



METEORIDS INTERACTION WITH THE EARTH ATMOSPHERE*

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ABSTRACT. In this study we evaluate meteoroid mass and its other properties based on the observed atmospheric trajectory. With account for aerodynamics, we formulate a problem by introducing key dimensionless parameters in the model, responsible for the drag, mass loss and rotation of meteoroid. The proposed model is suitable to categorize various impact events in terms of meteor survivability and impact damage and thus, to analyze consequences that accompany collisions of cosmic bodies with planetary atmosphere and surface. The different types of events, namely, formation of a massive single crater (Barringer, Lonar Lake), dispersion of craters and meteorites over a large area (Sikhote-Alin), absent of craters and meteorites, but huge damage (Tunguska) are considered as illustrative examples. The proposed approach helps to summarize the data on existing terrestrial impacts and to formulate recommendations for further studies valuable for planetary defence. It also significantly increases chances of successful meteorite recoveries in future. In other words, the study represents a 'cheap' possibility to probe cosmic matter reaching planetary surface and it complements results of sample-return missions bringing back pristine samples of the materials.

KEY WORDS: Meteoroid, meteor, meteorite, impact crater, inverse problem.

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1. Introduction

Techniques to determine the meteoroid's mass and bulk density values based on ground-based observations have been long discussed in the literature dedicated to meteor studies. These data are of current importance for the calculation of the cosmic matter influx onto the Earth, for more reliable and rapid meteorite recovery, for the studies of cometary nuclei composition and structure, for the calculations of orbital evolution timescale and have other important implications. These applications are quite sensitive to the initial mass and bulk density values and these values sometimes differ by more than an order of magnitude depending on the approach. For example, based on the model of Park (1978) [26] Turco et al. (1982) [31] employed in their calculation a value of 0.002 g/cm^3 as a bulk density of the Tunguska meteor body. At the same time Sekanina (1983) [27] concluded that Tunguska is described most consistently as Apollo-type asteroid with the bulk density of about 3 g/cm^3 resulting in three orders of magnitude bulk density difference between these two models for the same event. Our preference in such conditions should be given to most realistic physical models and to accessible experimental data, which avoid rough assumptions and uncertainties in the explanation. Some approaches are very inefficient if applied to studies of large bodies for which the major part of the luminous segment of their trajectory corresponds to the conditions of continuous medium flow around the meteoroid (Gritsevich and Stulov 2006 [15]; Gritsevich 2008 a [16]). As one of the consequences, only a few meteorites photographed by special cameras during their flight in the atmosphere were found using existing data processing techniques (Gritsevich 2008 b [16]).

In this study, we revise the problem of meteoroid mass determination based on multi-station meteor observations. We start from the drag and the mass-loss equations and the geometrical relation along the luminous part of a meteor trajectory in the atmosphere. We solve the equations and then compare obtained theoretical dependencies with experimental data using dimensionless parameters and assuming an isothermal atmosphere. It is shown that meteor events and their consequences can be classified with respect to values of the dimensionless parameters derived from the aerodynamic equations, based on further derivations.

2. Basic equations

In Meteor Physics the drag and mass loss equations are usually given

as (Bronshten, 1983 [4]; Stulov et al., 1995 [29]):

$$(1) \quad M \frac{dV}{dt} = -\frac{1}{2} c_d \rho_a V^2 S,$$

$$(2) \quad H * \frac{dM}{dt} = -\frac{1}{2} c_h \rho_a V^3 S.$$

The explanation of the used variables and parameters is given in the Appendix A of this paper. The Earth's gravity and variations in the entry angle γ for the meteoroid are negligible and usually they are not taken into consideration (Gritsevich, 2011 [18]). Therefore the change in height with time is given as:

$$(3) \quad \frac{dh}{dt} = -V \sin \gamma.$$

The Eq. (3) can be used to introduce the new variable h instead of t in Eqs. (1)–(2). Excluding t we have 5 variables, namely M , V , S , h , and ρ_a and thus, two additional dependencies between these quantities are necessary to obtain analytical solution. Here we use the assumption of an isothermal atmosphere:

$$(4) \quad \rho_a = \rho_0 \exp(-h/h_0).$$

We also introduce the parameter μ taking into account variations in the meteoroid shape according to Levin (1956) [25], as:

$$(5) \quad \frac{S}{S_e} = \left(\frac{M}{M_e} \right)^\mu.$$

To solve these equations, we introduce dimensionless parameters. Detailed definitions for used variables and parameters are given in the Appendix B. As a general rule, we use capital letters for the parameters with dimensions (e.g. M for the mass in kg) and small letters for the dimensionless parameters (e.g. m for the dimensionless mass varying between 0 and 1). Applying these assumptions together with initial conditions $y = \infty$, $v = 1$, $m = 1$, the first integrals of differential Eqs (1)–(2) appear as:

$$(6) \quad m = \exp \left(-\beta \frac{1 - v^2}{1 - \mu} \right).$$

$$(7) \quad y = \ln 2\alpha + \beta - \ln(\overline{E}i(\beta) - \overline{E}i(\beta v^2)).$$

In Eqs (6)–(7) the ballistic coefficient α characterizes the aerobraking efficiency. It is proportional to the ratio between the mass of the atmospheric column along the trajectory and with the cross section S_e , to the meteoroid initial mass. The mass loss parameter β is proportional to the ratio of the meteoroid initial kinetic energy to the energy which is required to fully destroy meteoroid. In the following sections we explain how to derive these parameters based on meteor observations, how to derive meteoroid mass, how to classify meteor events and how to predict consequences of their collisions with the Earth's atmosphere.

3. Determination of key parameters

Meteor observations are often documented as tabled data at certain time points ($i = 1, 2, \dots, n$) containing the values of the meteor height h_i and the velocity V_i , as well as the meteor stellar magnitude $Mpan_i$. Such kind of data has been published, for example, in the paper reporting the results from the Meteorite Observation and Recovery Project (MORP) by Halliday et al. (1996) [23].

For each given fireball, the parameters α and β can be found by comparing the theoretical curve (Eq. (7)) with the actual rate of meteoroid's deceleration in the atmosphere (Gritsevich, 2008) [16]. The problem is solved using the least square method:

$$(8) \quad Q_4(\alpha, \beta) = \sum_{i=1}^n (2\alpha \exp(-y_i) - \Delta_i \exp(-\beta))^2 \rightarrow \min,$$

where $\Delta_i = \overline{E}i(\beta) - \overline{E}i(\beta v_i^2)$.

The parameters α and β correspond to the minimum of expression (8) and thus, they are defined by the following conditions:

$$(9) \quad \sum_{i=1}^n \left[\left(\Delta_i \left(\sum_{i=1}^n \exp(-2y_i) \right) - \left(\sum_{i=1}^n \Delta_i \exp(-y_i) \right) \exp(-y_i) \right) \cdot (\Delta_i - (\Delta_i)') \right] = 0,$$

$$\alpha = \sum_{i=1}^n e^{-\beta - y_i} \cdot \Delta_i / \left(2 \sum_{i=1}^n e^{-2y_i} \right),$$

$$\frac{\sum_{i=1}^n e^{-2y_i} \sum_{i=1}^n (((\Delta_i)' - \Delta_i)^2 + (\Delta_i - 2\alpha \exp(\beta - y_i))((\Delta_i)'' - 2(\Delta_i)' + \Delta_i))}{\left(\sum_{i=1}^n \exp(-y_i)(\Delta_i - (\Delta_i)') \right)^2} > 1.$$

Here, $(\Delta_i)'$, $(\Delta_i)''$ are the first and second derivatives of the function Δ_i .

The solution found according to Eq. (9) is unique for each meteor observation with detectable deceleration. Also, the main advantage of this method is that we do not involve any other assumptions or empirical constants to derive α and β values. Thus, the exact values of the meteoroid shape factor, meteoroid density and shape change parameter are not required.

Now, the shape change coefficient μ and luminous efficiency coefficient τ can be derived from the luminosity equation, which according to (Gritsevich and Koschny, 2011[18]) can be then rewritten as:

$$(10) \quad I = -\tau \cdot \frac{dE}{dt} = \tau \cdot \frac{M_e V_e^3 \sin \gamma}{2h_0} f(v),$$

where:

$$f(v) = v^3 \cdot (\bar{E}i(\beta) - \bar{E}i(\beta v^2)) \cdot \left(\frac{\beta v^2}{1 - \mu} + 1 \right) \cdot \exp \left(\beta \frac{\mu v^2 - 1}{1 - \mu} \right).$$

Using the definitions of the ballistic coefficient α and pre-entry shape factor of meteoroid A_e (see Appendix B), the pre-entry mass value M_e can be derived from the following formula:

$$(11) \quad M_e = \left(\frac{1}{2} c_d \frac{\rho_0 h_0}{\alpha \sin \gamma} \frac{A_e}{\rho_m^{2/3}} \right)^3.$$

The right side of Eq. (11) contains several known parameters: the atmospheric density $\rho_0 = 0.00129 \text{ g/cm}^3$ at sea level, the atmospheric scale height $h_0 = 7160 \text{ m}$ for Earth, the initial meteoroid velocity V_e and the slope γ between horizon and trajectory. The value of the ballistic coefficient α is derived according to Eq. (9). Combining the Eqs (6) and (11), we derive expression for calculating the mass along luminous segment of a meteor trajectory:

$$(12) \quad M = \left(\frac{1}{2} c_d \frac{\rho_0 h_0}{\alpha \sin \gamma} \frac{A_e}{\rho_m^{2/3}} \right)^3 \exp \left(-\beta \frac{1 - v^2}{1 - \mu} \right).$$

Let us note, that no initial assumptions about the meteoroid's shape, mass, or bulk density have been involved in our study so far. The value of the product of the drag coefficient and the initial shape factor $c_d A_e$ is important to estimate more precisely the meteoroid mass. It is necessary to find the most probable initial shape of the body to define more exactly its value.

The majority of publications on Meteor Physics assume a constant meteoroid shape factor $A = S\rho_m^{2/3}/M^{2/3} = S_e\rho_m^{2/3}/M_e^{2/3} = A_e$ along the whole trajectory. In our work, this assumption is avoided because only the initial value of the shape factor is needed and subsequent calculations are still valid even the shape of the body changes during its motion through the atmosphere.

One obstacle kept in the suggested mass calculation is the uncertainty of the preentry body shape. As a rule of thumb, the spherical shape with $A_e = 1.209$ and $c_d = 1$ is mostly used (Bronshten 1983 [4]; Stulov et al. 1995 [29]). In the literature on meteors, it is sometimes assumed that $c_d A_e = 1.1$ (Wetherill and ReVelle 1981 [31]) or $c_d = 2.0$ (Bellot Rubio et al. 2002 [2]) for a spherical body. As a more general case, one can assume that the geometrical shape of the meteoroid has no preferred direction (Stulov et al. 1995 [29]), and that the real mass values are higher than those for spherical shape. This fact is connected to a more complex shape of the body, which results from its specific orientation during supersonic flight, where the body's maximal cross section is preferably oriented across the trajectory (Halliday et al. 1981 [22]; Zhdan et al. 2009 [35]). Therefore, on average, the shape factor exceeds the analogous value for a sphere. This result agrees well with the opinion of the outstanding American astronomer F. L. Whipple, who stated that the average value of the shape factor of a meteor body is $A = 1.5$ (the original citation can be found in the paper by Jacchia et al. (1965) [24]). After calculations by Jacchia et al. (1965) [24], the same value was used by Verniani (1966) [32], who writes: "The most reasonable average value for the shape factor A is 1.5, with an uncertainty on the order of 10 percent". A brick-like shape with the value of the shape factor $A \sim 1.55$ is used by Halliday et al. (1984 [21], 1996 [23]). In the studies by Ceplecha and McCrosky (1992) [5], Spurný et al. (2000) [28], Ceplecha and ReVelle (2005) [6] and by Borovička and Kalenda (2003) [3], the calculations are performed with $c_d A_e = 2.2$ and 2.0, respectively. During the study of the 12 stony micrometeoroids recovered from the stratosphere, Flynn and Sutton (1991) [7] found that 9 of them have a rectangular, box-like shape, while the remaining 3 micrometeoroids have spherical, cylindrical and ellipsoidal shape, respectively. In many cases the shapes of meteorites or their fragments are also close to rectangular parallelepipeds (Gritsevich and Stulov 2008 [16]; Gritsevich 2008c [11]).

One more argument in favour of increasing the product $c_d A_e$ is the result of the numerical modelling by Zhdan et al. (2007) [34], where the drag coefficient $c_d \sim 1.6$ was obtained for the flow around the plate shapes. Drag coefficients were calculated from the following formula:

$$(13) \quad c_d = \frac{1}{0.5\rho_\infty V_\infty^2 S} \int_{S_t} p ds.$$

Here, S_t is the total surface area. The flow of a non-viscous perfect gas around a body was calculated for the Mach number $Ma = 6$ and the adiabatic index 1.4. The basic stream properties almost do not change quantitatively at $Ma > 6$, because of a principle of independence on Mach number. At $Ma < 6$ quantitative changes are possible; however, qualitative features should be kept within the whole range of high-flow velocities.

Zhdan et al. (2007) [34] emphasizes that the drag coefficient only slightly varies with an increasing longitudinal size of the brick shaped body. At the same time, the drag coefficient drops to 1.2–1.3 for the rounded vertexes and edges of such bodies.

Thus, a realistic range of the initial shape factor is $1 < A_e < 2$, while drag coefficient c_d may be ranging from 0.9 to 1.6, leading to a range of the $A_e c_d$ product 0.9–3.2.

4. General statistics and consequences

Meteor events and their consequences can be classified with respect to corresponding values of the dimensionless parameters derived from the aerodynamic equations (Fig. 1). Below, we propose several examples of collisions of cosmic bodies with the Earth and their consequences. These examples are supplemented by the brief analysis of the actual events and the corresponding criteria have been strictly analytically derived (see Gritsevich et al., 2009 [17], 2011 [18], 2012 [19], 2013 [20] for the exact formulas and derivations):

1. The range $\alpha \ll 1$, $\beta \ll 1$: the impact of a unified massive body with the Earth's surface results in the formation of a vast crater. The large body's mass minimizes or entirely excludes the effect of the atmosphere. Almost certainly, the atmosphere is penetrated by a cosmic body without its fracture. Illustrative examples are the Barringer crater in the state of Arizona, United States and crater Lonar Lake, India.

2. The range $\alpha < 1$, $\beta < 1$: fracture of the meteor body in the atmosphere and deposition of a fragments cloud onto the Earth's surface take place

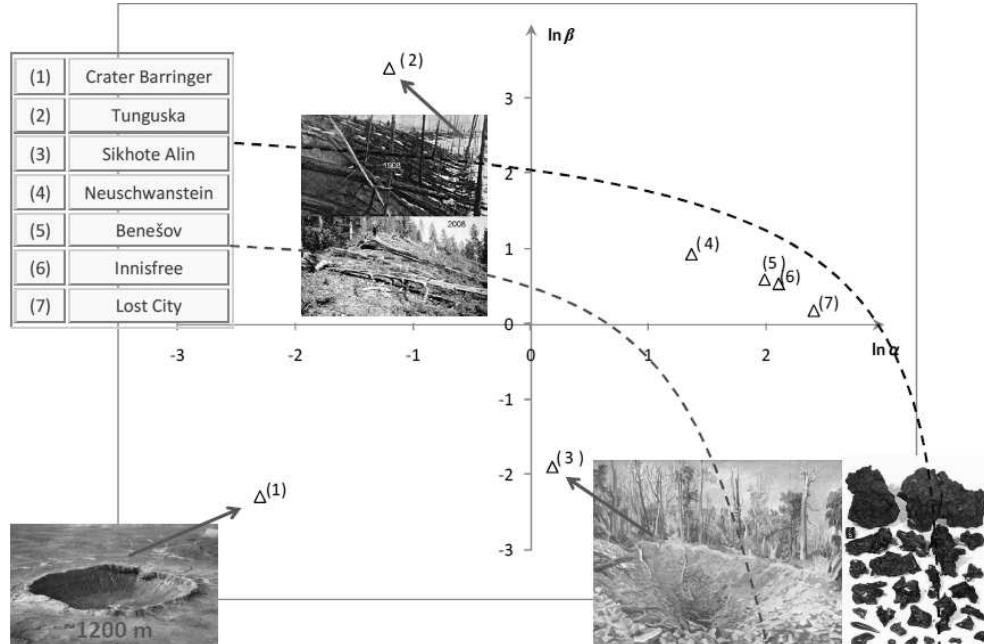


Fig. 1. The upper curve shows the margin for the region with crater formation, the lower curve shows the margin for meteorite survivors

with the formation of a crater strew field with corresponding meteorite fragments. Modern mathematical models describing the motion of the fragments cloud in the atmosphere allow us to predict basic geographic and other features of these fields. The motion ablation effect of the fragments is of minor importance. An illustrative example is the Sikhote-Alin meteorite shower (1947, Russia).

3. The range $\alpha \sim 1$, $\beta \sim 1$. These conditions are close to those of the preceding section. However, they are characterized by more significant role of ablation. As a result, we have group of meteorite-producing fireballs, with kinetic energy being not sufficient to produce significant craters during impact. Here, we can indicate reliably documented fireballs for which luminous segment of atmospheric trajectory were observed, meteorite fragments being also found in a number of cases. Among them, there are famous bolides Chelyabinsk (2013, Russia), Kosice (2010, Slovakia), Neuschwanstein (2002, Germany), Innisfree (1977, Canada), Lost City (1970, United States).

4. The range $\alpha < 1$, $\beta \gg 1$: fracture and complete evaporation of a meteoroid in the atmosphere take place at the low velocity loss. The character-

istic consequence of these events is the fall of a high-speed air-vapour jet onto the Earth's surface. Descending in the atmosphere, the gas volume expands (Turchak, 1980 [80]). Then, the gas cloud arrives at the Earth's surface, which is accompanied by the formation of a high-pressure region, and flows around its relief. As a result, the characteristic size of the impact region exceeds the characteristic size of the original meteoroid by several orders of magnitude. The Tunguska Event (1908, Russia) serves as a real example of an event of this type.

5. Conclusions

Meteorite studies represent a low-cost opportunity for probing the cosmic matter that reaches the Earth's surface and thus, for revealing the nature and origin of our Solar System. The correct interpretation of meteor observations could promptly confirm fresh meteorite fall, and furthermore provide a link to its parent body. We have classified results that might accompany collisions of cosmic bodies with the Earth's atmosphere and surface, based on the analysis of the fireball aerodynamic equations. After integrating, these equations well characterize the body's trajectory in the atmosphere, while the exact derived dependency of body's velocity on the height of the fireball can be further compared to the observations. The solution depends on three key dimensionless parameters defining the meteoroid drag, mass loss and rotation rate in the atmosphere. We recommend systematic evaluation of these parameters for the real-time observations of meteors.

REFERENCES

- [1] ABRAMOVITZ, M., I. A. STEGUN, eds. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, New York, Dover Publications, 1972.
- [2] BELLOT, RUBIO, L. R., M. J. MARTÍNEZ GONZÁLEZ, L. RUIZ HERRERA, J. LICANDRO, D. MARTÍNEZ-DELGADO, P. RODRÍGUEZ-GIL, M. SERRA-RICART. Modelling the Photometric and Dynamic behaviour of Super-Schmidt Meteors in the Earth's Atmosphere. *Astron. Astrophys.*, **389** (2002), 680–691.
- [3] BOROVIČKA, J., P. KALENDA. The Morávka Meteorite Fall: 4 Meteoroid Dynamics and Fragmentation in the Atmosphere. *Meteorit. Planet. Sci.*, **38** (2003), No. 7, 1023–1043.
- [4] BRONSHTEN, V. A. Physics of Meteoric Phenomena, Moscow, Nauka, 1981 and D. Reidel Publishing Company, 1983.

- [5] CEPLECHA, Z., R. E. MCCROSKY. Gross-fragmentation of Meteoroids and Bulk Density of Geminids from Photographic Fireball Records, Proc. of the Asteroids, Comets, Meteors 1991 Conf., 1992, 109–112.
- [6] CEPLECHA, Z., D. O. REVELLE. Fragmentation Model of Meteoroid Motion, Mass Loss, and Radiation in the Atmosphere. *Meteorit. Planet. Sci.*, **40** (2005), No. 1, 35–54.
- [7] FLYNN, G. J., S. R. SUTTON. Cosmic Dust Particle Densities – Evidence for Two Populations of Stony Micrometeorites, Proc. of the Lunar and Planetary Science Conf., 1991, 541–547.
- [8] GRITSEVICH, M. I. Approximation of the Observed Motion of Bolides by the Analytical Solution of the Equations of Meteor Physics. *Solar System Research*, **41** (2007), No. 6, 509–514.
- [9] GRITSEVICH, M. I. Validity of the Photometric Formula for Estimating the Mass of a Fireball Projectile. *Doklady Physics*, **53** (2008a), No. 2, 97–102.
- [10] GRITSEVICH, M. I. The Příbram, Lost City, Innisfree, and Neuschwanstein Falls: An Analysis of the Atmospheric Trajectories. *Solar System Research*, **42** (2008b), No. 5, 372–390.
- [11] GRITSEVICH, M. I. Estimating the Terminal Mass of Large Meteoroids. *Doklady Physics*, **53** (2008c), No. 11, 588–594.
- [12] GRITSEVICH, M. I. Determination of Parameters of Meteor Bodies Based on Flight Observational Data. *Advances in Space Research*, **44** (2009), No. 3, 323–334.
- [13] GRITSEVICH, M. I. On a Formulation of Meteor Physics Problems. *Moscow University Mechanics Bulletin*, **65** (2010), No. 4, 94–95.
- [14] GRITSEVICH, M., D. KOSCHNY. Constraining the Luminous Efficiency of Meteors. *Icarus*, **212** (2011), No. 2, 877–884.
- [15] GRITSEVICH, M. I., V. P. STULOV. Extra-Atmospheric Masses of the Canadian Network Bolides. *Solar System Research*, **40** (2006), No. 6, 477–484.
- [16] GRITSEVICH, M. I., V. P. STULOV. A Model of the Motion of the Neuschwanstein Bolide in the Atmosphere. *Solar System Research*, **42** (2008), No. 2, 118–123.
- [17] GRITSEVICH, M. I., V. P. STULOV, L. I. TURCHAK. Classification of the Consequences of the Impacts of the Natural Space Bodies with the Earth. *Doklady Physics*, **54** (2009), No. 11, 499–503.
- [18] GRITSEVICH, M. I., V. P. STULOV, L. I. TURCHAK. Standards for Crater Formation and Meteorite Fallout by the Light Sector of an Atmospheric Trajectory. *Doklady Physics*, **56** (2011), No. 3, 199–203.
- [19] GRITSEVICH, M. I., V. P. STULOV, L. I. TURCHAK. Consequences for Collisions of Natural Cosmic Bodies with the Earth Atmosphere and Surface. *Cosmic Research*, **50** (2012), No. 1, 56–64.

- [20] GRITSEVICH, M. I., V. P. STULOV, L. I. TURCHAK. Formation of Large Craters on the Earth as a Result of Impacts of Natural Cosmic Bodies. *Doklady Physics*, **58** (2013), No. 1, 37–39.
- [21] HALLIDAY, I., A. T. BLACKWELL, A. A. GRIFFIN. The Frequency of Meteorite Falls on the Earth. *Science*, **223** (1984), 1405–1407.
- [22] HALLIDAY, I., A. A. GRIFFIN, A. T. BLACKWELL. The Innisfree Meteorite Fall – A Photographic Analysis of Fragmentation, Dynamics and Luminosity. *Meteoritics*, **16** (1981), No. 2, 153–170.
- [23] HALLIDAY, I., A. A. GRIFFIN, A. T. BLACKWELL. Detailed Data for 259 Fireballs from Canadian Camera Network and Inferences Concerning the Influx of Large Meteoroids. *Meteoritics & Planetary Science*, **31** (1996), 185–217.
- [24] JACCHIA, L. G., F. VERNIANI, R. E. BRIGGS. An Analysis of the Trajectories of 413 Precisely Photographic Meteors, SAO Spec. Rep. No. 175, 1965.
- [25] LEVIN, B. YU. Physical Theory of Meteors and Meteoroid Substance in the Solar System, AN SSSR, Moscow, 1956 (in Russian).
- [26] PARK, C. Nitric Oxide Production by Tunguska Meteor. *Acta Astronautica*, **5** (1978), 523–542.
- [27] SEKANINA, Z. The Tunguska Event – No Cometary Signature in Evidence. *Astronomical Journal*, **88** (1983), 1382–1413.
- [28] SPURNÝ, P., H. BETLEM, J. VAN’T LEVEN, P. JENNISKENS. Atmospheric Behaviour and Extreme Beginning Heights of the 13 Brightest Photographic Leonids from the Ground-based Expedition to China. *Meteorit. Planet. Sci.*, **35** (2000), No. 2, 243–249.
- [29] STULOV, V. P., V. N. MIRSKII, A. I. VISLYI. Aerodynamics of Bolides, Moscow, Nauka, 1995 (in Russian).
- [30] TURCHAK, L. I. Braking of Bodies upon Entry into an Atmosphere. *Cosmic Research*, **17** (1980), No. 6, 778–782.
- [31] TURCO, R. P., O. B. TOON, C. PARK, R. C. WHITTEN, J. B. POLLACK, P. NOERDLINGER. An Analysis of the Physical, Chemical, Optical, and Historical Impacts of the 1908 Tunguska Meteor Fall. *Icarus*, **50** (1982), 1–52.
- [32] VERNIANI, F. Meteor Masses and Luminosity. *Smithson. Contrib. Astrophys.*, **10** (1967), No. 3, 181–195, 1967, SAO Spec. Rep. No. 219, 1966.
- [33] WETHERILL, G. W., D. O. REVELLE. Which Fireballs are Meteorites— A Study of the Prairie Network Photographic Meteor Data. *Icarus*, **48** (1981), 308–328.
- [34] ZHDAN, I. A., V. P. STULOV, P. V. STULOV, L. I. TURCHAK. Drag Coefficients for Bodies of Meteorite-Like Shapes. *Solar System Research*, **41** (2007), No. 6, 505–508.
- [35] ZHDAN, I. A., V. P. STULOV, P. V. STULOV, L. I. TURCHAK. Motion of Meteor form Bodies at an Arbitrary Angle of Attack. *Solar System Research*, **43** (2009), No. 5, 434–437.

Appendix A**List of symbols:**

- α = ballistic coefficient;
 β = mass loss parameter;
 γ = slope between horizon and trajectory;
 ρ_0 = gas density at sea level;
 ρ_a = gas density;
 ρ_m = meteoroid bulk density;
 A = shape factor;
 A_e = pre-entry shape factor of meteoroid;
 c_d = drag coefficient;
 c_h = heat-transfer coefficient;
 h = height;
 H^* = effective destruction enthalpy;
 h_0 = scale height;
 I = meteor luminosity;
 E = meteoroid kinetic energy;
 Ma = Mach number;
 M = meteoroid mass;
 M_e = pre-entry meteoroid mass;
 n = number of observational points;
 S = middle section area;
 S_t = total surface area;
 S_e = pre-entry middle section area of meteoroid;
 t = time;
 V = velocity;

- V_e = pre-entry velocity;
 μ = shape change coefficient;
 τ = luminous efficiency coefficient.

Appendix B

Dimensionless parameters used:

- $y = h/h_0,$
 $v = V/V_e,$
 $m = M/M_e,$
 $A_e = \frac{S_e \rho_m^{2/3}}{M_e^{2/3}},$
 $\alpha = 0.5c_d \frac{\rho_0 h_0 S_e}{M_e \sin \gamma},$
 $\beta = 0.5(1 - \mu) \frac{c_h V_e^2}{c_d H^*},$
 $\mu = \log_m \frac{S}{S_e},$
 $\Delta = \bar{E}i(\beta) - \bar{E}i(\beta v^2).$

Appendix C

Special mathematical function used

The exponential integral $Ei(x)$, which for real, nonzero values of x , can be defined as:

$$\bar{E}i(x) = \int_{-\infty}^x \frac{e^z dz}{z}.$$

The integral has to be understood in terms of the Cauchy principal value, due to the singularity in the integrand at zero.

Integrating the Taylor series for the function $\frac{e^{-z}}{z}$, and extracting the logarithmic singularity, we can derive the following series representation for $Ei(x)$ for real values of x (see e. g. Abramovitz and Stegun, 1972 [1]):

$$\bar{E}i(x) = c + \ln x + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}, \quad x > 0,$$

where: c is the Euler–Mascheroni constant (also called Euler’s constant). It is defined as the limiting difference between the harmonic series and the natural logarithm:

$$c = \lim_{n \rightarrow \infty} \left(\sum_{k=1}^n \frac{1}{k} - \ln n \right) \approx 0.5772.$$