Method for the determination of the index of refraction of particles suspended in the ocean*

J. Ronald V. Zaneveld and Hasong Pak

Department of Oceanography, Oregon State University, Corvallis, Oregon 97331

(Received 18 May 1972)

It is shown that the complex index of refraction of a given particle-size distribution may be calculated if the particle extinction coefficient and the particle absorption coefficient are known. If the particles are assumed to be nonabsorbing, a real index of refraction may be calculated from the ratio of light scattering at 45° from the forward for two wavelengths. Application of the method to two stations off Ecuador indicates that the particle index of refraction can be determined with sufficient accuracy to become an important parameter in the study of the oceans.

Index Headings: Oceanography; Refractive index; Scattering.

The index of refraction of suspended particles in the ocean is a most interesting but elusive parameter. The index of refraction of suspended particles has not been measured directly. Nevertheless, many calculations of volume scattering functions using the Mie theory have been carried out using estimated indices of refraction. The index of refraction can also be a useful oceanic parameter describing the origin of particles. Pavlov and Grechushnikov[1] have estimated that the relative index of refraction of minerals in the sea is 1.17, whereas the relative index of refraction of living matter in the sea is probably closer to unity. Pak *et al.*[2] have characterized the suspensoids in the ocean by means of "light-scattering vectors." These vectors use the average size of a suspended particle in a sample and the average scattering at 45°. The method could probably be improved by using the particulate index of refraction rather than the average scattering.

The only method currently available for the calculation of a weighted average index of refraction is that of Gordon and Brown.[3] Using the theory developed by Mie[4] to calculate a number of volume scattering functions, they found the closest fit to experimental data. The corresponding index of refraction is that of the scattering sample.

This method, although accurate, is not deterministic. In this paper, a theory will be developed that will give an index of refraction directly from the particle-size distribution and the inherent optical properties (light attenuation and scattering).

Caution must be exercised when the term index of refraction is used for a collection of particles. The average index of refraction such as determined by Gordon and Brown[3] and by ourselves is really the value of the index of refraction that reproduces the bulk scattering properties of the particles. The individual indices of refraction are weighted by the scattering cross sections of the particles and then averaged. This process tends to favor the larger particles, as the scattering cross section is related to the cross-sectional area of the particles. In order to distinguish it from a true average index of refraction, the term significant index of refraction will be applied to the value of the index of refraction that will reproduce the bulk optical properties of the particle distribution.

GENERAL CONSIDERATIONS

The ocean will be considered to be a collection of spherical particles suspended in pure water. The index of refraction of water (real) will be given by m_w, and the (complex) significant index of refraction of the particles will be given by $m_p=n_p-in_p'$, following Van de Hulst's[5] notation.

The particles will further be characterized by a particle-size distribution $f(D)dD$, giving the number of particles with diameters between D and $D+dD$. Mie[4] theory permits us to calculate extinction, scattering, and absorption cross sections for suspended spherical particles if the indices of refraction of the particles and the suspending medium are known. The expressions for the cross sections may be greatly simplified if the index of refraction of the particles is close to that of the suspending medium

$$|m-1|\ll 1,$$

where $m=m_p/m_w$.

This implies both $|n_p/m_w-1|\ll 1$ and $n_p'/m_w\ll 1$. In the ocean, the assumption $|m-1|\ll 1$ is reasonable. Van de Hulst[5] has derived approximate formulas for the extinction (Q_{ext}) and absorption efficiencies (Q_{abs}) derived from Mie theory for the case $|m-1|\ll 1$. The efficiencies are obtained by dividing the cross sections for the extinction or absorption by the geometrical cross section of the particle. Burt[6] has presented a useful scattering diagram that shows the efficiency factor as a function of size, wavelength, and relative refractive index. The relations are

$$Q_{ext}=2-4e^{-\rho\tan\beta}\frac{\cos\beta}{\rho}\sin(\rho-\beta)-4e^{-\rho\tan\beta}\left(\frac{\cos\beta}{\rho}\right)^2\cos(\rho-2\beta)+4\left(\frac{\cos\beta}{\rho}\right)^2\cos 2\beta \quad (1)$$

and

$$Q_{\text{abs}}=1+\frac{e^{-2\rho \tan\beta}}{\rho \tan\beta}+\frac{e^{-2\rho \tan\beta}-1}{2(\rho \tan\beta)^2}, \tag{2}$$

where

$$\rho=\frac{2\pi D}{\lambda}\left|\frac{n_p}{m_w}-1\right|,$$

but λ is the wavelength of light in the medium so that $\lambda=\lambda_{\text{vac}}/m_w$. Hence

$$\rho=\frac{2\pi D}{\lambda_{\text{vac}}}|n_p-m_w|.$$

For convenience the factor

$$k=\frac{2\pi}{\lambda_{\text{vac}}}|n_p-m_w|$$

is introduced, so that $\rho=kD$. Furthermore,

$$\tan\beta=n_p'/(n_p-m_w).$$

By integrating the extinction cross section over the particle-size distribution, we obtain the total extinction coefficient for particles (c_p) in the medium

$$c_p=\int_0^\infty f(D)Q_{\text{ext}}(k,D,\beta)\frac{\pi D^2}{4}dD. \tag{3}$$

Similarly, the total absorption coefficient for particles (a_p) is given by

$$a_p=\int_0^\infty f(D)Q_{\text{abs}}(k,D,\beta)\frac{\pi D^2}{4}dD. \tag{4}$$

If the particle-size distribution is known, Eqs. (3) and (4) permit us, in principle, to solve for k and β, and hence n_p and n_p'.

THE TWO-PARAMETER PARTICLE-SIZE DISTRIBUTION

The integrations indicated by Eqs. (3) and (4) may be performed numerically or analytically using any measured or assumed particle-size distribution. One form of the particle-size distribution that lends itself readily to analytical integration is the exponential distribution with two parameters A and N,

$$f(D)\,dD=NAe^{-AD}\,dD. \tag{5}$$

This form of the distribution permits the cumulative particle-size distribution $g(D)$ to be written as

$$g(D)=\int_D^\infty NAe^{-AD'}dD'=Ne^{-AD}. \tag{6}$$

The cumulative particle-size distribution (the number of particles per unit volume with diameters larger than D) is the distribution usually measured. Almost all samples obtained in the eastern Pacific during the Yaloc 1969 and Yaloc 1971 cruises of Oregon State University[7] can be fitted within experimental error by a distribution like that in Eq. (6).

For the remainder of this paper, we will employ the distributions of Eqs. (5) and (6), although similar calculations may be performed using different size distributions.

Substitution of Eqs. (5) and (1) into Eq. (3) results in

$$c_p=\int_0^\infty NAe^{-AD}\frac{\pi D^2}{4}\left\{2-4e^{-kD\tan\beta}\frac{\cos\beta}{kD}\sin(kD-\beta)-4e^{-kD\tan\beta}\left(\frac{\cos\beta}{kD}\right)^2\cos(kD-2\beta)+4\left(\frac{\cos\beta}{kD}\right)^2\cos2\beta\right\}dD. \tag{7}$$

Equation (7) may be integrated to express the particulate extinction coefficient entirely in terms of N, A, k, and β,

$$c_p=NA\pi\left\{\frac{1}{A^3}-2\cos^2\beta\frac{A+k\tan\beta}{[(A+k\tan\beta)^2+k^2]^2}+\frac{\sin2\beta}{2k}\frac{[(A+k\tan\beta)^2-k^2]}{[(A+k\tan\beta)^2+k^2]^2}-\left(\frac{\cos\beta}{k}\right)^2\cos2\beta\frac{A+k\tan\beta}{(A+k\tan\beta)^2+k^2}-\cos^2\beta\frac{\sin2\beta}{k}\frac{1}{(A+k\tan\beta)^2+k^2}+\left(\frac{\cos\beta}{k}\right)^2\frac{\cos2\beta}{A}\right\}. \tag{8}$$

Substitution of Eqs. (5) and (2) into Eq. (4) results in

$$a_p=\int_0^\infty NAe^{-AD}\frac{\pi D^2}{4}\left[1+\frac{e^{-2kD\tan\beta}}{kD\tan\beta}+\frac{e^{-2kD\tan\beta}-1}{2(kD\tan\beta)^2}\right]dD \tag{9}$$

or

$$a_p=\frac{N\pi}{2}\left[\frac{1}{A^2}-\frac{1}{(A+2k\tan\beta)^2}\right]. \tag{10}$$

Equations (7) and (9) overestimate the contribution of particles for which $\pi D/\lambda\gg1$ is not satisfied. By use of Eqs. (1) and (2) rather than the exact expressions for the extinction and absorption coefficients of small particles, an error of not more than 2% is introduced. Equation (10) expresses

$$k\tan\beta=\frac{2\pi}{\lambda}n_p'$$

as a function of a_p, N, and A. Once n_p' is calculated from Eq. (10), n_p may be obtained from Eq. (8). Equation (8) is a quartic equation in $(n_p-m_w)^2$ and may be solved directly. A graphical solution may be obtained by plotting

$$k=\frac{2\pi}{\lambda}|n_p-m_w|$$

as a function of $c_p/N\pi$ and A for various values of n_p'. Figure 1 shows such a graph for the case $n_p'=0$ (nonabsorbing spheres). Similar graphs may be constructed for all values of n_p'.

APPLICATION TO OCEANIC DATA

The accurate and routine determination of c_p and a_p in the ocean is difficult, owing to the influence of absorption by yellow matter and the difficulty of determining the total scattering coefficient for particles with reasonable accuracy.

If large sections of the ocean are to be analyzed for the significant index of refraction, a slightly different approach can be used. We will make the (dangerous) assumption that $n_p'=0$, that is, that the particles are nonabsorbing. The great majority of scattering calculations for oceanic suspensions have been based on this approximation. This, however, is no justification for ignoring particulate absorption. It is merely an expedient, as most oceanic data currently available do not lend themselves to direct application of the complete theory. By use of data usually obtained from optical measurements, a significant real index of refraction may be obtained, however.

The total particulate scattering coefficient can be approximated by assuming it to be proportional to β_{45}, the volume scattering function due to particles at 45° from the forward direction. This assumption was first postulated by Jerlov during the Swedish Deep-Sea Expedition.[8] Subsequent theoretical calculations by Deirmendjian[9] have confirmed the experimental observations. Further recent observations[10] also show a high correlation between β_{45} and b_p. For the ratio of two scattering coefficients at different wavelengths, we obtain

$$\frac{b_p(\lambda_1)}{b_p(\lambda_2)}=\frac{\beta_{45}(\lambda_1)}{\beta_{45}(\lambda_2)}. \tag{11}$$

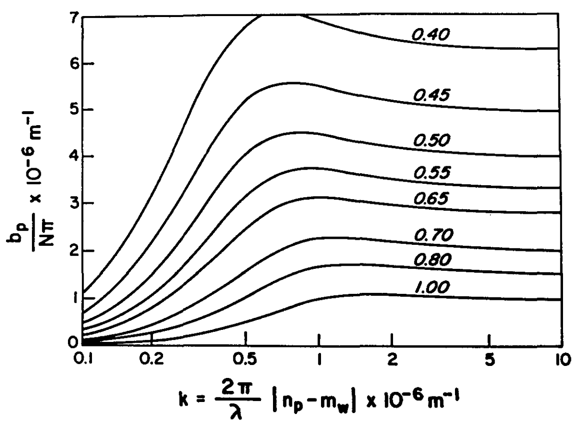

FIG. 1. The total scattering coefficient per particle $b_p/N\pi\times10^{-6}$ m^{-1} as a function of the index-of-refraction parameter $k=(2\pi/\lambda)|n_p-m_w|\times10^{-6}$ m^{-1} and the parameter A of the exponential particle-size distribution for the case of nonabsorbing particles ($n_p'=0$).

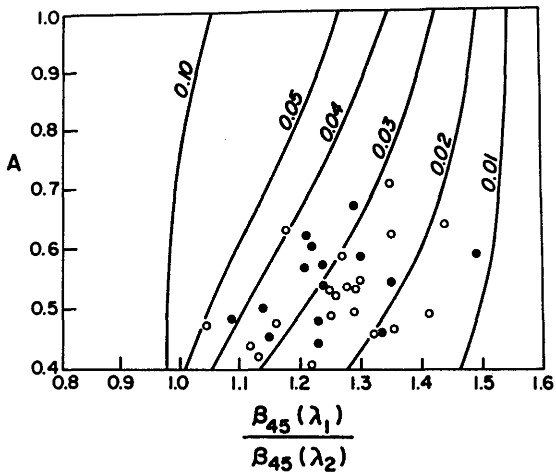

FIG. 2. The exponential particle-size distribution parameter A as a function of the ratio of light scattered at 45° for two wavelengths $\beta_{45}(\lambda_1)/\beta_{45}(\lambda_2)$ with the difference between the particulate and water indices of refraction $|n_p-m_w|$ as a parameter. Dots and circles mark samples taken off the coast of Ecuador from stations 4-2 and 4-8, respectively. $\lambda_1=436$ nm, $\lambda_2=546$ nm.

Equation (11) makes use of the fact that even if the constant of proportionality between b_p and β_{45} is unknown, the ratio of β_{45} at two wavelengths should be equal to the ratio of b_p at two wavelengths, provided that the constant of proportionality is independent of wavelength. Substituting $n_p'=0$ into Eq. (8) and noting that for nonabsorbing particles $b_p=c_p$, we obtain

$$\frac{b_p(\lambda_1)}{b_p(\lambda_2)}=\frac{\beta_{45}(\lambda_1)}{\beta_{45}(\lambda_2)}=\frac{1/A^2+(k_1^2-A^2)/(A^2+k_1^2)^2}{1/A^2+(k_2^2-A^2)/(A^2+k_2^2)^2}, \tag{12}$$

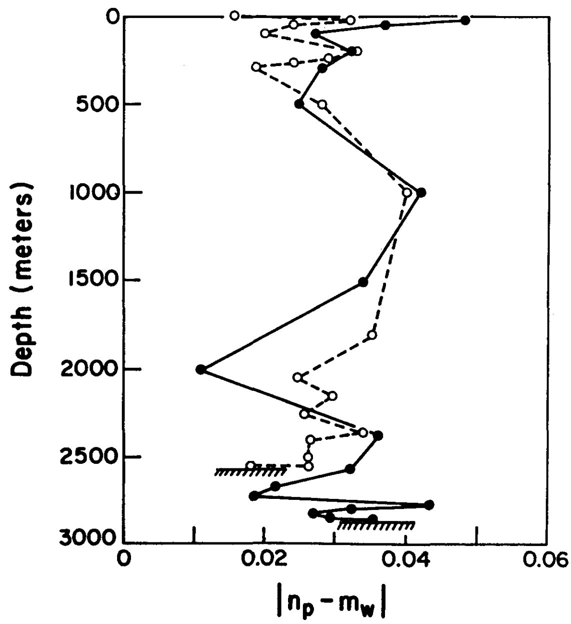

FIG. 3. The difference between the particulate and water indices of refraction $|n_p-m_w|$ as a function of depth for the two stations of Fig. 2.

where

$$k_1=\frac{2\pi}{\lambda_1}|n_p-m_w| \quad \text{and} \quad k_2=\frac{2\pi}{\lambda_2}|n_p-m_w|.$$

Equation (12) may be solved for $|n_p-m_w|$ as a function of $\beta_{45}(\lambda_1)/\beta_{45}(\lambda_2)$ and A if λ_1/λ_2 is given. Such a solution is shown in Fig. 2 for the case $\lambda_1=436$ nm and $\lambda_2=546$ nm. In order to provide some insight into actual measurements, results from two stations 210 km apart in the trench off Ecuador have been plotted in Fig. 2. The significant real indices of refraction thus obtained are replotted as a function of depth in Fig. 3. Figure 3 shows that in the simplified case $n_p'=0$, the depth dependence of the index of refraction of the suspended particles is roughly similar for the two stations.

CONCLUSIONS

The significant complex index of refraction for a known particle-size distribution can be obtained by use of Van de Hulst's[5] approximation for the extinction and absorption efficiencies. Accurate determination of the significant index of refraction is possible if both the particle extinction coefficient and particle absorption coefficient are known. In practice, these are currently difficult to measure.

If the particles are assumed to be nonabsorbing, a significant (real) index of refraction can be calculated from the ratio of light scattering at 45° for two different wavelengths. Application of this method to samples taken off Ecuador indicates that the significant index of refraction can currently be determined with sufficient accuracy to become an important parameter in the study of the oceans. The difference between the water index of refraction and the significant real index of refraction $|n_p-m_w|$ for these samples lies in the range 0.01–0.05.

REFERENCES

*Research supported by the Office of Naval Research through contract No. NOOO 14-67-A-0369-0007 under project No. NR 083-102.

[1] V. M. Pavlov and B. N. Grechushnikov, *Some Aspects of the Theory of Daylight Polarization Indices,* U. S. Dept. of Commerce, Joint Publication Research Session, Report No. 36 (U.S. Dept. of Commerce, Washington, D.C., 1966), p.25.

[2] H. Pak, G. F. Beardsley, Jr., G. R. Heath, and H. Curl, Limnol. Oceanogr. **15**, 683 (1970).

[3] H. R. Gordon and O. B. Brown, Trans. Am. Geophys. Union **52**, 245 (1971).

[4] G. Mie, Ann. Phys. (Leipz.) **25**, 377 (1908).

[5] H. C. van de Hulst, *Light Scattering by Small Particles* (Wiley, New York, 1957), pp. 172–199.

[6] W. V. Burt, J. Mar. Res. **15**, 76 (1956).

[7] K. L. Carder, Ph.D. thesis, Oregon State University, Corvallis (1970).

[8] N. G. Jerlov, *Report of the Swedish Deep-Sea Expedition, 1947–48, Vol. 3* (Elanders Boktryckeri Aktiebolag, Göteborg, 1953), pp. 73–97.

[9] G. D. Deirmendjian, in *Electromagnetic Scattering,* edited by M. Kerker (Macmillan, New York, 1963), p. 171.

[10] G. F. Beardsley, Jr., H. Pak, K. L. Carder, and B. Lundgren, J. Geophys. Res. **75**, 2837 (1970).