Memory Type Ratio and Product Estimators in Stratified Sampling

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Received 16 December 2019; Accepted 05 April 2020;
Publication 07 October 2020

Abstract

The exponential weighted moving average (EWMA) statistic is utilized the past information along with the present to enhance the efficiency of the estimators of the population parameters. In this study, the EWMA statistic is used to estimate the population mean with auxiliary information. The memory type ratio and product estimators are proposed under stratified sampling (*StS*). Mean square errors (*MSE*) expressions and relative efficiencies of the proposed estimators are derived. An extensive simulation study is conducted to evaluate the performance of the proposed estimators. An empirical study is presented based on real-life data that supports the findings of the simulation study.

Keywords: Stratified sampling, memory type, EWMA, ratio estimator, product estimator.

Journal of Reliability and Statistical Studies, Vol. 13, Issue 1 (2020), 1–20.
doi: 10.13052/jrss0974-8024.1311

1 Introduction

Estimation of the parameters for the heterogeneous population has remained a keen interest for survey statistician. In this context, the use of auxiliary information is considered incredibly supportive for the estimation procedure. If auxiliary information is linearly positive correlated with the study variable, the ratio estimator is employed. The ratio estimator for population mean using stratified sampling (*StS*) suggested by (Cochran, 1977) as

$$\bar{y}_{rs} = \frac{\bar{y}_s}{\bar{x}_s}\bar{X}, \tag{1}$$

and if the relationship of the auxiliary variable is linearly negative with the variable of interest then it is healthier to use product estimator. Robson (1957) used such auxiliary information for the population mean estimation and is given by

$$\bar{y}_{ps} = \frac{\bar{y}_s}{\bar{X}}\bar{x}_s \tag{2}$$

where $\bar{y}_s = \sum_{h=1}^{L} W_h\bar{y}_h$ is the *StS* mean of study variable y, $\bar{x}_s = \sum_{h=1}^{L} W_h\bar{x}_h$ is the *StS* mean of auxiliary variable x and $\bar{X}$ is the known population mean of auxiliary variable.

The *MSE*s of (1) and (2) are as follows

$$MSE(\bar{y}_{rs}) = \sum_{h=1}^{L} W_h^2\theta_h(S_{yh}^2 + R^2S_{xh}^2 - 2RS_{yxh}) \tag{3}$$

$$MSE(\bar{y}_{ps}) = \sum_{h=1}^{L} W_h^2\theta_h(S_{yh}^2 + R^2S_{xh}^2 + 2RS_{yxh}) \tag{4}$$

where $\theta_h = \frac{1-n_n/N_h}{n_h}, W_h = \frac{N_h}{N}, R = \frac{\bar{Y}}{\bar{X}}$ is the population ratio of means, n_h is the sample size of stratum $h \cdot S_{yh}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)^2$ is the population variance of h^{th} stratum of study variable, $S_{xh}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_h)^2$ is the population variance of auxiliary variable of h^{th} stratum, $S_{yxh} = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_h)(y_{hi} - \bar{Y}_h)$ is the population covariance between auxiliary variables and study variable of h^{th} stratum and $\overline{Y}_h, \overline{X}_h$ are the means of h^{th} stratum of y and x.

Many authors improved the precision of ratio and product estimators given in (1–2) by modifying them in different scenarios. For example, (Kadilar and Cingi, 2003) gave the stratified version of (Sisodia and Dwivedi, 1981; Upadhyaya and Singh, 1999), similarly (Kadilar and Cingi, 2005) extended the version of (Prasad, 1989) from simple random sampling (SRS) to *StS*, (Koyuncu and Kadilar, 2009) adopted the general family of estimators of (Khoshnevisan, et al., 2007; Searls, 1964) to *StS*, (Yasmeen, Noor-ul-Amin, and Hanif, 2015) gave a generalized exponential estimator for population mean using transformed auxiliary variable, (Malik and Singh, 2017) suggested exponential-type estimators with two auxiliary variables in *StS*, (Kumar, Trehan, and Joorel, 2018) gave the population mean estimate using two auxiliary variables by considering the impact of meansurment error and in the presence of non-response, (Noor-ul-Amin, Asghar, Sanaullah, and Shehzad, 2018) proposed a robust estimator using redescending M-estimator using auxiliary information. (Raza, Noor-ul-Amin, and Hanif, 2019) proposed a regression in ratio estimator in the existence of outliers using redescending M-Esimator, (Saini and Kumar, 2019) gave the transformed version of *StS* and stratified ranked set sampling to estimate the mean of the population.

Aforementioned studies of estimators are only useful for the surveys which are cross-sectional in nature. Therefore, the competence of the usual ratio and product estimators may be affected for surveys, in which collected information is based on time scaled, see for example (Noor-ul-Amin, 2019). To overcome this situation (Noor-ul-Amin, 2019) proposed population mean ratio and product estimators using SRS for time series data, namely memory type ratio and product estimators. (Noor-ul-Amin, 2019) suggested memory type estimators are efficient only in the case when under study population is homogenous. The problem arises for the case where under study population is heterogeneous. i.e. under study population is divided into different strata, in which each stratum is relatively heterogeneous with other strata and homogenous within itself. To overcome this problem, in this article we are suggesting the new memory type estimators for population mean that is heterogeneous in nature. A brief discussion about the memory type estimator is provided in the next section.

2 Memory Type Estimator

The EWMA statistic was first used by the (Roberts, 1959) for the purpose to make use of past information and current information simultaneously to enhance the efficiency of the estimators and is given below

$$Z_t = \lambda \bar{y}_t + (1-\lambda) Z_{t-1}, \quad \text{where } t > 0 \tag{5}$$

where $\bar{y}_t$ is the sample mean at time *t*, λ is the weight parameter/smoothing constant of observations. Its value ranges from $0 < \lambda \leq 1$. As λ moves from 0 to 1 current information gains more weight and at the same time past information goes down its weight. For $\lambda = 1$, the statistic provided in (5) becomes usual sample mean. Where Z_{t-1} indicate the past observations of the statistic. The initial value (Z_0) of Z_{t-1} is the expected mean. Expected mean value along with variance of (5) is given as

$$E(Z_t) = \bar{Y} \quad \text{and} \quad Var(Z_t) = \frac{S_y^2}{n}\left[\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2t})\right],$$

where, $\bar{Y}$ and S_y^2 are the mean and variance of y, the study variable. As $t \to \infty$ the variance of (5) is given as

$$Var(Z_t) = \frac{S_y^2}{n}\left[\frac{\lambda}{2-\lambda}\right]. \tag{6}$$

3 Proposed Memory Type *StS* Estimators

This section include proposed memory type ratio and product estimators. Suppose that the study variable is denoted by *y* and the auxiliary variable is denoted by *x*. The EWMA statistic is utilized to propose memory type estimators as given in (5). For easiness, we are once more bringing up the EWMA memroty base statistic for study variable (*y*) as provided in (5), represented by Z_{st} such that

$$Z_{st} = \lambda \bar{y}_{st} + (1-\lambda) Z_{s(t-1)} \tag{7}$$

and EWMA statistic for auxiliary variable (*x*) represented by Q_{st} such that

$$Q_{st} = \lambda \bar{x}_{st} + (1-\lambda) Q_{s(t-1)}, \tag{8}$$

where $\bar{y}_{st}$ and $\bar{x}_{st}$ representes the means of sample data under *StS* scheme at time *t*. The proposed memory type EWMA ratio and product estimators under *StS* design as

$$\bar{y}_{rst}^{M} = \frac{Z_{st}}{Q_{st}} \bar{X}, \tag{9}$$

and

$$\bar{y}_{pst}^{M} = \frac{Z_{st}}{\bar{X}} Q_{st}, \tag{10}$$

to obtain the *MSE* expression for proposed ratio estimator, we define the following notations such as

$$\left.\begin{aligned} &e_{yst} = \frac{Z_{st} - \bar{Y}}{\bar{Y}} \quad \text{and} \quad e_{xst} = \frac{Q_{st} - \bar{X}}{\bar{X}} \\ &E(e_{yst}) = 0,\ E(e_{yst}^2) = \frac{Var(Z_{st})}{\bar{Y}^2} = \frac{1}{\bar{Y}^2}\left[\frac{\lambda}{2-\lambda}\right] Var(\bar{y}_s) \end{aligned}\right\} \tag{11}$$

Further we have,

$$E(e_{yst}e_{xst}) = \frac{1}{\bar{X}\bar{Y}} Cov(Z_{st}, Q_{st}) = \frac{1}{\bar{X}\bar{Y}}\left[\frac{\lambda}{2-\lambda}\right] Cov(\bar{y}_s, \bar{x}_s), \tag{12}$$

where

$$Var(\bar{y}_s) = \sum_{h=1}^{L} \theta_h W_h^2 S_{yh}^2, \quad Var(\bar{x}_s) = \sum_{h=1}^{L} \theta_h W_h^2 S_{xh}^2$$

and

$$Cov(\bar{y}_s, \bar{x}_s) = \sum_{h=1}^{L} \theta_h W_h^2 S_{yxh}.$$

To derive the *MSE* expression put value of Z_{st} and Q_{st} from (11) into (9) and then simplifying by Taylor series approximation, we have

$$\bar{y}_{rst}^M = \bar{Y}(1 + e_{yst})(1 + e_{xst})^{-1},$$

further simplifying and ignoring the higher order terms, the approximate expression is obtained as

$$\bar{y}_{rst}^M \approx \bar{Y}(1 + e_{yst})(1 - e_{xst}),$$

the approximate MSE expression is given by

$$MSE(\bar{y}_{rst}^M) \approx \bar{Y}^2[Var(Z_{st}) + Var(Q_{st}) - 2Cov(Z_{st}, Q_{st})] \tag{13}$$

$$MSE(\bar{y}_{rst}^M) \approx \frac{\lambda}{\lambda - 2}[Var(\bar{y}_s) + Var(\bar{x}_s) - 2Cov(\bar{y}_s, \bar{x}_s)] \tag{14}$$

after simplification, we have

$$MSE(\bar{y}_{rst}^M) \approx \frac{\lambda}{2-\lambda} \sum_{h=1}^{L} W_h^2 \theta_h [S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh}] \tag{15}$$

In similar manner, the approximate *MSE* expression for the proposed memory type product estimator is specified by

$$MSE(\bar{y}_{pst}^{M}) \approx \frac{\lambda}{2-\lambda} \sum_{h=1}^{L} W_h^2 \theta_h [S_{yh}^2 + R^2 S_{xh}^2 + 2RS_{yxh}] \quad (16)$$

4 Simulation Study

To evaluate the performance of the proposed memory type estimator an extensive simulation study is carried out of the proposed ratio and product estimators. The presented *MSE*s are based on 50,000 replicates for various *StS*. The *MSE* is computed for each *StS* scheme as

$$MSE(\bar{y}_{rst}) = \frac{1}{50,000} \sum_{i=1}^{50000} (a_i - \bar{Y})^2, \quad (17)$$

where $a = \bar{y}_{rs}, \bar{y}_{rs}^{M}, \bar{y}_{ps}, \bar{y}_{ps}^{M}$. The relative efficiencies (*RE*s) are obtained as

$$RE(a) = \frac{MSE(\bar{y}_{st})}{MSE(a)}. \quad (18)$$

where d $= \bar{y}_{rs}, \bar{y}_{rs}^{M}, \bar{y}_{ps}, \bar{y}_{ps}^{M}$ and μ_d is the mean of *d*.

The *MSE* values are presented in Tables 1 and 2 the results regarding the *RE*s are given in Tables 3 and 4. The results of *MSE*s and *RE*s are acquired for different choices of correlation coefficients, i.e. 0.05, 0.25, 0.50, 0.75, 0.95. To see the impact of smoothing constant, we have used the various values of λ, i.e. 0.05, 0.10, 0.25, 0.50, 0.75, 1.0.

The following are steps that have been used to compute the *MSE*s and *RE*s of the proposed EWMA type ratio and product estimators under stratified sampling:

(i) A population of size $N = 10{,}000$ is breeded from the bivariate normal distribution and divided into two strata as, half of the size is generated with parameters $(Y, X) \sim N_2(2, 10, 1, 1, \rho)$ for stratum 1 and remaining half with parameters $(Y, X) \sim N_2(40, 50, 1, 1, \rho)$ for stratum 2.

(ii) Units from each stratum are selected to make sample sizes as $n = 10$, 20, 30, 50, 200 and 500 using SRS.

(iii) Compute the EWMA statistic given in (7–8) with a different choice of smoothing constant.

Table 1 *MSEs* of usual and proposed memory type ratio estimators

			$\lambda = 0.05$	$\lambda = 0.10$	$\lambda = 0.25$	$\lambda = 0.50$	$\lambda = 0.75$	$\lambda = 1.0$
ρ	n	$\bar{y}_{rst}$	$\bar{y}^M_{rst}$	$\bar{y}^M_{rst}$	$\bar{y}^M_{rst}$	$\bar{y}^M_{rst}$	$\bar{y}^M_{rst}$	$\bar{y}^M_{rst}$
0.05	10	0.1443	0.0038	0.0076	0.0205	0.0480	0.0860	0.1450
	20	0.0720	0.0018	0.0038	0.0102	0.0230	0.0430	0.0720
	30	0.0483	0.0012	0.0025	0.0069	0.0160	0.0290	0.0480
	50	0.0290	0.0007	0.0015	0.0040	0.0090	0.0170	0.0290
	200	0.0072	0.0002	0.0003	0.0010	0.0020	0.0040	0.0070
	500	0.0029	0.0001	0.0001	0.0004	0.0000	0.0010	0.0020
0.25	10	0.1150	0.0038	0.0061	0.0164	0.0380	0.0690	0.1150
	20	0.0518	0.0015	0.0030	0.0081	0.0190	0.0340	0.0580
	30	0.0390	0.0010	0.0020	0.0054	0.0120	0.0230	0.0380
	50	0.0232	0.0006	0.0012	0.0033	0.0070	0.0130	0.0230
	200	0.0057	0.0001	0.0003	0.0008	0.0010	0.0030	0.0050
	500	0.0023	0.0001	0.0001	0.0003	0.0000	0.0010	0.0020
0.50	10	0.0799	0.0021	0.0041	0.1150	0.0260	0.0480	0.0800
	20	0.0402	0.0010	0.0020	0.0037	0.0130	0.0240	0.0400
	30	0.0269	0.0007	0.0014	0.0038	0.0080	0.6300	0.0270
	50	0.0162	0.0004	0.0008	0.0023	0.0050	0.6180	0.0160
	200	0.0040	0.0004	0.0002	0.0005	0.0010	0.6210	0.0040
	500	0.0016	0.0001	0.0002	0.0002	0.0000	0.6230	0.0010
0.75	10	0.0447	0.0010	0.0023	0.0064	0.0140	0.0260	0.0450
	20	0.0223	0.0005	0.0012	0.0032	0.0070	0.0130	0.0220
	30	0.0149	0.0003	0.0007	0.0021	0.0040	0.0080	0.0140
	50	0.0090	0.0002	0.0004	0.0012	0.0020	0.0050	0.0090
	200	0.0022	0.0001	0.0001	0.0003	0.0000	0.0010	0.0020
	500	0.0008	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
0.95	10	0.0163	0.0004	0.0000	0.0020	0.0060	0.0090	0.0160
	20	0.0082	0.0002	0.0000	0.0011	0.0020	0.0040	0.0080
	30	0.0054	0.0001	0.0000	0.0000	0.0010	0.0030	0.0050
	50	0.0032	0.0000	0.0000	0.0000	0.0010	0.0010	0.0030
	200	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	500	0.0003	0.000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 2 *MSEs* of usual and proposed memory type product estimators

			$\lambda=0.05$	$\lambda=0.10$	$\lambda=0.25$	$\lambda=0.50$	$\lambda=0.75$	$\lambda=1.0$
ρ	n	$\bar{y}_{pst}$	$\bar{y}_{pst}^{M}$	$\bar{y}_{pst}^{M}$	$\bar{y}_{pst}^{M}$	$\bar{y}_{pst}^{M}$	$\bar{y}_{pst}^{M}$	$\bar{y}_{pst}^{M}$
−0.05	10	0.1463	0.0038	0.0078	0.0214	0.0480	0.0875	0.1451
	20	0.0724	0.0018	0.0037	0.0102	0.0244	0.0434	0.0721
	30	0.0480	0.0013	0.0025	0.0069	0.0161	0.0292	0.0481
	50	0.0291	0.0007	0.0015	0.0041	0.0096	0.0172	0.0288
	200	0.0072	0.0002	0.0004	0.0010	0.0024	0.0044	0.0073
	500	0.0029	0.0001	0.0002	0.0004	0.0010	0.0018	0.0029
−0.25	10	0.1162	0.0030	0.0063	0.0165	0.0385	0.0699	0.1164
	20	0.0578	0.0015	0.0030	0.0083	0.0195	0.0348	0.0578
	30	0.0386	0.0010	0.0020	0.0056	0.0132	0.0234	0.0389
	50	0.0232	0.0006	0.0012	0.0034	0.0079	0.0139	0.0236
	200	0.0059	0.0001	0.0003	0.0008	0.0020	0.0035	0.0059
	500	0.0023	0.0001	0.0001	0.0003	0.0008	0.0014	0.0023
−0.50	10	0.0805	0.0020	0.0042	0.0116	0.0268	0.0479	0.0802
	20	0.0406	0.0010	0.0021	0.0058	0.0135	0.0241	0.0402
	30	0.0271	0.0007	0.0014	0.0038	0.0089	0.0161	0.0267
	50	0.0161	0.0004	0.0008	0.0024	0.0054	0.0096	0.0159
	200	0.0040	0.0001	0.0002	0.0006	0.0013	0.0024	0.0040
	500	0.0016	0.0000	0.0001	0.0002	0.0006	0.0010	0.0016
−0.75	10	0.0447	0.0011	0.0023	0.0064	0.0150	0.0268	0.0446
	20	0.0223	0.0006	0.0012	0.0032	0.0075	0.0135	0.0224
	30	0.0149	0.0004	0.0008	0.0021	0.0050	0.0090	0.0149
	50	0.0089	0.0002	0.0005	0.0013	0.0030	0.0054	0.0090
	200	0.0022	0.0001	0.0001	0.0003	0.0008	0.0014	0.0023
	500	0.0009	0.0000	0.0000	0.0001	0.0003	0.0005	0.0009
−0.95	10	0.0163	0.0004	0.0009	0.0024	0.0054	0.0097	0.0162
	20	0.0081	0.0002	0.0004	0.0011	0.0027	0.0049	0.0082
	30	0.0055	0.0001	0.0003	0.0008	0.0018	0.0033	0.0055
	50	0.0033	0.0001	0.0002	0.0005	0.0011	0.0020	0.0032
	200	0.0008	0.0000	0.0000	0.0001	0.0003	0.0005	0.0008
	500	0.0003	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003

Table 3 *REs* of usual and proposed memory type ratio estimator

			$\lambda = 0.05$	$\lambda = 0.10$	$\lambda = 0.25$	$\lambda = 0.50$	$\lambda = 0.75$	$\lambda = 1.0$
ρ	n	$\bar{y}_{rst}$	$\bar{y}_{rst}^{M}$	$\bar{y}_{rst}^{M}$	$\bar{y}_{rst}^{M}$	$\bar{y}_{rst}^{M}$	$\bar{y}_{rst}^{M}$	$\bar{y}_{rst}^{M}$
0.05	10	0.3470	13.1600	6.5410	2.4133	1.0390	0.5800	0.3440
	20	0.3462	13.4110	6.4000	2.4360	1.0480	0.5740	0.3430
	30	0.3466	13.9360	6.4460	2.3870	1.0520	0.5730	0.3480
	50	0.3441	13.5930	6.5780	2.4350	1.0470	0.5760	0.3420
	200	0.3421	12.6590	6.9070	2.3780	1.0370	0.5720	0.3430
	500	0.3460	13.8380	6.2940	2.4480	1.0250	0.5730	0.3450
0.25	10	0.4510	16.2200	8.2400	3.0200	1.2900	0.7110	0.4302
	20	0.4210	16.4000	8.1000	3.0240	1.3000	0.7200	0.4307
	30	0.4320	17.1800	8.2500	3.0700	1.2900	0.7040	0.4341
	50	0.4280	16.6300	7.9500	2.9600	1.2800	0.7210	0.4295
	200	0.4260	17.1400	8.1300	2.9800	1.2980	0.7220	0.4349
	500	0.4290	16.5500	8.3300	3.0500	1.2990	0.7160	0.4300
0.50	10	0.6240	24.0040	12.1400	4.3200	1.8640	1.0230	0.6254
	20	0.6290	24.3800	11.9200	4.4000	1.8560	1.0300	0.6095
	30	0.6170	23.4230	11.7900	4.3500	1.8640	1.0480	0.6137
	50	0.6120	24.0060	11.7000	4.3400	1.8540	1.0360	0.6161
	200	0.6220	23.5100	12.1600	4.3000	1.8500	1.0290	0.6180
	500	0.6040	23.6230	11.7100	4.3500	1.8240	1.0450	0.6241
0.75	10	1.1295	44.4650	21.1800	7.8350	3.3840	1.8410	1.0994
	20	1.1145	43.7736	20.9190	7.7290	3.3160	1.8380	1.1188
	30	1.1155	43.6897	21.7500	7.7910	3.3500	1.8620	1.1122
	50	1.1065	41.7361	21.7100	7.8130	3.3980	1.8800	1.1163
	200	1.1138	43.3487	21.1900	7.7320	3.3520	1.8360	1.1128
	500	1.1158	41.6332	21.2200	7.9120	3.3240	1.8410	1.1128
0.95	10	3.0200	121.7600	60.1400	21.9500	9.1960	5.0700	3.0408
	20	3.0000	112.9100	58.7000	21.6300	9.2520	5.1000	3.0946
	30	3.0300	118.6000	58.7000	21.4100	9.2980	5.1330	3.0989
	50	3.0600	118.6500	58.9900	21.3900	9.2130	5.0990	3.0420
	200	3.0400	119.9000	59.1700	21.1600	9.2000	5.1330	3.1346
	500	3.0754	115.8383	57.2356	21.7014	9.3282	5.1587	3.0780

Table 4 *REs* of usual and proposed memory type product estimators

			$\lambda = 0.05$	$\lambda = 0.10$	$\lambda = 0.25$	$\lambda = 0.50$	$\lambda = 0.75$	$\lambda = 1.0$
ρ	n	$\bar{y}_{pst}$	$\bar{y}^M_{pst}$	$\bar{y}^M_{pst}$	$\bar{y}^M_{pst}$	$\bar{y}^M_{pst}$	$\bar{y}^M_{pst}$	$\bar{y}^M_{pst}$
−0.05	10	0.3435	13.2232	6.3732	2.3632	1.0376	0.5735	0.3475
	20	0.3431	13.4697	6.6597	2.4502	1.0190	0.5766	0.3445
	30	0.3465	12.9030	6.6374	2.4120	1.0319	0.5692	0.3486
	50	0.3421	14.0460	6.6663	2.4332	1.0398	0.5766	0.3468
	200	0.3467	12.9845	6.3458	2.3874	1.0229	0.5724	0.3428
	500	0.3471	13.9675	6.5946	2.3853	1.0336	0.5680	0.3434
−0.25	10	0.4267	16.6829	7.8544	3.0119	1.2879	0.7180	0.4324
	20	0.4304	17.0404	8.3323	3.0080	1.2796	0.7088	0.4323
	30	0.4297	17.3095	8.3542	2.9640	1.2614	0.7158	0.4267
	50	0.4320	17.2784	8.3305	2.9379	1.2636	0.7196	0.4204
	200	0.4266	17.0044	8.0705	3.0803	1.2883	0.7135	0.4257
	500	0.4242	16.1858	8.2002	3.0031	1.2761	0.7175	0.4289
−0.50	10	0.6160	24.9066	11.9546	4.3494	1.8636	1.0393	0.6226
	20	0.6171	24.0858	11.8185	4.3106	1.8408	1.0238	0.6286
	30	0.6086	23.8897	11.6560	4.3591	1.8535	1.0375	0.6152
	50	0.6173	23.8366	11.9841	4.1903	1.8678	1.0473	0.6250
	200	0.6222	24.1115	11.6746	4.2693	1.8669	1.0369	0.6201
	500	0.6194	23.8225	11.8118	4.2322	1.8262	1.0329	0.6253
−0.75	10	1.1033	43.4641	21.5880	7.8199	3.3253	1.8607	1.1189
	20	1.1227	44.0298	21.2748	7.6801	3.3052	1.8562	1.1184
	30	1.1313	41.4012	20.7627	7.8146	3.3316	1.8628	1.1158
	50	1.1226	45.8385	20.6880	7.7081	3.3333	1.8787	1.1199
	200	1.1090	42.5829	20.2710	7.8833	3.3342	1.8392	1.0847
	500	1.1102	42.9247	20.9144	7.7987	3.3018	1.8213	1.1157
−0.95	10	3.0487	117.4114	56.8925	20.9934	9.1926	5.1134	3.0829
	20	3.0113	123.1020	57.1116	21.9927	9.2291	5.1361	3.0418
	30	3.0592	112.4043	59.9413	21.2540	9.2102	5.1279	3.0765
	50	3.0416	118.3387	59.6482	21.6707	9.3233	5.1562	3.0720
	200	3.0854	121.3365	59.5081	20.9629	9.0135	5.0850	3.0435
	500	3.0123	113.8820	58.8907	21.4038	9.1840	5.1075	3.0562

(iv) Compute the ratio and product estimators given in (9–10).
(v) Repeat the steps (i) to (iv) 50,000 times.
(vi) The simulated *MSE* of proposed estimators are computed using the formula (17) and results are produced in Tables 1–2.
(vii) *RE*s of each sample size for proposed estimators are obtained by using equation (18) and are reported in Tables 3–4.

5 Main Findings

The calculated results of *MSE*s and *RE*s for the proposed scheme are presented in Tables 1–4. The comparative study in terms of *MSE*s and *RE*s of proposed memory type ratio estimator based on stratified sampling scheme are given in Tables 1 and 3. The comparisons of the proposed memory type product estimator based on stratified sampling scheme are presented in Tables 2 and 4. Imperative findings of the proposed estimators are:

(i) We can observe from Table 1 that *MSEs* of the suggested EWMA ratio estimator for population mean is smaller as compared with the usual *StS* estimator for all values of λ. This shows the performance of proposed EWMA ratio estimator than usual *StS*. Similar results are observed for the proposed memory type product estimator which can be observed in Table 2.
(ii) The *RE*s of the proposed ratio estimator are shown in Tables 3 and 4. In Table 3 the *RE*s are higher than the usual *StS* estimator. This proves that the efficiency of the proposed ratio estimator is good with respect to the usual stratified sample estimator. Similar results are observed from the proposed product estimator which is given in Table 4.
(iii) As ρ_{xy} i.e. coefficient of correlation increases from zero to 0.95, we observed the decrease in the values of *MSE*s with the increase of relative efficiency for the suggested estimator. As we ρ_{xy} decreases from zero to -0.95, the relative efficiency of the proposed product EWMA estimator improved, this fact can be see from Table 4. This conclude that the use of auxiliary information has positive impact of the estimator in term of efficiency.
(iv) Fixing λ and ρ_{xy}, increases the sample size i.e. $n = 10$, 20, 30, 50, 100, 200, 500 causes to reduced the *MSEs* with increase of sample size.
(v) Smoothing constant λ is used to utilize the past information and current information. The impact of λ in term of efficiency can be viewed from

Tables 3 and 4. Moreover, $\lambda = 1$ means only the current information is utilized and no previous information is going to use. Then in this situation memory type proposed estimators will depend only on current information to estimate the population mean just like usual ratio and product estimators. Hence, for $\lambda = 1$ the proposed *StS* EWMA estimator and usual *StS* estimator will be equally good. On the other hand, as value of λ decreased this causes the increased in the efficiency of proposed EWMA ratio and product estimator, which can also be see from Tables 3 and 4.

6 Mathematical Comparison

In real life application proposed EWMA type ratio and product estimators are preferable to use as compared with usual *StS* ratio estimator; if it is provide evidence that proposed estimator has lesser *MSE*. This purpose can be achieved by obtaining the mathematical condition. Consequently, the proposed EWMA type ratio estimator in *StS* considered to be more efficient than usual ratio estimator in *StS* if the following condition hold:

$$
\begin{aligned}
&MSE(\bar{y}_{rst}^{M}) < MSE(\bar{y}_{rst}),\\
&\frac{\lambda}{2-\lambda}\sum_{h=1}^{L} W_h^2\theta_h[S_{yh}^2 + R^2S_{xh}^2 - 2RS_{yxh}]\\
&\qquad < \sum_{h=1}^{L} W_h^2\theta_h(S_{yh}^2 + R^2S_{xh}^2 - 2RS_{yxh}),\\
&\qquad \frac{\lambda}{2-\lambda} < 1,\\
&\qquad \lambda < 1.
\end{aligned}
\tag{19}
$$

This shows the efficiency of proposed EWMA type estimator over usual ratio estimator for all values of $\lambda < 1$. As we have previously pointed out in Section 1 for EWMA statistic that λ range from 0 to 1. So, the proposed

EWMA ratio estimator will always more efficient than usual *StS* ratio estimator for all values of $\lambda < 1$. Further, the execution of both proposed and usual *StS* estimator will be identical for $\lambda = 1$. Simulation study presented in Section 4 also support this fact.

Similarly, proposed product estimator considered to be more efficient as compared with comparative product estimator if

$$MSE(\bar{y}_{pst}^{M}) < MSE(\bar{y}_{pst})$$

$$\lambda < 1$$

7 Application

In this section, to demonstrate the relevance of proposed estimators a real life example is presented.

The time scaled data of wheat consisting of 28 years from 1981–82 to 2008–09 is taken from Agriculture Census Wing, Pakistan Bureau of Statistics, Government of Pakistan. The study variable *y* is production (in “000” Tonnes) and area (in “000” Hectares) is taken as auxiliary variable *x*. Data consisting of four provinces of Pakistan, theses provinces are used as strata. The linear correlation between production and area is found to be 0.9723 and $\lambda = 0.10$. From Table 5 it is clear that the proposed estimator is performing better as compared to the usual ratio estimator given in (1). *MSE* of the proposed ratio estimator is found to be 13011.83 which is lesser than the *MSE* of corresponding usual ratio estimator which is 346098.80. The relative efficiency of the proposed estimator is 2091.393 and usual ratio estimator is 78.6274 showing that the proposed estimator outclass the available usual ratio estimator. Figure 1 represents the performance of the proposed estimator with the usual ratio estimator. From this figure, we can observe that the proposed estimator is close to the population mean in the case of stratified sample and has less variation as compared to the usual ratio estimator. So, from Table 5 and Figure 1 we can conclude that the proposed memory type estimator performing better than the available usual ratio estimator.

Table 5 Calculations of usual and proposed memory type estimator

	Stratified Mean		EWMA Stratified Mean		Ratio Mean Estimator Usual	Ratio Mean Estimator Proposed
Crop Year	$\bar{y}_s$	$\bar{x}_s$	Z_{st}	Q_{st}	$\bar{y}_{rst}$	$\bar{y}^M_{rst}$
1981–82	3038.08	1807.98	4085.48	1984.25	3367.19	4125.80
1982–83	4132.63	1972.70	4090.19	1983.09	4197.85	4132.97
1983–84	3753.00	1918.88	4056.47	1976.67	3919.17	4112.21
1984–85	4960.00	2162.35	4146.82	1995.24	4596.4	4164.69
1985–86	3832.55	2010.63	4115.40	1996.78	3819.61	4129.94
1986–87	3355.53	1903.15	4039.41	1987.42	3533.05	4072.78
1987–88	4888.15	2099.08	4124.28	1998.58	4666.36	4135.12
1988–89	4886.05	2056.53	4200.46	2004.38	4760.86	4199.33
1989–90	5081.33	2101.03	4288.55	2014.04	4846.27	4266.81
1990–91	5723.68	2146.10	4432.06	2027.25	5344.25	4380.87
1991–92	4315.23	2051.73	4420.38	2029.69	4214.50	4364.06
1992–93	5205.00	2138.78	4498.84	2040.60	4876.60	4417.78
1993–94	4078.03	2014.55	4456.76	2038.00	4056.33	4382.05
1994–95	5208.33	2192.85	4531.91	2053.48	4759.39	4422.34
1995–96	4950.85	2096.98	4573.81	2057.83	4730.95	4453.79
1996–97	4948.10	2133.78	4611.24	2065.43	4646.77	4473.73
1997–98	3047.43	1861.85	4454.86	2045.07	3279.82	4365.03
1998–99	3152.80	1959.15	4324.65	2036.48	3224.71	4255.33
1999–00	2987.70	1802.98	4190.96	2013.13	3320.54	4171.61
2000–01	4813.60	2108.45	4253.22	2022.66	4574.76	4213.64
2001–02	3585.30	1957.58	4186.43	2016.15	3670.02	4160.85
2002–03	4929.28	2126.70	4260.71	2027.21	4644.50	4211.59
2003–04	4821.08	2067.63	4316.75	2031.25	4672.33	4258.49
2004–05	3764.63	1958.33	4261.54	2023.96	3852.11	4219.17
2005–06	4408.13	1989.58	4276.20	2020.52	4439.72	4240.89
2006–07	4711.35	2010.63	4319.71	2019.53	4695.44	4286.14
2007–08	3314.23	1832.63	4219.16	2000.84	3623.85	4225.48
2008–09	3692.00	1930.75	4166.45	1993.83	3831.75	4187.35

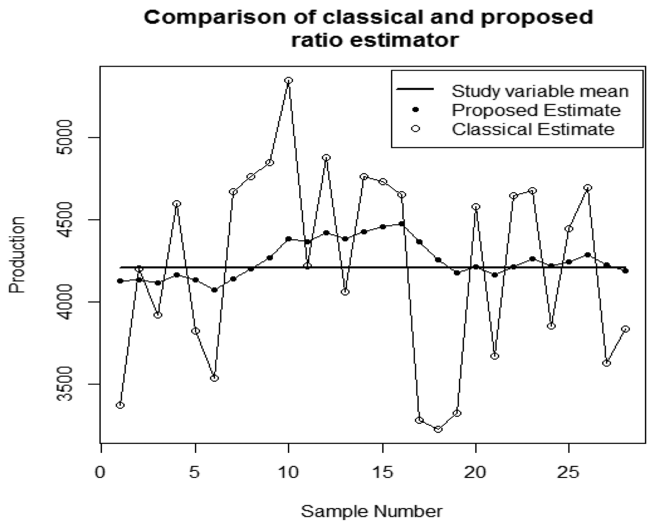

Figure 1 Comparison of proposed with usual ratio estimator for production of wheat.

8 Concluding Remarks

When we conduct time scaled surveys, it is important to use the available information as well as the current information through the sample. To deal with this issue (Noor-ul-Amin, 2019) advised product and ratio estimators using EWMA statistic for the surveys based on time under simple random sampling. In the current study, we made an attempt to improve the estimator of (Noor-ul-Amin, 2019) by using stratified sampling scheme. Therefore, we proposed product and ratio estimator using the EWMA statistic for *StS*. We examine that the proposed estimator by conducting an extensive simulation study and the results are given in Tables 1–4. From the simulation study and real life application, we observed that the proposed estimators are showing better results than the usual ratio and product estimators. Hence, we can conclude that the proposed memory type estimators under stratified sampling are more efficient than the usual ratio and product estimators.

References

Cochran, W. G. (1977). Sampling Techniques, 3rd ed. New York, N.Y. (USA) Wiley.

Kadilar, C. and Cingi, H. (2003). Ratio Estimators in Stratified Random Sampling, Biometrical Journal, 45(2), pp. 218–225.

Kadilar, C., and Cingi, H. (2005). A new ratio estimator in stratified random sampling, Communications in Statistics – Theory and Methods, 34(3), pp. 597–602.

Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N. and Smarandache, F. (2007). A general family of estimators for estmating population mean using known value of some population parameter(s), Far East Journal of Theoretical Statistics, 22(2), pp. 1–4.

Koyuncu, N. and Kadilar, C. (2009). Ratio and product estimators in stratified random sampling, Journal of Statistical Planning and Inference, 139(8), pp. 2552–2558.

Kumar, S., Trehan, M., and Joorel, J. P. S. (2018). A simulation study: estimation of population mean using two auxiliary variables in stratified random sampling, Journal of Statistical Computation and Simulation, 88(18), pp. 3694–3707.

Malik, S. and Singh, R. (2017). A new estimator for population mean using two auxiliary variables in stratified random sampling, Journal of Information and Optimization Sciences, 38(8), pp. 1243–1252.

Noor-ul-Amin, M. (2019). Memory type estimators of population mean using exponentially weighted moving averages for time scaled surveys, Communications in Statistics – Theory and Methods, pp. 1–12. https://doi.org/10.1080/03610926.2019.1670850

Noor-ul-Amin, M., Asghar, S. U. D., Sanaullah, A. and Shehzad, M. A. (2018). Redescending M-Estimator for Robust Regression, Journal of Reliability and Statistical Studies, 11(2), pp. 69–80.

Prasad, B. (1989). Some improved ratio type estimators of population mean and ratio in finite population sample surveys, Communications in Statistics – Theory and Methods, 18(1), pp. 379–392.

Raza, A., Noor-ul-Amin, M. and Hanif, M. (2019). Regression-in-ratio estimators in the redescending M-estimator, Journal of Reliability and Statistical Studies, 12(2), pp. 1–10.

Roberts, S. W. (1959). Control chart tests based on geometric moving averages, Technometrics, 1(3), pp. 239–250.

Robson, D. S. (1957). Applications of multivariate polykays to the theory of unbiased ratio-type estimation, Journal of the American Statistical Association, 52(280), pp. 511–522.

Saini, M. and Kumar, A. (2019). Ratio estimators using stratified random sampling and stratified ranked set sampling, Life Cycle Reliability and Safety Engineering, 8(1), pp. 85–89.

Searls, D. T. (1964). The utilization of a known coefficient of variation in the estimation procedure, Journal of the American Statistical Association, 59(308), pp. 1225–1226.

Sisodia, B. V. S. and Dwivedi, V. K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable, Journal of Indian Society of Agricultural Statistics, 33, pp. 13–18.

Upadhyaya, L. N. and Singh, H. P. (1999). Use of transformed auxiliary variable in estimating the finite population mean, Biometrical Journal, 41(5), pp. 627–636.

Yasmeen, U., Noor-ul-Amin, M. and Hanif, M. (2015). Generalized exponential estimators of finite population mean using transformed auxiliary variables. International Journal of Applied and Computational Mathematics, 1(4), pp. 589–598.

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