1972

Memory dependent search processes: evidence for a fast self-terminating scan.

Karl Dieter Gutschera
University of Massachusetts Amherst

R-0142

MEMORY DEPENDENT SEARCH PROCESSES: EVIDENCE FOR A FAST SELF-TERMINATING SCAN

A Thesis Presented

by

Karl Dieter Gutschera

Submitted to the Graduate School of the
University of Massachusetts in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

August, 1972

PSYCHOLOGY

MEMORY DEPENDENT SEARCH PROCESSES:

EVIDENCE FOR A FAST SELF-TERMINATING SCAN

A Thesis by

Karl D. Gutschera

Approved as to style and content by:

(Chairman of Committee)

(Head of Department)

(Member)

Alexander Pollatsek

(Member)

August	1972
(Month)	(Year)

Acknowledgements

I wish to express my appreciation to my committee chairman, Charles Clifton, Jr., for his helpful assistance in the development of this thesis, and to Alexander Pollatsek and Bill Eichelman, the other members of my committee, for their helpful comments and criticism. Further, I want to thank Jerome L. Myers and Jim Chumbley for their assistance in developing the mathematical equations.

This research was supported in part by a research grant from the National Institute of Mental Health which was awarded to Charles Clifton, Jr., and the funds for data analysis were provided by a grant from the University of Massachusetts computing Center.

Table of Contents

List of Tables

List of Figures

Abstract

Latency of S's response was measured in a task involving recognition memory for short lists of numbers or letters. Subjects saw a list of one to five items presented simultaneously, followed by a string of identical probe items, matched spatially to the locations of the memory set. The S indicated whether the probe item was the same as an item in the memory set. The results showed that under these conditions Ss are able to engage in a fast self-terminating search. The second experiment demonstrated that a self-terminating model, which assumes that the S starts his scan at the middle position of the probe array, proceeding in a self-terminating fashion to either side, fits the major aspects of the data.

Introduction

Serial-exhaustive scanning processes

Evidence that a serial-exhaustive scanning mechanism operates when one retrieves information from short-term memory has been reported several times in the literature. Sternberg (1966) presented subjects (*Ss*) with lists of one to six digits (the positive or memory set) to remember. The *S*'s task was to indicate by pulling one of two levers whether a following test stimulus matched an item in the previously memorized set. If a match occurred the *S* made a positive response; otherwise, he made a negative response. Sternberg found that reaction time (RT) was a linear increasing function of the memory set and that the slopes of positive and negative responses were identical. Sternberg proposed that in a first stage the test stimulus is processed and encoded in such a way that it is comparable to the representation of the memorized set. Then in a second stage the *S* serially compares each memory representation of the memorized set to the memory representation of the test stimulus, to determine whether a match occurs. If after comparing all elements of the memory set no match occurs, the *S* must exit with a negative response. If, on the other hand, a match is found during this scanning process, the *S* could either terminate his search immediately (self-terminating scan), or he could continue his search until all the items have been scanned and only then make a positive response (exhaustive scan).

For each model (exhaustive or self-terminating) equations can be derived, which relate mean RT to the size of the positive set:

For the serial exhaustive scanning model:

$$\overline{RT}_{Neg} = a + sN \qquad \text{for a negative response} \qquad (1)$$

$$\overline{RT}_{Pos} = a' + sN \qquad \text{for a positive response} \qquad (2)$$

For the serial self-terminating scanning model:

$$\overline{RT}_{Neg} = a + sN \qquad \text{for a negative response} \qquad (3)$$

$$\overline{RT}_{Pos} = a' + s(\frac{N+1}{2}) \qquad \text{for a positive response} \qquad (4)$$

In these equations $\underline{N}$ is the number of items in the memorized list (the set size, SS), $\underline{s}$ is the comparison time for a single item, and $\underline{a}$ is a constant (not necessarily the same for positive and negative responses) which consists of the encoding time of the test stimulus, the decision time, the time to execute the response, etc.

In one of Sternberg's experiments (1966) which used single digits as stimuli, he found support for a serial exhaustive scanning model. The serial search notion was supported by the linear relationship between $\overline{RT}$ and SS, while the exhaustiveness of the comparison process was indicated by the identity of the slopes for positive and negative responses.

These results have been replicated and extended by Sternberg and other investigators using somewhat different procedures and different stimulus material. Sternberg (1966, Expt. 2) also employed what he called the fixed set procedure.

Under this procedure, the S receives a large number of test trials on one list before a new list of numbers to memorize is presented. The results were almost identical to the ones reported above.

The exhaustive scanning model has also been verified when other kinds of stimulus material were used (Sternberg, 1969, Expt. 4, using nonsense forms and photographed faces as stimuli; also Klatzky, Juola and Atkinson, 1971, using letters and pictures as probe stimuli); or when a visual detection task was employed, in which the S first sees a positive target stimulus and then, after an interval, sees a display of several items presented simultaneously to search (Atkinson, Holmgren, and Juola, 1969).

Briggs and Blaha (1969), in a task similar to Sternberg (1966), varied both the size of the positive set of stimuli and the display load, the latter being either one, two, or four stimuli presented simultaneously. The stimuli consisted of 23 eight-sided random figures. The items in the positive set were chosen from seven of the figures, while the negative test stimuli were chosen from the remaining sixteen. Briggs and Blaha used a fixed set procedure, giving Ss 96 trials following a practice session of 96 trials on a particular set size. On one day Ss were tested on only one set size, but on all three display sizes. The S had to respond "yes" if any one of the one, two, or four presented figures on a given trial matched a member of the positive set, otherwise he had to respond "no". Briggs and Blaha obtained linear increasing RT-functions when they plotted $\overline{RT}$ against size of the positive set with display load as parameter. Positive responses were 40-50 msec. faster than negative responses with equal slopes for display size one and two. The obtained values were: For display size one: $RT_{Pos} = 468 + 31(N)$ and $RT_{Neg} = 502 + 34(N)$, and for display size two: $RT_{Pos} = 472 + 54(N)$ and $RT_{Neg} = 516 + 59(N)$. However for display size four the slope of negative responses was much steeper than that for positive responses: $RT_{Pos} = 529 + 92(N)$ and $RT_{Neg} = 566 + 141(N)$, the ratio of positive to negative responses was 1 : .65. This could indicate a self-terminating process but the authors want to argue that Ss under this relatively high input load condition double check for a negative response.

Serial self-terminating scanning processes

Evidence for the existence of a self-terminating process also comes from several authors (Sternberg, 1967; Nickerson, 1966; Klatzky and Atkinson, 1970; Klatzky, et al., 1971; Smith and Nielson, 1970).

Sternberg (1967) presented Ss with from one to four digits successively at a one second rate. After a two second delay interval the S saw from one to three items presented simultaneously for 70 msec. and he responded "yes" if one of the digits matched a digit in the memorized set, otherwise "no". From his results Sternberg concluded that the S takes an element from the display set and compares it serially and exhaustively to the memory set. He then terminates the comparison process if a match is found, but otherwise takes a new element from the display set and repeats the comparison process until he obtains a positive outcome or completes scanning both sets exhaustively with a negative outcome.

Sternberg plotted RT against the size of the memory set with the number of items in the display set (d) as parameter. By looking just at the case where only one item was in the display set (d=1), comparable to the common Sternberg paradigm, he found identical slopes for positive and negative responses, supporting the notion that Ss retrieve information from the memorized set in a serial exhaustive fashion. But he also found that the slopes for negative responses for each display size increased with d at a faster rate (1:2:3) than the slopes for positive responses (1:1.5:2), ruling out a search process which is exhaustive both at the memory set and the display set. His data were reasonably well fit by two equations assuming an exhaustive search through the memorized list and a self-terminating one through the visual image of the display set. ($RT_{Neg} = \alpha + sd\beta$; $RT_{Pos} = \alpha' + s\frac{(d+1)}{2}\beta$; where α = zero-intercept; s = size of the memorized set, d = size of the visual display and β = scanning rate, the mean time per comparison estimated from the data.)

But a closer look at his data reveals that for the condition where the memory set contained only one item and the display set varied from one to three items the increase in RT for positive and negative responses was about equal indicating an exhaustive search under this condition. Atkinson, Holmgren, and Juola (1969) noticed that for this special condition the data were not fitted by a self-terminating model and showed that when presenting a single item first followed by a variable display size Ss engage in a serial exhaustive search.

Nickerson (1966) in an experiment similar to the one of Sternberg discussed above used all combinations of size of display d = 1, 2, and 4, and size of memorized list s = 1, 2, and 4, resulting in 9 conditions. Sternberg analyzed Nickerson's data in the same way as his own (looking at RT as a function of memory size with display size as a parameter) and the data were approximately fitted by assuming an exhaustive search through the memorized list and a self-terminating one through the visual display. But again the RT for a memory list of one when the display size varies increased about the same amount for the positive and negative responses contrary to a self-terminating model.

Klatzky and Atkinson (1970) using letters, pictures, and words as probe stimuli also found deviations from an exhaustive search model. In their task, the Ss responded "yes" in the case of letter probes if the probe letter matched one of the letters from the memory set, for word probes if the first letter of the word matched one of the letters in the memory set, and for picture probes if the first letter of the name of the picture was a member of the memory set. They obtained slopes and intercepts of the RT-function greater than those reported by Sternberg _et al_. (Their method deviated from those of most other investigations in that they did not use speed instructions. This difference could account to some extent for their slow RTs.) Their ratio of negative to positive responses was not 1.0 as an exhaustive search model would predict but approximately 1.75 for all three types of probe stimuli, which is also not quite congruent with a self-terminating model which would require a ratio of 2.0. Klatzky and Atkinson argued that the _S_s, when presented with a word or a picture, transformed the stimulus into a verbal code and made the comparison on the basis of the verbal representation instead of the visual one, such resulting in slower but self-terminating comparisons. But in the case of letter stimuli _S_s would use a mixed strategy, sometimes applying a verbal code and sometimes a visual code, so on part of the trials performing a self-terminating, and on part of the trials performing an exhaustive search, resulting in a smaller slope for letter material.

In an additional study (Klatzky _et al_., 1971) the experimenters used letters and pictures as test stimulus material but they used blocked as well as mixed representation.

Contrary to their earlier study, they found evidence for a self-terminating search, but limited to the third block in the mixed session when pictures were used as a probe. In all other conditions, the slopes for positive and negative responses did not differ significantly from each other indicating an exhaustive search process. But they also obtained linear increasing serial position functions in all conditions which in Sternberg's model would argue for a self-terminating process with constant starting point. Both notions together, however, identity of slopes for positive and negative responses and linear increasing serial positions curves, are incompatible with a simple scanning theory.

Dual-Process theories

Smith and Nielson (1970) had Ss decide whether a "test-face" was the same as or different from a previously presented original. They varied the number of relevant features on which the S had to base his decision using RT as their dependent variable. A "same" response was required if the test face was the same as the original and a "different" response if the test face differed from the original in any relevant feature. They found that the number of features which could be relevant to a S's judgment (r) had no effect on "different" RT at a given level of d (d, the number of features actually different on a given trial). They argued that the feature comparison process is done in parallel; RT is always determined by the fastest of the $\underline{d}$ comparisons.

In addition they found that RT for different responses decreased for all levels of $\underline{r}$ as the number of features which distinguished the test from the original increased, implying that the feature comparison process was self-terminating.

But "same" responses were also relatively unaffected by $\underline{r}$ (at least at the short retention intervals), which is not predicted by either a sequential or a simultaneous feature comparison process. So the authors want to argue for a dual process, underlying "same" and "different" responses. They conclude that for "same" responses the Ss assess and compare stimulus and probe as a unit, while for "different" responses they engage in a self-terminating search.

Bamber (1969) presented two horizontal rows of letters, containing the same number of elements, successively to Ss and had Ss judge whether the two letter strings were identical or differed in one or more letters. In the former case, Ss made a "same" response, in the latter a "different" response. His results for "different" responses were fitted quite well by a self-terminating model: RT decreased as the number of different letters in the array increased for all set sizes. However, his observed "same" responses were considerably faster than a self-terminating model would predict and futhermore, the slope of the observed "same" responses as a function of set size was considerably less than the predicted one. To account for these results Bamber proposed a two-process model.

On each trial, S engages simultaneously in two stimulus comparison processes. One of the processes is a fast identity reporter which checks physical identity and can emit only a "same" signal. The second process is a serial self-terminating comparison process which emits either a "same" or a "different" signal. When the test stimulus requires a "same" response, both comparison processes emit "same" signals. But because the identity reporter is faster, its signal has already initiated the "same" response by the time the serial processor emits its signal, which would lead to faster "same" than "different" responses.

From the experiments discussed above some generalizations can be made. It seems that for simple stimulus and test material, results compatible with an exhaustive search process are obtained if any one of the stimulus strings (either the memory set or the display set) consists only of a single element. On the other hand, if both stimulus strings contain more than one element, or if the scanning rate is relatively slow, results are obtained which seem to argue for a self-terminating process on the display set.

If in fact the condition necessary for a fast self-terminating scan is the presence of two separate lists displayed at identical locations, then it might not be necessary that the visual display has different items in it. What might be necessary is that each item in the memorized list is compared against a newly inputted comparison item. If one presents Ss with the items to be memorized simultaneously, and follows them with a string of identical probe items displayed simultaneously at the same locations as the memorized list, Ss should engage in a fast self-terminating search. However, Ss might be able to compare all the items of the memory set in parallel against all the items of the probe string. In this case, negative accelerated RT-functions for both positive and negative responses would be expected, with no slope difference if the comparison process is exhaustive, but with a steeper increase for negative responses than for positive ones, if the comparison process is self-terminating. It is conceivable that a parallel comparison process may only be obtained under short retention intervals when Ss can make their comparisons on the physical features of stimulus and probe string, but that the comparison process would change its nature under longer retention intervals when the S presumably operates on the name level. Experiment 1 was designed to test these notions using intervals of 100, 300, and 1000 msec. between the memorized list and the probe display.

Experiment 1

Method

Subjects. Four volunteer college students served for 7 consecutive daily sessions of approximately one hour each as paid subjects. The first day was considered practice and Ss were run in groups of two. All four Ss had participated in similar experiments before.

Design. A five factor repeated measurement design with Ss as a random-effect variable was used. The remaining fixed effect variables (5 x 3 x 2 x 6) determined each trial condition and the trial-block condition. The first factor determined the size of the positive set (from one to five items). The second factor specified the retention interval (RI), -- defined as the time from the offset of the first stimulus (the set size) to the onset of the second stimulus (the probe), -- which was either 100 msec., 300 msec. or 1000 msec. The next factor determined the kind of response required, either positive or negative, and the last factor the day of testing (one to six).

Apparatus. A Dec PDP-8/I computer controlled the experiment and displayed the stimuli to be remembered, and the probe stimuli on Burroughs Nixie tubes (Alpha-numeric). Digits or letters displayed on a single tube subtended a visual angle of approximately 0° 30' high by up to 0° 22' wide. Ss were seated behind a table approximately 2.5m from the display.

On the table in front of each S was a console with the response buttons. For all Ss the button on the right represented the positive response, and the button on the left the negative response.

The Ss were instructed to hold their right and left index fingers respectively just slightly above the response buttons at all times during a trial block.

Procedure. On each day Ss received 9 series of 50 trials each, following an initial 10 practice trials. The 9 series were divided into three units of three blocks each, each unit associated with a different RI. The order of the units was balanced over the two subject groups.

At the start of each series the word "READY" appeared on the display tubes followed by a 2.5 sec blank interval before the first trial. On each trial a warning signal - a 400 hz tone lasting 250 msec. - sounded, followed by a silent interval of 250 msec. Then one to five digits randomly selected without replacement, from the digits zero to nine were presented simultaneously and centered on the Nixies, for 200 msec. After an interval of either 100, 300, or 1000 msec. the probe was presented for 200 msec in such a way that each location which contained previously a stimulus item now was occupied by the probe item. As an example, the set size and probe display for a positive trial, SS = 5, looked like this:

	5	7	3	8	2	

SS-display

	3	3	3	3	3	

probe-display

In each series of 50 trials, each of the 10 conditions (five different set sizes, 2 response-types) appeared five times. The order was randomized.

At the end of each series of trials, Ss were shown a score based on correct responses faster than 500 msec. and on the number of errors the Ss made. Speed and accuracy was stressed and the Ss were informed that making an error subtracted twice as many points as making a fast correct response added. Each time a S made an error the word "ERROR" plus the subject's number was flashed for 1 sec on the display tubes.

RESULTS

Error rates. The proportions of trials on which errors were made are shown in Table 1. The overall error rate was 5.7%, with 8.2% for the retention interval (RI) of 100 msec., 5.5% for RI = 300 msec., and 3.5% for RI = 1000 msec. A strong increase in errors occurred for SS = 4 and SS = 5, especially for the two short RIs, which poses limitations on the interpretation of the RT data, since the hypotheses regarding RTs assume nearly error free performance.

RT as a function of SS. The results are shown in Figure 1, which shows the relationship between $\overline{RT}$ and SS for positive and negative trials and for each retention interval separately.

Mean RT increases with SS for all conditions, but $\overline{RT}$ for positive trials does not increase to the same extent as $\overline{RT}$ for negative trials, contradicting a serial exhaustive scanning hypothesis. Straight lines were fit (least square criterion) to the $\overline{RT}$ data (for SS two to five) for each S within each condition defined by type of probe, retention interval and blocks. (SS = 1 was excluded from the analysis because RT for positive responses were extremely fast, indicating perhaps that Ss were able to perform a template match under this condition.) The mean slopes (over blocks and subjects) and the zero-intercepts of these functions relating $\overline{RT}$ to SS are presented in Table 2 together with a measure

Table 1

Proportions of trials on which errors occurred

for Experiment 1

	Retention Interval									Average over Retention Intervals		
	100			300			1000					
SS	Miss	F.A.	Total	Miss	F.A.	Total	Miss	F.A.	Total	Miss	F.A.	Total
1	.003	.047	.025	.005	.019	.012	.011	.011	.011	.006	.025	.016
2	.039	.017	.028	.033	.022	.027	.033	.014	.024	.035	.017	.026
3	.047	.014	.030	.044	.017	.030	.011	.025	.018	.034	.018	.026
4	.100	.011	.106	.103	.033	.068	.039	.042	.040	.080	.062	.071
5	.125	.319	.222	.125	.153	.139	.117	.044	.080	.122	.172	.147
Σ/n	.063	.102	.082	.062	.049	.055	.042	.027	.035	.056	.059	.057

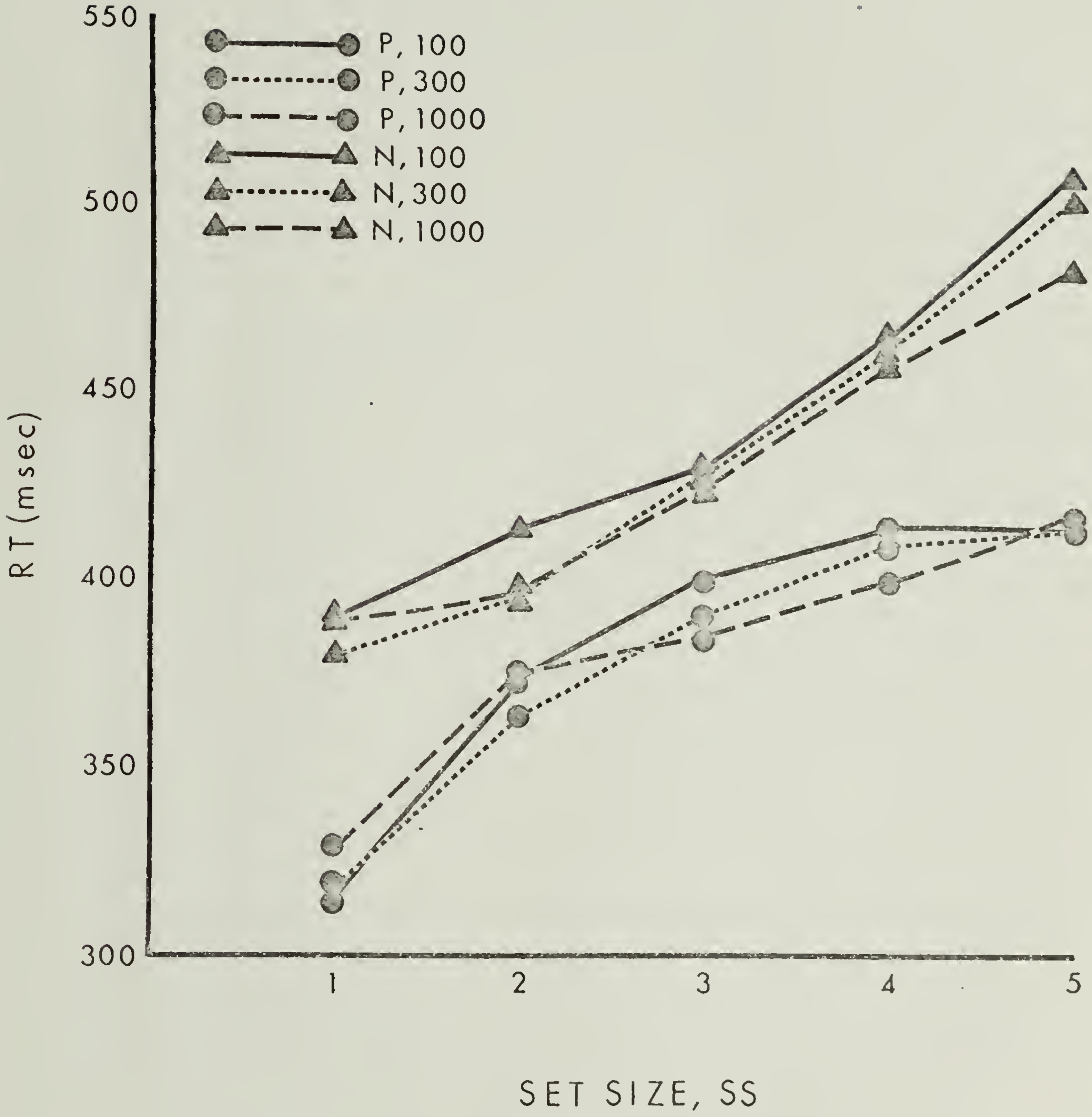

Fig. 1. Expt. I: $\overline{RT}$ as a function of SS.

Table 2

Parameter of straight lines relating RT to SS

(SS 2-5) for Experiment 1

	Slope		0-Intercept		% Variance	
RI	POS	NEG	POS	NEG	POS	NEG
100	13.9	32.9	352	340	83	96
300	17.5	35.5	334	323	92	100
1000	13.0	29.9	349	337	97	100

of how well the data were fit by these straight lines. The linear component of the increase in set size accounted for at least 96% of the between set size variance for negative responses and between 83% and 97% for positive responses.

The slopes and zero-intercept values were subjected to an analysis of variance with blocks, retention interval and response as factors. Analysis of the slopes showed that only the response main effect reached the level of significance [$F(1,3) = 29.3$, $p < .025$]; the slopes for positive responses had a lower value than the slopes for negative responses. The ratio for negative and positive slopes averaged over Ss were 2.4:1, 2.0:1, and 2.3:1 for the 100, 300, and 1000 msec. RI respectively. The ratio of negative to positive slopes for individual subjects averaged over the three RI were 2.1:1, 2.5:1, 1.7:1. and 3.1:1. The effects of blocks and retention intervals were not quite significant [$F(2,6) = 3.6$, $p < .10$ and $F(2,6) = 3.6$, $p < .10$, respectively]. None of the interactions were significant.

Analysis of the zero-intercepts relating $\overline{RT}$ to the different conditions showed that intercepts for negative responses were 12 msec. lower than those for positive responses and that the 300 msec. RI condition had a lower intercept than the 100 and 1000 msec. RI conditions, which were about the same. But neither the response main effect, the RI main effect, nor the response by RI interaction reached the .05 level of significance [$F(1,3) = 2.7$, $p \approx .20$; $F(2,6) = 4.4$,

$p < .10$ and $F(2,6) = .02$, respectively]. None of the other effects were significant.

RT as a function of serial position (SP). A serial self-terminating model predicts linear increasing serial position curves which are superimposed (for the different set sizes) if one assumes that the subject always starts with the left-most item and then proceeds to the right. On the other hand, if the self-terminating search occurred in random order or started at a random point, flat serial position curves would be expected. Although the obtained SP curves were relatively flat, a tendency existed for Ss to be faster if the probe occurred in the middle of the array than on either side. The results are shown in Figure 2, separately for each RI. This pattern of results is consistent with a search strategy in which Ss start their search from the center of the array, sometimes proceeding to the right and sometimes to the left. But an analysis of variance with serial position and retention interval as factors carried out separately for SS = 4 and SS = 5 did not show a significant main effect of serial position [$F(3,9) = .6$, and $F(4,12) = 2.3$, $p < .20$, respectively]. The Position x Delay interaction was significant for SS = 5 at the .05 level, reflecting the tendency that the effect was more pronounced for the two longer RI than for the shortest RI [$F(8,24) = 3.06$, $p < .025$].

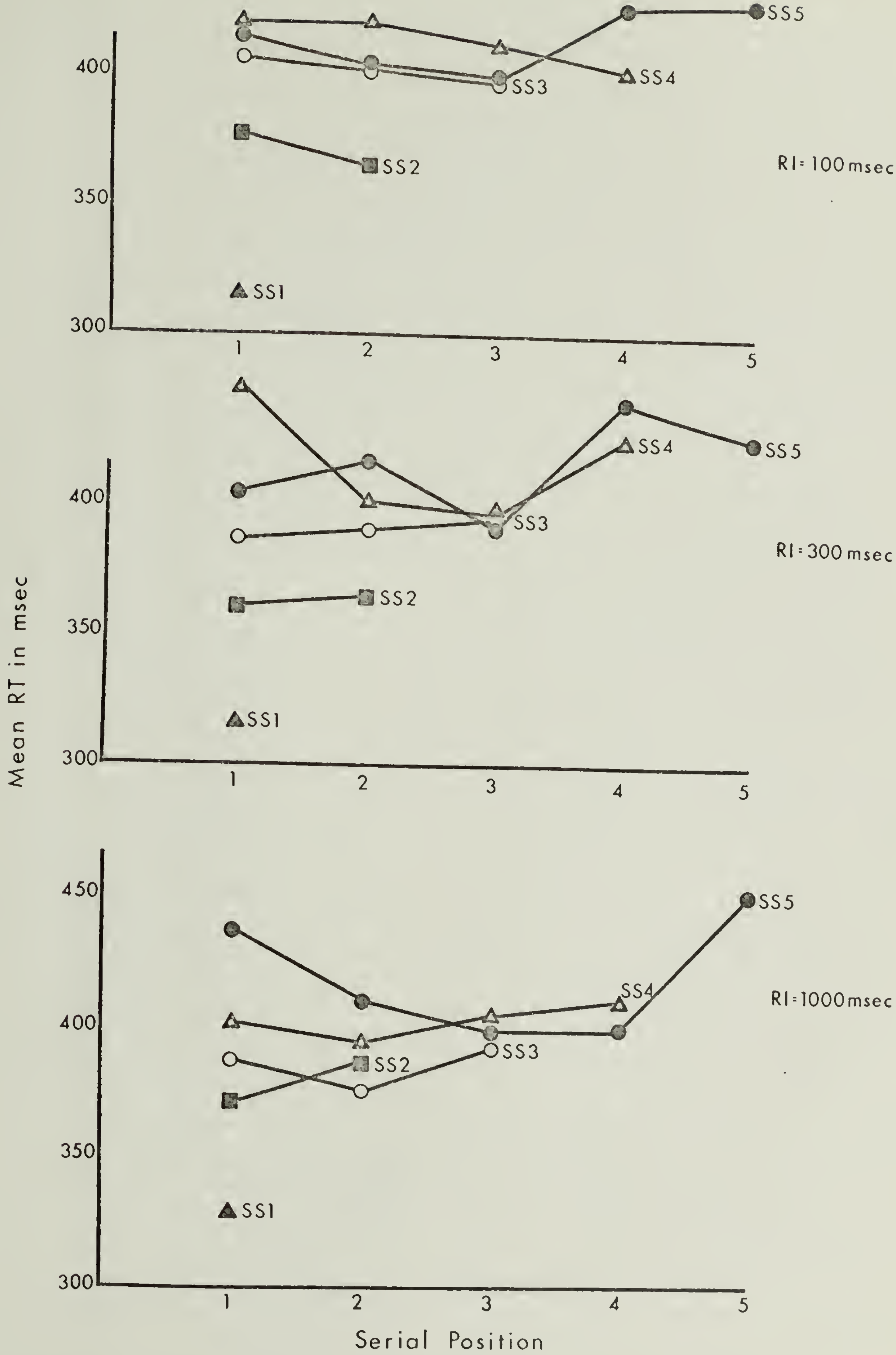

Fig. 2. Expt. I. $\overline{RT}$ as a function of SP.

DISCUSSION

A serial exhaustive scanning hypothesis is clearly rejected by the data. The slopes of the functions relating $\overline{RT}$ to SS are not parallel for positive and negative responses as a serial exhaustive model would predict. On the contrary, the data seem to argue for a self-terminating model; the obtained slopes for positive responses were about half the negatives. However, two aspects of the data call for caution: First, the slopes are not exactly half the negatives and second, mean RT for positive responses were not very well fit by straight lines, especially for the 100 and 300 msec. RI conditions. In addition, the error rate for SS = 4 and SS = 5 was extremely high, making it possible that Ss followed other kinds of strategies. For example, it could be argued that Ss do not always have all the items in memory and therefore search a list which is inconsistently shorter than it is assumed to be. Also it could be argued that a sizeable number of the correct responses are due to a guessing process. If the latency of correct guesses deviates strongly from that of true correct responses, estimates of the latter might be biased. The data show that misses were about equally fast as true negatives but false alarms were significantly slower than true positives for the 300 and 100 msec. RI. (430 vs. 378 msec., $t(4) = 4.06$, $p < .02$ and 431 vs. 382 msec., $t(4) = 3.22$, $p < .05$), while the difference for the 1000 msec. RI condition did not reach significance (401 vs. 383 msec., t(4) = 1.29). (The t was computed for the five set size conditions, separately for each delay, so the population generalized to was conditions, not Ss.) This seems to cast doubt on the hypothesis that Ss dropped items from memory because in that case one might expect misses to be shorter than other responses. (For a more extensive discussion on the problem of high error rate in RT-experiments see Clifton and Gutschera, 1970.)

The obtained results, so best described by a serial self-terminating scanning hypothesis, do not allow one to rule out certain kinds of parallel models or a dual process theory, such as Bamber's (1969). Bamber's theory would likewise predict flatter slopes for positive responses since it assumes that "same" and "different" judgments are two distinct comparison processes which occur simultaneously and where the process which emits the "same" signal is the faster one. But it does not necessarily predict that the slope for positive response functions should be 1/2 the slope of negative response functions. Therefore, a second experiment was designed with the goal (1) to lower the error rate by displaying the stimuli for 400 msec. instead of 200 msec., thus giving Ss more time to encode the stimulus material, and (2) to make possible a distinction between a serial self-terminating hypothesis and a dual process theory. The self-terminating scanning hypothesis could be tested by determining how Ss scan a memorized list, where one of its items is repeated at two positions. The self-terminating theory predicts that the slope of the positive response function for probes of a nonrepeated item should be 1/2 the slope of the negative response function. However, if one assumes that the S self-terminates when he compares the first occurrence of a repeated item against a probe, the slope for the positive response function for probes of the repeated item should be 1/3 the slope of the negative response function. That is, equation 3 holds for negative probes, and equation 4 for positive probes of nonrepeated items, but

$$RT_{Pos} = a' + s\left(\frac{N+1}{3}\right) \qquad (5)$$

for positive probes of repeated items.

On the other hand, dual process theories, as they have been developed, would not seem to make any clear prediction about the effect of repeating an item in a memorized list. While it would be simple to develop a dual-process theory which predicts that RT to probes of repeated items should be faster than RT to probes of nonrepeated items, special and *ad hoc* assumptions about the distribution of comparison times which result in a positive match would have to be made to predict the same results predicted in a natural fashion by the self-terminating model.

However, an experiment which adequately tests the prediction of the self-terminating model must have very small error variance. In particular, since the slope for negative response functions is typically around 36 msec/item, the predicted slope of the function for positive probes of nonrepeated items is 18 msec/item, and the predicted slope for probes of repeated items 12 msec/item. Thus, the experiment had to be designed so that a difference in slope of 6 msec/item had a reasonably high probability of being detected.

In Experiment 1, no significant difference between the three RI had been found [$F(2,6) = 3.6$, $p < .10$ for slopes and $F(2,6) = 4.43$, $p < .10$ for intercepts]. However, it could be argued that the longest interval, RI = 1000 msec., was still too short to bring about a switch in Ss retrieval strategy. For example, Posner, Boies, Eichelman, & Taylor (1969) showed that the advantage of a PI-match over a NI-match is lost after about two seconds. So in Experiment 2, two RI were used, a short one, SRI = 500 msec. and a long one, LRI = 2500 msec.

Experiment 2

Method

Subjects. Six paid students were tested for 13 consecutive daily sessions of approximately one hour each. The first day was considered practice and *Ss* were run in groups of two.

Design. A six factor repeated measurement design with *Ss* as a random effect variable was used. The remaining fixed-effect variables determined the size of the positive set, the repetition factor (repeated vs. nonrepeated element in the memory set), the type of response (N = negative, $P_{\overline{R}}$ = positive and probing for a nonrepeated element and P_R = positive and probing for the repeated element), and the number of blocks (four blocks of three days each).

Apparatus. The same apparatus as in Experiment 1, a Dec PDP-8/I computer was used, except that the stimuli were letters rather than digits.

Procedure. On each day, *Ss* received 12 series of 28 trials each, following an initial 12 practice trials. A daily session was divided into two main parts. Each part consisting of six series was associated with one of the two RI (short retention interval, SRI = 500 msec.; or long retention interval, LRI = 2500 msec.), balanced over days and subject groups.

At the start of each series the word "GO" appeared for two seconds. The subject then indicated that he was ready by pushing one of the buttons. After both Ss had done so, the word "READY" appeared for 2 sec, indicating that the first trial was about to start. 2.5 sec later a warning signal, a 300 hz tone sounded for 70 msec., followed by a silent interval of 330 msec. Then two to five letters randomly selected from the letters of the English alphabet with the exclusion of the letters, C, G, M, S, T, V, W, X, Y, and Z were presented simultaneously for 400 msec. After an interval of either 500 or 2500 msec. the probe letters were presented also for 400 msec. List and probe letters were presented in the same way as in Experiment 1, except that the list letters were framed by 2 dashes. Lists containing either a repeated element or no repeated element were presented equally often, and on half the trials a positive response and on the other half a negative response was required. If a positive response was required, the probe item could either be identical with one of the nonrepeated elements in the memory set ($P_{\bar{R}}$) or with the repeated element (P_R). Consider the lists with repeated elements: One quarter of these lists (half the positive probe trials) were tested by presenting the repeated element as a probe (R, P_R Condition), and one quarter by presenting a nonrepeated element (R, $P_{\bar{R}}$ Condition). In the former case, set size could vary from 2 to 5, while in the latter, it could only vary from 3 to 5. Thus, lists of set size 2 were presented less frequently than lists of set size 3 to 5. The same inequality in the frequency with which different set sizes were tested was maintained on negative probe trials, and on trials on which lists with no repeated items were presented.

S's payoff, feedback and instructions were similar to Experiment 1.

RESULTS

Error rates. The overall error rate was 4%, somewhat lower than in Experiment 1. However, a sharp increase in errors occurred for SS = 5, resulting in error rates of over 10% for some conditions. On the other hand the two conditions where the memory set contained a repeated element and that element was also probed for, resulted in extremely low error rates, varying from zero to 2.1% for the different set sizes. The proportion of trials on which errors occurred for each condition are shown in Table 3.

RT as a function of SS. The functions relating $\overline{RT}$ to SS are presented in Figures 3 and 4. Straight lines were fit to set sizes 3 to 5 in the same way as in Experiment 1. (SS = 2 was excluded for the same reasons that SS = 1 was excluded in Experiment 1.) Mean slopes, zero-intercepts and a measure stating % variance accounted for by fitting straight lines are presented in Table 4. Table 4 shows that the data in Figures 3 and 4 are well described by straight lines, supporting the inference that Ss engage in a serial comparison process.

Slopes (SS 3 to 5) and zero-intercepts were subjected to an analysis of variance with blocks (4), retention interval (2), and condition (5) as factors. Analysis of the slopes showed that the condition main effect was highly significant [$F(4,20) = 33.8$, $p < .001$], while none of the other main ef-

Table 3

Proportion of trials on which errors occurred for Experiment 2

	Short Retention Interval							
	Nonrepeated			Repeated				
SS	Neg	Pos	Total	Neg	$P_{\overline{R}}$	P_R	Total	Overall
2	1.16	1.62	1.39	1.16	----	.93	1.04	1.21
3	2.89	2.89	2.89	1.39	4.63	.00	2.00	2.36
4	4.17	4.74	4.45	2.66	5.79	.46	2.97	3.56
5	10.76	11.80	11.28	9.49	10.88	.93	7.10	8.77
Σ/n	4.74	5.26	5.00	3.67	7.10	.58	3.48	4.12

Long Retention Interval

	Nonrepeated			Repeated				
SS	Neg	Pos	Total	Neg	$P_{\overline{R}}$	P_R	Total	Overall
2	.69	2.31	1.50	1.62	----	1.16	1.39	1.44
3	2.67	3.82	3.24	3.01	4.63	.46	2.70	2.91
4	3.36	3.70	3.53	5.56	6.02	2.08	4.55	4.14
5	6.94	6.48	6.71	10.88	9.26	1.39	7.17	6.99
Σ/n	3.41	4.07	3.74	5.26	6.63	1.27	4.18	4.00

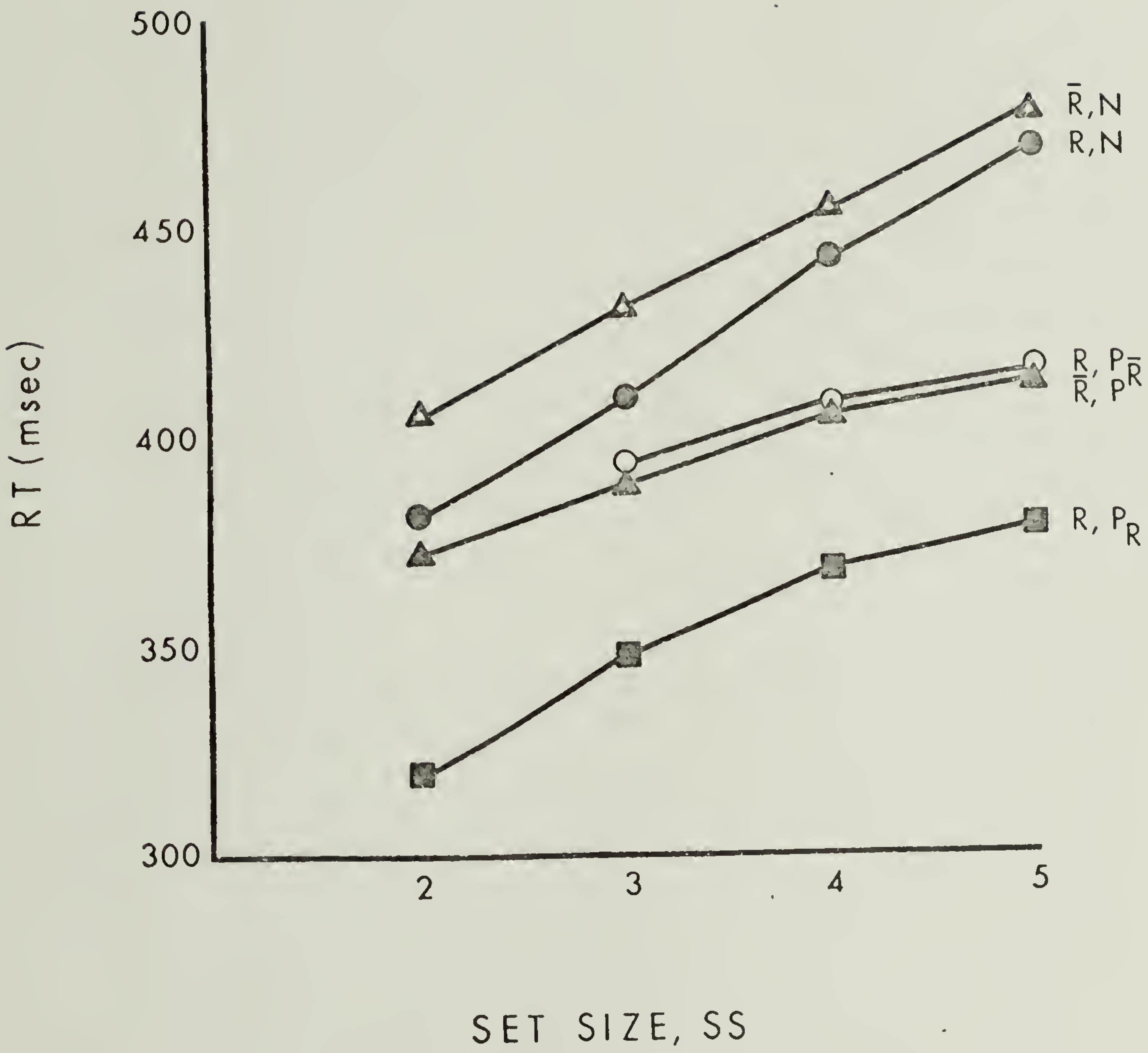

Fig. 3. Expt. II: $\overline{RT}$ as a function of SS; SRI.

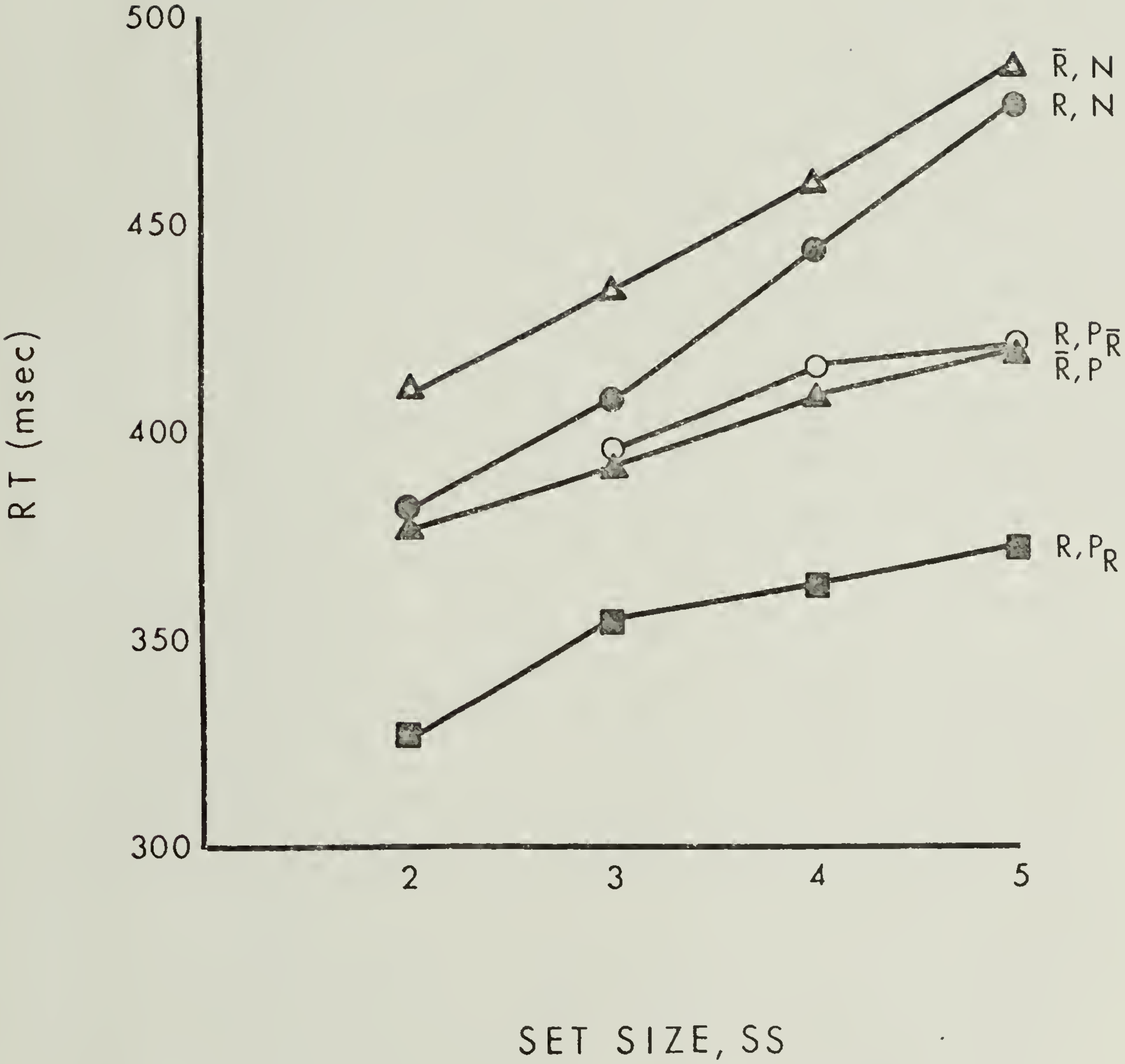

Fig. 4. Expt. II: $\overline{RT}$ as a function of SS; LRI.

Table 4

Parameters of straight lines relating RT to SS

(SS 3-5) for Experiment 2

Condition	Short Retention Interval			Long Retention Interval		
	Slope	0-Int.	% Var.	Slope	0-Int.	% Var.
$\overline{R}$, N	24.1	359	100	28.0	350	100
R, N	30.8	319	100	36.5	298	100
$\overline{R}$, P	12.9	352	97	14.8	348	99
R, $P_{\overline{R}}$	10.8	363	98	12.4	363	89
R, P_R	14.8	307	96	8.9	328	100

fects and none of the interactions reached the .05 level of significance. Especially there was no difference with respect to the two retention intervals. Neither the RI-main effect nor any of the interactions involving RI reached the .05 level of significance. Scheffé tests showed further that overall negatives had a steeper slope than positives [$F_{obt} = 124.2 > F_{crit} = 9.0$] and that the slope for the R, N condition was steeper than the slope for the $\bar{R}$, N condition [$F_{obt} = 9.86 > F_{crit} = 9.0$].

The fact that negatives have a steeper slope than positives excludes a serial exhaustive scanning hypothesis but is not contradictory to a self-terminating one. The ratio of negative and positive slopes averaged over Ss for the condition where there was no repeated element present in the list to be memorized was 1.9:1 for both retention intervals. This is in good agreement with a serial self-terminating scanning hypothesis. However, a self-terminating model further predicted a difference in slope for the two conditions containing a repeated element in the memory set and requiring a positive response. The slope of the condition when probing for the repeated item should be 2/3 of the slope when probing for a member of the set but not for the repeated item. The slope of the condition in which repeated items were probed should be 5.1 msec. (1/6 of the scanning rate estimated from negative responses to lists containing a repeated element) less than the slope of the condition in which nonrepeated items were probed for the SRI, and 6.1 msec. less for the LRI. The experiment was possibly sensitive enough to detect differences of this magnitude, since the 95% confidence interval of the slopes was $\pm$ 5.78 msec. However, the results were not in accord with the prediction. The slopes of the repeated probe function was 4 msec/item greater than the slope of the non-repeated probe function for the SRI, and 3.5 msec/item less for the LRI.

Analysis of the zero-intercepts showed a significant effect due to condition [$F(4,20) = 11.0$, $p < .001$], and revealed that Ss became faster over blocks of trials [$F(3,15) = 3.7$, $p < .05$]. None of the other main effects or interactions reached the .05 level of significance. Scheffé follow ups showed that there was no difference in intercepts between positive and negative responses but that intercepts for lists containing a repeated item were lower than for those without [$F_{obt} = 11.9 > F_{crit} = 9.0$]. The R, P_R condition, where the repeated element was the probed one, had a lower intercept than both of the conditions where the probe was positive but not the repeated element, regardless whether the list contained a repeated element or not [$F_{obt} = 18.8 > F_{crit} = 9.0$, and $F_{obt} = 10.2 > F_{crit} = 9.0$]. Comparing only the negatives, lists containing a repeated item had a lower intercept than those without [$F_{obt} = 20.2 > F_{crit} = 9.0$].

<u>RT as a function of serial position (SP)</u>. As in Experi-

ment 1 but even more so, serial position curves were u-shaped. RT as a function of SP for each positive condition, separately for each RI, are shown in Figures 5 to 10. Analysis of variance on the SP-curves for different SS with SP, RI and Ss as factors showed only for the two conditions, $\bar{R}$, P, SS = 4 and R, $P_{\bar{R}}$, SS = 5, a significant main effect due to SP at the .05 level. However, because each cell mean is based on a small and variable number of observations, this has to be regarded as a weak test.

Mean values of the first four cumulants as a function of SS. Sternberg's model of memory assumes that if s-elements are in memory then RT is given by the sum of s-comparison times (T_i) plus the base time (T_b); $RT_{(s)} = T_b + T_1 + T_2 + \ldots\ldots + T_s$. If one is willing to make the assumption that those component random variables are independent and furthermore that the T_i are identically distributed, then the r^{th} - cumulant should be a linear function of set size, where the slopes of the first four cumulants provide an estimate of the first four cumulants of the comparison-time distribution and the four intercepts provide an estimate of the cumulants of the base time distribution (Sternberg, 1964). The first four cumulants computed for each condition, for each subject and each block and then averaged over blocks and subjects for Experiment 2 are shown in Figures 11 and 12. Although the variance and higher cumulants increase more or less linearily it proved impossible to proceed further with an attempt of

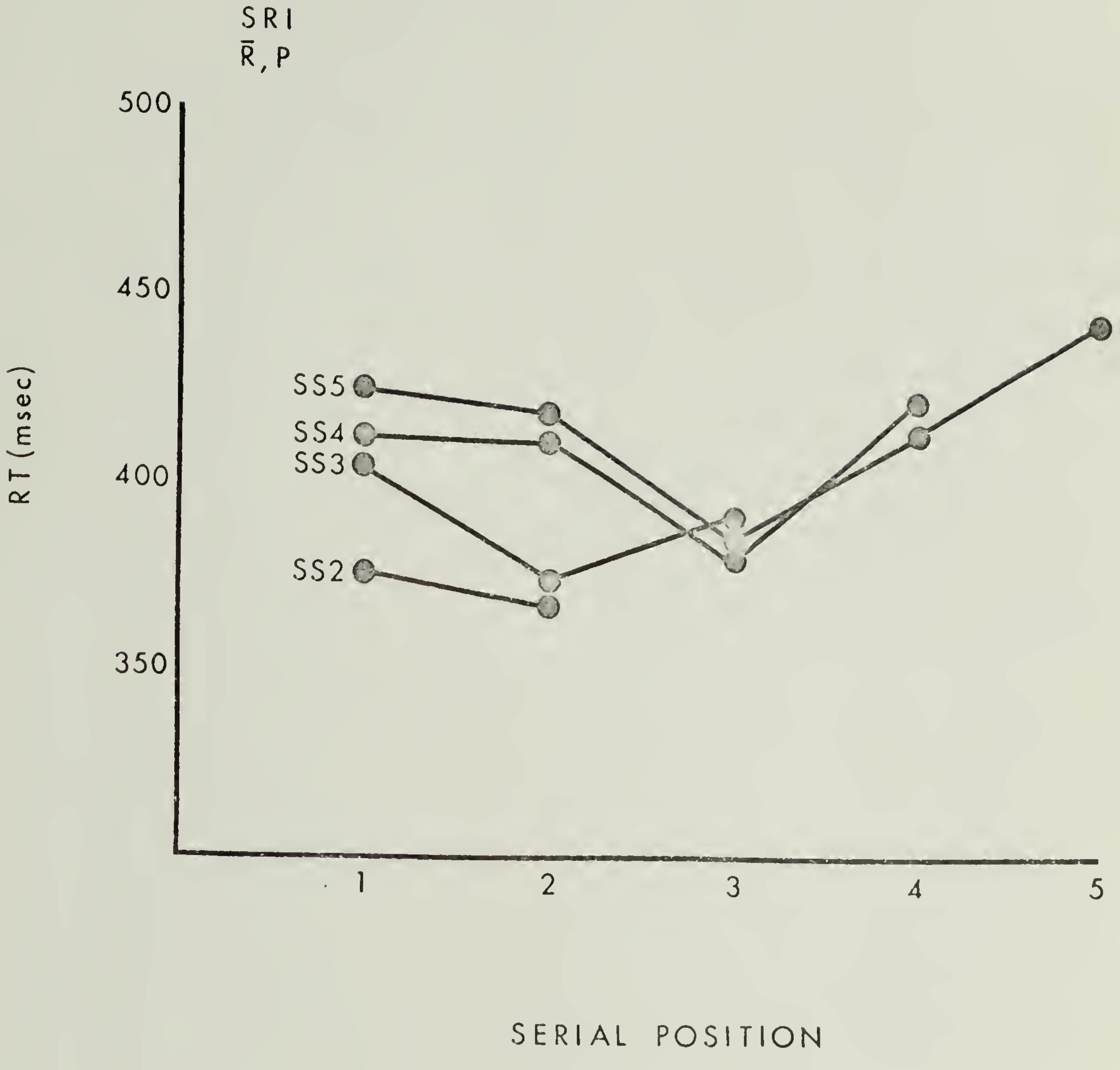

Fig. 5. Expt. II: $\overline{RT}$ as a function of SP for the $\bar{R}$,P condition; SRI.

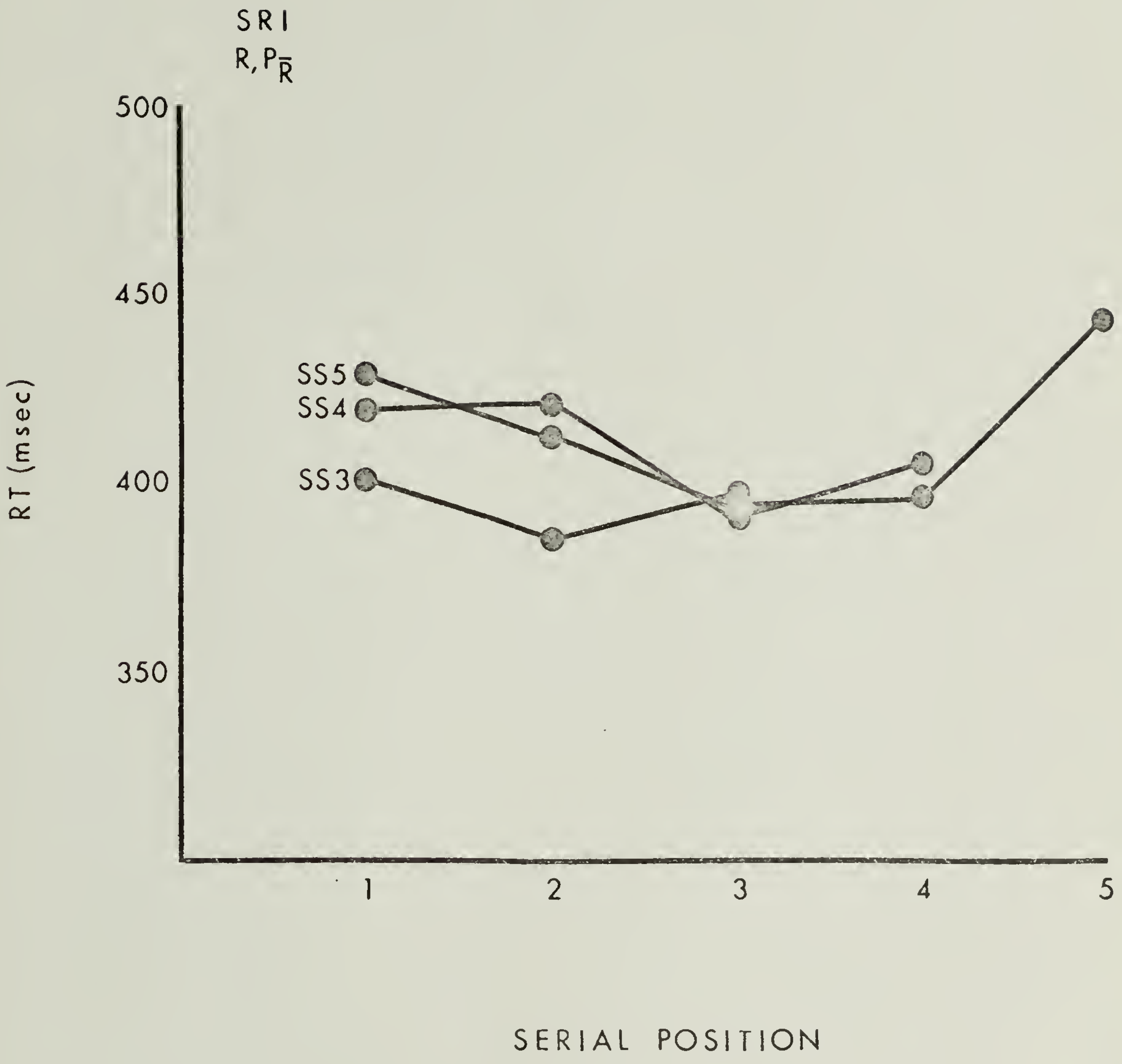

Fig. 6. Expt. II: $\overline{RT}$ as a function of SP for the R,$P_{\bar{R}}$ condition: SRI.

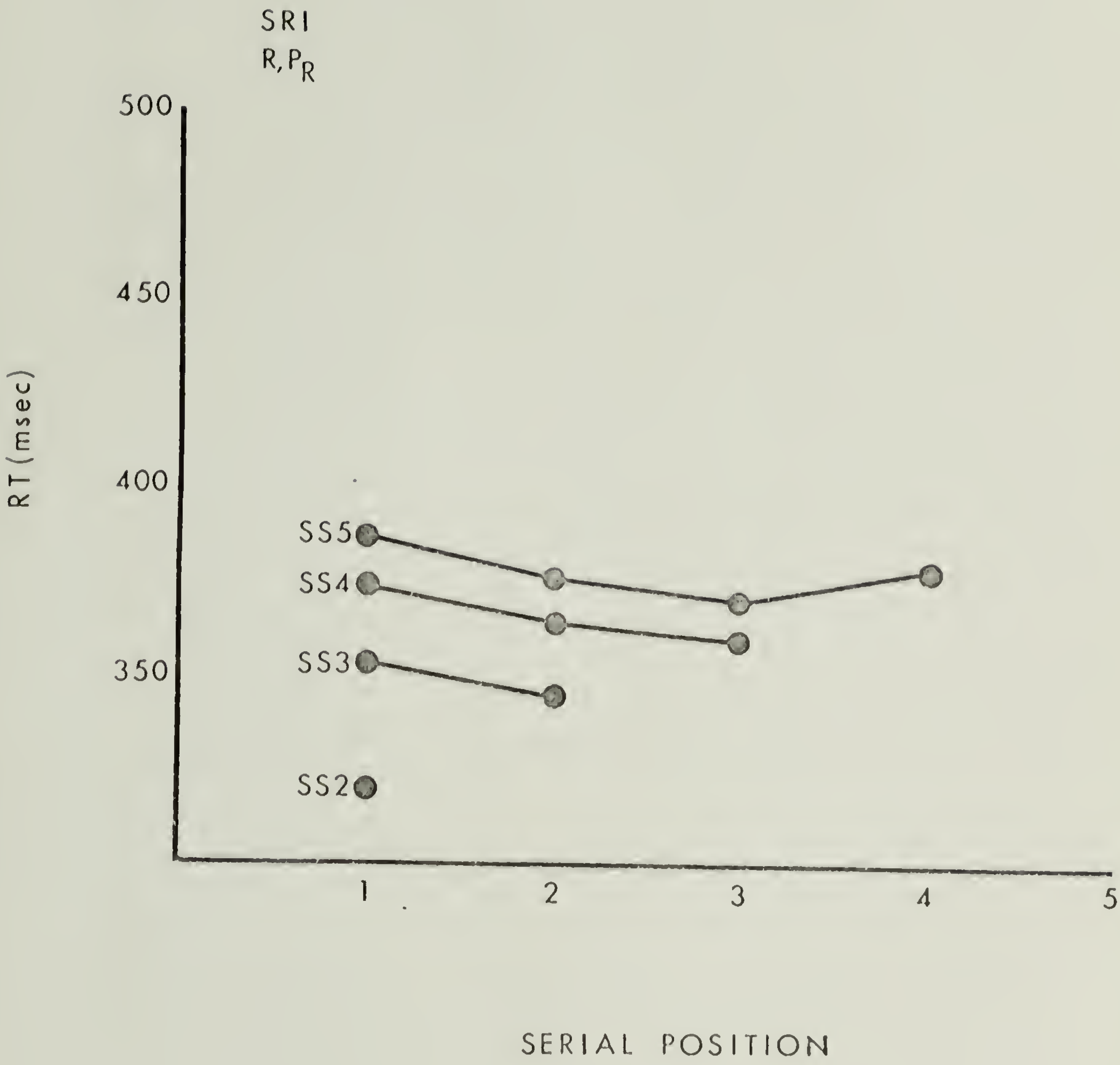

Fig. 7. Expt. II: $\overline{RT}$ as a function of SP for the R,P_R condition; SRI.

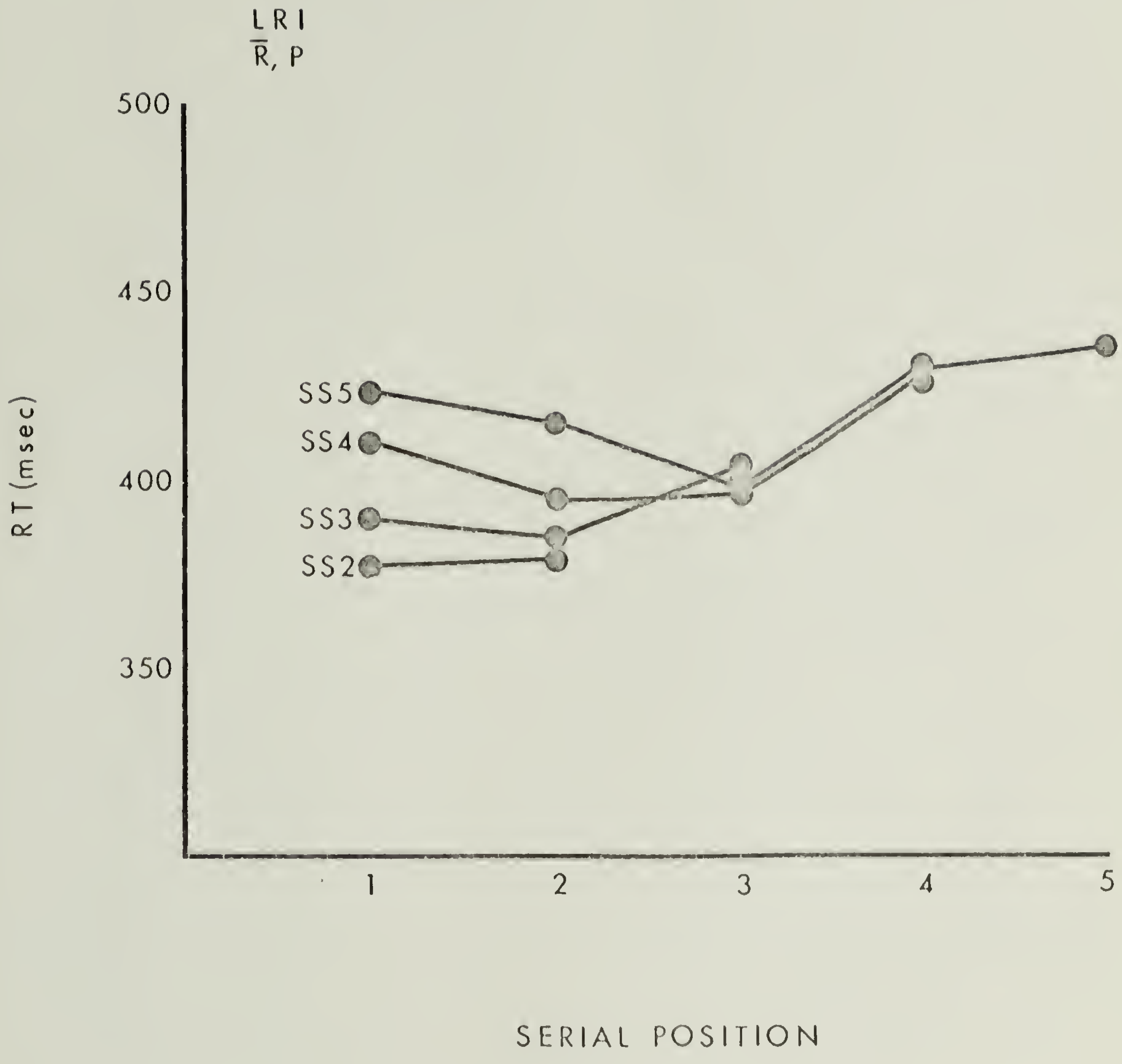

Fig. 8. Expt. II: $\overline{RT}$ as a function of SP for the $\overline{R}$,P condition: LRI.

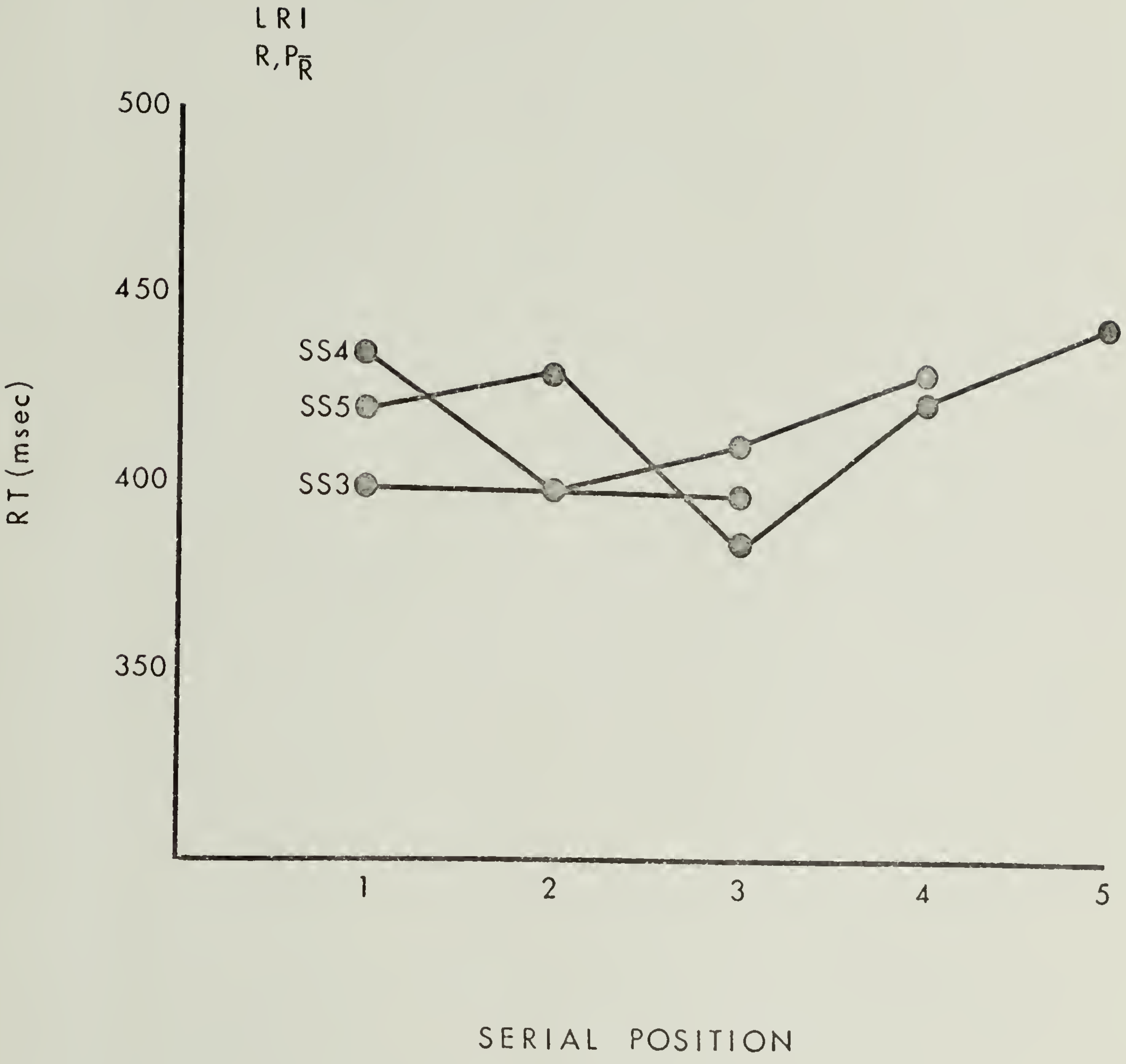

Fig. 9. Expt. II: $\overline{RT}$ as a function of SP for the R,$P_{\bar{R}}$ condition; LRI.

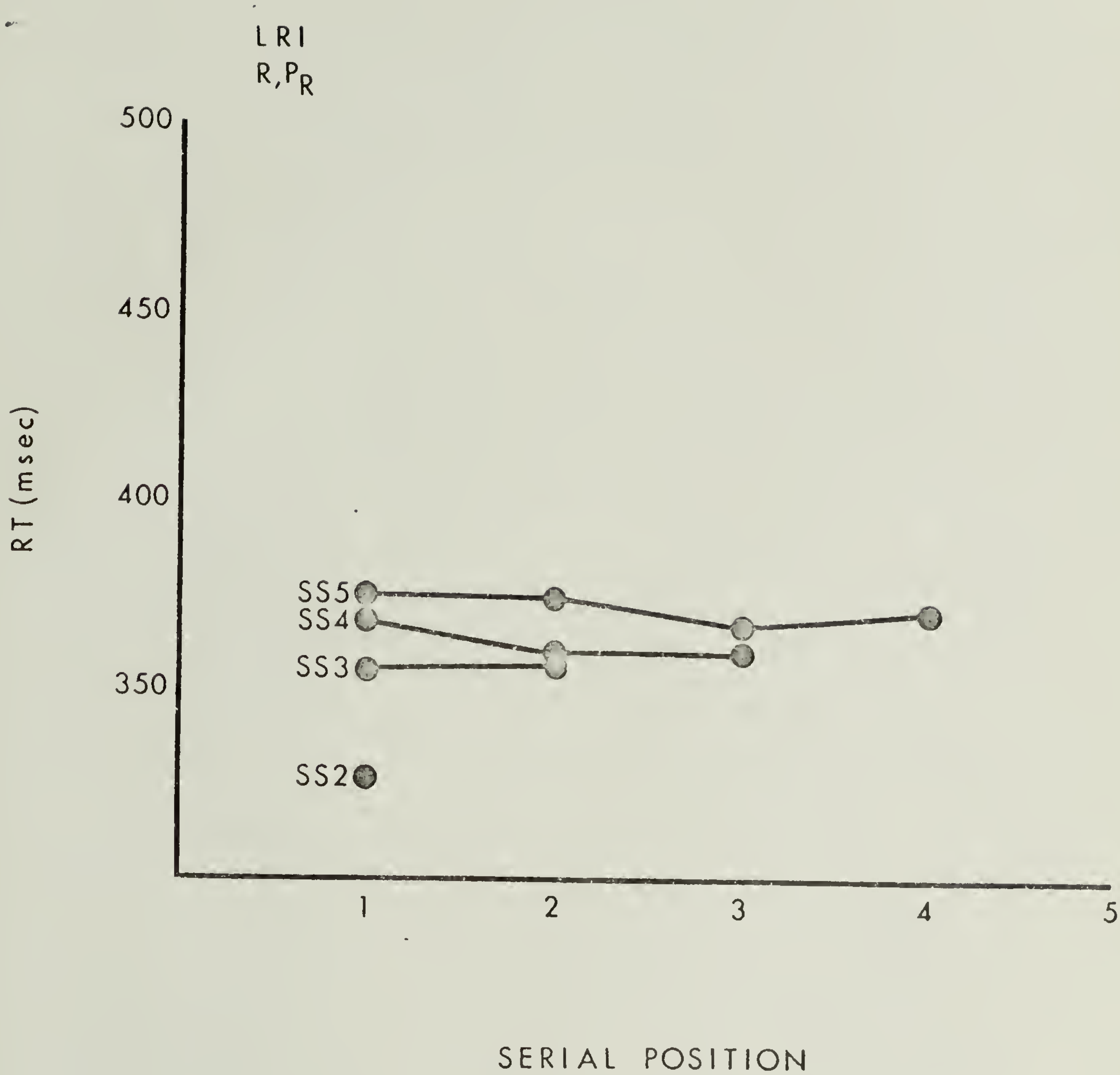

Fig. 10. Expt. II: $\overline{RT}$ as a function of SP for the R,P_R condition: LRI.

estimating the form of base and comparison distribution because after fitting straight lines (least square criterion) to the data negative intercepts were obtained for the higher cumulants.

RT-distributions. By assuming a self-terminating scanning hypothesis the RT-distributions of positive responses ($\bar{R}$, P) were predicted for each SS from the negative responses ($\bar{R}$, N) for both experiments and all RI (see Appendix A). (SS = 1 for Experiment 2 was obtained from the data of Experiment 1 with a rough adjustment for the intercept difference between experiments.) A nonparametric test (Kolmogorov - Smirnov) of goodness of fit was performed, which compared the observed vs. the predicted cumulative frequency distribution and evaluates the point of greatest divergence. From the 12 comparisons made, only one difference exceeded the critical value at the .01 level, while 7 comparisons didn't exceed the .05 level of significance, giving further support to a serial self-terminating scanning hypothesis.

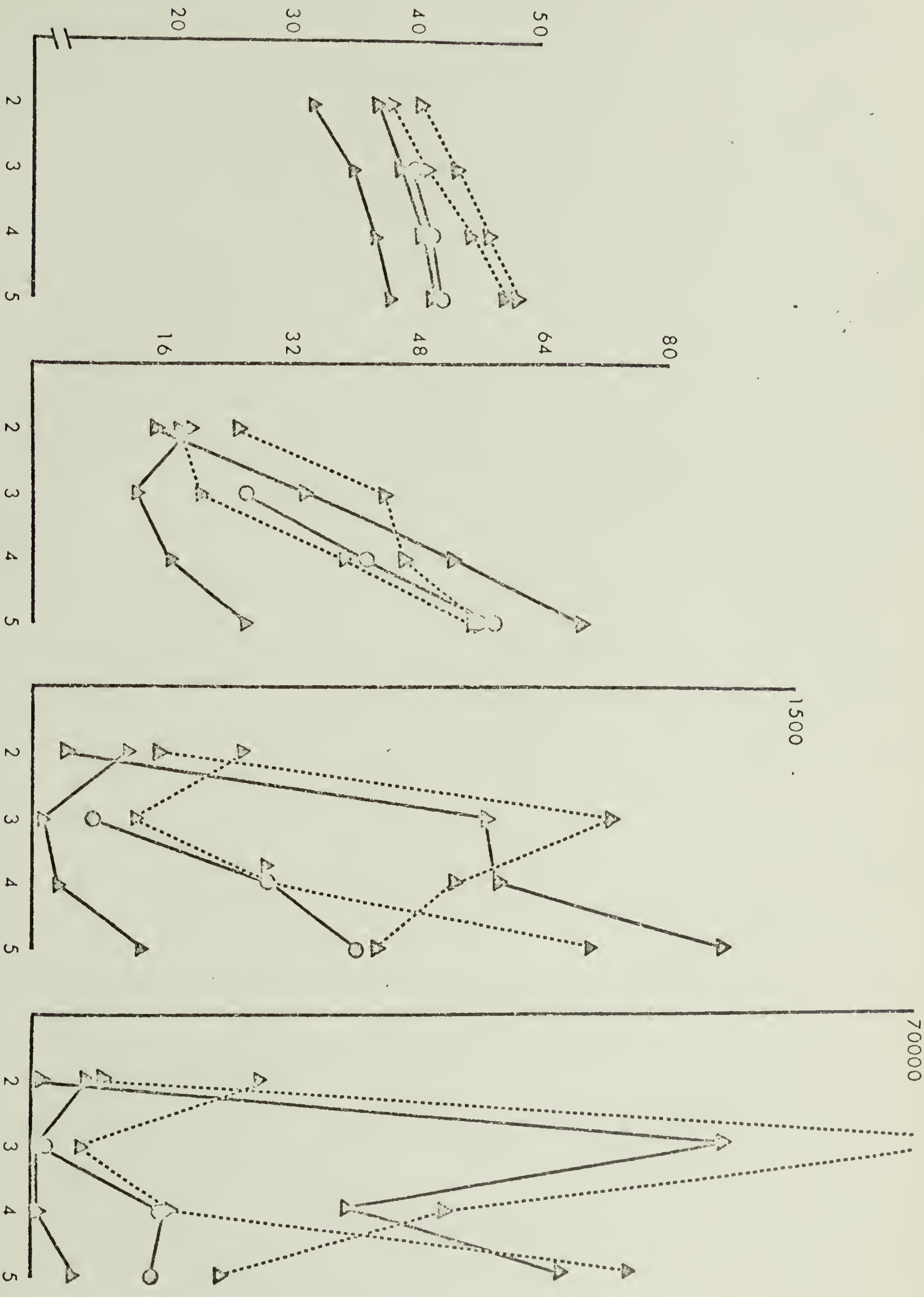

Fig. 11. Expt. II: Mean values of the first four k-statistics versus setsize; SRI.

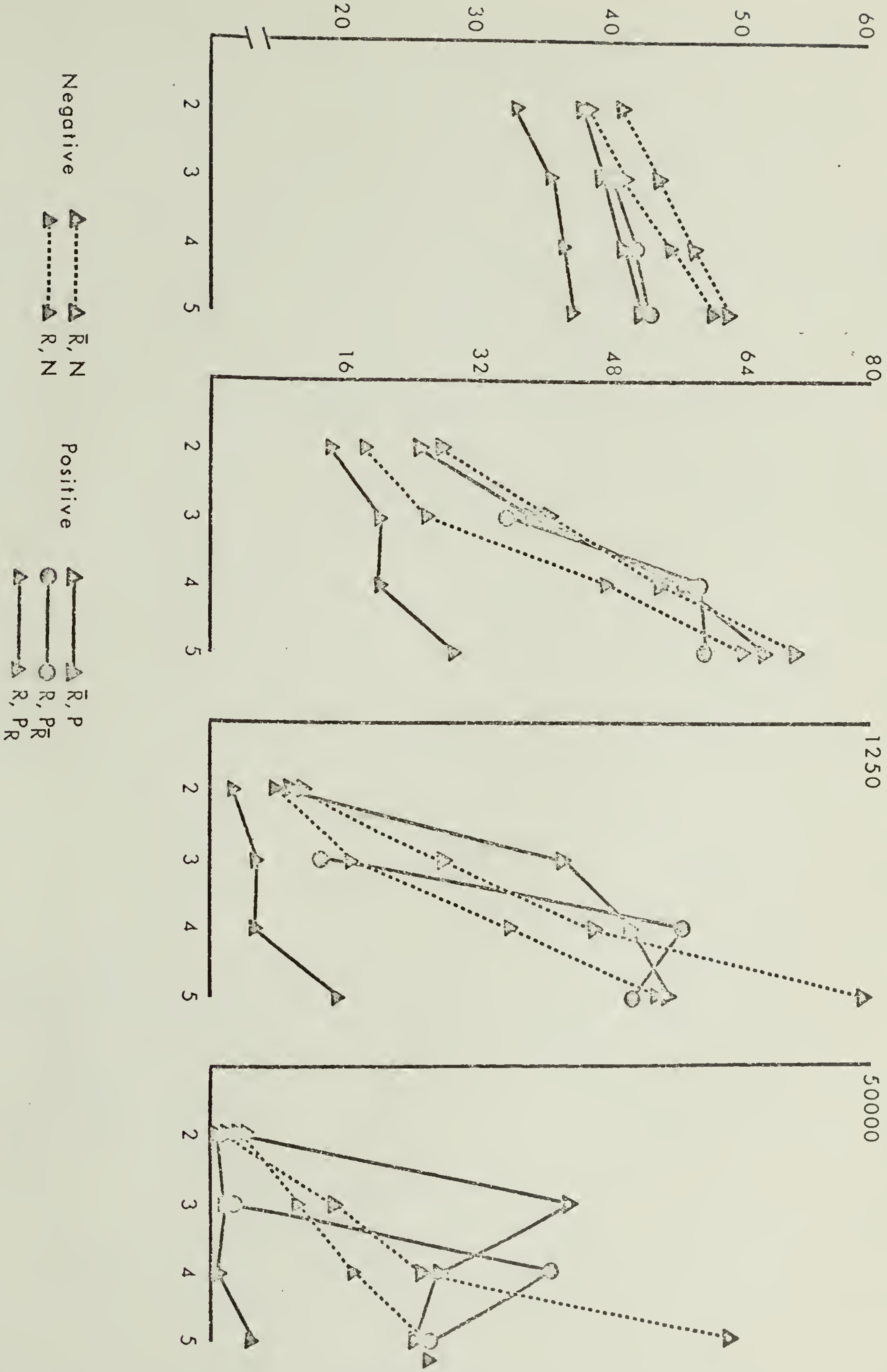

Fig. 12. Expt. II: Mean values of the first four k-statistics versus setsize; LRI.

DISCUSSION

Both experiments clearly reject a serial exhaustive scanning hypothesis. However, some aspects of the data seem to be in good agreement with a serial self-terminating scanning hypothesis. The slopes for positive responses of nonrepeated probes were about half the slopes of negative responses for both experiments as the theory predicts. And second, a good fit of the RT-distributions for positive responses was obtained when predicting those from negative responses assuming a self-terminating search. However, some aspects of the data superficially contradict the self-terminating hypothesis. First, the slope of the functions for probes of repeated items was not 2/3 the value of the slope of the functions for probes of nonrepeated items. Second, the zero-intercepts of the former functions were a great deal lower than those of the latter functions, while the self-terminating hypothesis would predict equality. And third, the serial position functions were neither flat nor linearly increasing, but appeared to be bowed.

An alternative self-terminating model can be constructed by making a very natural interpretation of the shape of the serial position functions. This model proves to account adequately for the other discrepancies from the predictions of the original self-terminating model. Imagine that S begins his scan at the middle position of the probe array, and with probability p scans to the right and with probability 1-p to the left. If he finds a match, he terminates the scan. However, if he reaches one end of the list before finding a match, he shifts his attention back to the middle of the list, taking time B, and scans the other half of the list in a self-terminating fashion. Such a strategy would predict bowed serial position curves. Furthermore, it is in agreement with introspective reports of the Ss. Finally, the model predicts the following functions relating $\overline{RT}$ to set size:

$$\overline{RT}_{Neg} = a + s(N) + B \qquad (6)$$

$$\overline{RT}_{Pos,nonrep} = a' + s\left(\frac{N+1}{2}\right) + 1/2(B) \qquad (7)$$

$$\overline{RT}_{Pos,rep} = a' + s\left(\frac{N+1}{3}\right) + x(B) \qquad (8)$$

where the value of x depends upon set size, B is the time taken to shift the scanning process back to the middle of the list after reaching an end, and the other terms have the same interpretations they had earlier (see Appendix B).

The mean RT for the 2 RI of Experiment 2 were collapsed, the value of s was computed from the $\overline{R}$, N conditions as 26 msec. and estimates for a' and B were obtained. Estimates of B and a' were obtained by minimizing the squared deviations between observed and predicted mean RT. The minimization was accomplished by using an iterative search routine, STEPIT, (Chandler, 1965) which manipulated the parameters until the minimum was found. The optimal value of a' was 303.7 and that of B was 76.76 msec. The results of observed and predicted RT as well as the 95% confidence interval for each positive condition are presented in Table 5. The model appears to be a good fit to the data. It does underestimate the slope of the R, P_R condition somewhat; however, all the predicted values (with the exception of SS = 2 for the R, P_R condition) are within the limits of the confidence intervals around the observed values.

To further test the model, the variances for the positive conditions were predicted from the negatives assuming the outlined self-terminating model (see Appendix C). As an example, the variances for SS = 2 for positive responses can be viewed as a mixture of 1/2 the variance of SS = 1 plus 1/2 the variance of SS = 2 plus 1/4 the squared differences between the means of SS = 2 and SS = 1 of the negative responses. The observed variances for the $\overline{R}$, N conditions were averaged over the two RI, and since no SS = 1 value was available for Experiment 2, the slope (least square criterion) and zero intercept of the function relating variance to SS was calculated and the variance value for SS = 1 estimated. The predicted and observed variances are shown in Table 6. As it can be seen the model does not predict the obtained variances very well. It underestimates the slopes for the $\overline{R}$, P and R, $P_{\overline{R}}$ condition and overestimates the slope for the R, P_R condition. (The actual value of the predicted slope for the latter condition is even higher, because the slope value predicted has been obtained without taking the increasing value of B with SS into account.) However, it has to be pointed out that the

Table 5

Observed (O) and Predicted (P) RT; 95% Confidence Intervals (CI); Experiment 2

	R, P_R				$R, P_{\bar{R}}$				$\bar{R}, P$			
SS	O	P	D	95% CI	O	P	D	95% CI	O	P	D	95% CI
2	322	330	+8	±6.8					374	381	+7	+8.4
3	352	351	-1	±6.8	396	394	-2	±10.0	391	394	+3	+8.4
4	366	360	-6	±6.8	413	407	-6	±10.0	408	407	-1	±8.4
5	376	371	-5	±6.8	419	420	+1	±10.0	418	420	+2	±8.4

Table 6

Slopes of observed and predicted variance functions

	Condition		
Slopes	$\bar{R}, P$	$R, P_{\bar{R}}$	R, P_R
observed	1560	1251	330
predicted	1076	1076	641

slopes of the variances for the $\bar{R}$, N conditions differ for each RI (1007.3 for the SRI, and 2272.1 for the LRI), and second, if slopes are fitted to the $\bar{R}$, N and R, N condition separately for each RI and the 1-intercepts are calculated only one results in a positive value.

Finally, the model in its present form does not make differential predictions for the different slopes and intercepts of the two distinguished negative conditions ($\bar{R}$, N and R, N) relating $\overline{RT}$ to SS. One might conceive that Ss some of the time search a list of only N-1 items for the case where there was a repeated item in the list, which would account for the negatives, but would not explain why no difference was found for the corresponding positive conditions ($\bar{R}$, P and R, $P_{\bar{R}}$).

CONCLUSION

The two experiments demonstrated that a fast self-terminating search is possible under certain conditions. The data of Experiment 1 exhibited the basic characteristics of a self-terminating search: The slope for the positive response function was 1/2 the slope of the negative response function. However, since the positive response function in Experiment 1 were somewhat negatively accelerated (as a parallel model might predict), a second experiment was conducted which confirmed the self-terminating scanning hypothesis and revealed further details about the nature of the self-terminating scanning process under the present condition. The data were in accord with a self-terminating model which assumes that the S starts to scan at the middle position of the probe array, proceeding with probability p to the left and with probability 1-p to the right. The S terminates his scan if he finds a match; however, if he reaches one end of the array without finding a match he shifts back to the middle of the list with time B and then scans the other half of the array in a self-terminating fashion.

Two questions still remain open. First, how is it possible for Ss to engage in a self-terminating search under the present condition. An attempt to answer this question was made by indicating that perhaps each item in the memorized list is compared against a newly-inputted (but identical) item from the probe list, and by suggesting that Ss are able to terminate their search when they shift from one list to another in selecting items to compare. If the presence of two separate lists is the necessary condition for self-termination, then a task where the memorized list is presented simultaneously, followed by a single probe item should result in an exhaustive search. And, in fact, Klatzky, Juola, and Atkinson (1971) did not find a significant difference between negative and positive slopes (with the exception of one condition on the third block) when presenting the memory set simultaneously and following it with a single probe item. However, a similar, earlier study (Klatzky and Atkinsin, 1970) gave different results. Also, the reverse, presenting the probe item first, followed by the memorized list, presented simultaneously, should result in an exhaustive search. This experiment has been discussed earlier (Atkinson et. al. 1969).

Second, it is not clear why when using multiple replications of the probe item, a fast self-terminating process occurs which is more efficient than an exhaustive one. Sternberg (1969) developed an explanation for his finding of exhaustiveness where he showed that under certain circumstances (relative long switching time compared to scanning rate) an exhaustive scan may be more efficient than a self-terminating one. While in the present experiments the slope and intercept values for negative responses are comparable to those obtained from experiments resulting in an exhaustive search process, the Ss in the present experiments were more efficient in handling positive responses, due to the fact that they engaged in a self-terminating search.

References

Atkinson, R. C., Holmgren, J. E., and Juola, H. F. Processing time as influenced by the number of elements in a visual display. Perception and Psychophysics, 1969, 6, 321-326.

Bamber, D. Reaction times and error rates for "same-different" judgments of multidimensional stimuli. Perception and Psychophysics, 1969, 6, 169-174.

Briggs, G. E., and Blaha, J. Memory retrieval and central comparison time in information processing. Journal of Experimental Psychology, 1969, 79, 395-402.

Chandler, J. P. Subroutine Stepit. Program QCPE66. Quantum Chemistry Program Exchange, Indiana University, Bloomington, Indiana, 1965.

Clifton, C., Jr., and Gutschera, K. D. Evidence for hierarchical search processes in a recognition memory task. Report 70-4, Cognitive Processes Laboratory, Department of Psychology, University of Massachusetts, November, 1970.

Klatzky, R. L., and Atkinson, R. C. Memory scans based on alternative test stimulus representation. Perception and Psychophysics, 1970, 8, 113-117.

Klatzky, R. L., Juola, J. F., and Atkinson, R. C. Test stimulus representations and experimental context effects in memory scanning. Journal of Experimental Psychology, 1971, 87, 281-288.

Nickerson, R. S. Response times with a memory dependent decision task. Journal of Experimental Psychology, 1966, 72, 761-769.

Smith, E. E., and Nielson, G. D. Representations and retrieval processes in short-term memory: Recognition and recall of faces. Journal of Experimental Psychology, 1970, 85, 397-405.

Sternberg, S. Estimating the distribution of additive reaction-time components. Paper presented at Psychometric Society, October, 1964.

Sternberg, S. High-speed scanning in human memory. Science, 1966, 153, 652-654.

Sternberg, S. Scanning a persisting visual image versus a memorized list. Eastern Psychological Association, 1967.

Sternberg, S. Memory-scanning: Mental Processes revealed by reaction time experiments. American Scientist, 1969, 57, 421-457.

Appendix A

Estimating the RT distribution for positive responses from negative responses assuming a self-terminating model. RT for positives can be thought of as a mixture of the response of the different set sizes for negatives

e.g. SS 5 RT for a positive response

(RT = 400/S.T. search with SS = 5)

= (RT = 400/1 comp)$\cdot$ p(1 comp)

+ (RT = 400/2 comp)$\cdot$ p(2 comp)

+ (RT = 400/3 comp)$\cdot$ p(3 comp)

+ (RT = 400/4 comp)$\cdot$ p(4 comp)

+ (RT = 400/5 comp)$\cdot$ p(5 comp)

where p for SS = 5 equals 1/5

Appendix B

Equations relating $\overline{RT}$ to SS assuming a self-terminating search with starting point in the middle of the array (for N = even, it was assumed that the starting point is the [(N/2)+1] item, for N = odd it was assumed that half of the time the S started with the [(N+1/2)+1] item and half of the time with the [N+1/2] item.)

I. R,P_R condition (it was assumed S always searches to the right)

SS 2	No. of items to search	prob.	E(I)
RR	1	1	1

SS 3			
xRR	1	1/3	1/3
RxR	1/2(2)+1/2(1)=1.5	1/3	1/2
RRx	1/2(1)+1/2(1+B+1)=1.5	1/3	1/2+1/6(B)
			Σ=4/3+1/6(B)

SS 4			
xxRR	1	1/6	1/6
xRxR	2	1/6	2/6
RxxR	2	1/6	2/6
xRRx	1	1/6	1/6
RxRx	1	1/6	1/6
RRxx	3+B	1/6	3/6+1/6(B)
			Σ=5/3+1/6(B)

SS 5	No. of items to search		prob.	E(I)
xxxRR	1/2(2)+1/2(1)	= 1.5	1/10	.15
xxRxR	1/2(1)+1/2(2)	= 1.5	"	.15
xRxxR	1/2(3)+1/2(2)	= 2.5	"	.25
RxxxR	1/2(3)+1/2(2)	= 2.5	"	.25
xxRRx	1/2(1)+1/2(1)	= 1.0	"	.10
xRxRx	1/2(2)+1/2(1)	= 1.5	"	.15
RxxRx	1/2(2)+1/2(1)	= 1.5	"	.15
xRRxx	1/2(1)+1/2[2+B+2]	= 2.5+.5B	"	.25+.05(B)
RxRxx	1/2(1)+1/2[2+B+1]	= 2.0+.5B	"	.20+.05(B)
RRxxx	1/2[3+B+1]+1/2[2+B+1]	= 3.5+B	1/10	.35+.10(B)
				Σ=2.0+1/5(B)

SS	
2	RT = a' + 3/3(s)
3	RT = a' + 4/3(s) + 1/6(B)
4	RT = a' + 5/3(s) + 1/6(B)
5	RT = a' + 6/3(s) + 1/5(B)

II. $\overline{R}$, P and R, $P_{\overline{R}}$

RT = a' + p[(N/2+1)/2]s + [(N/2)s+B+(N/2+1)/2)s](1-p)

if p = 1/2

RT = a' + [(N+1)/2]s + 1/2(B)

III. $\overline{R}$, N and R, N

RT = a' + p[(N/2)s+B+(N/2)s] + (1-p)[(N/2)s+B+(N/2)s]

if p = 1/2

RT = a' + (N)s + B

Appendix C

Estimation of the variances for positive responses from the variances of negative responses assuming a self-terminating model.

$$Var(X) = E(X^2) - [E(X)]^2$$ by definition of a variance

SS = 2

$$E(X) = pE(X_1) + qE(X_2)$$ $E(X_1)$ = mean of SS1, neg.

$$E(X^2) = pE(X_1{}^2) + qE(X_2{}^2)$$ $p = q = 1/2$

SS = 3

$$E(X) = 1/3E(X_1) + 1/3E(X_2) + 1/3E(X_3)$$
$$= 2/3E(X_{1+2}) + 1/3E(X_3)$$
$$E(X^2) = 1/3E(X_1{}^2) + 1/3E(X_2{}^2) + 1/3E(X_3{}^2)$$
$$= 2/3E(X_{1+2}{}^2) + 1/3E(X_3{}^2)$$

SS = 4

$$E(X) = 3/4E(X_{1+2+3}) + 1/4E(X_4)$$
$$E(X^2) = 3/4E(X_{1+2+3}{}^2) + 1/4E(X_4{}^2)$$

SS = 5

$$E(X) = 4/5E(X_{1+2+3+4}) + 1/5E(X_5)$$
$$E(X^2) = 4/5E(X_{1+2+3+4}{}^2) + 1/5E(X_5{}^2)$$