

Memristor-Based Reactance-Less Oscillator

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In this letter, the first reactance-less oscillator is introduced. By using memristor, the oscillator can be fully implemented on-chip without the need of any capacitors or inductors, which results in an area-efficient fully integrated solution. The concept of operation of the proposed oscillator is explained and detailed mathematical analysis is introduced. Closed-form expressions for the oscillation frequency and oscillation conditions are derived. Finally, the derived equations are verified with circuit simulations showing excellent agreement.

Introduction: Memristor, the fourth basic circuit element, was first proposed by L. Chua in 1971 [1], but it was not until 2008 when a passive physical implementation was realized [2]. Since then, memristor attracted researchers from multiple disciplines with its unique characteristics and showed potential importance in many fields, including non-volatile memory, reconfigurable logic and circuit design [3, 4, 5].

In this letter, we introduce a reactance-less memristor-based oscillator. The increase/decrease of memristor resistance according to the applied voltage resembles the charging/discharging of a reactive element. The inherent delay in the memristor response is exploited to realize the oscillator function. The “resistance-storage” property of memristor eliminates the need for an energy-storing reactive element, i.e., capacitor or inductor. It should be noted that even ring oscillators are formed of delay stages which depend on the charging/discharging of intrinsic and extrinsic capacitances.

Using the mathematical model of HP memristor in [6], the memristor resistance as a function of time is given by

$$R_m^2(t) = R_o^2 \pm 2k' \int_o^t V_m(\tau) d\tau \quad (1)$$

where R_m is the memristor resistance, R_o is the initial resistance of the memristor, V_m is the voltage across the memristor, and $k' = \mu_v R_{on} (R_{off} - R_{on}) / d^2$, where μ_v is the dopant

drift mobility, R_{on} and R_{off} are the minimum and maximum memristor resistances respectively, and d is the length of the device. The dopant drift mobility is the physical limit that determines the response time of the memristor. For current values of μ_v , the proposed oscillator is suitable for low frequency applications. However, the introduced concept is general and can be extended to higher frequencies given that technology advancement improves the response speed. Low frequency oscillators find usage in biomedical applications and embedded systems [7, 8]. However, off-chip components are usually used because large capacitances are required [7]. In [8], a novel technique was used to implement the oscillator on-chip, but the capacitor consumed 77.8% of the total chip area. By using memristor, the implementation proposed in this letter eliminates the need for capacitors or inductors allowing a fully integrated implementation in a very small area.

Proposed Circuit: The proposed circuit is based on a voltage divider between a resistor and a memristor, and a feedback function ($F(V_i)$) as shown in Fig. 1 and Fig. 2. The voltage on the memristor is given by

$$V_i(t) = V_o(t) \frac{R_m}{R_m + R_a} \quad (2)$$

The memristor is connected in a polarity such that R_m increases for positive V_i and decreases for negative V_i . The threshold voltages V_p and V_n should be selected such that $V_n < 0 < V_p$. A simple implementation of $F(V_i)$ using two comparators and an AND gate is shown in Fig. 1, while other implementations are also possible.

Using Fig. 2, the operation of the oscillator can be traced assuming that we start at 'a':

a→b: At 'a' a positive voltage is applied on the memristor since $V_o = V_{oh}$. The memristor resistance will increase, and so will V_i until the operating point reaches 'b'.

b→c: At 'b' the value of V_i will just pass V_p , thus V_o will switch to V_{ol} , and the operating point will jump to 'c'.

c→d: At 'c' a negative voltage is applied on the memristor. The memristor resistance will decrease, and so will $|V_i|$ until the operating point reaches 'd'.

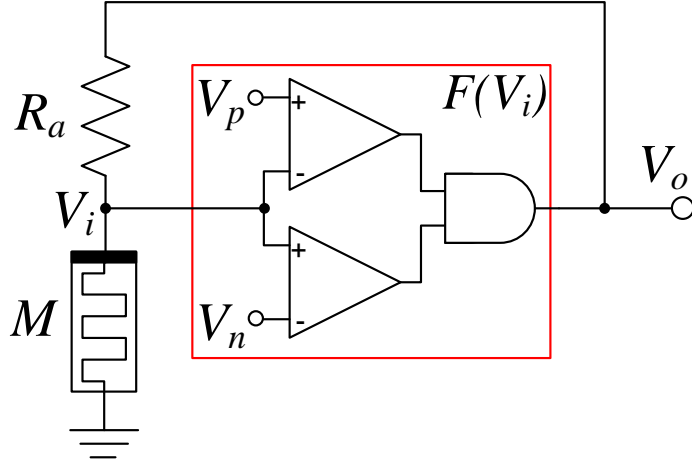


Figure 1: The proposed memristor-based reactance-less oscillator.

d→a: At 'd' the value of V_i will just pass V_n , thus V_o will switch to V_{oh} , and the operating point will jump to 'a'.

The circuit will oscillate independent of the initial memristor resistance (R_o). If the initial point is at ($V_i < V_n$) the memristor resistance will decrease until reaching 'd'. If it starts at ($0 < V_i < V_p$) the memristor resistance will increase until reaching 'b'. In both cases, the circuit will start oscillation from 'b' or 'd' and will follow the path described previously. The cases ($V_n < V_i < 0$) and ($V_i > V_p$) will not happen since V_i and V_o must have the same polarity.

Oscillation Condition: Based on (2), there is a single equivalent memristor resistance for a

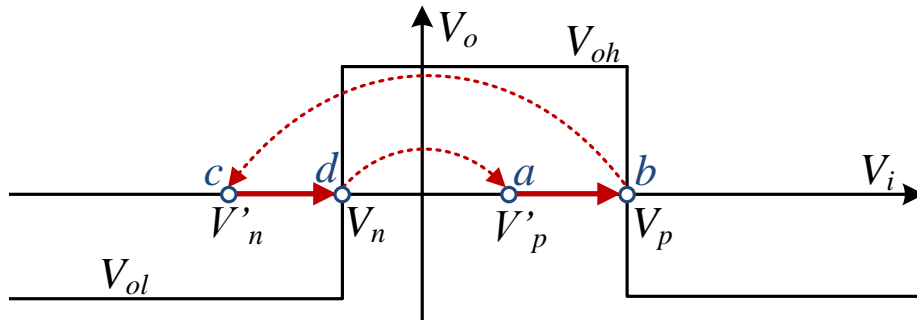


Figure 2: Transfer function of $F(V_i)$ showing transition between different operating points.

given value of V_i . The resistances at $V_i = V_p$ and $V_i = V_n$ are given by

$$R_{mp} = R_a \frac{V_p}{V_{oh} - V_p}, \quad R_{mn} = R_a \frac{V_n}{V_{ol} - V_n} \quad (3)$$

R_{mp} and R_{mn} have to be selected such that $R_{on} < R_{mn} < R_{mp} < R_{off}$. Based on the circuit tracing given in the previous section, the oscillation will occur if V_p and V_n are selected such that

$$V_p - V_n \frac{V_{oh}}{V_{ol}} > 0 \quad (4)$$

Oscillation Frequency: The time required by the circuit to change its state from 'a' to 'b' is determined by the time required for the memristor to change its resistance from R_{mn} to R_{mp} . The derivative of (1) with respect to time is given by

$$R_m dR_m = k' V_i(t) dt \quad (5)$$

By substituting (2) into (5) then integrating, the equation can be written as

$$\int_0^{T_H} dt = \frac{1}{k' V_{oh}} \int_{R_{mn}}^{R_{mp}} (R_m + R_a) dR_m \quad (6)$$

By solving the integration, the time of the positive half cycle is

$$T_H = \frac{R_{mp}^2 - R_{mn}^2 + 2R_a (R_{mp} - R_{mn})}{2k' V_{oh}} \quad (7)$$

Similarly, the time of the negative half cycle is

$$T_L = \frac{R_{mn}^2 - R_{mp}^2 + 2R_a (R_{mn} - R_{mp})}{2k' V_{ol}} \quad (8)$$

The duty cycle expression can be derived in a simple form by substituting (3) into (7) and (8)

$$D = \frac{|V_{ol}|}{V_{oh} - V_{ol}} \quad (9)$$

Using the same substitutions and after simplification, the frequency of oscillation is given

by

$$f_o = \frac{2Dk'V_{oh}(V_{oh} - V_p)^2(V_n - V_{ol})^2}{R_a^2(V_pV_{ol} - V_nV_{oh})(2V_{oh}V_{ol} - V_{ol}V_p - V_{oh}V_n)} \quad (10)$$

Circuit Simulation: As the access to the experimental realization of memristor is very limited, researchers resort to SPICE and behavioral models of HP memristor [9], or emulate the memristor model using active circuitry [3]. The proposed oscillator was simulated using both SPICE and Verilog-A models, both giving similar results. We chose R_{on} , R_{off} , d , and μ_v to be 100Ω , $38k\Omega$, $10nm$, and $10^{-10}cm^2s^{-1}V^{-1}$ respectively. Fig. 3 shows the transient simulation results. The memristor resistance oscillates between R_{mn} and R_{mp} , which defines the location of the operating points. By substituting the circuit parameters into (3) and (10): $R_{mn} = 3k\Omega$, $R_{mp} = 9k\Omega$, and $f_o = 3.51Hz$, which shows excellent match to the simulation results. For further verification of the mathematical analysis presented, the oscillation frequency was tuned by sweeping R_a from $2k\Omega$ to $12k\Omega$. The maximum error was 2.13% and 0.1% for SPICE and Verilog-A simulations respectively.

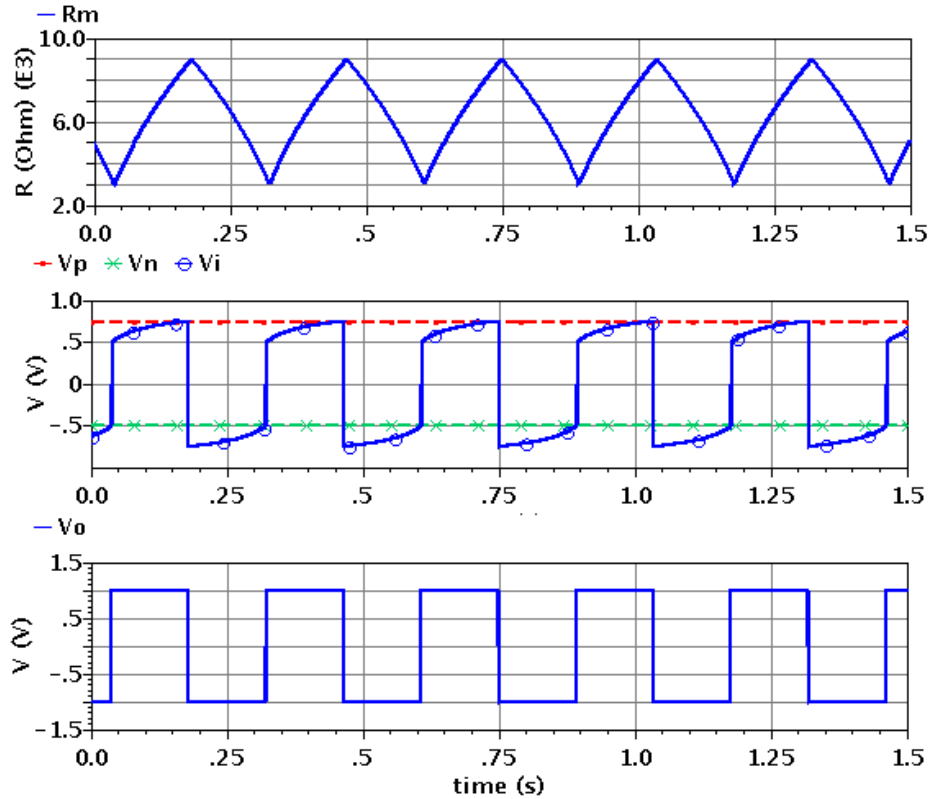


Figure 3: Transient simulation results for R_m (upper), V_i (middle), and V_o (lower), for $R_a = 3k\Omega$, $V_{oh} = 1V$, $V_{ol} = -1V$, $V_p = 0.75V$, and $V_n = -0.5V$.

Conclusion: A memristor-based oscillator without using any capacitors or inductors was presented. The introduced reactance-less oscillator enables an area efficient implementation for low frequency oscillators, which find applications in biomedical and embedded systems. Mathematical analysis for the new oscillator is presented and verified using circuit simulation.

References

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