Meson-nucleon vertex form factors at finite temperature using a soft pion form factor

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The temperature and density dependence of the meson-nucleon vertex form factors is studied in the framework of thermofield dynamics. Results are obtained for two rather different nucleon-nucleon potentials: the usual Bonn potential and the variation with a softer πNN form factor, due to Holinde and Thomas. In general, the results show only a modest degree of sensitivity to the choice of interaction.

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Recently [1], the temperature dependence of the meson-nucleon form factors has been calculated in thermofield dynamics using the Bonn $N-N$ interaction [2] that includes exchanges of π,σ,ρ, and ω mesons, with the πNN vertex form factor having a monopole form

$$G_{\pi NN}(q^2) = g_{\pi NN}\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + q^2}, \tag{1}$$

where $\Lambda = 1.3$ GeV for the πNN vertex. It is anticipated that in low energy N-N scattering $q^0 \approx 0$ so that the four-momentum transfer squared is usually minus the three-momentum transfer squared. However, it has been argued [3] that there are several factors that point to a significantly softer form factor with $\Lambda \approx 700-800$ MeV. It was concluded that the πNN form factor for a free nucleon is close to the measured axial-vector form factor, thus giving $\Lambda \approx 500-800$ MeV. It has been suggested that a soft πNN form factor would make it difficult to fit the N-N scattering and, in particular, the properties of the deuteron. However, Holinde and Thomas [3,4] established that a good fit can be obtained for $\Lambda = 800$ MeV.

The study of the temperature and density dependence of the coupling constants, form factors, and the critical temperature, where the coupling constant goes to zero, is needed for this softer πNN form factor. This would clarify any dependence on the choice of the interaction and give us a better understanding of the equation of state that may be relevant for heavy-ion collisions. The behavior of nuclear matter at finite temperature and density and the phase transition from the hadronic to the quark-gluon phase with subsequent hadronization can provide valuable information about the nature of confinement in QCD.

In this Brief Communication we present results for the temperature and density dependence of the meson-nucleon couplings, using the OBEPTI N-N interaction [3,4], and compare them to the results obtained earlier with the Bonn potential, OBEP [2]. The main calculational procedure has been described earlier [1] and we concentrate on discussing the results in the present note.

The temperature dependence is calculated within thermofield dynamics, a finite temperature field theory. The finite temperature modifications of the vertex functions are calculated from the one-meson-exchange Feynman diagrams shown in Fig. 1. The mathematical representation was explained earlier [1]. To initiate the calculations, zero temperature coupling constants and vertex form factors are chosen to be either (i) the Bonn potential, OBEP, or (ii) the modified Bonn potential with a soft πNN form factor, OBEPTI, due to Holinde and Thomas (see Table I).

It is important to note that results for π', appearing in OBEPTI, can be obtained from those of the π meson by the following procedure

$$f(\vec{q}^2) = \frac{G_{\pi' NN}(\vec{q}^2, T, \rho)}{G_{\pi NN}(\vec{q}^2, T, \rho)} = \frac{G_{\pi' NN}(\vec{q}^2, 0, 0)}{G_{\pi NN}(\vec{q}^2, 0, 0)} = \frac{g_{\pi' NN}}{g_{\pi NN}}\left(\frac{\Lambda_{\pi'}^2 - m_{\pi'}^2}{\Lambda_\pi^2 - m_\pi^2}\right)\left(\frac{\Lambda_\pi^2 + \vec{q}^2}{\Lambda_{\pi'}^2 + \vec{q}^2}\right). \tag{2}$$

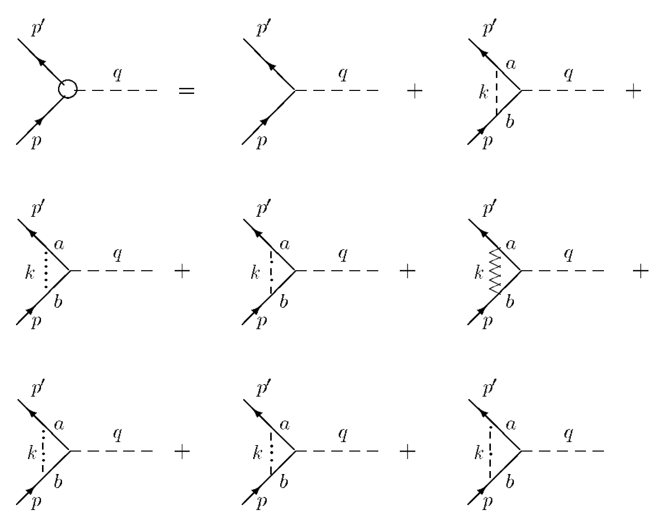

FIG. 1. Feynman diagrams for the pion-nucleon vertex. The solid line indicates the nucleon, while the dashed, dotted, dot-dashed, wavy, dot-dot-dashed, dot-dot-dot-dashed, and dashed-dashed-dot lines indicate the $\pi-$, ρ, ω, σ, π', η, and δ mesons, respectively.

TABLE I. Parameters for the Bonn potential, OBEP, [2] and for the modified Bonn potential, OBEPTI [4].

Meson	$I(J^P)$	Mass (GeV)	Λ_α(GeV)	$g_\alpha^2/4\pi$	f_α/g_α
		OBEP parametrization			
π	$1(0^-)$	0.138	1.05	14.9	
ρ	$1(1^-)$	0.769	1.3	0.99	6.1
ω	$0(1^-)$	0.783	1.5	20.0	
σ	$0(0^+)$	0.550	2.0	8.383	
		OBEPTI parametrization			
π	$1(0^-)$	0.138	0.8	14.6	
π'	$1(0^-)$	1.200	2.0	100	
η	$0(0^-)$	0.549	1.5	5.0	
ρ	$1(1^-)$	0.769	1.3	0.92	6.6
ω	$0(1^-)$	0.783	1.5	20.0	
δ	$1(0^+)$	0.983	2.0	2.881	
σ	$0(0^+)$	0.550	2.0	8.383	

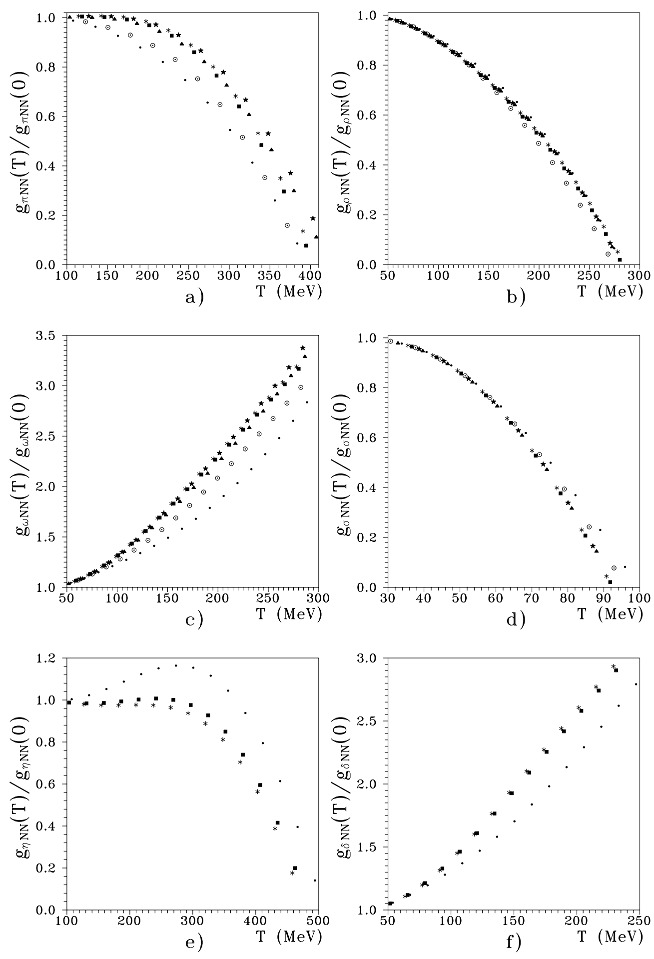

FIG. 2. The ratio of meson-nucleon coupling constants at finite temperature T to those at T=0, as a function of temperature at ρ =0 (stars and five stars), ρ=ρ_0 (squares and triangles), ρ=5ρ_0 (dotted circles and simple dots). Stars, squares, and dotted circles correspond to calculations using the OBEP potential [2]. Five stars, triangles, and dots correspond to calculations using the OBEPTI potential [4]. Results for the π' coincide with the OBEPTI part of (a) [see Eq. (2)].

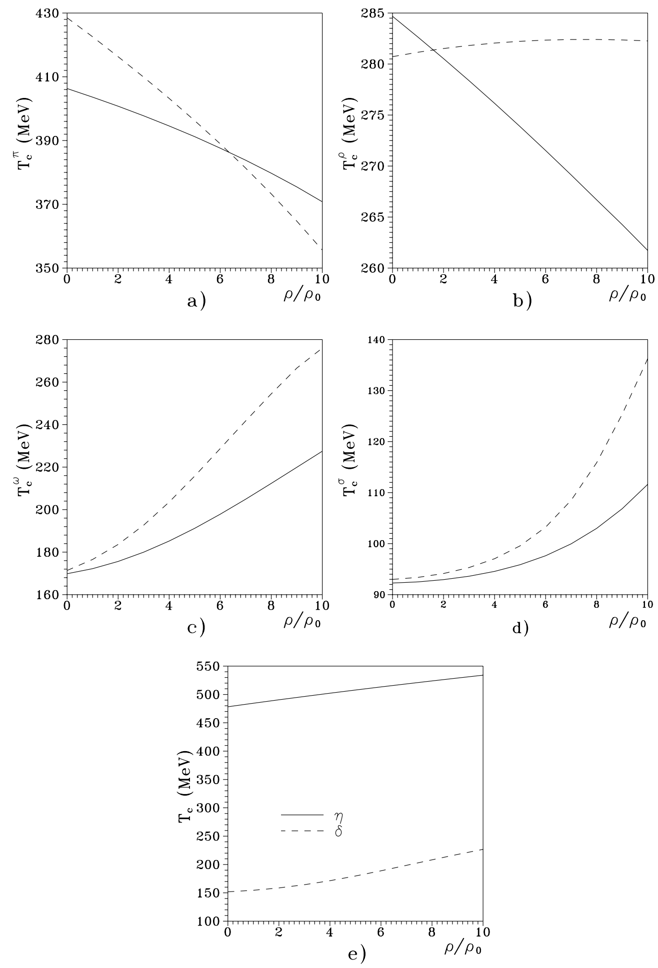

FIG. 3. Density dependence of the critical temperature for several [(a) π, (b) ρ, (c) ω, (d)σ, (e)η,δ] mesons. In (a)–(d) the solid lines correspond to the calculation based on the OBEP potential [2], while the dashed lines correspond to calculations using the OBEPTI potential [4]. In (e) the solid line corresponds to the η and the dashed line to the δ. Calculations in (e) are made using the OBEPTI potential. The results for the π' coincide with the OBEPTI part of (a).

The ratios of the coupling constants, $g_{BNN}(T)/g_{BNN}(0)$, are plotted in Fig. 2. Note that the η,δ, and π' mesons are present only for the potential of Holinde and Thomas. We observe that for the π,π',ρ,σ, and η mesons this ratio goes through zero at different temperature depending on the density. This temperature is called the critical temperature, T_C^B. For ω and δ mesons the behavior of the ratio is opposite—i.e., it increases instead of going through zero. For π mesons the difference between the two potentials is smallest at a density of order $\sim 5\rho_0$, where ρ_0 is normal nuclear matter density.

The changes with temperature for the two sets of parameters are usually quite similar, though the actual numbers differ. In Fig. 3 we show the critical temperatures as a function of the density of nuclear matter. The behavior of T_c^B for various mesons using the two potentials is quite different. For the π and σ mesons, T_c^B changes more rapidly for the softer pion vertex, while for the ρ meson T_c^B is practically

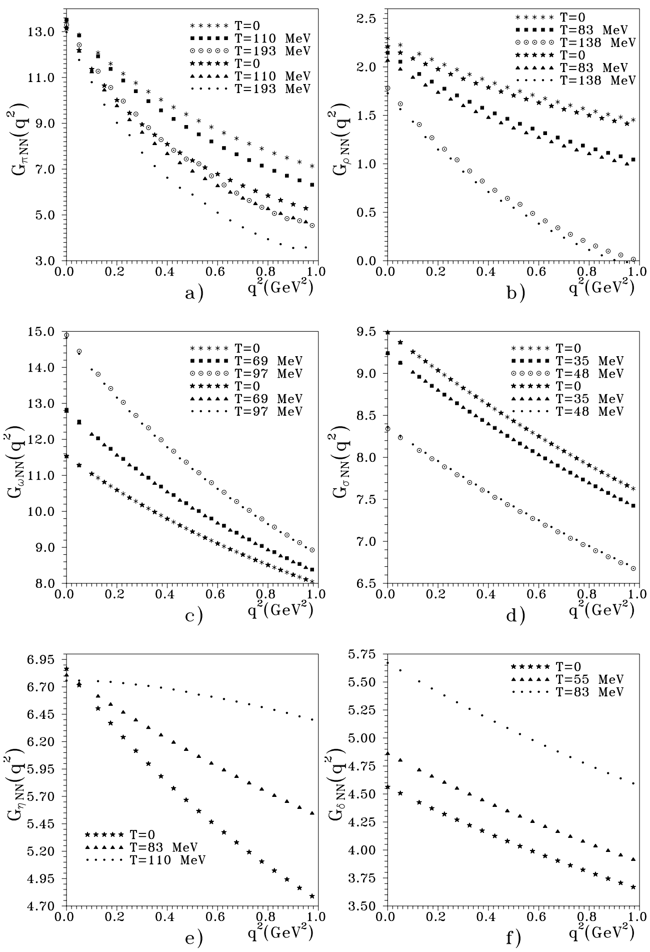

FIG. 4. Meson-nucleon form factors at several temperatures as a function of q^2 using OBEP [2] (stars, squares, and dotted circles) and OBEPTI potentials [4] (five stars, triangles, and dots) at ρ=0. Results for the π' can be extracted from the OBEPTI part of (a), taking into account the scaling function $f(q^2)$ [see Eq. (2)].

constant—in contrast with the OBEP potential, where it changes rapidly. The changes in T_c^{ω} show quite a significant dependence on the choice of potential.

We have also investigated the q^2 dependence of the meson-nucleon form factors with temperature. The results of these calculations are presented in Fig. 4. In this case the pion form factor shows a large change and a stronger dependence on the choice of T=0 parameter. For other mesons the form factors show very little dependence on a hard or soft pion vertex function.

It is clear that at density ρ_0, the behavior of the critical temperature for the ρ meson is dramatically changed when the parameters of the OBE potential are changed. In other words, $g_{\rho NN}$ is sensitive to parameter changes. For the OBEPTI potential, $T_c^{\rho}(\rho)$ is almost constant, while for the OBEP potential $T_c^{\rho}(\rho)$ decreases rapidly. For the π meson, $T_c(\rho)$ decreases even more rapidly with increasing density. For other mesons, $T_c(\rho)$ has the same behavior, increasing with increasing density. For all mesons except the pion, the critical temperature increases when one uses the OBEPTI parametrization. Nevertheless, at moderate densities the critical temperature for the π is also somewhat higher.

In Fig. 4 we present the form factors at several temperatures at zero density. The form factors of the πNN and ρNN vertices are more sensitive to parameter changes, while $G_{\omega NN}$ and $G_{\sigma NN}$ are not. This is also seen in Fig. 3, where the critical temperature for the ω and σ mesons is almost the same for both parameter sets at zero density. At high densities $G_{\pi NN}$, and $G_{\rho NN}$ fall off more rapidly at high momentum transfer, constituting quenching of the cut off parameters for corresponding meson form factors.

To summarize our results we present a practical parametrization of the temperature and density dependence of the form factors. This may be useful for other applications. At small momentum transfers we can parametrize them by a monopole form

$$G_{BNN}(\vec{q}^{\,2},T,\rho)=g_B(T,\rho)\frac{\Lambda_B^2(T,\rho)-m_B^2}{\Lambda_B^2(T,\rho)+\vec{q}^{\,2}}, \tag{3}$$

where we still use the mass of the corresponding meson at zero temperature and density. In general, the effective mass of a meson m_B is also temperature dependent, but that has not been considered here. In Fig. 3 we see that T_c has an almost linear dependence on density, except for the case of σ meson. However, even for this case, at moderate densities there is an almost linear dependence. Conse-

TABLE II. Parameters of vertex form factors in Eqs. (4) and (5). The last column is the critical temperature T_c at zero density, ρ=0.

Meson	α^g	β^g	C^g	α^Λ	β^Λ	C^Λ	D	T_c
				OBEP parametrization				
π	1.329	−2.390	0.033	−0.230	−0.400	−0.008	−0.008	406
ρ	0.066	−1.058	−0.002	−0.070	−1.976	−0.007	−0.008	285
ω	0.138	0.808	−0.008	0.008	−0.524	0.006	0.031	170
σ	0.559	−1.547	0.010	−0.018	−0.024	−0.003	0.005	93
				OBEPTI parametrization				
π	1.018	−2.029	0.031	−0.142	−0.369	−0.007	−0.016	429
ρ	0.108	−1.079	−0.001	−0.081	−2.012	−0.006	0.001	281
ω	0.062	0.904	−0.013	0.005	−0.547	0.008	0.058	171
σ	0.554	−1.528	0.016	−0.008	−0.043	−0.002	0.007	93
η	1.449	−2.442	−0.020	−0.963	14.468	−0.003	0.012	478
δ	0.211	0.724	−0.014	0.233	−0.364	0.019	0.045	152

quently, we can write the following density dependence of the critical temperature:

$$T_c = T_0\left(1 + D_B\frac{\rho}{\rho_0}\right). \tag{4}$$

Furthermore, using a polynomial form for the ratios

$$\frac{g_B(T,\rho)}{g_B(T=0,\rho=0)} = \frac{1 + (T/T_c)\alpha_B^g + (T/T_c)^2\beta_B^g}{1 + (\rho/\rho_0)C_B^g},$$

$$\frac{\Lambda_B(T,\rho)}{\Lambda_B(T=0,\rho=0)} = \frac{1 + (T/T_C)\alpha_B^\Lambda + (T/T_c)^2\beta_B^\Lambda}{1 + (\rho/\rho_0)C_B^\Lambda}, \tag{5}$$

we find the values of the parameters given in Table II. Again it is clear that $G_{\pi' NN}$ can be restored from $G_{\pi NN}$ as given in Eq. (2).

In conclusion, the choice of hard and soft πNN form factors does change the behavior of the critical temperature with density. While the changes are not, in general, dramatic, there is a qualitative change for the ρ meson. When it comes to the q^2 dependence of the form factors, the changes for all mesons are similar. The parametrization of the changes with temperature and density of the boson coupling constants to the nucleon, as well as the cutoff parameters in the form factors, provides a useful way to summarize the variation of these parameters. We note that this study clearly suggests that it would be worthwhile to investigate the effect of finite temperature and density on the masses of the exchanged bosons [5,6] and to feed this information back into the calculation of the meson-nucleon vertices. Finally, it is important to realize that while the couplings of the mesons to nucleons decrease to zero at a critical temperature, it is possible that before we reach that temperature, the system may have already undergone a phase transition, leading to a deconfined quark-gluon state.

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[1] A. M. Rakhimov, U. T. Yakhshiev, and F. C. Khanna, Phys. Rev. C **61**, 024907 (2000).
[2] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[3] A. W. Thomas and K. Holinde, Phys. Rev. Lett. **63**, 2025 (1989); P. A. Guichon, G. A. Miller, and A. W. Thomas, Phys. Lett. **124B**, 109 (1983).
[4] K. Holinde and A. W. Thomas, Phys. Rev. C **42**, R1195 (1990).
[5] E. Marco, R. Hofmann, and W. Weise, Phys. Lett. B **530**, 88 (2002).
[6] K. Saito, K. Tsushima, A. W. Thomas, and A. G. Williams, Phys. Lett. B **433**, 243 (1998).