesis and growth of a historical canard that can be encountered in numerous popular as well as some scholarly publications devoted to the history of mathematics. According to one of the core elements of this story, the number or symbol for zero was the cause of much anxiety in medieval Europe, as its unusual properties caused it to be associated with the Devil or with black magic. This anxiety is supposed to have extended to the entire system of Hindu-Arabic numerals, such that the use of these numerals was banned by the Church or by other powerful institutions. It is also commonly claimed that prohibitions of this sort explain the modern meaning of ‘cipher’ as ‘secret code’, as zero (*cifra*) and Hindu-Arabic numerals (*cifrae*) were once only used in hiding. I shall argue that this narrative is false or unsubstantiated at nearly every level of analysis. Some elements arose from an unwarranted interpretation of medieval sources, while others are based on mere supposition or the unbridled imagination of certain modern authors. The narrative’s main foundations were laid in 1905 by Leo Jordan, a German scholar of Romance languages, whose claims regarding a supposed quarrel between ‘abacists’ and ‘algorists’ in later medieval Europe exerted a strong influence on the modern historiography. Its more extreme contemporary manifestations are the result of a series of creative elaborations

contributed since the late 1980s by widely read authors such as Brian Rotman, Georges Ifrah, and Charles Seife.

Keywords

medieval mathematics; medievalism; pseudo-history; Hindu-Arabic numerals; zero

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Introduction

To historians of medieval Europe, it remains a barely escapable fact that the popular perception of their period of study has been severely deformed by modern myth-making about the past. Created at different times and for different purposes, these myths continue to affect representations of most areas of medieval life, from sexuality to war and from religion to science. Their combined upshot is a bizarre funhouse mirror image of the Middle Ages, which has ensured that the very term ‘medieval’ is still widely associated with barbarism, cruelty, unreason, and deranged cultural mores. The resistance of academic historians to this constant tide of false information has taken a variety of forms. Some have written monographs or organized collective volumes aimed at correcting popular misconceptions,¹ while others have made efforts trace the origin of specific myths or to reconstruct the processes by which they are disseminated.² Work of the latter category can be especially valuable for the way it holds up a mirror to the myth-makers. Chris Bishop, in an excellent recent deconstruction of a supposedly medieval torture instrument (the *poire d’angoisse*), has coined the term ‘dark medievalism’ to describe a widespread tendency among self-consciously modern individuals to view the Middle Ages in dystopian terms—as ‘a time-scape in which any excess can be imagined, in which any cruelty might seem credible’ (Bishop 2014, 600). All too often, of course, the cruelties imagined by the modern observer have no counterpart in the historical record, but instead are ‘indicative only of our dark desires and secret fears’ (ibid.).

¹ Pernoud 1977; Harris and Grigsby 2008; Weill-Parot and Sales 2017. See also the relevant chapters in Numbers 2009.

² This sort of reconstructive work has been carried out with particular thoroughness for the myth of medieval belief in a flat Earth. See, for example, Russell 1991; Wolf 2004; Reinhardt 2007; Roland 2013. See also Classen 2007, for the myth of the medieval chastity belt.

My modest goal in this article is to extend the project of deconstructing the products of modern-day ‘dark medievalism’ to the seemingly sterile area of mathematics, more specifically to the way the introduction of the number zero as well as the whole system of Hindu-Arabic numerals into medieval Europe has been represented in modern scholarship and popular media. To my knowledge, the mythical narrative I am about to discuss has never been scrutinized at any serious level of detail. A representative example of it may be drawn from a history of mathematical proof published in 2011 by the mathematician Steven G. Krantz. ‘In the Middle Ages’, Krantz asserts

zero took on religious overtones. The concept of nothing seemed to have connections to the soul and to spirituality. In many contexts it was forbidden to discuss zero. People feared committing heresy. It was not until the sixteenth century that zero began to play a useful role in commerce. (Krantz 2011, 49)

What is more:

Much thought of the time was influenced by religion, and people feared that things they did not understand were the works of the devil. Also the notion of a symbol that stood for nothing raised specters of evil signs and works of Satan. At various times, and by various people, it was actually *forbidden* to give explicit mention to zero or negative numbers. They were sometimes referred to explicitly in print as ‘forbidden’ or ‘evil’. (Krantz 2011, 50–51).

The notion that zero was once a cause of great anxiety is by no means exclusive to Krantz. A particularly strong statement to the same effect can be encountered in a German guide to Reformed churches, published in 2007:

In der arabischen Ziffernreihe taucht, wie gesagt, eine Zahl auf, die es bei römischen Ziffern nicht gibt: die Null. Vor dieser Zahl hatten die Menschen damals eine panische Angst. Sie erblickten in ihr ein Symbol des Nichts, des Bösen, des Teufels. Denn die Null steht ja für nichts. (Rauhaus 2007, 21)

As mentioned, the Arabic series of numerals exhibits a number not included among the Roman numerals: zero. Back in the day, people were petrified of this number. They saw in it a symbol of nothing, of evil, of the devil. After all, zero stands for nothing.

While some writers emphasize the fear that zero supposedly instilled in the general population, others suggest that medieval rejection of this strange number was imposed from the top down. The late Christopher Hitchens, in his bestselling anti-theist polemic *God Is Not Great* (2007), alleged that

the papacy of the Middle Ages always resisted the idea of ‘zero’ as alien and heretical, perhaps because of its supposedly Arab (in fact Sanskrit) origin but perhaps also because it contained a frightening possibility. (Hitchens 2007, 89)

He may have taken this information from Charles C. Mann, author of *1491: New Revelations of the Americas Before Columbus* (2005), who informs us that the number zero

didn’t appear in Europe until the twelfth century. Even then European governments and the Vatican resisted zero—a something that stood for nothing—as foreign and un-Christian. (Mann 2005, 19)

According to popular science writer Alex Bellos, evidence of this medieval fear and institutional resistance against zero is encoded in the vocabulary of certain European languages. He states that Leonardo of Pisa's *Liber Abbaci*

appeared during the period of crusades against Islam, with the clergy suspicious of anything with Arab connotations. Some, in fact, considered the new arithmetic the devil's work precisely because it was so ingenious. A fear of Arabic numerals is revealed through the etymology of some modern words. From *zephyr* came 'zero' but also the Portuguese word *chifre*, which means [Devil] horns, and the English word 'cipher', meaning code. It has been argued that this was because using numbers with a *zephyr*, or zero, was done in hiding, against the wishes of the Church. (Bellos 2010, 81)

An effective way of documenting the currency of these historical beliefs is by sampling articles hosted by online platforms dedicated to education or popular science. Reuben Westmaas, a writer for *curiosity.com*, told his readers in 2017:

When the Moors conquered Spain, they brought their math along with them, and from there, zero made it to Italy. Where it was promptly outlawed. Yes, religious leaders of Europe saw the devil in that little blank circle, which they strongly associated with Islam. But the number didn't stop being useful, and merchants knew that very well. So when they'd include zeroes on their ledgers, they did so in secret—and the word 'cipher' came to be synonymous with 'code' in the process. (Westmaas 2017)

An article from the same year on *livescience.com* strongly suggests that this narrative is supported by academic scholarship, by quoting the Indologist Annette van der Hoek:

Medieval religious leaders in Europe did not support the use of zero, van der Hoek said. They saw it as satanic. ‘God was in everything that was. Everything that was not was of the devil’, she said. (Szalay 2017)

Van der Hoek’s alleged statement about the link between the concept of nothing and the Devil is rather similar to one contained in an online outreach article by the mathematician Ittay Weiss:

After the advent of Christianity, religious leaders in Europe argued that since God is in everything that exists, anything that represents nothing must be satanic. In an attempt to save humanity from the devil, they promptly banished zero from existence, though merchants continued secretly to use it. (Weiss 2017)

Weiss in turn relied on a piece by Nils-Bertil Wallin, written for *YaleGlobalOnline*, which makes no mention of the Devil, but has the following to say about the spread of zero among Italian merchants post-1200:

Accountants knew their books were balanced when the positive and negative amounts of their assets and liabilities equaled zero. But governments were still suspicious of Arabic numerals because of the ease in which it was possible to change one symbol into another. Though outlawed, merchants continued to use zero in encrypted messages, thus the derivation of the word cipher, meaning code, from the Arabic sifr. (Wallin 2002)³

³ Wallin’s article is based on the books by Kaplan (1999) and Seife (2000), which will be discussed in section 2.3 of this article.

As one can see from these examples, which are going to be increased in what follows, the narrative concerning the medieval reaction to zero involves a number of recurring elements, which can be encountered separately as well as in combination. Some of these, for example Krantz's undocumented assertion that zero and negative numbers 'were sometimes referred to explicitly in print as "forbidden" or "evil"' (Krantz 2011, 51), are rare, while others may be regarded as core components of the myth. The most important appear to be the following three: (i) the number zero was feared for being associated with the Devil; (ii) it was therefore outlawed by church or state (or both); (iii) owing to these prohibitions, zero and Hindu-Arabic numerals were for some time only used under the cover of secrecy, hence the etymology of the English noun 'cipher'.

All three of these claims must be rejected. The first and third are false outright in the sense that no compelling evidence to support them has ever surfaced and the existing evidence shows that they are extremely unlikely to be true. The second claim is likewise false if applied to the medieval Church or 'religious leaders'. It remains false if applied to governments and other institutions unless certain qualifications are made, as will be done below. My plan for the remainder of this article is to proceed in two parts. In the first, I shall briefly outline the current state of knowledge concerning the reception of Hindu-Arabic numerals and zero in medieval Europe, which is meant to give a sense of the counterfactual nature of the statements already cited as well as to provide clarification on some historical issues. In the second part, I shall attempt to reconstruct the major lines of influence and individual steps that allowed the counterfactual narrative concerning the medieval reaction to zero to take shape in the course of the twentieth century.

1. Historical background

The spread of Hindu-Arabic numerals (abbreviated as HAN in what follows) in medieval Latin Europe is a complex topic, one whose precise contours are still subject to revision based on further research and new textual discoveries. I shall here content myself with a few signposts concerning the earliest known evidence, which are there to indicate the degree to which decimal place-value notation and the concomitant use of HAN, and especially the symbol and concept of zero, were disseminated until c.1250.⁴

Following their development in India and subsequent spread to the near East (the first mention in Syriac dates from c.662), the numerals from 1 to 9 make their first attested appearance in Latin Europe in a manuscript from the monastery of San Martín de Albada (La Rioja), which is dated to 976.⁵ The accompanying text attributes the numerals to the Indians and praises them for their ‘most subtle talent’, stating that ‘all other nations yield to them in arithmetic and geometry and the other liberal arts’.⁶ An additional reason for believing that the ‘Indian numerals’ were already well known in that region of Christendom is the attested presence of Gerbert of Aurillac, a French monk (and later pope) who studied in Catalonia

⁴ For general orientation, see Lemay 1977; 1982; Allard 1990; Folkerts 2001a; Berggren 2002; Kunitzsch 2005; Burnett 2006; Ambrosetti 2008; Schärli 2010; Wedell 2015; Folkerts and Hughes 2016, and the articles assembled in Burnett 2010.

⁵ See MS El Escorial, Real Bibliotheca de San Lorenzo, d.I.2, fol. 12v. The numerals on this page were copied into MS El Escorial, Real Biblioteca de San Lorenzo, d.I.1, fol. 9v (from San Millán de la Cogolla), which dates from 994. For transcriptions and facsimiles, see Smith and Karpinski (1911, 138) and Burnam (1912–25, 89–96). On the dating of these codices, see Gómez Pallarès (1987, 29–32).

⁶ MS El Escorial, Real Bibliotheca de San Lorenzo, d.I.2, fol. 12v, ‘Scire debemus in Indos subtilissimum ingenium habere et ceteras gentes eis in arithmetica et geometrica et ceteris liberalibus disciplinis concedere’. I here follow the translation in Burnett (2006, 17).

from c.967 to 970. That Gerbert was exposed to HAN during this visit is suggested by the practice, common in the eleventh and early twelfth centuries, of inscribing such numerals on the counters (*apices*) used in conjunction with the abacus. Several sources associate this type of abacus with Gerbert, which is why modern scholars have christened it the ‘Gerbertian’ abacus.⁷ Contemporary texts and depictions also reveal that the basic set of counters for the numbers from 1 to 9 was at least occasionally supplemented by a tenth counter, known as *sipos*, which appears to have been used as a placeholder. In some sources, the *sipos* is represented by a circle or wheel reminiscent of the later symbol for zero.⁸

Written arithmetic on the basis of HAN entered Latin Europe in the first half of the twelfth century as part of a wave of translated Arabic works on mathematics and astronomy. The foundational text in this regard is al-Khwārizmī’s ninth-century *Book of Indian Reckoning*, which became available in a number of Latin adaptations known respectively as *Dixit Algorizmi* (transmitted in two manuscripts),⁹ *Liber Ysagogarum Alchorismi* (extant in different versions in at least 8 MSS), *Liber Alchorismi* (10 MSS), and *Liber pulveris* (3 MSS).¹⁰ These texts form the baseline of a tradition of Latin writing on HAN-based arithmetic known as ‘algorithm’ (*algorismus*), a word derived from a Latinization of al-Khwārizmī’s name. Further algorithms still written in the twelfth century or very soon after 1200 include two texts in manuscripts at the British Library first brought to light by

⁷ On the Gerbertian abacus, see Beaujouan 1948; 1996; Evans 1977a; 1979; Gibson and Newton 1995; Folkerts 1996; 2000; 2001b; Burnett 2002c; Otisk 2015.

⁸ The evidence is discussed by Burnett (1996, 228–229; 2002c, 106–107; 2003, 94; 2006, 25–27) and Folkerts (2000, 219–221).

⁹ The text was last edited by Folkerts (1997).

¹⁰ These three texts were last edited by Allard (1992). See also the overviews in Allard (1991) and Ambrosetti (2008, 197–212). For the so-called ‘second book’ of the *Liber Alchorismi*, see Burnett, Zhao and Lampe (2007).

Karpinski (1921), the *Ars algorismi* edited by Allard (1978),¹¹ and the Salem *algorismus* published by Cantor (1865), which adds theological interpretations to its exposition of arithmetical operations. They were followed early in the thirteenth century by two immensely popular textbooks, John of Sacrobosco's *Algorismus vulgaris* and the metrical *Carmen de algorismo* attributed to Alexandre de Villedieu, both of which played a crucial role in establishing HAN-based arithmetic as part of the curriculum of medieval Arts faculties.¹²

A second textual genre through which this knowledge was diffused in Europe from the thirteenth century onwards were the treatises on commercial arithmetic used in Italian schools. The most prominent example of this literature is Leonardo of Pisa's *Liber abbaci* (first redaction 1202, second redaction 1228), in which the term *abbacus* refers not to the counting board, but to the art of performing calculations using HAN. This terminology became standard in late medieval Italy, to the extent that schools teaching commercial arithmetic were known as *scuole* or *botteghe d'abbaco* (Van Egmond 1980; Franci 2003; Ulivi 2004; 2013; Spiesser 2004; Høyrup 2007, 27–44; Caianiello 2014).

The Arabic term for zero is *ṣifr*, which Latin algorithms were in the habit of transliterating as *cifra*, whence the English 'cipher'.¹³ Another popular Latin name, found especially in twelfth-century texts, is *circulus*, which was presumably inspired by the Arabic

¹¹ See also Herreman 2001.

¹² John of Sacrobosco's work was last edited by Pedersen (1983, 174–201). See also Pedersen (1985, 182–183, 195–201). On the *Carmen de algorismo*, see Ambrosetti (2016), who counts 161 manuscripts. On the topic of thirteenth-century algorithmic writing, see also the texts edited and discussed in Busard (2000) and Burnett (2002b).

¹³ On the nomenclature of zero in early algorithms, see Lemay (1977, 451–453); Evans (1977, 117); Allard (1990, 154); Burnett (1996, 236–237); Ambrosetti (2016, 80). On the relationship between *cifra* and words in various European vernaculars, see Tropfke (1921, 8–14).

da'ira saghir ('little circle') and the circular shape of the corresponding symbol.¹⁴ A noteworthy effort to collect names and symbols for this unusual number was made by a late-twelfth-century English scribe in MS Cambridge, Trinity College, R.15.16, on the verso-side of the front flyleaf. The different names of zero are here given in a note: *cifra vel solfra vel nichil t. 0 cimera sipos*. The most peculiar of these alternatives is *cimera*, which is not attested in any other known source (Burnett 1996, 251; 2002b, 16; 2006, 26). It may have been the scribe's own bemused reaction to the value-less nature of zero.

The reception of HAN in the Latin West after c.1150 resembles that in the Islamic world after c.800 in that there were several centuries during which the new numerals coexisted with other number-writing systems.¹⁵ Besides the old Roman numerals, which remained in widespread use until the end of the medieval period, the twelfth and subsequent centuries saw experiments with alphanumeric notation as well as the use of completely unrelated systems of ciphers.¹⁶ A certain scepticism towards HAN may be reflected in an algorithmic treatise written in the twelfth century by a certain H. Ocreatus, where the new style of place-value based arithmetic is taught using Roman rather than Arabic numerals.¹⁷ According to a quantitative study of dates in Latin manuscripts carried out by John Crossley,

¹⁴ See, for example, the *Liber Algorismi*, (ed. Folkerts 1997, 32–34). For an unconvincing attempt to trace the circular sign for zero back to a ninth-century Carolingian origin, see Stevens (2013–14), whose account is replete with inaccuracies.

¹⁵ In Arabic manuscripts prior to 1200, the use of HAN is mostly limited to texts on arithmetic, whereas in astronomical sources the *abjad* system of notation was preferred. HAN are rarely encountered in texts of a non-scientific nature. See Lemay (1982, 383–384); Kunitzsch (2005, 9–14, 28).

¹⁶ See on this plurality Burnett (2000; 2006, 20–21, 28–29); Berggren (2002, 351–354); King (2001); Kunitzsch (2005, 24–27).

¹⁷ See the edition, translation, and discussion of this text by Burnett (1996).

HAN began to be the predominant mode of writing numbers across all contexts only around 1500 (Crossley 2013, 97–99). Crossley himself judges the adoption of HAN in medieval Europe to have been ‘slow, problematic, and spasmodic’ (Crossley 2013, 80), arguing that those accustomed with Roman numerals experienced some degree of cognitive difficulty in learning the place-value system (Crossley 2013, 83–86, 105–106). This picture deserves to be nuanced insofar as some milieus were clearly much quicker to react to the new numerals than others. The first to adopt them as part of their own practices appear to have been writers on astronomy and astrology, the two branches of the science of the stars.¹⁸ Two manuscripts from mid-twelfth century Bavaria document that these early users could also include students of the *computus*, the discipline devoted to the mathematics of the ecclesiastical calendar (Nothaft 2014). A commitment to HAN ‘for the sake of economy in writing and calculating’ was expressly voiced in 1170/71 by the cathedral canon Reinher of Paderborn, who employed the new numerals in the calendrical tables included in his *Compotus emendatus*.¹⁹ Another example is the *Compotus* written in the 1220s by Robert Grosseteste, later Bishop of Lincoln (c.1170–1253), which employs HAN as a matter of course for numerical tables as well as for computational rules and parameters expressed in the text itself—a preference replicated across the work’s 38 extant copies (Lohr and Nothaft 2019, 9).

Despite this relatively mundane picture of HAN as a well-established, albeit not universally used, aspect of medieval written culture, modern authors have frequently advanced the claim that the ‘new numerals’ faced considerable opposition from authorities. In most cases, they have done so by bringing up two cases where HAN became the target of prohibitions in medieval Italy. Perhaps the first English-language commentator to mention

¹⁸ Some early examples of this are provided by Burnett (2002a).

¹⁹ See Reinher of Paderborn, *Compotus emendatus*, praef. (ed. Lohr 2015, 3, ll. 20–22): ‘In designatione numerorum figuris plerumque utimur aliis quam Latinis propter scribendi et computandi compendium’.

both prohibitions in one place was Isaac Taylor, author of a two-volume history of *The Alphabet* (1883). According to Taylor, HAN

were introduced by the Arabs into Spain, from whence during the 12th and 13th centuries they spread over Europe, not, however, without considerable opposition. The bankers of Florence, for example, were forbidden, in 1299, to use them in their transactions, and the Statutes of the University of Padua ordain that the stationer should keep a list of the books for sale with the prices marked ‘non per cifras, sed per literas claras’. (Taylor 1883, 263)

The year of the Paduan statutes was supplied a few years later by the British mathematician W. W. Rouse Ball, whose frequently reprinted *Short Account of the History of Mathematics* (1st edition 1888; 4th edition 1908; last reprint 1960) contains a passage conspicuously similar to that in Taylor’s *Alphabet*:

The trade of Europe during the thirteenth and fourteenth centuries was mostly in Italian hands, and the obvious advantages of the algoristic system led to its general adoption in Italy for mercantile purposes. This change was not effected, however, without considerable opposition; thus, an edict was issued at Florence in 1299 forbidding bankers to use Arabic numerals, and in 1348 the authorities of the university of Padua directed that a list should be kept of books for sale with the prices marked ‘non per cifras sed per literas claras’. (Ball 1908, 186)²⁰

The first of the two ordinances mentioned here appears in the statutes of the Florentine guild of money changers (*Arte del Cambio*), which barred its members from writing (or letting

²⁰ See also the quote from this passage in Cunningham (1904, 42).

others write) down credits or debits in their account books ‘by means of or in the letter of the abacus, but let him write it openly and in full by way of letters’ (*aliquod quod per modum vel licteram abbachi intelligatur, sed aperte et extense scribat per licteram*).²¹ The reasons behind this prohibition are a matter of speculation. A concern that the Hindu-Arabic numerals, which existed in a variety of forms, might lend themselves to forgery or fraud has been one of the more popular explanations,²² but other possibilities exist.²³

The second prohibition, the one allegedly affecting Paduan booksellers in 1348, seems more difficult to trace owing to a failure of virtually all scholars mentioning it to cite the relevant primary source. The chain of second-hand (as well as third-hand etc.) references to this Paduan ordinance is so thick that one might easily suspect one is dealing with a

²¹ Article no. 102 (*Quod nullus de arte scribat in suo libro per abbacum*) was last edited in Camerani Marri (1955, 72–73). For the appearance of the same article in the revised statutes of 1300, 1313, and 1316, see the comparative table at the end of Camerani Marri (1955, not paginated). For the English translation cited here, see Struik (1968, 292). The passage is translated differently in Murray (1978, 170–171). See the discussion in Lüneburg (2008, 106–110), which offers a valuable criticism of some of the previous literature. On the interpretation of the phrase ‘openly and in full by way of letters’ (*aperte et extense scribere per literam*), see Nagl (1889, 162–167).

²² See, for example, Smith and Ginsburg (1937, 17); Neill Wright (1952, 126); Menninger (1969, 426–427); Tucci (1989, 556); Burton (2011, 280); King (2001, 315); Cherubini (2006, 332); Chrisomalis (2010, 123). According to Gazalé (2000, 48) the guild ‘ruled that the old figures could not be as easily falsified as the new ones, which could be turned into different figures without difficulty.’ It should be emphasized that no such comment appears in the source text.

²³ According to Struik (1968, 292–293) the ordinance reflects a power struggle between merchant guilds in the context of the ongoing conflict between Guelphs and Ghibellines in Florence.

literary ‘ghost’ without a counterpart in the sources.²⁴ In reality, a version of the relevant injunction already appears in statutes that the University of Padua passed in 1331, where booksellers (*stationarii*) are required to indicate their own name and the price of their merchandise clearly visible on the outside cover, using ‘clear letters’ rather than ciphers.²⁵ Almost identical prescriptions are preserved in statutes passed for the University of Bologna in 1317/47 and 1432 (ed. Malagola 1888, 28, 87) and for the University of Florence in 1387 (ed. Gherardi 1881, 41).

None of these sources contain the specific wording reported by Taylor and Ball (*non per cifras sed per litteras claras*), which can instead be found in the University of Padua’s 1465 statutes.²⁶ The year 1348, which is ubiquitous in the literature, may have been a mistake on Ball’s part. What should be clear, at any rate, is that the constraints the Italian universities imposed on the booksellers working on their premises only concerned the way book prices were displayed to prospective customers—how these booksellers kept their own financial

²⁴ As already noticed by Crossley (2013, 92n42), who gives the example of Chrisomalis (2010, 123–124) citing Berggren (2002, 361), who cites Pullan (1968, 34). To give but two additional ones: Caianiello (2014, 229n61) cites Cherubini (2006, 331–332n55), who cites Tabarroni (1983, 148n5) citing Horn d’Arturo (1925, 211), who refers to Taylor (1883, 263), whose sources are unknown. Also, Ambrosetti (2008, 247) cites Pellegrini (1972, 27), who in turn cites Pareja (1951, 693), who fails to provide a source, as do many others, including Haskins (1904, 154–155) and King (2001, 316).

²⁵ See the edition by Denifle (1892, 453, ll. 25–28): ‘Ponat eciam in libro venali extrinsecus et in evidenti loco et claris litteris non [per] zyphras nomen venditoris cum ipsius congnomine [*sic*] et precium libri, sub pena viginti solidorum Universitati solvenda et per rectoris exigenda’. Murray (1978, 171), for unclear reasons, dates these statutes to 1305.

²⁶ *Statuta dominorum artistarum Achademiae Patauinae*, [sine loc.]: Pasquino di Roma, [after 1500], fol. 34v (lib. III, c. 24). These statutes were already cited by Kirchhoff (1853, 30).

records was left to them. Transparency may have been the chief concern here, as not everyone would have been trained in a *scuola d'abbaco*. A more restrictive policy was enacted in 1350 in Trieste, which barred both money changers and merchants from keeping books *per abachum* (Szombathely 1930, 293–294; Tucci 1989, 555–556). There is also the case of the city council of Frankfurt am Main, which in 1494 ordered its accounting clerks to desist from using the numerals in public books (Lüneburg 2008, 109). Of course, neither of these examples documents a blanket ‘ban’ of the sort imagined by some modern authors. To these authors I shall now turn.

2. Anatomy of a myth

2.1. Origins

The notion that medieval resistance against HAN or the number zero was spearheaded by religious authorities, as taken for granted by several of the authors already cited in the introduction, resonates with the ‘warfare’ or ‘conflict thesis’ with regard to the relationship between science and religion. As far as the Anglophone world is concerned, this thesis is frequently traced back to the anti-Catholic polemics of John William Draper and Andrew Dickson White, first published respectively in 1874 and 1896 (Principe 2018). While they are indeed a source for numerous popular falsehoods about medieval science, Draper and White bear no responsibility for any of the above-cited claims. It generally appears that nineteenth-century books on the history of mathematics have little to say on the resistance or opposition the introduction of HAN supposedly faced in ‘Dark Age’ Europe. Even authors such as Taylor and Ball, who mention the prohibitions affecting Florentine bankers or Paduan booksellers, make no further attempt to deny that the numerals were widely used. The same is true for Florian Cajori’s popular *History of Mathematics*, first published in 1894 (and

reprinted in 1895, 1897, 1901, 1906, and 1909), which brings up the Florentine ordinance of 1299, falsely associating it with ‘merchants’ rather than bankers, but otherwise concedes that

[t]he calculation with the zero was the portion of Arabic mathematics earliest adopted by the Christians. The minds of men had been prepared for the reception of this by the use of the abacus and the apices. (Cajori 1894, 129)

The first author significantly to depart from this unexciting narrative was Leo Jordan (1874–1940), a scholar of Romance languages based at the University of Munich.²⁷ In 1905, the year of his habilitation, Jordan published an article presenting *Materialien zur Geschichte der arabischen Zahlzeichen in Frankreich* (‘Materials on the history of the Arabic numerals in France’), which was the result of manuscript studies carried out at the Bibliothèque Nationale in Paris three years earlier.²⁸ Using leaps of logic that are difficult to comprehend even after repeated readings, he presented an array of source texts as evidence of a clash between two arithmetical camps in thirteenth-century and later medieval Europe: reactionary ‘abacists’, who defended the old ways of reckoning, and the followers of the algorithm, whose symbol for zero was either mercilessly ridiculed or anxiously suspected to constitute harmful pagan magic (Jordan 1905, 164).²⁹

²⁷ For biographical details, see Lebsanft 1989.

²⁸ Some results of this article are summarized in Jordan (1906, 64–68).

²⁹ Jordan’s idea of two opposing camps of calculators was to have a remarkably long afterlife, as seen from the popular articles by Stone (1972) and Reynolds (1993). As recently as 2010 a serious history of mathematics (Chrisomalis 2010, 123) declared that Leonardo of Pisa’s *Liber abbaci* ‘sparked an important debate between two camps, the *abacists*, those who preferred computation with the medieval abacus, and the *algorithmists*, who preferred pen-and-paper calculations using the Western ciphered-positional numerals. The history of this debate is well documented, as it involved many important commercial families, renowned mathematicians and

That Jordan's reconstruction is far from reliable already becomes clear from the beginning of his article, which refers to William of Malmesbury's account of Gerbert of Aurillac's alleged sojourn in Muslim Spain. According to William, Gerbert was 'the first to seize the abacus from the Saracens, and he handed down the rules which calculators for all their efforts hardly understand'.³⁰ This brief remark is all the English historian has to say about Gerbert the mathematician,³¹ as his main focus is on Gerbert's rumoured exploits as a diviner and necromancer.³² Having told the story of Gerbert's theft of a 'forbidden' book from his Saracen teacher, which forced him to flee and call upon the Devil's aid in order to escape overseas, William concludes:

Some people may think this mere popular fiction; for public opinion often wounds the reputation of learned men, maintaining that one whom they have seen to excel in some department converses with the Devil.³³

clergymen, and even state authorities (Menninger 1969, 422–445; Evans 1977b; Murray 1978, 163–175). None of the studies cited here provide robust documentation of an actual debate between two factions. See also Caianiello (2014, 229).

³⁰ William of Malmesbury, *Gesta regum Anglorum*, 2.167.3, ed. Mynors (1998, 280): 'Abacum certe primum a Saracenis rapiens, regulas dedit quae a sudantibus abacistis uix intelliguntur'. For the English translation see *ibid.*, 281.

³¹ The notion that William regarded Gerbert's mathematics as 'dangerous Saracen magic' was introduced into the literature by Cochrane (1994, 7, 43), who masked her incorrect interpretation of the text as a quote taken from it. From Cochrane's book, the fake quote passed into Lyons (2012, 86) and Kaplan (1999, 66, 75), who repeated it in an appearance on BBC Radio 4's *In Our Time* (13 May 2004).

³² On the 'black legend' surrounding Gerbert of Aurillac, see Truitt 2012; Ricklin 2015.

³³ William of Malmesbury, *Gesta regum Anglorum*, 2.167.5, ed. Mynors 1998, 282, 'Sed haec uulgariter ficta crediderit aliquis, quod soleat populus litteratorum famam ledere, dicens illum loqui cum demone quem in aliquo uiderint excellentem opere'. English translation *ibid.*, p. 283.

Even though the context of this passage is entirely non-mathematical, Jordan commented on it in a way that was bound to mislead any reader not already familiar with the source into thinking that the rumours reported by William concerned the introduction of HAN:

Niemand anders soll nämlich dem späteren Papste die Kunst, mit arabischen Zeichen zu rechnen, gezeigt haben, als der Teufel. Und man nennt sie ja auch heute noch die ‘Teufelskunst’. (Jordan 1905, 156)

For none other than the Devil is supposed to have shown the art of reckoning with Arabic ciphers to the later pope. After all, even today it is still called the ‘Devil’s art’.

Jordan’s tendency to read very liberally into the material available to him manifests even more strongly on following pages of his article, where the number zero is associated with magic—not by any medieval source text, but by Jordan himself:

Eine Zahl, die Nichts bedeutete,—eine *Contradictio in adjecto*. Mußte dies den Romanen und Germanen als etwas Abgeschmacktes erscheinen, so war die orientalische Manier, an diesen scheinbaren Widerspruch mystisch-philosophische Betrachtungen zu knüpfen, bei den Abendländern geeignet, geradezu Verdacht erwecken: ein Zauber sei im Spiele. Der Kodex des Klosters Salem schreibt nämlich: ‘Jede Zahl entsteht aus der Eins, jene aber aus der Null’. (Jordan 1905, 159)

A number signifying nothing—a *contradictio in adjecto*. If this was bound to appear absurd to Romans and Germans, the oriental manner of attaching to this apparent contradiction mystical-philosophical reflections was positively apt to make occidentals suspect the involvement of magic. For the codex from Salem Abbey reads: ‘Every number arises from one, but this one from zero’.

How this harmless remark in the Salem *algorismus* (ed. Cantor 1865, 2: *Sic omnis numerus ab una generatur, ipsa a nullo*), which Jordan suspected to have been translated from an ‘oriental language’ (1905, 160–161), could possibly document his claims about zero’s ‘magical’ associations remains unclear. The same sort of puzzlement is occasioned by Jordan’s take on the works of Alain de Lille (1128–1202/3), whom he labels a staunch opponent of HAN (Jordan 1905, 157). His attempts to justify this claim (Jordan 1905, 165–167) do not proffer more than a handful of poetic or humorous quips on Alain’s part, as when he compares the role of zero among numbers to that of bats among birds and *H* among the letters of the alphabet—all being odd ones out relative to their respective genus.³⁴ Another medieval poet whom Jordan subjects to this sort of forced interpretation (Jordan 1905, 168–171) is Gautier de Coinci (1177–1236), who used ‘algorithm-cipher’ as a metaphor for worthlessness or nullity, as in the following example:

Beste cornue est et moutonz
Et s’est chiffres en augorisme
Clers qui ce jour de lui meïsmes
*Ne festoie la mere Dieu.*³⁵

One may note in passing that the concept of ‘zero’ possesses self-evident utility as a way of expressing the utter worthlessness of a thing or person and is for this very reason still present as an insult in various languages. The speakers of these language are not normally suspected

³⁴ Alain de Lille, *Anticlaudianus*, 2.436–438 (ed. Bossuat 1955, 85); Alain de Lille, *De planctu naturae*, 2.192–193 (ed. Häring 1978, 816).

³⁵ Gautier de Coinci, *Miracles de Nostre Dame*, II Mir 32, ll. 224–227 (ed. Koenig 1970, 426–427).

to harbour unexpressed hostilities towards HAN.³⁶ It seems reasonable to extend the same benefit of the doubt to the various other metaphorical uses cited in Jordan's survey (1905, 169–171), which cannot by themselves document any serious aversion to 'Indian' arithmetic or the use of zero as a place-holder in calculations. Jordan thought differently, having convinced himself that large parts of later medieval society must have felt a deep-seated hostility to HAN. What looked like tangible evidence of such hostility came from the Florentine 'prohibition' of 1299, known to Jordan from the earlier work of Nagl (1889, 161–170), which called for an explanation. Eager to provide one, Jordan allowed his imagination and rhetoric free rein:

Was war es nun, das die Regierungen veranlaßte, mit Verboten gegen diese unschuldigen Zeichen vorzugehen? War es die Sorge, daß im Falle eines Prozesses die Ziffern dem Richter unbekannt sein und so zu Verwicklungen Anlaß geben könnten? Nein! Bedenken wir doch, wir sind im 14. Jahrhundert, in der Blütezeit der Astrologen und Alchimisten, bedenken wir, daß die Zeichen von den Arabern stammten, den Zauberern par excellence, daß sich um ihren Import schon im 12. Jahrhundert eine Reihe von Fabeln gebildet hatten—kurz die Ziffern waren in aller Munde und doch kannte sie eigentlich keiner, und so ist es kein Wunder, daß sie dem Zeitgeiste gemäß allmählich den Stempel des Unheimlichen, Zauberformelartigen erhielten. (Jordan 1905, 186)

³⁶ Compare, for example, the Modern Hebrew slang term *dapar efes* or the German insult *Du Null!* Gazalé (2000, 46) notes that '[i]n contemporary Egypt, an individual despised by his or her peers, is metaphorically referred to as *sifr* 'ala al-yassaar, literally meaning "zero on the left-hand side", or as we would say today, "nonsignificant zero"'.

What was it, then, that caused governments to take action against these innocent symbols by imposing prohibitions? Was it a worry that the ciphers might be unknown to the judge in a trial and thereby cause confusions? No! We have to bear in mind here that this is the fourteenth century, the heyday of astrologers and alchemists. We have to bear in mind that the symbols came from the Arabs, the magicians *par excellence*, that already in the twelfth century their introduction had been the subject of a series of legends. With other words: the ciphers were on everybody's lips—and yet, nobody really knew them. It is hence no miracle that, in accordance with the spirit of the times, they gradually received the mark of the uncanny, of something akin to a magic spell.

Perhaps the most remarkable element in this hodgepodge of wild speculations is the claim that, already in the twelfth century, 'a series of legends' had sprung up surrounding the Arabic origin of HAN. What were these legends? Based on Jordan's article, one might be tempted to bring up William of Malmesbury, but we have already seen that this would be to rely on a misinterpretation. Jordan's only other concrete example is a passage in Alain de Lille's *Anticlaudianus* (3.378; ed. Bossuat 1955, 100), which states that a mathematician named Gilbert 'transgressed the art in a fallacious leap' (*Gilbertus saltu fallaci transilit artem*). Jordan took it for granted that this remark targeted Gerbert of Aurillac's adoption of HAN, which assumption is not borne out by the text itself. He could have known from a note published by Bubnov (1899, 391) that Alan's own pupil and commentator, Raoul de Longchamp (c.1212/13), understood it to mean that Gilbert (i.e., Gerbert) had called upon the aid of a demon in interpreting a difficult passage in Boethius's *De institutione arithmetica* (ed. Sulowski 1972, 187). The chapter in *De institutione* Raoul had in mind here was bk. II.1, which dealt with the reduction of every kind of inequality to equality. Gerbert had written a

famous scholium on this chapter, in which he proposed a method of reduction later known as the *saltus Gerberti* (Frova 1974, 344–350; Evans 1980; Otisk 2011, 46–47; Folkerts and Hughes 2016, 36–39). Contrary to what Jordan believed, this ‘leap’ had nothing whatsoever to do with HAN or zero.

2.2: Consolidation

Despite its flimsy evidentiary basis, the narrative concocted by Jordan was to have an almost immediate impact on the historiography of mathematics. The effects are already visible in David Eugene Smith’s and Louis Charles Karpinski’s book on the *The Hindu-Arabic Numerals* (1911), which became a standard work on the topic. In a passage that was recognizably inspired by Jordan’s article, the two authors chose to characterize the entire period between Gerbert of Aurillac and Leonardo of Pisa as the ‘period of the abacists’, asserting that even after the appearance of Indian reckoning early in the twelfth century there ‘was strife between the abacists, the advocates of the abacus, and the algorists, those who favored the new numerals’ (Smith and Karpinski 1911, 120). They went on to note that the words ‘*cifra* and *algorismus cifra* were used with a somewhat derisive significance, indicative of absolute uselessness’, citing Gautier de Coinci as their witness. As a result of these recalcitrant attitudes, ‘the abacus held the field for a long time, even against the new algorism employing the new numerals’ (Smith and Karpinski 1911, 120).

From Smith and Karpinski, the above-cited lines from Gautier de Coinci’s *Miracles de Nostre Dame* made their way into a brief and heavily Anglocentric volume on *The Story of Reckoning in the Middle Ages* (1926), written by the British school teacher Florence Annie Yeldham, in which the poet’s reference to an ‘algorismus-cipher’ is adduced to illustrate ‘[a]n abacist’s contempt for the little o which was to prove the undoing of his instrument’ (Yeldham 1926, 89). The dreary picture of medieval numeracy that Yeldham’s volume

offered was reinforced by a preface contributed by the eminent science historian Charles Singer, in whose eyes the whole book was proof that '[t]he peoples of Europe started their careers in the Dark Ages in a mentally incoherent state' (Yeldham 1926, 11). Commenting on Yeldham's account of 'the slow and painful process by which the simple rules of arithmetic have come into use' (Yeldham 1926, 15), Singer opined that this process was

an illustration, in a particular field, of the labour and effort required to raise mankind out of the intellectual trough of the Dark and Middle Ages. Of late there as flourished a school that would exalt mediaeval over modern civilization. One disposed to take this despondent view of our present state may be commended to think himself back, if he can, into the mathematical ineptitude of the twelfth century. A simple multiplication sum, such as can now be mastered in the nursery, then required the employment of a highly trained specialist! The solution of these simple mathematical problems was held to be so wonderful that those who achieved it were regarded as tainted with magic! (Yeldham 1926, 15–16)

Singer's comment on the supposed association between mathematics and magic received seeming validation from Yeldham's remarks later in the same book about the survival rate of medieval algorithms:

A number of these writings are preserved, and this is fortunate considering the vicissitudes England had passed through in civil war and the destruction of houses of religion and learning. A work on mathematics had a particularly bad chance because of outcries raised at intervals against the 'black art', with which mathematics was supposed to be closely linked. Arithmetic too is the subsidiary science to astronomy, and astronomy led to astrology, and thence to much fraud. Wherever attempts to put down these recurrent troubles were in the hands of ignorant men, as was often the

case, anything containing astronomical signs or mathematical figures and symbols was in danger of destruction. (Yeldham 1926, 69)

Despite the alleged frequency of attempts to quell the spread of astrology by force, Yeldham does not provide a single example, let alone one where this suppression went hand in hand with the destruction of non-astrological astronomical or mathematical texts. Claims of this nature nevertheless continued to be made, for example in Vera Sanford's *Short of History of Mathematics* of 1930, a book equipped with a foreword by David Eugene Smith. 'The algorisms', claimed Sanford

are an interesting commentary on the slow progress of Europeans in adopting the new numerals. This was due in part to inertia, but an important factor was the unfortunate association of these numerals with soothsaying through their use by astrologers.

(Sanford 1930, 28–29)

No sources are given in her text, although later the Florentine statutes of 1299 are brought up as an indicator of the slow 'recognition of the value of the symbols' (Sanford 1930, 95). A significant step towards misinterpreting this and other Italian ordinances as blanket 'bans' of the use of HAN was taken by the American mathematician Tobias Dantzig, author of the highly influential book *Number* (1930), which was reprinted as recently as 2005. He readily repeated Smith's and Karpinski's assertion about a supposed 'struggle between the *Abacists*, who defended the old traditions, and the *Algorists*, who advocated the reform'. According to him, this struggle 'lasted from the eleventh to the fifteenth century and went through all the usual stage of obscurantism and reaction' (Dantzig 2005, 33). 'In some places', Dantzig revealed to his readers

Arabic numerals were banned from official documents; in others, the art was prohibited altogether. And, as usual, *prohibition* did not succeed in abolishing, but merely served to spread *bootlegging*, ample evidence of which is found in the thirteenth century archives of Italy, where, it appears, merchants were using the Arabic numerals as a sort of secret code. (Dantzig 2005, 33).

The claim that ‘thirteenth century archives of Italy’ contain sources documenting the use of HAN as a secret code must be among the more baffling assertions encountered in the literature on this topic. What these archives were or where they might be found, Dantzig did not say, nor is it clear where he derived his opinion that in certain places ‘the art was prohibited altogether’. The only real hint comes from an earlier passage of his book, where the attitude of the ‘common people’ towards HAN is linked to ‘the fact that soon after its introduction into Europe, the word *cifra* was used as a secret sign; but this connotation was altogether lost in the succeeding centuries’ (Dantzig 2005, 32). It seems likely that, rather than consulting medieval archival material, Dantzig simply relied on the modern meaning of ‘cipher’, which he assumed must be explicable by medieval behaviour. The same idea is already present *in nuce* in Jordan’s 1905 article, which declared on no evidence whatsoever that HAN were seized upon by astrologers and alchemists who sought to make their books more mysterious, and that this led to *cifrae* gaining the additional meaning of ‘secret code’ (Jordan 1905, 186–188).

Another author who contributed to muddying the waters during the 1930s was the British life scientist Lancelot Hogben, whose *Mathematics for the Million* (1936) received several dozen printings during the twentieth century, with the last paperback edition dating from 1993. Commenting on the ‘obstruction’ against new ways of reckoning exercised by medieval ‘representatives of custom thought’, he asserted that

an edict of AD 1259 [*sic*] forbade the bankers of Florence to use the infidel symbols, and the ecclesiastical authorities of the University of Padua in AD 1348 ordered that the price list of books should be prepared not in ‘ciphers’, but in ‘plain letters’.

(Hogben 1936, 288)

The obvious mistakes in these few words (the former edict was passed in 1299 and nowhere mentions ‘infidel symbols’, the latter was passed by a university, not by ‘ecclesiastical authorities’) are emblematic of the work as a whole, whose degree of historical reliability is indicated by its way of referring to Luca Pacioli as ‘[t]he heretical ecclesiastic Paciulo, who had the good fortune not to be burned at the stake’ and who allegedly ‘translated the arithmetic of Bhaskara, and introduced Theon’s method of getting square roots’ (Hogben 1936, 288).

Neither Hogbart nor most of the other authors echoing Jordan’s conclusions—with the notable exception of Smith and Karpinski—show any signs of first-hand familiarity with his article. It is different with the German mathematics educator Karl Menninger, whose ‘cultural history’ of number words and symbols (*Zahlwort und Ziffer*), although already published in 1934, exerted its greatest influence after the Second World War, starting with the augmented second edition of 1958 and especially with its translation into English in 1969. Following in the footsteps of Jordan’s 1905 article, whose sources and conclusions he utilized liberally, Menninger declared from the outset that zero’s special role as a signifier of nothing must have made it ‘the great stumbling block for the medieval arithmeticians in the West’ (Menninger 1969, 400). The reasons, he insisted, were cognitive in nature:

What kind of crazy symbol is this, which means nothing at all? Is it a digit, or isn’t it?

1, 2, 3, 4, 5, 6, 7, 8, and 9 all stand for numbers one can understand and grasp—but 0?

If it is nothing, then it should be nothing. But sometimes it is nothing, and then at

other times it is something: $3 + 0 = 3$ and $3 - 0 = 3$, so here the zero is nothing, it is not expressed, and when it is placed in front of a number it does not change it: $03 = 3$, so the zero is still nothing, *nulla figura*! But write the zero *after* a number, and it suddenly multiplies the number by ten: $30 = 3 \times 10$. So now it is something—something incomprehensible but powerful, if a few “nothings” can raise a small number to an immeasurably vast magnitude. Who could understand such a thing? (Menninger 1969, 422)

Inspired by Jordan, who had referred to reckoning with HAN as *Teufelskunst* (Jordan 1905, 156, 172, 182), Menninger foregrounded this element of the narrative even more strongly, by stating: ‘[t]he zero, of course, is no Devil, but during the Middle Ages it was often regarded as the creation of the Devil’ (Menninger 1969, 400). And again: ‘the resistance to the Indian numerals by those who used the counting board took two forms: some regarded them as the creation of the Devil, while the others made fun and ridiculed them’. (Menninger 1969, 422). Turning to Alexandre de Villedieu’s *Carmen de algorismo*, Menninger emphasized not the great success this work evidently enjoyed as a teaching tool and means of spreading HAN-based arithmetic, but instead read signs of stubborn resistance into its metrical form.

According to him, the

popular disinclination to use the new numerals was also behind the attempt to make these strange new concepts, the zero and the place-value principle, comprehensible by presenting them in verse form. (Menninger 1969, 423)³⁷

³⁷ The questionable value of this argument becomes clear once one considers that that Alexandre de Villedieu authored (or is believed to have authored) equally popular didactic poems on the ecclesiastical calendar (*Massa compoti*) and grammar (*Doctrinale puerorum*). Would anyone claim that their verse form was indicative of a ‘popular disinclination’ towards these subjects?

Menninger's emphasis on the cognitive barriers that explain the supposedly slow adoption of the 'Arabic system' in medieval Europe also characterizes a slew of other works published on the subject in the second half of the twentieth century. A particularly laconic take on this theme can be spotted in Morris Kline's massive compendium on *Mathematical Thought from Ancient to Modern Times*: 'Arabic notation for numbers and Hindu methods of calculation were already known to some extent' before Leonardo of Pisa, 'but only in the monasteries. People in general used Roman numerals and avoided zero because they did not understand it' (Kline 1972, 209). In a similar vein, David M. Burton's *History of Mathematics*, first printed in 1985, briefly mentions the 'the confusion and insecurity that the zero produced in the minds of ordinary people (who could understand a symbol that meant nothing at all?)' (Burton 2011, 280). Another text supporting this particular point of view is J. M. Pullan's *History of the Abacus*, published in 1968, which makes its familiar case by pointing to the Italian 'bans' of 1299 and 1348 and the etymology of 'cipher' (Pullan 1968, 34–35). Ten years later, the idea of widespread resistance to HAN was given greater scholarly respectability by the historian Alexander Murray, whose *Reason and Society in the Middle Ages* (1978) singled out the 'literate tradition' as possessing an 'innate prejudice against arithmetic':

Latin official tradition [...] insulated its floor against Arabic numerals: they were a suspect novelty from below, unworthy to be allowed among the hallowed customs of literacy. (Murray 1978, 169)

His evidence for this hostility was in large parts an argument from silence, based on the continuing absence of Arabic numerals from many written sources, although Murray also cited 'the fact that our best indirect evidence of the early use of Arabic numerals comes from

official attempts to block them' (Murray 1978, 169). What he had in mind here were, unsurprisingly, the notorious Florentine prohibition of 1299 and the restrictions imposed on Paduan booksellers, which Murray dated to 1305 rather than 1348 (Murray 1978, 170–172).

2.3 Untrammelled growth

Prior to the 1980s, most authors who contributed to the spread of Leo Jordan's narrative regarding the controversial nature of zero in medieval Europe did so more or less unwittingly, as they uncritically repeated assertions already made by their predecessors. Even in cases where false information was conjured up out of thin air, these additions were relatively brief and do not necessarily expose their inventors as fantasists. This picture begins to change with two influential 'contributions' to the historiography made in 1987 by the mathematician and philosopher Brian Rotman and in 1994 by the mathematics teacher Georges Ifrah, which jointly ushered in a new and still-ongoing phase in the development of the myth. It is characterized by the repeated invention and rapid spread of ever more outlandish claims about the fate of zero and HAN during the European Middle Ages, claims which are now no longer tethered to even a shred of source evidence. This refusal to accompany bold assertions with references to primary or secondary sources already characterizes some of the relevant parts of Rotman's *Signifying Nothing* (1987), a free-floating exploration of the semotics of zero and nothingness. Having opened his book with the inaccurate statement that the mathematical sign zero entered 'Western consciousness' only in the thirteenth century (Rotman 1987, 1), the author later goes on to proclaim that it

entered European consciousness with difficulty and incomprehension. [...] Between the tenth and thirteenth century the sign stayed within the confines of Arab culture, resisted by Christian Europe, and dismissed by those whose function it was to handle numbers as an incomprehensible and unnecessary symbol. (Rotman 1987, 7)

What might have occasioned this sort of resistance? Rotman reaches deep to find an intellectually satisfying solution: ‘One answer would be that zero, being somehow about “nothing”, became therefore the object of a hostility to “nothing” already entrenched within Christian orthodoxy’ (Rotman 1987, 8). This orthodoxy, he claims, involved an Aristotelian *horror vacui* supported by the likes of St Augustine, who ‘assigned an eschatological status to “nothing”—it was the devil—which neatly Christianises the sort of horrific object Aristotle was at such pains to deny’ (Rotman 1987, 63). With one big leap across the history of Christian thought Rotman arrives at Thomas Aquinas and his ‘Aristotelian-based negativism’ in which ‘God [...] abhors nothing, and indeed destroys and nullifies “nothing” in order to create the world’ (Rotman 1987, 64). Rotman’s preference for historical conclusions based on sheer supposition rather than the careful citation of sources is manifest from the following summary, which may be regarded as emblematic of the ways in which some twentieth-century authors have managed to create intellectual history out of nothing:

Perhaps if zero had not made its appearance within Christian Europe, much of this larger interest in ‘nothing’ would not have occurred, and ‘nothing’ might have stayed within the writings of Aquinas and the Schoolmen as a remote theological issue. But this was not the case, and if from the tenth to the thirteenth century the church’s hostility to ‘nothing’ was successful in confining zero to Arab mathematics, and so staving off the threat to nullity it presented, the neo-Aristotelian apparatus of concepts inherited by Christian theology was too fixated against absence to even impinge on the larger issue of zero as a sign. Christian theology ignored the source and attacked the nihilistic consequences, the heretical and atheistic dangers of believing in and talking about ‘nothing’. (Rotman 1987, 64)

Rotman's narrative inventiveness pales in comparison to that brought to bear by Georges Ifrah, whose universal history of numbering systems, first published in French in 1981, went on to become one of the most impactful and celebrated books on the history of mathematics ever to appear in print.³⁸ In the original *Histoire universelle des chiffres*, the English version of which appeared in 1985, Jordan's narrative with regard to zero is present only in relatively faint traces. Explaining the shift in meaning undergone by various European derivatives of the word *cifra*, Ifrah resorts to the idea that

[b]ecause of the great simplification and speed that it brought to written computation, the zero was regarded as a mysterious, almost magic sign. That attitude gradually faded away (though a vestige of it can still be found in the use of 'cipher' to mean 'secret writing' or 'code') when written computation with the Hindu-Arabic numerals became commonplace. But since the zero sign had such an important place in that revolutionary numeration, the word for it came to designate any numeral of the system, and the French *chiffre*, for example, took on the meaning it still has today. (Ifrah 1985, 484)

Whatever restraint Ifrah may have shown in his first account of the universal history of numbers had completely disappeared by 1994, the year in which he published a dramatically enlarged version of his *Histoire universelle* in two volumes that covered not only the history of numbers, but also that of computing. Upon its French release in 1994, and again upon the publication of an English version in 1998, the popular press hailed the work as a monument to Ifrah's erudition, even as some experts expressed serious misgivings about his treatment of Mesopotamian, Chinese, Mayan, and Indian mathematics (Dauben 2002). Their negative

³⁸ Morrison and Morrison (1999, 546), even saw fit to include it among '100 or so books that shaped a century of science'.

verdict on Ifrah's reliability in these and other areas deserves to be extended to his treatment of the introduction of HAN into Europe, which offers a textbook example of the use of hyperbole and biased rhetoric in the service of lionizing one civilization (Islamic) while downgrading the other (Christian Europe). Ifrah's disdainful attitude towards Christian efforts to assimilate HAN is already apparent from the opening paragraphs of one of the relevant chapters, tendentiously titled 'The Slow Progress of Indo-Arabic Numerals in Western Europe':

When they first encountered numeral systems and computational methods of Indian origin, Europeans proved so attached to their archaic customs, so extremely reluctant to engage in novel ideas, that many centuries passed before written arithmetic scored its decisive and total victory in the West. (Ifrah 1998, 577)

Following a grossly exaggerated account of the difficulties Europeans experienced in carrying out simple calculations, Ifrah concludes: 'It is now perhaps easier to understand why skilled abacists were long regarded in Europe as magicians enjoying supernatural powers' (Ifrah 1998, 578). No source is provided, but Gerbert of Aurillac is singled out as the hero who could have introduced HAN early on, only to meet 'fierce resistance' as '[t]he time was simply not ripe for a great revolution of the mind' (Ifrah 1998, 579):

Some arithmeticians even put up a solid resistance to the new-fangled figures from the East by inscribing their *apices* with the Greek letter-numerals [...] or the Roman figures I to IX. Anything was better than having recourse to the 'diabolical signs' or the 'satanic accomplices' that the Arabs were supposed to be! (Ifrah 1998, 586)

After wrongly claiming that the second introduction of HAN was a result of the Crusades, such that the 'first European "algorists" were born at the gates of Jerusalem' (Ifrah 1998,

587), Ifrah goes on to depict their abacus-wielding opponents as ‘a powerful caste, enjoying the protection of the Church’:

They were inclined to keep the secrets of their art to themselves; they necessarily saw algorism, which brought arithmetic within everyone’s grasp, as a threat to their livelihood. [...] Knowledge, though it may now seem rudimentary, brought power and privilege when it represented the state of the art, and the prospect of seeing it shared seemed fearful, perhaps even sacrilegious, for its practitioners. But there was another, more properly ideological reason for European resistance to Indo-Arabic numerals. Even whilst learning was reborn in the West, the Church maintained a climate of dogmatism, of mysticism, and of submission to the holy scriptures, through doctrines of sin, hell and the salvation of the soul. Science and philosophy were under ecclesiastical control, were obliged to remain in accordance with religious dogma, and to support, not to contradict, theological teachings. The control of knowledge served not to liberate the intellect, but to restrict its scope for several centuries, and was the cause of several tragedies. Some ecclesiastical authorities thus pit it about that arithmetic in the Arabic manner, precisely because it was so easy and ingenious, reeked of magic and of the diabolical: it must have come from Satan himself! It was only a short step from there to sending over-keen algorists to the stake, along with witches and heretics. And many did indeed suffer that fate at the hands of Inquisition. The very etymology of the words ‘cypher’ and ‘zero’ provides evidence of this. (Ifrah 1998, 588–589)

Ifrah’s absurd statement that mathematicians were burned at the stake for using prohibited numerals makes him a trailblazer in the history of falsifying the record on medieval mathematics. No one before him had taken the implications of Jordan’s false narrative quite

as far. That this was more than a momentary slip of reason is clear from his final remarks of the matter, which once again depict the medieval Church as a monolithic and oppressive force with a firm grasp on all aspects of society:

The Church effectively issued a veto [on HAN], for it did not favour a democratisation of arithmetical calculation that would loosen its hold on education and thus weaken its power and influence; the corporation of accountants raised its own drawbridges against the ‘foreign’ invasion; and in any case the Church preferred the abacists—who were most often clerics as well—to keep their monopoly on arithmetic. ‘Arabic’ numerals and written calculation were thus for a long while almost underground activities. Algorists plied their skills in hiding, as if they were using a secret code. [...] So the history of words for zero also tell the history of our culture: each time we use the word ‘cipher’, we are also reviving a linguistic memory of the time when a *zero* was a dangerous *secret* that could have got you burned at the stake. (Ifrah 1998, 590)

The bold fictions created and divulged by Rotman and Ifrah were bound to encourage other authors to paint equally farcical pictures of the medieval Church as zero’s worst enemy. A comparatively moderate example of this tendency in the literature since 1994 may be spotted in Robert Kaplan’s bestselling history of zero, *The Nothing That Is*, which sets the stage for its particular version of the narrative by invoking Gerbert of Aurillac’s supposed use of an abacus-counter for zero (the so-called *sipos*) as a possible explanation of why the tenth-century monk was accused of conversing with ‘evil spirits’

since dealing with mathematics is bad enough, but letting nothingness loose in what passed for a civilized world just wouldn’t do. (Kaplan 1999, 52)

True to this negative framing of medieval European culture, Kaplan later expands on his opinion that opposition to zero stemmed from the triple threat of ‘superstition, bafflement and distrust’ (Kaplan 1999, 93):

Anything imported into what was still largely a peasant culture in the West would likely have been looked at askance; anything from the East was especially dangerous, seat as it was of old and still potent heresies. Most hated and feared of these was Manichaeism, that third-century AD mixture of Persian mythology and Gnostic theology, which lasted in various forms through the Middle Ages. It saw good and evil in equal struggle, God and the Devil fighting it out on the battlefield of Man. As conclusions accumulated to a system, two features stayed constant that matter for us: the first, that the void was identified with evil; the second, that forces and beings would be *evoked* into existence [...]. To the extent, therefore, that zero was connected in shape or meaning with the void, it had to be dealt with gingerly, if at all. [...] Superstition made zero abhorrent to the godly, while bringing it into the arcana of those who crossed over to the occult. (Kaplan 1999, 93–96)

Having vaguely associated the circular symbol for zero with alchemy, Jungian psychology, and tarot cards (Kaplan 1999, 96–97), Kaplan closes his account by echoing Jordan’s and later authors’ misuse of Gautier de Coinci:

We laugh at those who can’t count—but in the thirteenth century they laughed at those who could, making ‘ciper’ and ‘the zero of algorismus’ terms of derision, because of their uselessness. (Kaplan 1999, 102)

Kaplan’s insinuations that the very thought of zero was anathema to the medieval mind were rather subtle compared to the way they were expanded upon in writing two years later by the

journalist Dick Teresi, whose best-selling account of the *Lost Discoveries* made (supposedly) by non-European civilizations asks the reader to imagine themselves in fifteenth-century Italy:

You are, let's say, a bookseller. You need to keep track of sales and inventory. [...] How do you get your accounts to balance? Like other merchants, you keep a secret set of books, in the gobar, or Gwalior, numerals, the so-called Hindu-Arabic numerals [...]. You would keep these books secret because in 1348 the ecclesiastical authorities of the University of Padua prohibited the use of 'ciphers' in the price listing of books, ruling the prices must be stated in 'plain' letters. A century earlier, a Florentine edict had forbidden bankers to use the 'infidel' symbols. (Teresi 2002, 24)

Teresi evidently derived some of his misleading diction ('ecclesiastical authorities' and 'infidel numbers') from Lancelot Hogben, although he goes much further than the latter in suggesting that the famous prohibitions were so severe that they forced merchants to perform their book-keeping in hiding:

Numbers were dangerous; at least these Indian numbers were. They were contraband. The zero was the most unholy: a symbol for nothingness, a Hindu concept, influenced by Buddhism and transplanted to Christian Europe. It became a secret sign, a signal between fellow travelers. [...] Flashing a zero to another merchant let him know that you were a user of Hindu-Arabic numerals. In many principalities, Arabic numerals were banned from official documents; in others, the numbers were prohibited altogether. Math was sometimes exported to the West by 'bootleggers' in Hindu-Arabic numerals. There is plentiful evidence of such illicit number use in thirteenth-century archives in Italy, where merchants used Gwalior numbers as a secret code. (Teresi 2002, 25).

Teresi's failure to cite any of this 'plentiful evidence' is unsurprising given his sole reliance on Tobias Dantzig, whose inexplicable reference to 'the thirteenth century archives of Italy' has already been mentioned. The citation practices in Teresi's book become even less transparent one paragraph later, as he fantasizes about the ways in which an 'out-of-work mathematician in Italy in the late Middle Ages' would have been able support themselves by becoming 'an itinerant math performer':

Traveling from town to town, you would set up in the village square and perform 'magic' tricks for the public. Multiplying 27 by 14 was considered as entertaining in that era as sword swallowing or juggling, and fewer people could do it. The public would toss coins in your cup. You would count your take at the end of each performance—secretly using Hindu-Arabic numerals, of course. (Teresi 2002, 25)

An endnote added to the passage in question alleges that Teresi's informant on this otherwise unheard-of aspect of medieval mathematical culture was an 'Interview with Robert Kaplan, January 1, 2000' (Teresi 2002, 371).³⁹ If the success of Kaplan's and Teresi's monographs bears some responsibility for the continuing popularity of the myth, an even bigger share ought probably to be accorded to Charles Seife's bestseller *Zero: Biography of Dangerous Idea*, which earned its author the 2001 PEN/Martha Albrand Award for First Nonfiction. Medieval Christianity's alleged fear of nothingness and zero is here more than just a historical footnote but provides a guiding theme of the entire book. This irrational fear, Seife alleges, was already there at the dawn of civilization:

³⁹ Asked to corroborate the reference in an email exchange of 1 June 2019, Kaplan responded to me with the following: 'Rather than misquoting me, let's say [Teresi] well exercised his considerable powers of summing up'.

It is hard to imagine being afraid of a number. Yet zero was inexorably linked with the void—with nothing. There was a primal fear of void and chaos. There was also a fear of zero. Most ancient peoples believed that only emptiness and chaos were present before the universe came to be. [...] Zero represented that void. But the fear of zero went deeper than unease about the void. To the ancients, zero's mathematical properties were inexplicable, as shrouded in mystery as the birth of the universe.

(Seife 2000, 19–20)

According to the yarn spun by Seife, which owes a lot to Rotman's *Signifying Nothing*, medieval aversion towards zero was in part the continuation of two ancient Greek anxieties cultivated by the likes of Pythagoras and Aristotle, namely, 'a fear of the infinite and a horror of the void'.

Medieval scholars branded void as evil—and evil as void. Satan was quite literally nothing. Boethius made the argument as follows: God is omnipotent. There is nothing God cannot do. But God, the ultimate goodness, cannot do evil. Therefore evil is nothing. It made perfect sense to the medieval mind. (Seife 2000, 61)

An example of how the undocumented assertions in 'popular science' literature can feed back into ostensibly serious scholarship is offered by an article by Jürgen Fröhlich (2003), who seeks to reflect on the symbolism of the number zero in an early modern antisemitic caricature, but in doing so derives much of his understanding of the medieval backstory (concerning the 'demonization' of zero) from the German translations of Ifrah, Rotman, Kaplan, and Seife (Fröhlich 2003, 136 n. 5, 143–152; see also Caianiello 2014, 229 n. 59). The resulting account is nevertheless far more reliable than that offered by Ronald Green, who in his 2011 book *Nothing Matters* not just eagerly repeats Seife's glib generalisations

about Western attitudes toward zero, but ends up raising them to cartoonish new levels. The narrative of *Nothing Matters* is premised on the assertion that the concept of *nothing* stifled Western progress for six centuries (up to Galileo Galilei!), as Christian hostility towards it led to the suppression of the number zero. Already on the very first page readers are informed that

there was a time when merely mentioning *nothing* would have been a very big deal. So big in fact that zero [...] was absolutely banned on pain of death. Dying for zero would have been dying for *nothing*. (Green 2011, 1)

The institution administering this pointless death was, or so Green's account suggests, 'the Church', an institution that did 'everything in its power' to eradicate the dreaded number zero, including 'ridicule, bribery, anti-zero laws and strong-arm tactics, in line with the general policy against anything modern, which meant anything that could be construed as heresy' (Green 2011, 8; see also p. 35). As for the Church's motivations, Green offers an unexpected one: zero had 'religious significance in India—it signified the uniquely Buddhist “no-thing-ness” paradigm of enlightenment' (Green 2011, 14). While he concedes that it is 'doubtful whether Fibonacci knew about spirituality in India', he is willing to affirm that

it is not unlikely that word from the East had reached the ears of Pope Innocent III from his emissaries. Did they know that in Sanskrit, the word for 'enlightenment as no-thing-ness' happened to also be the word for 'zero as a number'? This was a serious point, as far as the Church was concerned, for it backed up their theological arguments that *nothing* was a heretical notion and it reinforced what the Church had said about the ungodliness of the concept; here, after all, was proof of its use by idol-worshipping cults, as seen from the viewpoint of the Church. (Green 2011, 14)

As with other contemporary authors, Green seems either blissfully unaware or criminally insouciant about the fact that his entire story is but a *creatio ex nihilo*.

Conclusion

As the foregoing investigation has shown, the false narrative surrounding the medieval suppression of zero is almost entirely a product of the twentieth century, the most extreme versions of which have been circulating only since the 1990s. The ‘patient zero’ in its development appears to have been an article published in 1905 by Leo Jordan, who read into the available sources signs of a protracted struggle between abacists and algorists, whose advocacy of HAN-based arithmetic faced opposition as long as the number zero was ridiculed for its strangeness or feared for its mysterious, ‘satanic’, and ‘magical’ associations. Jordan’s arguments, as indicated above, are in large parts rhetorical and do not withstand simple scrutiny. This has not kept them from influencing the subsequent historiography in rather profound ways, to the extent that his narrative has been echoed, adapted, and sometimes even expanded by some widely read authors.

What nearly all these authors have in common is that they made no discernible attempt to add new sources to the picture, or to subject the existing ones to an independent evaluation. The way Jordan’s narrative evolved over time is instead more reminiscent of a game of telephone, albeit one aided by certain external factors. One of them is the widely felt temptation to explain the etymology of ‘cipher’ through a just-so story involving the enforced secrecy in the use of HAN. Another is the recorded existence of local prohibitions of the use of HAN in late medieval Italy. Two of these prohibitions (concerning Florentine bankers and Paduan booksellers) are reported in the literature with such frequency that constant repetition, decontextualization, and confirmation bias eventually turned them into ‘evidence’ that the medieval Church banned zero. The sensationalistic narrative that could be concocted from

these elements reached maturation in the astonishingly successful ‘universal history’ of Georges Ifrah, who augmented claims made by earlier authors with a hefty dose of his own imagination. While few writers since Ifrah have dared to set the bar any higher, some have added further layers to the narrative, as with the contention—found in books by Rotman, Kaplan, Seife, and Green—that ‘the West’ feared zero because it projected on it its much older fear of the void.

On the whole the literature surveyed in this article offers a rather depressing case study of the way careless scholarship and failure to check one’s sources can allow false narratives and made-up ‘facts’ to proliferate in the modern world. In the specific case of the medieval reception of HAN and zero, historical accuracy has not been helped by the fact that the history of mathematics is still often written by mathematicians rather than individuals trained in the techniques of historical research. Lack of such training makes it all the easier to resort to simple fallacies, as when conclusions about the past are derived from schematic assumptions about the parameters of medieval thought. To the historically uninformed, but self-consciously ‘modern’ observer, it may seem obvious that these parameters can be derived by inverting familiar Enlightenment principles. The cognitive ‘medieval’ is then tantamount to succumbing to any kind of superstition and intolerance, and to see the Devil lurking beneath the surface of the unfamiliar (including the number zero). Medieval historians are instead aware that the true abode of the Devil is found elsewhere: in the detail.

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References

- Allard, André, ‘A propos d’un algorithme latin de Frankenthal: une méthode de recherche’, *Janus*, 65 (1978), 119–141.
- Allard, André, ‘La formation du vocabulaire latin de l’arithmétique médiévale’, in Olga Weijers (ed), *Méthodes et instruments du travail intellectuel au moyen âge: études sur le vocabulaire*, Turnhout Brepols, 1990, 137–181.
- Allard, André, ‘The Arabic origins and development of Latin algorisms in the twelfth century’, *Arabic sciences and philosophy*, 1 (1991), 233–283.
- Allard, André (ed), *Muhammad Ibn Mūsā al-Khwārizmī: Le calcul indien (Algorismus)*, Paris: Blanchard, 1992.
- Ambrosetti, Nadia, *L’eredità arabo-islamica nelle scienze e nelle arti del calcolo dell’Europa medievale*, Milan: Edizioni Universitarie di Lettere Economia Diritto, 2008.
- Ambrosetti, Nadia, ‘Algorithmic in the 12th century: the Carmen de Algorismo by Alexander de Villa Dei’, in Fabio Gadducci and Mirko Tamosanis (eds), *History and philosophy of computing*, Cham: Springer, 2016, 71–86.
- Ball, W. W. Rouse, *A short account of the history of mathematics*, London: Macmillan, 4th edition, 1908.
- Beaujouan, Guy, ‘Etude paléographique sur la “rotation” des chiffres et l’emploi des *apices* du X^e au XII^e siècle’, *Revue d’histoire des sciences*, 1, no. 4 (1948), 301–313.
- Beaujouan, Guy, ‘Les chiffres arabes selon Gerbert: l’abaque du Pseudo-Boèce, XI^e siècle’, in Olivier Guyotjeannin and Emmanuel Poulle (eds), *Autour de Gerbert d’Aurillac: le pape de l’an mil*, Paris: École des Chartes, 1996, 322–328.
- Bellos, Alex, *Here’s looking at Euclid: a surprising excursion through the astonishing world of maths*, New York: Free Press, 2010.

- Berggren, J. Lennart, ‘Medieval arithmetic: Arabic texts and European motivations’, in John J. Contreni and Santa Casciani (eds), *Word, image, number: communication in the Middle Ages*, Florence: SISMEL, 2002, 351–365.
- Bishop, Chris, ‘The “Pear of Anguish”: truth, torture and dark medievalism’, *International journal of cultural studies*, 17 (2014), 591–602.
- Bossuat, Robert (ed), *Alain de Lille: Anticlaudianus*, Paris: Vrin, 1955.
- Bubnov, Nicolaus (ed), *Gerberti Opera mathematica (972–1003)*, Berlin: Friedländer, 1899.
- Burnam, John M., *Palaeographia iberica: fac-similés de manuscrits espagnols et portugais (IX^e–XV^e siècles) avec notices et transcriptions*. Paris: Champion, 1912–25.
- Burnett, Charles, ‘*Algorismi vel helcep decentior est diligentia*: the arithmetic of Adelard of Bath and his circle’, in Menso Folkerts (ed), *Mathematische Probleme im Mittelalter: Der lateinische und arabische Sprachbereich*, Wiesbaden: Harrassowitz, 1996, 221–331. Reprinted as Chapter 3 in Burnett 2010.
- Burnett, Charles, ‘Latin alphanumerical notation, and annotation in Italian, in the twelfth century: MS London, British Library, Harley, 5402’, in Menso Folkerts and Richard Lorch (eds), *Sic itur ad astra: Studien zur Geschichte der Mathematik und Naturwissenschaften*, Wiesbaden: Harrassowitz, 2000, 77–90. Reprinted as Chapter 10 in Burnett 2010.
- Burnett, Charles, ‘Indian numerals in the Mediterranean Basin in the twelfth century, with special reference to the “eastern forms”’, in Yvonne Dold-Samplonius, Joseph W. Dauben, Menso Folkerts, and Benno van Dalen (eds), *From China to Paris: 2000 years transmission of mathematical ideas*, Stuttgart: Steiner, 2002a, 237–288. Reprinted as Chapter 5 in Burnett 2010.

- Burnett, Charles, ‘Learning Indian arithmetic in the early thirteenth century’, *Boletín de la Asociación Matemática Venezolana*, 9 (2002b), 15–26. Reprinted as Chapter 9 in Burnett 2010.
- Burnett, Charles, ‘The abacus at Echternach in ca. 1000 A.D.’, *Sciamus*, 3 (2002c), 91–108. Reprinted as Chapter 1 in Burnett 2010.
- Burnett, Charles, ‘Fibonacci’s “Method of the Indians”’, *Bollettino di storia delle scienze matematiche*, 23, no. 2 (2003), 87–97. Reprinted as Chapter 11 in Burnett 2010.
- Burnett, Charles, ‘The semantics of Indian numerals in Arabic, Greek and Latin’, *Journal of Indian philosophy*, 34 (2006), 15–30.
- Burnett, Charles, *Numerals and arithmetic in the Middle Ages*, Farnham: Ashgate, 2010.
- Burnett, Charles, Ji-Wei Zhao, and Kurt Lampe, ‘The Toledan *Regule* (*Liber Alchorismi*, part II): a twelfth-century arithmetical miscellany’, *Sciamus*, 8 (2007), 141–231. Reprinted as Chapter 8 in Burnett 2010.
- Burton, David M., *The history of mathematics: an introduction*, New York: McGraw-Hill, 7th edition, 2011 (first published in 1985).
- Busard, H. L. L., ‘Über zwei Algorithmus-Schriften aus dem 13. Jahrhundert’, in Menso Folkerts and Richard Lorch (eds), *Sic itur ad astra: Studien zur Geschichte der Mathematik und Naturwissenschaften*, Wiesbaden: Harrassowitz, 2000, 91–137.
- Caianiello, Eva, ‘Leonardo of Pisa and the *Liber Abaci*: biographical elements and the project of the work’, in Alain Bernard and Christine Proust (eds), *Scientific sources and teaching contexts throughout history: problems and perspectives*, Dordrecht: Springer, 2014, 217–246.
- Cajori, Florian, *A history of mathematics*, New York: Macmillan, 1894.
- Camerani Marri, Giulia (ed), *Statuti dell’Arte del Cambio di Firenze (1299–1316)*, Florence: Olschki, 1955.

- Cantor, Moritz, 'Ueber einen Codex des Klosters Salem', *Zeitschrift für Mathematik und Physik*, 10 (1865), 1–16.
- Cherubini, Paolo, 'Il numero come elemento di disturbo', in Rita Librandi and Rosa Piro (eds), *Lo scaffale della biblioteca scientifica in volgare (secoli XIII–XVI)*, Florence: SISMEL, 2006, 313–339.
- Chrisomalis, Stephen, *Numerical notation: a comparative history*, Cambridge: Cambridge University Press, 2010.
- Classen, Albrecht, *The medieval chastity belt: a myth-making process*, New York: Palgrave Macmillan, 2007.
- Cochrane, Louise, *Adelard of Bath: the first English scientist*, London: British Museum Press, 1994.
- Crossley, John N., 'Old-fashioned versus newfangled: reading and writing numbers, 1200–1500', *Studies in medieval and Renaissance history*, 3rd ser., 10 (2013), 79–109.
- Cunnington, Susan, *The story of arithmetic: a short history of its origin and development*, London: Sonnenschein & Co, 1904.
- Dantzig, Tobias, *Number: the language of science*, New York: Pi Press, republication of 4th edition, 2005 (first published in 1930).
- Dauben, Joseph, 'The universal history of numbers and The universal history of computing', *Notices of the American Mathematical Society*, 49 (2002), 32–38, 211–216.
- Denifle, Heinrich, 'Die Statuten der Juristen-Universität Padua vom Jahre 1331', *Archiv für Literatur- und Kirchengeschichte des Mittelalters*, 6 (1892), 309–562.
- Evans, Gillian R. 'Difficillima et ardua: theory and practice in treatises on the abacus, 950–1150', *Journal of medieval history*, 3 (1977a), 21–38.
- Evans, Gillian R., 'From abacus to algorism: theory and practice in medieval arithmetic', *British journal for the history of science*, 10 (1977b), 114–131.

Evans, Gillian R., ‘Schools and scholars: the study of the abacus in English schools c. 980–c. 1150’, *English historical review*, 94 (1979), 71–89.

Evans, Gillian R., ‘The *saltus Gerberti*: the problem of the “leap”’, *Janus*, 67 (1980), 261–268.

Folkerts, Menso, ‘Frühe Darstellungen des Gerbertschen Abakus’, Raffaella Franci, Paolo Pagli, and Laura Toti Rigatelli (eds), *Itinera mathematica: studi in onore di Gino Arrighi per il suo 90° compleanno*, Siena: Centro studi sulla matematica medioevale, Università di Siena, 1996, 23–43.

Folkerts, Menso (ed), *Die älteste lateinische Schrift über das indische Rechnen nach al-Hwārizmī* (Abhandlungen der Bayerischen Akademie der Wissenschaften, phil.-hist. Kl., n.s., 113.), Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1997.

Folkerts, Menso, ‘Frühe westliche Benennungen der indisch-arabischen Ziffern und ihr Vorkommen’, in Menso Folkerts and Richard Lorch (eds), *Sic itur ad astra: Studien zur Geschichte der Mathematik und Naturwissenschaften*, Wiesbaden: Harrassowitz, 2000, 216–233.

Folkerts, Menso, ‘Early texts on Hindu-Arabic calculation’, *Science in Context*, 14 (2001a), 13–38.

Folkerts, Menso, ‘The names and forms of the numerals on the abacus in the Gerbert tradition’, in Fabio G. Nuvolone (ed), *Gerberto d’Aurillac, da Abate di Bobbio a Papa dell’Anno 1000*, Bobbio: Associazione culturale Amici di *Archivum Bobiense*, 2001b, 245–265. Modified reprint as Chapter 6 of Folkerts, *Essays on early medieval mathematics: the Latin tradition*, Aldershot: Ashgate, 2003.

Folkerts, Menso, and Barnabas Hughes, ‘The Latin mathematics of medieval Europe’, in Victor J. Katz (ed), *Sourcebook in the mathematics of medieval Europe and North Africa*, Princeton, NJ: Princeton University Press, 2016, 4–223.

- Franci, Raffaella, ‘Leonardo Pisano e la trattatistica dell’abaco in Italia nei secoli XIV e XV’, *Bollettino di storia delle scienze matematiche*, 23, no. 2 (2003), 33–54.
- Fröhlich, Jürgen, ‘*Meßkram* oder Die Einwanderung der Null in den modernen Schaltkreislauf über das spätmittelalterliche Rechnungsbuch’, in Ulrich Schmitz and Horst Wenzel (eds), *Wissen und neue Medien: Bilder und Zeichen von 800 bis 2000*, Berlin: Schmidt, 2003, 135–158.
- Frova, Carla, ‘Le opere aritmetiche di Gerberto d’Aurillac’, in *Studi sul medioevo cristiano offerti a Raffaello Morghen per il 90° anniversario dell’Istituto Storico Italiano (1883–1973)*, Vol. 1, Rome: Istituto Storico Italiano, 1974, 323–353.
- Gazalé, Midhat, *Number: From Ahmes to Cantor*, Princeton, NJ: Princeton University Press, 2000.
- Gherardi, Alessandro (ed), *Statuti della università e Studio Fiorentino dell’anno MCCCLXXXVII*, Florence: Vieusseux, 1881.
- Gómez Pallarès, Juan, ‘Sobre manuscritos latinos de cómputo en escritura visigótica’, *Hispania Sacra*, 39 (1987), 25–48.
- Gibson, Craig A., and Francis Newton, ‘Pandulf of Capua’s *De calculatione*: An illustrated abacus treatise and some evidence for the Hindu-Arabic numerals in eleventh-century South Italy’, *Mediaeval studies*, 57 (1995), 293–335.
- Green, Ronald, *Nothing Matters*, Alresford: iff Books, 2011.
- Haskins, Charles Waldo, *Business Education and Accountancy*, New York: Harper & Brothers, 1904.
- Herreman, Alain, ‘La mise en texte mathématique: une analyse de l’“Algorithme de Frankenthal”’, *Methodos* 1 (2001), <http://journals.openedition.org/methodos/45>; DOI: 10.4000/methodos.45. Accessed on 10 June 2019.
- Hitchens, Christopher, *God is not great: the case against religion*, London: Atlantic, 2007.

- Hogben, Lancelot, *Mathematics for the million: a popular self educator*, London: George Allen & Unwin, 1936.
- Horn-d'Arturo, Guido, 'Numeri arabici e simboli celesti', *Memorie della Società astronomica italiana*, n.s., 3 (1925) 195–212.
- Høyrup, Jens, *Jacopo da Firenze's Tractatus algorismi and early Italian abacus culture*, Basel: Birkhäuser, 2007.
- Häring, Nikolaus M., 'Alan of Lille: *De planctu Naturae*', *Studi medievali*, 3rd ser., 19 (1978), 797–879.
- Harris, Stephen J., and Byron Lee Grigsby (eds), *Misconceptions about the Middle Ages*, New York: Routledge, 2008.
- Ifrah, Georges, *From one to zero: a universal history of numbers* (trans Lowell Bair), New York: Viking Penguin, 1985 (original work published 1981).
- Ifrah, Georges, *The universal history of numbers: from prehistory to the invention of the computer* (trans David Bellos, E. F. Harding, Sophie Wood, and Ian Monk), London: Harvill Press, 1998 (original work published 1994).
- Jordan, Leo, 'Materialien zur Geschichte der arabischen Zahlzeichen in Frankreich', *Archiv für Kulturgeschichte*, 3 (1905), 155–195.
- Jordan, Leo, 'Wortgeschichtliches', in E. Stollreither (ed), *Festschrift zum XII. Allgemeinen deutschen Neuphilologentage in München, Pfingsten 1906*, Erlangen: Junge, 1906, 61–80.
- Kaplan, Robert, *The nothing that is: a natural history of zero*, London: Allen Lane, 1999.
- Karpinski, Louis C., 'Two Twelfth-Century Algorisms', *Isis*, 3 (1921), 396–413.
- King, David A., *The ciphers of the monks: a forgotten number-notation of the Middle Ages* (Boethius, 44), Stuttgart: Steiner, 2001.
- Kirchhoff, Albrecht, *Die Handschriftenhändler des Mittelalters*, Leipzig: Kirchhoff, 2nd ed., 1853.

- Kline, Morris, *Mathematical thought from ancient to modern times*, New York: Oxford University Press, 1972.
- Koenig, V. Frederic (ed), *Les Miracles de Nostre Dame par Gautier de Coinci*, Vol. 4, Geneva: Droz, 1970.
- Krantz, Steven G., *The proof is in the pudding: the changing nature of mathematical proof*, New York: Springer, 2011.
- Kunitzsch, Paul, *Zur Geschichte der 'arabischen' Ziffern* (Sitzungsberichte der Bayerischen Akademie der Wissenschaften, phil.-hist. Kl., 2005.3.), Munich: Verlag der Bayerischen Akademie der Wissenschaften, 2005.
- Lebsanft, Franz, 'Ein deutsch-jüdisches Schicksal: Der Philologe und Linguist Leo Jordan (1874–1940)', in Hans Helmut Christmann, Frank-Rutger Hausmann, and Manfred Briegel (eds), *Deutsche und österreichische Romanisten als Verfolgte des Nationalsozialismus*, Tübingen: Stauffenburg, 1989, 157–175, 287–288.
- Lemay, Richard, 'The Hispanic origin of our present numeral forms', *Viator*, 8 (1977), 435–477.
- Lemay, Richard, 'Arabic numerals', in Joseph R. Strayer (ed), *Dictionary of the Middle Ages*, Vol. 1, *Aachen–Augustinism*, New York: Charles Scribner's Sons, 1982, 382–398.
- Lohr, Alfred (ed), *Opera de computo saeculi duodecimi* (Corpus Christianorum Continuatio Mediaevalis, 272), Turnhout: Brepols, 2015.
- Lohr, Alfred, and C. Philipp E. Nothaft (eds), *Robert Grosseteste's Computus*, Oxford: Oxford University Press, 2019.
- Lüneburg, Heinz, *Von Zahlen und Größen: Dritthalbtausend Jahre Theorie und Praxis*, Vol. 1, Basel: Birkhäuser, 2008.
- Lyons, Jonathan, *Islam through Western eyes: from the Crusades to the war on terrorism*. New York: Columbia University Press, 2012.

- Malagola, Carlo (ed), *Statuti delle università e dei collegi dello Studio Bolognese*, Bologna: Zanichelli, 1888.
- Menninger, Karl, *Number words and number symbols: a cultural history of numbers* (trans Paul Broneer), Cambridge, Mass.: M.I.T. Press, 1969 (original work published 1934).
- Morrison, Philip, and Phylis Morrison, '100 or so books that shaped a century of science', *American Scientist*, 87 (1999), 542–544, 546, 549–550, 553.
- Murray, Alexander, *Reason and society in the Middle Ages*, Oxford: Clarendon Press, 1978.
- Mynors, R. A. B. (ed), *William of Malmesbury: Gesta regum Anglorum; the History of the English kings*, Vol. 1, completed by R. M. Thomson and M. Winterbottom, Oxford: Clarendon Press, 1998.
- Nagl, Alfred, 'Ueber eine Algorismus-Schrift des XII. Jahrhunderts und über die Verbreitung der indisch-arabischen Rechenkunst und Zahlzeichen im christl. Abendlande', *Zeitschrift für Mathematik und Physik*, hist.-lit. Abt., 34 (1889), 129–146, 161–170.
- Neill Wright, G. G., *The writing of Arabic numerals*, London: University of London Press, 1952.
- Nothaft, C. Philipp E., 'The reception and application of Arabic science in twelfth-century computistics: new evidence from Bavaria', *Journal for the history of astronomy*, 45 (2014), 35–60.
- Numbers, Ronald L. (ed), *Galileo goes to jail and other myths about science and religion*, Cambridge, Mass.: Harvard University Press, 2009.
- Otisk, Marek, 'The interpretations and applications of Boethius's *Introduction to the Arithmetic* II, 1 at the end of the 10th century', *GERBERTVS*, 2 (2011), 33–56.
- Otisk, Marek, 'Descriptions and images of the early medieval Latin abacus', *Średniowiecze Polskie i Powszechne*, 11 (2015), 13–35.
- Pareja, Felix M., *Islamologia*, Rome: Orbis Catholicus, 1951.

- Pedersen, Fritz Saaby (ed), *Petri Philomenae de Dacia et Petri de S. Audomaro Opera quadrivialia*, Vol. 1, *Opera Petri Philomenae*. Copenhagen: Gad, 1983.
- Pedersen, Olaf, 'In quest of Sacrobosco', *Journal for the history of astronomy*, 16 (1985), 175–221.
- Pellegrini, Giovan Battista, *Gli arabismi nelle lingue neolatine con speciale riguardo all'Italia*, Brescia: Paideia, 1972.
- Pernoud, Régine, *Pour en finir avec le Moyen Âge*, Paris: Éditions du Seuil, 1977.
- Principe, Lawrence M., "The warfare thesis," in Jeff Hardin, Ronald L. Numbers, and Ronald A. Binzley (eds), *The warfare between science and religion: the idea that wouldn't die*, Baltimore, Md: Johns Hopkins University Press, 2018, 6–26.
- Pullan, J. M., *The history of the abacus*. London: Hutchinson, 1968.
- Rauhaus, Alfred, *Kleine Kirchenkunde: Reformierte Kirchen von innen und außen*, Göttingen: Vandenhoeck & Ruprecht, 2007.
- Reinhardt, Thomas, 'Die Erfindung der flachen Erde: Der Mythos Kolumbus und die Konstruktion der Epochenschwelle zwischen Mittelalter und Neuzeit', *Paideuma*, 53 (2007), 161–180.
- Reynolds, Barbara E., 'The algorists vs. the abacists: an ancient controversy on the use of calculators', *The college mathematics journal*, 24 (1993), 218–223.
- Ricklin, Thomas, 'Der Philosoph als Nekromant: Gerbert von Aurillac (Silvester II.) und Vergil im europäischen Hochmittelalter', *Interfaces*, 1 (2015), 236–264.
- Roland, Bernhard, 'Der Eingang des "Mythos der flachen Erde" in deutsche und österreichische Geschichtsschulbücher im 20. Jahrhundert', *Geschichte in Wissenschaft und Unterricht*, 64 (2013), 687–701.
- Rotman, Brian, *Signifying nothing: the semiotics of zero*, Basingstoke: Macmillan, 1987.

- Russell, Jeffrey Burton, *Inventing the flat Earth: Columbus and modern historians*, New York: Praeger, 1991.
- Sanford, Vera, *A short history of mathematics*, London: George G. Harrap & Co., 1930.
- Schärlig, Alain. *Du zéro à la virgule: les chiffres arabes à la conquête de l'Europe, 1143–1585*. Lausanne: Presses polytechniques et universitaires romandes.
- Seife, Charles, *Zero: the biography of a dangerous idea*, London: Souvenir, 2000.
- Smith, David Eugene, and Jekuthiel Ginsburg, *Numbers and numerals: a story book for young and old*, Washington, DC: National Council of Teachers of Mathematics, 1937.
- Smith, David Eugene, and Louis Charles Karpinski, *The Hindu-Arabic numerals*, Boston, Mass.: Ginn and Company, 1911.
- Spiesser, Maryvonne, 'Questions sur la diffusion du *Liber Abbaci* en France au XV^e siècle à travers l'étude des traités commerciaux', *Bollettino di storia delle scienze matematiche* 24, no. 1 (2004), 115–135.
- Stevens, Wesley M., 'A small circle in the *Tabula Paschalis* of Murbach, A.D. 814–820', *Physis*, n.s., 49 (2013–14), 1–19.
- Stone, Williard E., 'Abacists versus algorists', *Journal of accounting research*, 10 (1972), 345–350.
- Struik, Dirk J., 'The prohibition of the use of Arabic numerals in Florence', *Archives internationales d'histoire des sciences*, 21 (1968), 291–294.
- Sulowski, Jan (ed), *Radulphus de Longo Campo: In Anticlaudianum Alani commentum* (Źródła do dziejów nauki i techniki, 13), Wrocław: Zakład Narodowy im. Ossolińskich, 1972.
- Szalay, Jessie, 'Who invented zero?', *Live Science*, 18 September 2017, <https://www.livescience.com/27853-who-invented-zero.html>. Accessed on 10 June 2019.
- Szombathely, Marino (ed), *Statuti di Trieste del 1350*, Trieste: Cappelli, 1930.

- Tabarroni, Giorgio, 'La matematica occidentale dopo il Mille: sua interazione con la vita quotidiana e la cultura', in *'Imago mundi': la conoscenza scientifica nel pensiero basomedievale* (Convegni del Centro di studi sulla spiritualità medievale, Università degli studi di Perugia, 22), Todi: Presso l'Accademia tudertina, 1983, 141–153.
- Taylor, Isaac, *The alphabet: an account of the origin and development of letters*, Vol. 2, *Aryan alphabets*, London: Kegan Paul, 1883.
- Teresi, Dick, *Lost discoveries: the ancient roots of modern science—from the Babylonians to the Maya*, New York: Simon & Schuster, 2002.
- Tropfke, Johannes, *Geschichte der Elementar-Mathematik in systematischer Darstellung mit besonderer Berücksichtigung der Fachwörter*, Vol. 1, *Rechnen*, Berlin: de Gruyter, 2nd ed., 1921.
- Truitt, Elly R., 'Celestial divination and Arabic science in twelfth-century England: the history of Gerbert of Aurillac's talking head', *Journal of the history of ideas*, 73 (2012), 201–222.
- Tucci, Ugo, 'Il documento del mercante', in *Civiltà comunale: libro, scrittura, documento; atti del convegno, Genova, 8–11 novembre 1988*, Genova: Società ligure di storia patria, 1989, 541–565.
- Ulivì, Elisabetta, 'Maestri e scuole d'Abaco a Firenze: la Bottega di Santa Trinita', *Bollettino di storia delle scienze matematiche* 24, no. 1 (2004), 43–91.
- Ulivì, Elisabetta, *Gli abacisti fiorentini delle famiglie 'del maestro Luca', Calandri e Micceri e le loro scuole d'abaco (secc. XIV–XVI)*, Florence: Olschki, 2013.
- Van Egmond, Warren, *Practical mathematics in the Italian Renaissance: a catalog of Italian abacus manuscripts and printed books to 1600*, Florence: Istituto e museo di storia della scienza, 1980.

- Wallin, Nils-Bertil, 'The history of zero', *YaleGlobal Online*, 19 November 2002, <https://yaleglobal.yale.edu/history-zero>. Accessed on 10 June 2019.
- Wedell, Moritz, 'Numbers', in Albrecht Classen (ed), *Handbook of medieval culture: fundamental aspects and conditions of the European Middle Ages*, Vol. 2, Berlin: de Gruyter, 2015, 1205–1260.
- Weill-Parot, Nicolas, and Véronique Sales (eds), *Le vrai visage du Moyen Âge: au-delà des idées reçues*, Paris: Vendémiaire, 2017.
- Weiss, Ittay, 'Nothing matters: how the invention of zero helped create modern mathematics', *The Conversation*, 20 September 2017, <https://theconversation.com/nothing-matters-how-the-invention-of-zero-helped-create-modern-mathematics-84232>. Accessed on 10 June 2019.
- Westmaas, Reuben, 'The concept of zero is newer than you'd expect', *Curiosity.com*, 7 September 2017, <https://curiosity.com/topics/the-concept-of-zero-is-newer-than-you-d-expect-curiosity>. Accessed on 10 June 2019.
- Wolf, Jürgen, *Die Moderne erfindet sich ihr Mittelalter—oder wie aus der 'mittelalterlichen Erdkugel' eine 'neuzeitliche Erdscheibe' wurde* (Akademie der Wissenschaften und der Literatur in Mainz, Abhandlungen der geistes- und sozialwissenschaftlichen Klasse, 2004.5), Stuttgart: Steiner, 2004.
- Yeldham, Florence A., *The story of reckoning in the Middle Ages*, London: George G. Harrap & Co., 1926.