E C O N O M I C S B U L L E T I N

Mergers among leaders and mergers among followers

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Abstract

We are the first to confirm that sufficient cost convexity in a Stackelberg model generates profitable mergers between two leaders and between two followers. Moreover, the degree of convexity required for leaders to merge is generally far smaller than that required for followers. Most importantly, the structure of the stage game means that the convexity required for either two followers or two leaders to merge is less than that required for two Cournot competitors.

Derivations and simulations rely on Maple 8 and all programs are availabe from the authors.

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1. Introduction

Salant et al. (1983) examined a model of *n* identical Cournot competitors with linear costs of which *m* merge. They demonstrate that only in the unlikely event that more than 80 percent of the market merges could the participants earn profits as a result of the merger. This demonstration gave rise to a literature on "the merger paradox" and to a prolonged effort to identify alternative models in which mergers can be profitable.[1] Huck et al. (2001) retain linear costs but adopt a Stackelberg model with *m* leaders and *n-m* followers. They show that if a leader merges with a follower, the merged firm earns more profit than its two pre-merger component firms. Yet, for all other types of mergers, followers merging with each other or leaders merging with each other, two firm mergers will never be profitable if there remains even a single excluded firm of the type merging (leaders or followers). In this paper, we modify the Stackelberg model of Huck et al. (2001) to allow for convex costs and focus on the possibility of profitable mergers between two followers and between two leaders.

Perry and Porter (1985) show that with sufficiently convex costs two Cournot competitors out of *n* can profitably merge. Yet, Heywood and McGinty (2007b) emphasize that the required degree of convexity typically remains unrealistically large. For example, in order for two Cournot firms out of ten to profitably merge, a linear marginal cost curve must have a slope more than 22 times as steep as the demand curve. In the results that follow, we are the first to show that in a Stackelberg model with *n* firms, sufficient convexity generates profitable mergers between two leaders and between two followers. Moreover, the degree of convexity required for two leaders to merge is generally far smaller than that required for two followers to merge. Most importantly, the structure of the stage game means that the degrees of convexity required for either two followers to merge or for two leaders to merge are each less than that required for two of *n* Cournot competitors to merger. Thus, given convex costs, profitable merger between similar firms is more likely in a market characterized by Stackelberg leadership. This implication does not emerge from comparing Stackelberg to Cournot under the assumption of linear costs.

In what follows, the next section introduces the model presenting the equilibria. The third section compares the influence of merger on profit isolating the central regularities for mergers of two firms. A final section concludes.

2. Model Set-up and Equilibrium

Following Huck et al. (2001), we imagine an industry of *n* firms facing a linear demand curve: $P = 1 - Q$, where $Q = q_1 + q_2 + ... + q_n$. The firms play a quantity Stackelberg game with *m* leaders and $n - m$ followers. Following Perry and Porter (1985), all firms share the same convex cost schedule: $C_i = f + (1/2)cq_i^2$ generating linear marginal cost curves with slope *c*. We consider a merger of two firms, either two leaders or two followers, resulting in *n* -1 post merger firms. We do not consider the case of a follower merging with a leader because Huck et al. (2001) have

[1]These attempts to resolve the paradox include adopting Bertrand competition (Deneckere and Davidson 1985), assuming two firms merging creates a leader (Daugherty 1990), adopting spatial competition (Rothschild et al. 2000), and considering merged firms that sequence output decisions across plants (Huck et al. 2004; Creane and Davidson 2004).

shown it to be profitable in the linear case, a result that carries over to the case with convex costs. We take the original number of firms *n* to be exogenous which allows us to ignore the fixed cost and set *f*=0 in the cost schedules. Indeed, as Perry and Porter (1985) make clear, adopting a positive fixed cost does not change in any way the incentives for merger because the merged firm would retain the fixed costs from each of its constituent parts.

The critical comparison determining the profitability of merger in our model will be the sum of profits earned by two of the *n* pre merger firms and the profit earned by the post merger firm. The pre-merger equilibrium price, quantities and profits are:

$$q_i^L = \frac{(1+c)}{m+1+2c+cn+c^2} \qquad \forall i \;\; i=1 \text{ to } m$$

$$q_i^F = \frac{(1+2c+cn+c^2-mc)}{(m+1+2c+cn+c^2)(1+n-m+c)} \qquad \forall i \;\; i=m+1 \text{ to } n$$

$$P = \frac{(1+3c+cn+3c^2+nc^2+c^3-mc-mc^2)}{(m+1+2c+cn+c^2)(1+n-m+c)} \tag{1}$$

$$\pi_i^L = \frac{(1+c)(2+5c+cn+(4+n)c^2+c^3-mc(1+c))}{2(m+1+2c+cn+c^2)^2(1+n-m+c)} \qquad \forall i \;\; i=1 \text{ to } m$$

$$\pi_i^F = \frac{(1+2c+cn+c^2-mc)(2+5c+2cn+(4+n)c^2+c^3-mc(2+c))}{2(m+1+2c+cn+c^2)^2(1+n-m+c)^2} \quad \forall i \; i=m+1 \text{ to } n$$

In the post-merger equilibrium, the *n-2* firms excluded from the merger have cost functions $C_i = (1/2)cq_i^2$ but the merged firm retains 2 plants each with that same cost function. The resulting composite cost function of the two-plant firm is $C_{merged} = (1/4)cq_{merged}^2$. This function reflects the underlying advantage of being able to direct output across multiple plants. Note, however, that if the output of the merged firm remained identical to that of its constituent pre-merger firms, $q_{merged} = 2q_i$, the total cost to produce that output would be unchanged. The merger by itself does not immediately provide cost savings.

The point of the merger remains to reduce output to exploit market power. We first consider the case of two leaders merging. There will now be three types of firms: the merged leader, the *m* - 2 excluded leaders and the *n* - *m* followers. The equilibrium resulting from *n*-2 firms with C_i and one firm with C_{merged} can be easily characterized but the expressions are very long (available from the authors upon request). As a point of reference, if *c*=1 the pre-merger profit of a leader from (1) can be compared with the profit of the merged leader:

$$\pi_i^L(c=1) = \frac{2(n+6-m)}{(m+4+n)^2(n+2-m)}$$

$$\pi_{merged}^L(c=1) = \frac{4(n+4-m)(40+14(n-m)-2mn+n^2+m^2)}{(16-m^2+2m+10n+n^2)(n+2-m)} \tag{2}$$

While it can be established that the merged profit is larger, the critical comparison for any *c* recognizes that the profit of two pre-merger leaders stands as the opportunity cost:

$$g^L(n,m,c) = \pi_{merged}^L - 2\pi_i^L \tag{3}$$

Only when this expression is positive is there profit from merger. While the full expression for (2) for any *c* is also available from the authors, we note that $g^L(n,m,c=0)$ collapses to exactly the profit expression for a merger among leaders as presented by Huck et al. (2001) for the case of linear costs: $g^L(n,m,c=0)=(1+2m-m^2)/m^2(n-m+1)(m+1)^2$. To take an example with convex costs that can be calculated from (2) and (3), when *n*=6, *m*=3, *c*=1, the pre-merger leader profit, π_i^L, is .0213 and the profit of the merged leader, π_{merged}^L, is .0429. Thus g^L equals .003 and the merger is profitable.

An analogous equilibrium and expression can be derived for the profit associated with two followers merging. The three types of firms now include the merged follower, the *n-m-2* excluded followers and the *m* leaders. The critical difference is:

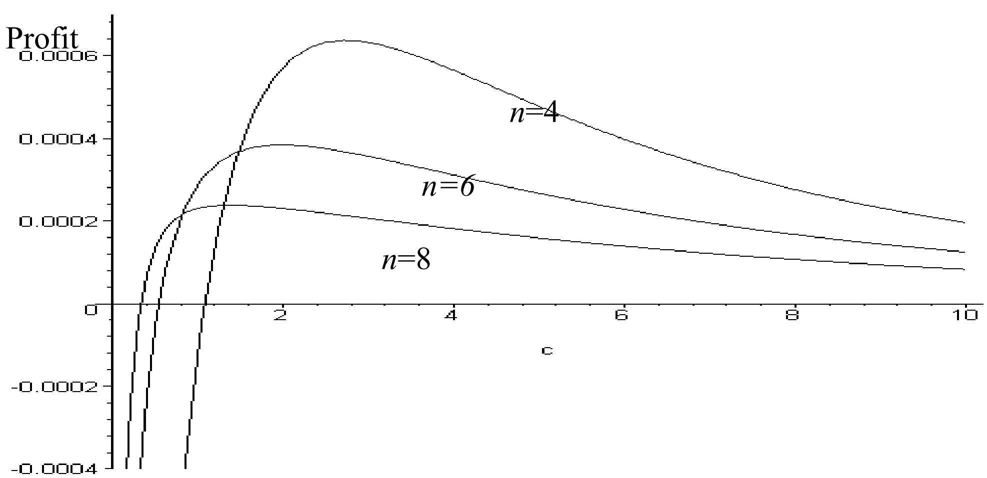
$$g^F(n,m,c)=\pi_{merged}^F-2\pi_i^F \qquad (4)$$

While the full expression for (3) is available from the authors, $g^F(n,m,c=0)$ collapses to the expression for a merger among followers as presented by Huck et al. (2001) for the case of linear costs: $g^F(n,m,c=0)=(2n-n^2+2mn-m^2-2m+1/(m+1)^2(n-m)^2(n+1-m)^2$.

3. Identifying Critical Levels of Cost Convexity

By plugging in specific values of *n* and *m*, (2) can be solved for the critical value of *c* such that the profit gain from the merger of two leaders is zero. For all of the values of *n* and *m* there exists only a single positive root. Values above that root return positive profit and values below that root return negative profit. As an illustration, $g^L(n=6,m=3,c)$ crosses the X-axis from negative to positive only once at the value *c* = 0.55. This is shown in Figure 1. Also shown in Figure 1 are the graphs retaining three leaders but decreasing *n* to 4 (increasing the critical *c*) and increasing *n* to 8 (decreasing the critical *c*). In both both cases m=3.While such simulation lacks elegance, it is required by the complexity of the expression in (2) which involves powers of *c* up to tenth order.[2]

Figure 1: The profit from two leaders merging, $g^L(n,m,c)$.



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[2] As an alternative, $g^L(n,m,c)=0$ can be solved for *c* in general as a function of *n* and *m*. This yields five roots and while the relevant root is not analytically tractable, it can be plotted in three dimensions and shown to be positive.

Similarly, plugging in specific values of *n* and *m* into (3) allows solving for the critical value of *c* such that the profit gain from the merger of two followers is zero. There exists only a single positive root for all values of *n* and *m* except when *n* - *m* =2. In the case of only two followers there are two roots requiring a bit of care in identifying critical regions.

The results of our simulations are presented in Table I. The Maple program that generated the table and other results is available upon request. Table entries are the values of *c* sufficiently high that a two firm merger will be profitable. The top entry is for a merger between leaders and bottom entry is for a merger between followers. We collect our observations into a series of regularities that flow from the simulations illustrated in the table.

Table I: Minimum Conditions for a Profitable Simultaneous Merger

	Critical Cost Parameter, *c*, Stackelberg				
	m=2	*m*=4	*M*=6	*m*=8	*m*=10
n-m=2	0 0.15 3.60*	3.00 0.10 9.30*	8.63 0.09 15.2*	14.53 0.09 21.2*	20.47 0.08 27.1*
n-m=4	0 10.10	1.88 15.84	6.93 21.67	12.69 27.56	18.59 33.47
n-m=6	0 16.20	1.21 22.01	 27.87	10.90 33.76	16.72 39.67
n-m=8	0 22.4	0.84 28.10	4.02 33.98	9.18 39.88	14.89 45.80
n-m=10	0 28.26	0.63 34.15	2.95 40.05	7.56 45.96	13.09 51.89
	Critical Cost Parameter, *c*, Cournot**				
	n=2	*n*=4	*n*=6	*n*=8	*n*=10
	0	4.50	10.42	16.40	22.38

*Wherever there are two followers all values below the first entry and above the second entry are profitable.

**The values for Cournot come from Heywood and McGinty (2007b).

Regularity 1: As the number of followers (leaders) increases, the critical degree of convexity for two leaders to profitably merge decreases (increases).

Thus, leaders are more likely to be able to profitably merge when there are relatively few of them but relatively many followers. In these circumstances, the merged leaders have few same stage competitors (increasing output in response to their reduction) and they enjoy leadership over many firms. This regularity provides insight into merger dynamics as well. As the number of leaders falls because of merger, the critical degree of convexity falls with it. Thus, if the convexity allows the original merger, additional mergers between leaders will be profitable.

Regularity 2: As either the number of leaders increases, or as the number of followers increases, the degree of convexity for two followers to merge increases.

Followers are placed at a disadvantage for merging both my more same stage competitors and more leaders that incorporate their desired quantity reduction. Again, similar implications result for merger dynamics. If the convexity is sufficient to allow the original merger of two followers, this regularity suggests that further mergers between followers will be profitable as well.

Regularity 3: The convexity required for two leaders to profitably merge exceeds that necessary for two followers to profitably merge (with the exception of when there are only two followers).

This reflects the timing advantage of the leaders. Merged followers will have their desired output reduction taken into account in the first stage quantity setting of the leaders resulting in a smaller decline in market output than that associated with a merger among leaders.

Regularity 4: Holding the total number of firms constant, the degree of cost convexity for either two leaders or two followers to merge is lower than for two Cournot competitors to merge.

The results for Cournot in Table I show, for instance, a critical value of 16.40 for 2 of 8 firms to merge. This exceeds the critical level for followers or leaders when there are 8 firms in a market with Stackelberg leadership (for instance, compare to 4 leaders and 4 followers). Thus, separating the stages of quantity choice blunts the reaction to the quantity reduction undertaken by the merged firms resulting in a lower needed level of cost convexity.

An important portion of the merger paradox is the free-rider aspect. It is often more advantageous for firms to remain excluded rivals rather than merged. While it is possible to model mergers that harm rivals, these have been the exception rather than the rule (Farrell and Shapiro 1990; Heywood and McGinty 2007a). The introduction of the stage game even with convex costs does not change this as excluded rivals continue to benefit from merger.

Regularity 5: Mergers of either two leaders or of two followers will result in increased profits for the excluded firms at both stages.

The profit of an excluded leader post merger is subtracted from the pre-merger leader profit. This is set equal to zero and solved for a critical c in terms of m and n. While there are two real roots, both are negative and all values of the profit difference are positive for c above the critical level. This can be repeated for followers. As the degree of convexity is constrained to be non-negative, excluded firms benefit from the merger of two leaders. The merged leader reduces its output compared to its pre-merger constituent firms. While it can earn profit from doing so with sufficient convexity, excluded firms gain for any degree of convexity. Examining the merger of two followers required returning to our simulations. Nonetheless, all our trials show both excluded followers and excluded leaders benefiting from the merger of two followers.

4. Conclusions

Sufficient convexity generates profitable mergers between two leaders and between two followers. The degree of convexity required for two leaders to merge is generally far smaller than that required for two followers to merge. Most importantly, the structure of the stage game means that the degrees of convexity required for either two followers to merge or for two leaders to merge are each less than that required for two of n Cournot competitors to merger. Thus, given convex costs, profitable merger between similar firms is more likely in a market characterized by Stackelberg leadership.

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