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Meson-Nucleon σ-term and the $\overline{B}BS$-Coupling Constants

K. K. SINHA* and V. P. GAUTAM

*Regional Engineering College, Durgapur-9, W. Bengal, India
Indian Association for the Cultivation of Science, Jadavpur, Calcutta-32, India

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The study of the σ-term in meson-nucleon scattering is of prime importance in particle physics. It will help us in understanding i) whether $SU(2)\otimes SU(2)$ is a better symmetry than $SU(3)\otimes SU(3)$ and ii) how chiral $SU(3)\otimes SU(3)$ symmetry as well as scale invariance is broken. A number of estimations of the σ-term has already been done with different approaches. Von Hippel and Kim[1] calculated the values of the σ-term with symmetry breaking Hamiltonian transforming like $(3,3^*)\oplus(3^*,3)$ of $SU(3)\otimes SU(3)$ and got the values of $\sigma_{\pi N}$ and σ_{KN} to be 26 MeV and 184.87 MeV respectively. On the other hand Cheng and Dashen[2] obtained $\sigma_{\pi N}$ to be 110 MeV.

We have derived an expression for the σ-term. For its numerical estimation we require the values of $\overline{B}BS$ coupling constants which have not yet been determined (here B and S are the members of $\frac{1}{2}^+$ and 0^+ octets respectively). The purpose of the present work is to determine these coupling constants with different values of the σ-terms obtained by us and by others[1)~3)] as input. The importance of these coupling constants has been emphasised in the work of Carruthers[4] and others (cited in Ref. 4)). We have obtained the value of the σ-term from Eq. (22a),[5] at threshold, derived by Golestaneh and Gautam[5] by simple algebraic method with the scattering length as input. Throughout this paper we have followed their notations.

The Eq. (22a) of Ref. 5) dropping W^{00} reads

$$a^{K^+P}=\frac{1}{4\pi c^2(1+m/M)}\times[W^0+W^1-W_+{}^P]_{t=0}, \quad (1)$$

where m and M are the kaon and nucleon masses respectively. With $W^1=-2m$ and $W^P=-.0463m$ as obtained in Ref. 5)*) along with experimental value of K^+P scattering length, $-.29$F as input, we get W^0_{KN} (i.e., the σ-term) $1.2038m$ as against $W^0_{KN}=.3744m$ obtained by Von Hippel and Kim.[1]

Making use of our value for W^0, we have investigated the possibility of evaluating the coupling constants $g_{\overline{B}BS}$. Towards this goal we start with the σ-term W^0 whose expression is the following:[5]

$$W^0=-i\int d^4ze^{-iKz}\delta(-z_0)\times\langle p'|[J_b{}^0(0),\partial_\nu J_a{}^{\nu\dagger}(z)]|p\rangle. \quad (2)$$

Solving it further at threshold with $t=(p'-p)^2=0$, we make use of the relations[5]

$$\partial_\nu J_a{}^{\nu\dagger}(z)=c(\Box+m^2)\phi_a{}^\dagger(z) \quad (3)$$

and[6]

$$[Q_b(0),\phi_a{}^\dagger(0)]=id_{abc}\sigma_c(0), \quad (4)$$

which yield

$$W^0_{KN}=cm^2\frac{1}{2}\Big[\langle p'|\sigma_3(0)|p\rangle-\frac{1}{\sqrt{3}}\langle p'|\sigma_8(0)|p\rangle+2\sqrt{\frac{2}{3}}\langle p'|\sigma_0(0)|p\rangle\Big]. \quad (5)$$

Now, as the scaler field, σ_i satisfies the equation

$$(\Box+m^2_{\sigma i})\sigma_i=j_i(\text{current}).$$

*) There will be an extra sign in the r.h.s. of Eq. (7f) of Ref. 5). In Eq. (23c) the r.h.s. will be .0463m.

Table I.[8] Particle masses used.

Particle	K^+	$\pi^{\pm}$	σ_0	σ_3	σ_8	M
Mass in MeV	493.84	139.58	720	1016	1071	938.25

Table II. $\bar{B}BS$ Coupling Constants.

	Ref. for σ-terms	W^0 (σ=term) (m)	$g_{\bar{p}p\sigma_8}$	$g_{\bar{p}p\sigma_3}$
KN	a) Present work	1.2038	1.347	3.892
	b) Ref. 1)	0.3744	10.694	30.906
πN	a) Ref. 1)	0.1863	9.26	
	b) Ref. 2)	1.142	79.66	
	c) Ref. 3)	0.2436	0.80	

We finally get

$$W^0_{KN}=cm^2\frac{1}{2}\left[\frac{f_{\sigma_3}(t)\,g_{\bar{p}p\sigma_3}}{m^2_{\sigma_3}-t}-\frac{1}{\sqrt{3}}\frac{f_{\sigma_8}(t)g_{\bar{p}p\sigma_8}}{m^2_{\sigma_8}-t}+2\sqrt{\frac{2}{3}}\frac{f_{\sigma_0}(t)\,g_{\bar{p}p\sigma_0}}{m^2_{\sigma_0}-t}\right]$$

$$=cm^2\frac{1}{2}\left[\frac{g_{\bar{p}p\sigma_3}}{m^2_{\sigma_3}}-\frac{1}{\sqrt{3}}\frac{g_{\bar{p}p\sigma_8}}{m^2_{\sigma_8}}+2\sqrt{\frac{2}{3}}\frac{g_{\bar{p}p\sigma_0}}{m^2_{\sigma_0}}\right], \qquad (6)$$

after putting $f_{\sigma_3}(0)=f_{\sigma_8}(0)=f_{\sigma_0}(0)=1$ and $t=0$. Taking $L^s=F/(F+D)=0.4$ we have got $g_{\bar{p}p\sigma_3}=2.89g_{\bar{p}p\sigma_8}$. Using this relation along with $g_{\bar{p}p\sigma_0}=11.76$ and $c=0.246m$,[1] Eq. (6) can be used to evaluate $g_{\bar{p}p\sigma_8}$ for a given value of W^0. The required masses are recorded in Table I. Therefore, from Eq. (6) we get $g_{\bar{p}p\sigma_8}=1.31$ and $g_{\bar{p}p\sigma_3}=3.786$. From these coupling canstants one can easily get the whole set of $g_{\bar{B}BS}$ under $SU(3)$ symmetry limit. In the similar fashion we have made use of other available values of W^0_{KN}[1] and $W^0_{\pi N}$[2),3)] for the evaluation of the $g_{\bar{p}p\sigma_8}$ which are given in Table II. The expression for W^0_{KN} is derivation in the same way as Eq. (6) for W^0_{KN}.

From Table II we observed that the values of $\bar{B}BS$ coupling constants are varying widely. It is obvious, as the precise estimation of the σ-term has not yet been possible. It may be remarked that $g_{\bar{p}p\sigma_8}$ obtained in the present work differs with that obtained from the σ-terms of Ref. 1) by a factor of 10, but our value is of the same order as that obtained from $W^0_{\pi N}$ of Ref. 3). It is interesting to note that this coupling constant when calculated from the σ-term of Cheng and Dashen greatly differs with ours and with others. Our equation (6) and similar relation for $W^0_{\pi N}$ should be further verified by more accurate determination of the σ-terms and $\bar{B}BS$ coupling constants.

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