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Method of Synthesizing Orthogonal Beam-Forming Networks Using *QR* Decomposition

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ABSTRACT A synthesis method for orthogonal beam-forming networks (BFNs) with arbitrary N inputs and N outputs is presented. Compared to those formerly developed, the new method allows the design of a BFN in order to not only generate arbitrary N orthogonal beams and N inputs but also to make the 180° hybrids less. This skill is obtained by means of a new approach to decompose the matrices Q_1 and Q_2 which are mentioned by Sodin. The solution of such a design problem can be carried out by applying *QR* decomposition based on Givens transformations. Such a design method also takes into account the computer programming realization. Numerical results are obtained through the commercial simulator to prove the correctness of the method. The ease, accuracy, and efficiency of this synthesis method for the design of BFN make it very useful in modern applications of multi-beam antenna arrays.

INDEX TERMS Orthogonal beam-forming network, butler matrix, arbitrary N beams, *QR* decomposition, Givens transformations.

I. INTRODUCTION

Multiple-beam antennas (MBAs) are antenna arrays that connect beam-forming networks (BFNs) [1]. With the advantages of transmitting or receiving multiple beams simultaneously in prefixed directions, MBAs can mitigate multipath fading, increase channel capacity, and enhance system performance [2], [3]. Therefore, MBAs have been broadly applied in satellite communications, adaptive nulling, electronic countermeasures, multi-target radars and so on [4]–[7]. Once the antenna arrays are determined, the BFNs are the key point in designing MBAs. Thus, the need for designing a proper BFN has arisen.

The profitable BFN used for multiple beams with a linear array is the Butler matrix [8]. It can form orthogonal beams with the advantages of lossless property, high beam crossover and easy design. Compared to the Blass matrix [9] and the Nolen matrix [10], the Butler matrix requires less microwave couplers. However, the biggest problem of the Butler matrix is that it can only allow $N=2^m$ inputs and $N=2^m$ outputs, where m is a positive integer.

Recently, many refinements of Butler matrices have been reported to extend the number of beams/antenna elements to arbitrary number [11]–[14]. By adding a particular hybrid junction to the conventional Butler network, [11] increases the number of antenna ports from 2^n to any number. In [12], a reduced side-lobe four-beam N-element antenna array fed by $4\times N$ butler matrices is presented. However, the beams in [11] and [12] are not orthogonal. Reference [13] describes a new kind of Butler matrices with the number of $N=2^l 3^m 4^n$, where l, m and n are integers using 2×2, 3×3 and 4×4 junctions. Though the number of matrices is extended, it is limited.

A method of synthesizing an orthogonal BFN with arbitrary number of beams is presented in [14]. The BFN is represented by a cascade connection of elementary matrices containing one 180° hybrid or several 90° phase shifters. Unfortunately, as mentioned in [14], the decomposing method using elementary matrices may not be the optimum one. Hence, finding a new method to reduce the number of hybrids required for building BFN becomes a matter of significance.

Based on the mentioned problems above, this paper has proposed a new synthesis method to design orthogonal BFNs with arbitrary N inputs and N outputs. The main purpose is to decompose the matrices Q_1 and Q_2 which are mentioned in [14]. This new technique is based on *QR* decomposition which is computed with a series of Givens matrices. It has two primary advantages over [14]: one is that it can find the less number of 180° hybrids for building matrices Q_1 and Q_2; the other is that it can be realized by computer programs. By theory analysis and formula derivation, two key results are concluded: one is that a Givens matrix can be presented by a 180° hybrid; the other is that any square, real, symmetric, and orthogonal matrix can be expressed as a product of several transposed Givens matrices. Hence, matrices Q_1 and Q_2 can be represented by a cascade connection of Givens matrices.

By changing the order of *QR* decomposition, the reduced number of non-unity Givens matrices is found out. With the help of computer program, the procedure is illustrated using examples of synthesizing orthogonal BFN for 9 inputs and 9 outputs, which is one less component than [14]. With the increasing size of BFN, the less number of components is required comparing to [14].

The remainder of this paper is organized as follows. Two key conclusions are described in Section II. In Section III, the synthesis method of the BFN is introduced. An example of synthesizing the BFN for N=9 is proposed in Section IV. Section V validates the method described in Section IV using commercial simulation software Keysight Advanced Design System (ADS). Finally, a summary and conclusions are given in Section VI.

II. TWO KEY THEORIES

A. ANALYSIS OF THE GIVENS MATRICES

At first, let us consider the characteristic of the transmission matrix of a 180° hybrid. It is worth noting that the 180° hybrid is a four port network with two inputs and two outputs. As mentioned in [14], the transmission matrix of 180° hybrid is expressed as:

$$T_h = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \tag{1}$$

or

$$T_h = \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \tag{2}$$

where a and b are real and they satisfy $a^2 + b^2 = 1$.

Normally, according to [11], a Givens matrix can be expressed as:

$$G(i,j,\theta) = \begin{array}{cc} \left(\begin{array}{cccccccc} 1 & \cdots & 0 & \cdots & & 0 & \cdots & 0 \\ & \ddots & & & & & & \\ \vdots & 1 & & & & & & \vdots \\ 0 & & c & \cdots & & s & & 0 \\ & & & 1 & & & & \\ \vdots & & \vdots & \ddots & & \vdots & & \vdots \\ & & & & 1 & & & \\ 0 & & -s & \cdots & & c & & 0 \\ \vdots & & & & & & 1 & \vdots \\ & & & & & & \ddots & \\ 0 & \cdots & 0 & \cdots & & 0 & \cdots & 1 \end{array}\right) & \begin{array}{c} \\ \\ \\ (i) \\ \\ \\ \\ (j) \\ \\ \\ \\ \end{array} \\ \begin{array}{cccccccc} & & [i] & & & [j] & & \end{array} & \end{array} \tag{3}$$

where c=*cosθ* and s=*sinθ* appear at the intersections *i*th and *j*th rows and columns. That is, for fixed $i > j$, the non-zero elements of Givens matrix are given by:

$$\begin{aligned} g_{kk} &= 1 \quad for\ k \neq i,j \\ g_{kk} &= c \quad for\ k = i,j \\ g_{ji} &= -g_{ij} = -s \end{aligned} \tag{4}$$

Getting rid of the unity elements and zero elements in Eq. (3), the rest elements of g_{ii}, g_{jj}, g_{ij}, and g_{ji} satisfy the conditions of the 180° hybrid transmission matrix in Eq. (1). Thus, a Givens matrix can be presented by a 180° hybrid with output powers ratio of $p=(c/s)^2$.

On the other hand, if we rotate the position of g_{ii}, g_{jj}, g_{ij}, and g_{ji} in Eq. (3) clockwise, the Givens matrix is modified as:

$$G(i,j,\theta)' = \begin{array}{cc} \left(\begin{array}{cccccccc} 1 & \cdots & 0 & \cdots & & 0 & \cdots & 0 \\ & \ddots & & & & & & \\ \vdots & 1 & & & & & & \vdots \\ 0 & & c & \cdots & & s & & 0 \\ & & & 1 & & & & \\ \vdots & & \vdots & \ddots & & \vdots & & \vdots \\ & & & & 1 & & & \\ 0 & & s & \cdots & & -c & & 0 \\ \vdots & & & & & & 1 & \vdots \\ & & & & & & \ddots & \\ 0 & \cdots & 0 & \cdots & & 0 & \cdots & 1 \end{array}\right) & \begin{array}{c} \\ \\ \\ (i) \\ \\ \\ \\ (j) \\ \\ \\ \\ \end{array} \\ \begin{array}{cccccccc} & & [i] & & & [j] & & \end{array} & \end{array} \tag{5}$$

where c=*cosθ* and s=*sinθ* appear at the intersections *i*th and *j*th rows and columns. Likewise, for fixed *i*>*j*, the non-zero elements of the modified Givens matrix are given by:

$$\begin{aligned} g_{kk} &= 1 \quad for\ k \neq i,j \\ g_{ii} &= -g_{jj} = c \\ g_{ji} &= g_{ij} = s \end{aligned} \tag{6}$$

Similarly, the non-unity elements and non-zero elements in Eq. (5) have the same characteristics of the 180° hybrid transmission matrix in Eq. (2). Thus, the modified Givens matrix can be presented by a 180° hybrid with output powers ratio of $p=(c/s)^2$.

In particular case, when $s=0$ and $c=1$, Eq. (3) and (5) are degenerated into the unity matrix. Considering the four port network with two inputs and two outputs, it means inputs are directly connected to the corresponding outputs without any hybrids.

B. QR DECOMPOSITION OF A SQUARE, REAL, SYMMETRIC, AND ORTHOGONAL MATRIC USING GIVENS MATRICES

Based on *QR* decomposition solved by Givens matrices [15], any real square matrix A of size $N \times N$ can be decomposed as:

$$A = Q_0 R \tag{7}$$

where R is a nonsingular upper triangular matrix and Q_0 is an orthogonal matrix which satisfies

$$Q_0^T Q_0 = Q_0 Q_0^T = I \tag{8}$$

Then, Q_0 can be represented as a product of several Givens matrices

$$Q_0 = G^T = (G_M G_{M-1} \cdots G_i \cdots G_2 G_1)^T \tag{9}$$

with

$$M = (N-1)N/2 \tag{10}$$

where G_i is a Givens matrix of size $N \times N$ for $i=1-M$.

Owing to the symmetry and orthogonality of matrix Q_0, Matrix R can be expressed as

$$R = Q_0^T A \tag{11}$$

Let matrix A be a square, real, symmetric, and orthogonal matrix, it has the characteristic as

$$A^T A = AA^T = I \tag{12}$$

Applying Eq. (8), (11) and (12), one gets

$$\begin{aligned} R^T R &= (Q_0^T A)^T Q_0^T A \\ &= A^T Q_0 Q_0^T A \\ &= I \end{aligned} \tag{13}$$

Therefore, we conclude that R is an orthogonal matrix. Because R is an upper triangular matrix, it is demonstrated that R is a unit matrix.

$$R = I \tag{14}$$

Submitting Eq. (9) and (14) into (7), we have

$$\begin{aligned} A &= Q_0 R = Q_0 \\ &= (G_M G_{M-1} \cdots G_2 G_1)^T \\ &= G_1^T G_2^T \cdots G_{M-1}^T G_M^T \end{aligned} \tag{15}$$

Thus, by means of Eq. (15), it has been demonstrated that when matrix A is a square, real, symmetric, and orthogonal matrix, it can be expressed as a product of several transposed Givens matrices. The number of Givens matrices satisfies:

$$M = (N-1)N/2 \tag{16}$$

where N is the size of matrix A.

Here, we give an example of how compute QR decomposition of a square matrix. As shown in Fig.1, a matrix A with size of 4×4 is decomposed using Givens matrices. At first, we form a Givens matrix that will zero element a_{21}. We need to rotate the vector (a_{11}, a_{21}) to point along the X axis. This vector has an angle

$$\theta = \arctan\left(\frac{-a_{21}}{a_{11}}\right) \tag{17}$$

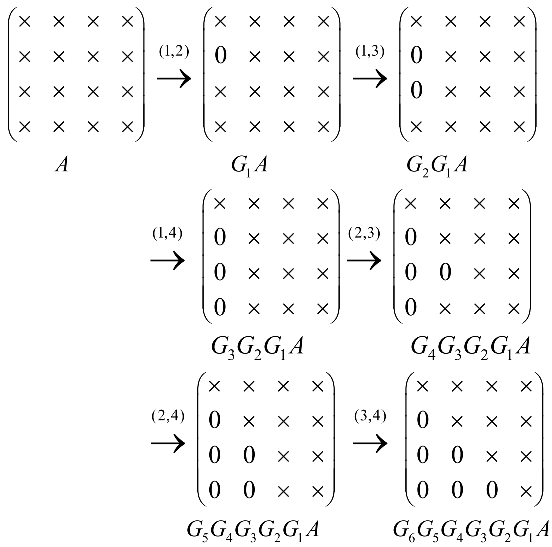

FIGURE 1. The general order of *QR* factorization at the 4 × 4 case.

Then, we create the orthogonal Givens matrix, G_1:

$$G_1 = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{18}$$

The result of $G_1 A$ has a zero in the a_{21} element.

$$G_1 A = \begin{pmatrix} \times & \times & \times & \times \\ 0 & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{pmatrix} \tag{19}$$

We can similarly form Givens matrices G_2 to G_6, which will zero the sub-diagonal elements of matrix A, forming a triangular matrix. It is worth noting that in each new zero element a_{ij} affects only the row with the element to be zeroed (i) and a row above (j).

III. A NEW SYNTHESIS METHOD OF THE BFN

Reference [14] has decomposed the transmission matrix of an orthogonal BFN into

$$T = YQY \tag{20}$$

where matrix Y is a square, real, symmetric, and orthogonal matrix. Q is a block-diagonal matrix and can be written as

$$Q = \begin{pmatrix} Q_1 & \\ & iQ_2 \end{pmatrix} \tag{21}$$

where matrices Q_1 and Q_2 are square, real, symmetric, and orthogonal matrices.

Based on the analysis above, matrices Q_1 of size $M \times M$ and Q_2 of size $N \times N$ can be decomposed into a product of several transposed Givens matrices:

$$Q_1 = G_1^T G_2^T \cdots G_{p-1}^T G_p^T \tag{22}$$
$$Q_2 = G_1^T G_2^T \cdots G_{q-1}^T G_q^T \tag{23}$$

where p and q are positive integers and they satisfy Eq. (16) with the order of matrices Q_1 and Q_2, respectively.

$$p = (M-1)M/2 \tag{24}$$
$$q = (N-1)N/2 \tag{25}$$

Recalling theories discussed in Section II, a Givens matrix can be presented by the corresponding 180° hybrid. Hence, matrices Q_1 and Q_2 can be realized by cascading a series of 180° hybrids which are determined by QR decomposition. The general order of QR decomposition is shown in Fig.1. Without any improvement, matrices Q_1 and Q_2 require p and q 180° hybrids, respectively.

To reduce the required 180° hybrids in matrices Q_1 and Q_2, the calculated Givens matrices must consist of several identity matrices or permutation matrices. Fortunately, that can be achieved by changing the order of zero elimination in QR decomposition. In order to decompose a matrix of size $N\times N$, there are q elements to be zeroed in the lower triangular area. Regardless of the realization of QR decomposition and repeatability, there is a factorial of q orders to zero elimination using Givens matrices.

$$q! = q \times (q-1) \times \cdots \times 2 \times 1 \tag{26}$$

Under some specific orders, QR decomposition can be computed with several identity matrices or permutation matrices. As the size of matrix Q becomes larger, the more identity matrices are obtained by specific orders of QR decomposition.

When the size of matrix is larger than 6×6, the better order required to fully exploit the algorithm. In our example, the maximum size of matrix is 5×5. Thus, the results can be easily solved by enumerating method using MATLAB. The detailed steps to find out the optimized solution are shown as follows:

1) Write out matrices T and Y of the BFN for any N based on [14].
2) Calculate the corresponding matrices Q_1 *and* Q_2 from Eq. (19) and (20).
3) Decompose matrices Q_1 and Q_2 using QR decomposition based on Givens matrices. The decomposition orders are at random and all cases are calculated.
4) Find out the better decompositions that the corresponding Givens matrices consist of more identity matrices or permutation matrices.

IV. SYNTHESIS OF THE BFN FOR 9 INPUTS AND 9 OUTPUTS

The proposed method is used for designing orthogonal BFNs with arbitrary N inputs and N outputs. In view of paper length limitations and simplicity, we do not show the detailed process for all N. As a result, an example of synthesizing an orthogonal BFN with N=9 is proposed, which can be compared with [14].

Matrices T and Y of the BFN are shown at the top of the next page.

This Y matrix represents four 180° hybrids with equal amplitude and one direct connection. Matrices Q_1 and Q_2 are calculated from Eq. (21)

$$Q_1 = \begin{pmatrix} 0.333 & 0.471 & 0.471 & 0.471 & 0.471 \\ 0.471 & 0.511 & 0.116 & -0.333 & -0.627 \\ 0.471 & 0.116 & -0.627 & -0.333 & 0.5111 \\ 0.471 & -0.333 & -0.333 & 0.667 & -0.333 \\ 0.471 & -0.627 & 0.511 & -0.333 & 0.116 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} -0.657 & 0.577 & -0.428 & 0.228 \\ 0.577 & 0 & -0.577 & 0.577 \\ -0.428 & -0.577 & 0.228 & 0.657 \\ 0.228 & 0.557 & 0.657 & 0.428 \end{pmatrix}$$

A. SYNTHESIS OF MATRIX Q_1

Matrix Q_1 of size 5×5 requires 10 Givens matrices to compute QR decomposition. By changing the sequence of zero elimination in the lower triangular elements, there are up to 3 unity matrices or permutation matrices among the calculated 10 Givens matrices. Thus, we can reduce the number of the 180° hybrid from 10 to 7. It is worth noting that the optimized solutions are not unique. Here we just give one solution.

The optimum solution for Q_1 is provided as

$$G_{54}^1 G_{21}^1 G_{51}^1 G_{43}^1 G_{31}^1 G_{42}^1 G_{53}^1 G_{52}^1 G_{32}^1 G_{41}^1 Q_1 = I \tag{27}$$

where

$$G_{41}^1 = \begin{pmatrix} 0.577 & 0 & 0 & 0.817 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.817 & 0 & 0 & -0.577 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$G_{32}^1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.975 & 0.221 & 0 & 0 \\ 0 & 0.221 & -0.975 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$G_{52}^1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -0.641 & 0 & 0 & 0.767 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.767 & 0 & 0 & 0.641 \end{pmatrix}$$

$$G_{53}^1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.900 & 0 & 0.435 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.435 & 0 & -0.900 \end{pmatrix}$$

$$G_{42}^1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -0.817 & 0 & 0.577 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0.577 & 0 & 0.817 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Y = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s & 0 & 0 & 0 & 0 & 0 & 0 & s \\ 0 & 0 & s & 0 & 0 & 0 & 0 & s & 0 \\ 0 & 0 & 0 & s & 0 & 0 & s & 0 & 0 \\ 0 & 0 & 0 & 0 & s & s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s & -s & 0 & 0 & 0 \\ 0 & 0 & 0 & s & 0 & 0 & -s & 0 & 0 \\ 0 & 0 & s & 0 & 0 & 0 & 0 & -s & 0 \\ 0 & s & 0 & 0 & 0 & 0 & 0 & 0 & -s \end{pmatrix}$$

$$T = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{j\frac{2}{9}\pi} & e^{j\frac{4}{9}\pi} & e^{j\frac{6}{9}\pi} & e^{j\frac{8}{9}\pi} & e^{j-\frac{8}{9}\pi} & e^{j-\frac{6}{9}\pi} & e^{j-\frac{4}{9}\pi} & e^{j-\frac{2}{9}\pi} \\ 1 & e^{j\frac{4}{9}\pi} & e^{j\frac{8}{9}\pi} & e^{j-\frac{6}{9}\pi} & e^{j-\frac{2}{9}\pi} & e^{j\frac{2}{9}\pi} & e^{j\frac{6}{9}\pi} & e^{j-\frac{8}{9}\pi} & e^{j-\frac{4}{9}\pi} \\ 1 & e^{j\frac{6}{9}\pi} & e^{j-\frac{6}{9}\pi} & 1 & e^{j\frac{6}{9}\pi} & e^{j-\frac{6}{9}\pi} & 1 & e^{j\frac{6}{9}\pi} & e^{j-\frac{6}{9}\pi} \\ 1 & e^{j\frac{8}{9}\pi} & e^{j-\frac{2}{9}\pi} & e^{j\frac{6}{9}\pi} & e^{j-\frac{4}{9}\pi} & e^{j\frac{4}{9}\pi} & e^{j-\frac{6}{9}\pi} & e^{j\frac{2}{9}\pi} & e^{j-\frac{8}{9}\pi} \\ 1 & e^{j-\frac{8}{9}\pi} & e^{j\frac{2}{9}\pi} & e^{j-\frac{6}{9}\pi} & e^{j\frac{4}{9}\pi} & e^{j-\frac{4}{9}\pi} & e^{j\frac{6}{9}\pi} & e^{j-\frac{2}{9}\pi} & e^{j\frac{8}{9}\pi} \\ 1 & e^{j-\frac{6}{9}\pi} & e^{j\frac{6}{9}\pi} & 1 & e^{j-\frac{6}{9}\pi} & e^{j\frac{6}{9}\pi} & 1 & e^{j-\frac{6}{9}\pi} & e^{j\frac{6}{9}\pi} \\ 1 & e^{j-\frac{4}{9}\pi} & e^{j-\frac{8}{9}\pi} & e^{j\frac{6}{9}\pi} & e^{j\frac{2}{9}\pi} & e^{j-\frac{2}{9}\pi} & e^{j-\frac{6}{9}\pi} & e^{j\frac{8}{9}\pi} & e^{j\frac{4}{9}\pi} \\ 1 & e^{j-\frac{2}{9}\pi} & e^{j-\frac{4}{9}\pi} & e^{j-\frac{6}{9}\pi} & e^{j-\frac{8}{9}\pi} & e^{j\frac{8}{9}\pi} & e^{j\frac{6}{9}\pi} & e^{j\frac{4}{9}\pi} & e^{j\frac{2}{9}\pi} \end{pmatrix}$$

$$G_{31}^1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$G_{43}^1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -0.707 & 0.707 & 0 \\ 0 & 0 & 0.707 & 0.707 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$G_{51}^1 = \begin{pmatrix} -0.577 & 0 & 0 & 0 & 0.817 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.817 & 0 & 0 & 0 & 0.577 \end{pmatrix}$$

$$G_{21}^1 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$G_{54}^1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Observing the calculated Givens matrices, it is seen that they are equal to the transposed matrices of themselves. Therefore, from the inverse transform of Eq. (27) we can obtain the required representation for Q_1:

$$\begin{aligned} Q_1 &= (G_{54}^1 G_{21}^1 G_{51}^1 G_{43}^1 G_{31}^1 G_{42}^1 G_{53}^1 G_{52}^1 G_{32}^1 G_{41}^1)^T \\ &= G_{41}^{1T} G_{32}^{1T} G_{52}^{1T} G_{53}^{1T} G_{42}^{1T} G_{31}^{1T} G_{43}^{1T} G_{51}^{1T} G_{21}^{1T} G_{54}^{1T} \\ &= G_{41}^1 G_{32}^1 G_{52}^1 G_{53}^1 G_{42}^1 G_{31}^1 G_{43}^1 G_{51}^1 G_{21}^1 G_{54}^1 \\ &= q_{11} q_{12} q_{13} q_{14} q_{15} q_{16} q_{17} p_{11} \end{aligned} \tag{28}$$

where

$$q_{11} = G_{41}^1$$
$$q_{12} = G_{32}^1$$
$$q_{13} = G_{52}^1$$
$$q_{14} = G_{53}^1$$
$$q_{15} = G_{42}^1$$
$$q_{16} = G_{31}^1 G_{43}^1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.707 & -0.707 & 0 \\ 0 & 0 & 0.707 & 0.707 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$
$$q_{17} = G_{51}^1 G_{21}^1 = \begin{pmatrix} 0.577 & 0 & 0 & 0 & 0.817 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -0.817 & 0 & 0 & 0 & 0.577 \end{pmatrix}$$
$$p_{11} = G_{54}^1$$

Therefore, matrix $q_{1i}(i{=}1-7)$ presents the corresponding 180° hybrid and matrix p_{11} is a permutation matrix. On the whole, for matrix Q_1, only seven 180° hybrids are required,

TABLE 1. Simulated amplitude and phase difference characteristics of the BFN for N=9 fed by imports 1∼9. Magnitude unit: dB, phase unit: degree.

input / output	1		2		3		4		5		6		7		8		9	
	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase
1	-9.54	0	-9.54	0	-9.54	0	-9.54	0	-9.54	0	-9.54	0	-9.54	0	-9.54	0	-9.54	0
2	-9.54	0	-9.54	40	-9.54	80	-9.54	120	-9.54	160	-9.54	-160	-9.54	-120	-9.54	-80	-9.54	-40
3	-9.54	0	-9.54	80	-9.54	160	-9.54	-120	-9.54	-40	-9.54	40	-9.54	120	-9.54	-160	-9.54	-80
4	-9.54	0	-9.54	120	-9.54	-120	-9.54	0	-9.54	120	-9.54	-120	-9.54	0	-9.54	120	-9.54	-120
5	-9.54	0	-9.54	160	-9.54	-40	-9.54	120	-9.54	-80	-9.54	80	-9.54	-120	-9.54	40	-9.54	-160
6	-9.54	0	-9.54	-160	-9.54	40	-9.54	-120	-9.54	80	-9.54	-80	-9.54	120	-9.54	-40	-9.54	160
7	-9.54	0	-9.54	-120	-9.54	120	-9.54	0	-9.54	-120	-9.54	120	-9.54	0	-9.54	-120	-9.54	120
8	-9.54	0	-9.54	-80	-9.54	-160	-9.54	120	-9.54	40	-9.54	-40	-9.54	-120	-9.54	160	-9.54	80
9	-9.54	0	-9.54	-40	-9.54	-80	-9.54	-120	-9.54	-160	-9.54	160	-9.54	120	-9.54	80	-9.54	40

with three (q_{11}, q_{15}, q_{17}) of 4.77dB, one (q_{16}) of 3dB, one (q_{13}) of 3.86dB, one (q_{14}) of 7.22dB, and one (q_{12}) of 13.1dB. As matrix p_{11} is a permutation matrix, input ports of 4 and 5 are directly connected to the output ports of 5 and 4, respectively.

B. SYNTHESIS OF MATRIX Q_2

By the similar actions for matrix Q_2, QR decomposition needs one unity or permutation matrix and 5 Givens matrices. Therefore, we can reduce the 180° hybrid from 6 to 5. Likewise, the optimized solutions are not unique. Here we just give one solution.

The optimum solution for synthesis of matrix Q_2 is provided as

$$G_{41}^2 G_{31}^2 G_{21}^2 G_{43}^2 G_{42}^2 G_{32}^2 Q_2 = I \tag{29}$$

where

$$G_{42}^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.707 & 0 & 0.707 \\ 0 & 0 & 1 & 0 \\ 0 & 0.707 & 0 & 0.707 \end{pmatrix}$$

$$G_{43}^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -0.678 & 0.735 \\ 0 & 0 & 0.735 & 0.678 \end{pmatrix}$$

$$G_{21}^2 = \begin{pmatrix} -0.817 & 0.577 & 0 & 0 \\ 0.577 & 0.817 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$G_{31}^2 = \begin{pmatrix} -0.851 & 0 & 0.525 & 0 \\ 0 & 1 & 0 & 0 \\ 0.525 & 0 & 0.851 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$G_{41}^2 = \begin{pmatrix} -0.945 & 0 & 0 & 0.328 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.328 & 0 & 0 & 0.945 \end{pmatrix}$$

$$G_{32}^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

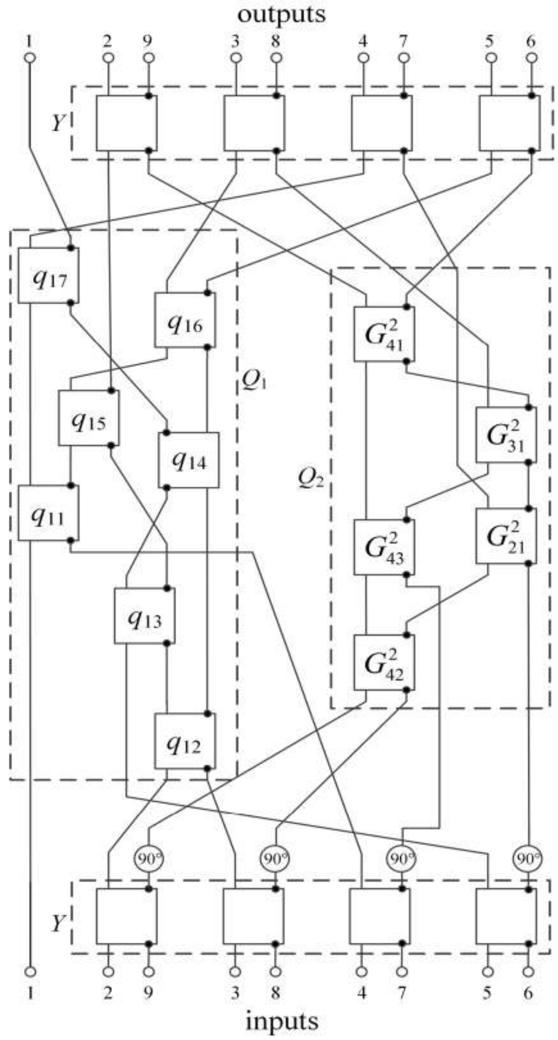

FIGURE 2. Beam-forming network for *N*=9.

From the inverse transform of this relationship we can obtain the required representation for Q_2:

$$\begin{aligned} Q_2 &= (G_{41}^2 G_{31}^2 G_{21}^2 G_{43}^2 G_{42}^2 G_{32}^2)^T \\ &= G_{32}^{2T} G_{42}^{2T} G_{43}^{2T} G_{21}^{2T} G_{31}^{2T} G_{41}^{2T} \\ &= G_{32}^2 G_{42}^2 G_{43}^2 G_{21}^2 G_{31}^2 G_{41}^2 \end{aligned} \tag{30}$$

For matrix Q_2, five 180° hybrids are required. As matrix G_{32}^2 is a permutation matrix, input ports of 2 and 3 are directly connected to the output ports of 3 and 2, respectively.

TABLE 2. Number of required hybrids for Q_1, Q_2, and Q for $N = 9$.

Ref.	Number of required hybrids for		
	Q_1	Q_2	Q
Optimal method	7	5	12
[14]	8	5	13

Thus, for the orthogonal BFN with 9 inputs and 9 outputs, twenty 180° hybrids and four 90° phase shifters are required by the proposed synthesis.

V. VERIFICATION AND COMPARISON

In order to validate the method, the Keysight Advanced Design System (ADS) simulator is used to simulate the proposed BFN obtained in Section IV. The full scheme of the device for $N=9$ is shown in Fig. 2. All the components are ideal.

Simulated results demonstrate the correctness of the proposed optimal solution. The simulated results in Table 1 show that the signal injected into one of the nine input ports is divided and transferred to the nine outputs with equal amplitude. The signals outputting from the nine output ports have constant phase difference, i.e., their phases are 0, ±40°, ±80°, ±120°, and ±160°, respectively.

The number of the required hybrids for Q_1, Q_2, and Q of our work and [14] for $N=9$ is summarized in Table 2. Seven hybrids and five hybrids are required using optimal method for matrices Q_1 and Q_2, respectively. In [14], eight hybrids and five hybrids are required for matrices Q_1 and Q_2, respectively. It is noted that our proposed method can find the better solution that uses one less hybrid than that of [14] for $N=9$.

VI. CONCLUSION

An improved method is presented to synthesize orthogonal BFNs for any beam number using Givens transformation. It reduces the components compared with method proposed in [14]. The procedure is illustrated using examples of synthesizing orthogonal BFN with 9 inputs and 9 outputs. As the BFN ports increase, the more hybrids are reduced. It is worth noting that as the size of matrix becomes larger, computer algorithm is needed. On the other hand, considering the power ratio of the hybrids and crossovers, it is important to find out the solution that is easier to be fabricated. Based on the synthesis method, optimal algorithm is going to be studied in the future.

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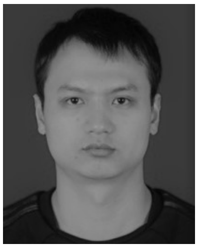

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