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Method of the Model Equation in the Theory of Unsteady Combustion of a Solid Propellant

S. A. Rashkovskii[a], Yu. M. Milekhin[b], A. N. Klyuchnikov[b], and A. V. Fedorychev[b]

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Abstract: A model equation for the unsteady burning rate of a solid propellant is proposed and justified. In the frequency range of interest for practice, the proposed model agrees with the phenomenological theory of unsteady combustion, but it is even more convenient for applications because it reduces to an ordinary differential equation of the second order with respect to the burning rate. A parametric study of the transitional process in the solid-propellant rocket motor is performed with variations of the nozzle throat area in a wide range of solid propellant parameters. The model predicts oscillatory combustion regimes and propellant extinction in the case of a decrease in pressure. The boundary of stability of the transitional process in the coordinates "sensitivity of the burning rate to changes in pressure—sensitivity of the burning rate to changes in initial temperature." It is demonstrated that the calculations performed with the use of this model are in qualitative and quantitative agreement with experimental data for a full-scale solid-propellant rocket motor.

Keywords: solid propellant, combustion, low-frequency instability, extinction, modeling.

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INTRODUCTION

One of the most complicated and hardly predictable phenomena in solid-propellant rocket motors (SRMs) is unsteady combustion. The phenomenological theory of unsteady combustion (PTUC) of the solid propellant (SP) [1, 2] formally predicts many phenomena being observed, but its practical application is usually constricted to the linear approximation. To calculate nonlinear phenomena, which are of the greatest interest for practice, it is necessary to solve a nonlinear partial differential equation or a nonlinear integral equation for the burning rate [1, 2]. This fact constrains PTUC application in engineering practice. Another constraint of this theory, in our opinion, is the conventional nature of the notions of the burning rate temperature and the temperature gradient near the burning surface, which cannot be used without significant simplifications of the real process for composite mixtures, because the zone of combustion of composite propellants is characterized by significant nonuniformity and local unsteadiness even under conditions of the so-called steady combustion.

For this reason, it became necessary to construct a simplified model of unsteady SP combustion, which would reflect the basic features of the real process on the basis of the PTUC [1, 2], but use measurable parameters only and be applicable for engineering calculations.

According to the PTUC, unsteady SP combustion is characterized by the parameters ν, k, μ, and r [1, 2], which are the sensitivities of the burning rate and burning surface temperature to changes in pressure and initial temperature. At $k > 1$, SP combustion is an oscillatory process with an eigenfrequency ω_0 and a damping factor λ [1, 2]:

[a]Ishlinskii Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow, 119526 Russia; rash@ipmnet.ru.
[b]Federal Center for Dual-Use Technologies "Soyuz," Dzerzhinskii, 140090 Russia.

$$\omega_0 = \Omega_0\left(\frac{u_0^2}{æ}\right), \quad \lambda = \Lambda\left(\frac{u_0^2}{æ}\right),$$
$$\Omega_0 = \frac{\sqrt{k}}{r}, \quad \Lambda = \frac{r(k+1)-(k-1)^2}{2r^2}. \tag{1}$$

Here Ω_0 and Λ are the dimensionless frequency and damping factor, u_0 is the steady burning rate, and æ is the thermal diffusivity of the propellant.

At $k < 1$, the transitional processes related to combustion are relaxation processes [1, 2]. In what follows, we consider SPs satisfying the condition $k > 1$, because the effects of unsteady combustion in SRMs are usually the governing factors for these propellants.

MODEL EQUATION

Taking into account Eqs. (1), we can describe the oscillatory process of SP combustion at a constant pressure p and small deviations of the burning rate u from the steady value u_0 by the equation

$$\frac{d^2u}{dt^2} + 2\lambda\frac{du}{dt} + \omega_0^2 u = \omega_0^2 u_0(p). \tag{2}$$

In the case with a variable pressure, the equation for small forced oscillations of the burning rate has the form

$$\frac{d^2u}{dt^2} + 2\lambda\frac{du}{dt} + \omega_0^2 u = Q\{p(t)\}, \tag{3}$$

where $Q\{p(t)\}$ is a certain real functional of $p(t)$.

For an unsteady process with p = const, Eq. (2) yields

$$Q\{p(t)\} = \omega_0^2 u_0(p). \tag{4}$$

Let us consider small harmonic oscillations of the pressure $p = p_0(1 + \eta\exp(i\omega t))$ and the burning rate $u = u_0(p_0)(1 + w\exp(i\omega t))$, where η and w are the dimensionless complex amplitudes, and ω is the frequency of pressure oscillations. The right side of the model equation (3) for small pressure oscillations with allowance for Eq. (4) can be written as

$$Q\{p(t)\} = \omega_0^2 u_0(p_0) + q(\omega)p_0\eta\exp(i\omega t), \tag{5}$$

where $q(\omega)$ is a complex function that describes the unsteady action induced by pressure oscillations on the combustion process.

For $|\eta| \ll 1$ and $|w| \ll 1$, the PTUC [1, 2] has a transfer function $F(\omega) = w/\eta$. The dependence $F(\omega)$ is irrational. The transfer function of the model equation (3), (5) has the form

$$F(\omega) = -\frac{(p_0/u_0)q(\omega)}{\omega^2 - \omega_0^2 - i2\lambda\omega}. \tag{6}$$

For the model equation (3), (5) to yield the same results as the PTUC, at least for small pressure oscillations, its transfer function should coincide with the PTUC transfer function [1, 2]. As it follows from the theory [1, 2], the transfer function has the form $F(\omega) = A(\omega) + i\omega D(\omega)$, where $A(\omega)$ and $D(\omega)$ are real functions depending on ω^2 only. Comparing the structure of the PTUC transfer function [1, 2] with solution (6), we conclude that $q(\omega) = (R(\omega) + i\omega P(\omega))(u_0/p_0)$, where $R(\omega)$ and $P(\omega)$ are real functions depending on ω^2 only.

In analyzing the functions $R(\omega)$ and $P(\omega)$, we considered the following limiting cases: 1) low frequencies, $\omega \to 0$; 2) regimes close to the resonance, $|\omega - \omega_0| \ll \omega_0$ and $\lambda \ll \omega_0$. For these cases, solution (6) of the model equation was compared with the PTUC transfer function [1, 2]. It was found that the functions $R(\omega)$ and $P(\omega)$ in a wide range of frequencies can be approximated by the polynomials

$$P(\omega) = \frac{u_0^2}{æ}P + \omega^2\frac{æ}{u_0^2}P_1, \; R(\omega) = \nu\omega_0^2 + \omega^2 Q, \tag{7}$$

where P, P_1, and Q are dimensionless parameters depending on ν, k, μ, and r. The relations of P, P_1, and Q to the parameters ν, k, r, and μ can be easily found by analyzing the limiting cases given above. The calculations show that the term $\omega^2 P_1$ in Eq. (7) has only a minor effect on the result and can be eliminated. Finally, the model transfer function (6) can be written as

$$F(\omega) = -\frac{\nu\omega_0^2 + \omega^2 Q + i\omega(u_0^2/æ)P}{\omega^2 - \omega_0^2 - i2\lambda\omega}. \tag{8}$$

If the parameters P and Q are calculated via ν, k, μ, and r by the formulas derived in the analysis of the limiting cases, then the model function (8) agrees well with the transfer function [1, 2] only at $\lambda \ll \omega_0$. Better agreement of Eq. (8) with the transfer function [1, 2] can be obtained if the parameters P and Q are considered as independent parameters of the propellant rather than if they are calculated via the constants ν, k, μ, and r. This approach is justified by the fact that the unsteady combustion theory [1, 2] has a phenomenological character, and its parameters r and μ should be considered as fitting coefficients because their physical meaning is rather conventional owing to the uncertainty of the notion of the burning surface temperature for composite propellants. If we assume that the parameters P and Q are certain characteristics of the propellant, like ν and k, we can reach reasonable agreement between the model function (8) and the PTUC transfer function [1, 2] by choosing appropriate values of these parameters.

Parametric studies were performed in wide ranges of the frequencies and parameters ν, k, r, and μ of the

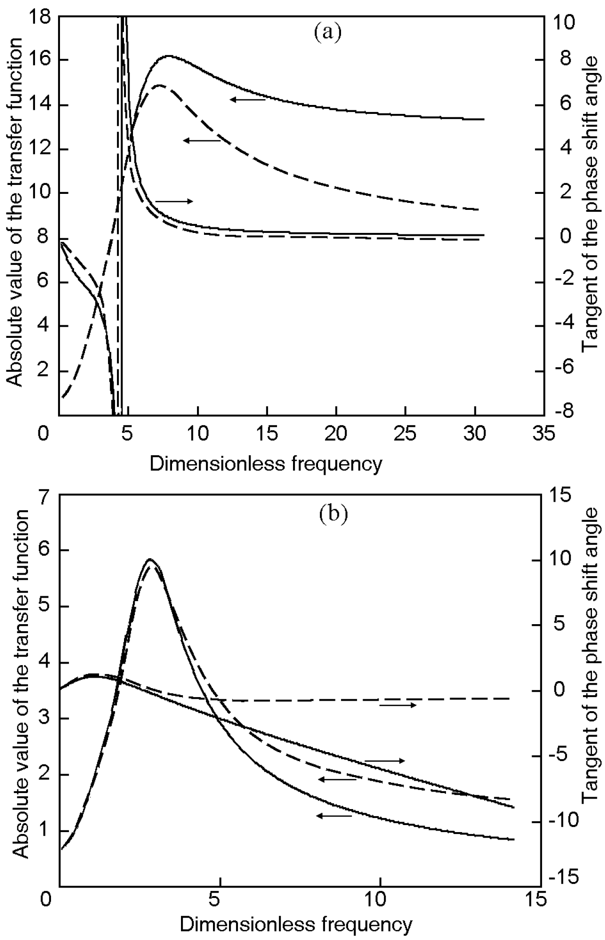

Fig. 1. Comparison of the exact PTUC transfer function (dashed curves) and the model transfer function [Eq. (8); solid curves]: (a) $\nu = 0.7$, $k = 1.5$, $r = 0.2$, $\mu = 1.0$, $P = -40$, and $Q = 13$ ($\omega_0 = 6.12$ and $\lambda = 3.13$); (b) $\nu = 0.7$, $k = 2.0$, $r = 0.5$, $\mu = 0$, $P = 11.5$, and $Q = 0$ ($\omega_0 = 2.83$ and $\lambda = 1.0$).

PTUC whose transfer function was approximated by function (8) with the constants P and Q being chosen from the condition of the best fitting between the exact PTUC solution and the model solution (8), at least in the frequency range $[0, \omega_0]$, which is of the greatest interest from the viewpoint of low-frequency instability of combustion, i.e., the most frequent phenomenon in SRMs. Typical results are presented in Fig. 1, which shows the absolute value and the tangent of the phase shift angle $\tan\theta$ for the exact PTUC transfer function and the model transfer function (8). The calculations show that the model function (8) reproduces even rather complicated PTUC transfer functions. It turned out that $Q = 0$ can be used in many cases of interest for practice. The parametric analysis performed allowed us to work out recommendations on choosing the parameter Q: it may be assumed that $Q = 0$ for those propellant parameters ν, k, μ, and r for which the phase shift point lies within the frequency range $\omega > \omega_0$; otherwise, $Q \neq 0$ should be chosen. In most cases, $P > 0$ if $Q = 0$ and $P < 0$ if $Q \neq 0$.

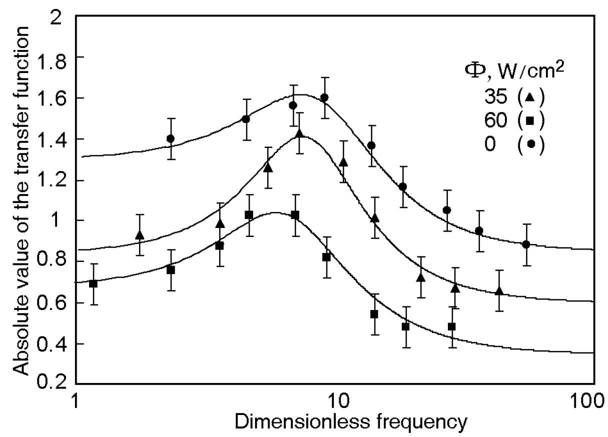

Fig. 2. Approximation of the experimental transfer functions for the MURI4 propellant [3] by the model function (8): the experimental data correspond to propellant combustion with a harmonically changing pressure and simultaneous irradiation of the burning surface by a constant-power laser beam; the points are the experimental data [3] for different powers of laser radiation; the curves show the calculations by Eq. (8).

In practice, the parameters r and μ for real propellants are unknown; moreover, as was indicated above, they are fitting coefficients. For this reason, the parameters P and Q of the model function (8) can be chosen on the basis of comparisons with available experimental data on unsteady combustion of propellants.

The analysis shows that the model function (8) approximates experimental transfer functions of composite SPs even under complicated conditions of combustion. Figure 2 shows the experimental data on unsteady combustion of the MURI4 propellant [3] (PA-based propellant with the HTPB binder) under conditions of a harmonically changing pressure with simultaneous irradiation of the burning surface by laser beams of different constant powers. With an appropriate choice of the constants ν, k, r, Q, and P, dependence (8) can describe the available experimental data in wide ranges of frequencies and propellant burning conditions (see the curves in Fig. 2).

Thus, the propellant is characterized in the considered approach by a set of empirical constants ν, k, r, Q, and P, which determine its unsteady combustion and can be obtained through comparisons either with the theoretical transfer function in the PTUC or with experimental transfer functions of real SPs.

Taking into account Eq. (8), we can rewrite the functional (5) in the form

$$Q\{p(t)\} = \omega_0^2 u_0(p_0) + (\nu\omega_0^2 + \omega^2 Q)u_0\eta\exp(i\omega t) + i\omega\frac{u_0^2}{æ}Pu_0\eta\exp(i\omega t)$$

or

$$Q\{p(t)\} = \omega_0^2 u_0(p(t)) - \frac{u_0}{p_0}Q\frac{d^2p}{dt^2} + \frac{u_0^2}{æ}\frac{u_0}{p_0}P\frac{dp}{dt}.$$

To eliminate p_0 from this expression, we take into account the relations

$$\frac{u_0}{p_0}\frac{dp}{dt} = \frac{1}{\nu}\frac{du_0}{dt}, \quad \frac{u_0}{p_0}\frac{d^2p}{dt^2} = \frac{1}{\nu}\frac{d^2u_0}{dt^2},$$

which are valid at small oscillations with $u_0(t) = u_0(p(t))$ in the right side of these equations. As a result, we obtain

$$Q\{p(t)\} = \omega_0^2 u_0(p(t)) + \frac{u_0^2}{æ}\frac{P}{\nu}\frac{du_0}{dt} - \frac{Q}{\nu}\frac{d^2u_0}{dt^2}. \quad (9)$$

Then, the model equation (3) for the unsteady burning rate takes the form

$$\frac{d^2u}{dt^2} + 2\lambda\frac{du}{dt} + \omega_0^2 u = \omega_0^2 u_0(p(t)) + \frac{u_0^2}{æ}\frac{P}{\nu}\frac{du_0}{dt} - \frac{Q}{\nu}\frac{d^2u_0}{dt^2}. \quad (10)$$

Note that the model transfer function was also proposed in [4, 5], but it was irrational, which prevented the authors from passing to a differential equation in calculating processes with finite changes in pressure and, moreover, processes accompanies by propellant extinction. The area of application of the model transfer function [4, 5] was limited to small pressure oscillations.

MODELING RESULT

Though Eq. (10) was derived for small oscillations of the burning rate, the calculations show that it is also applicable for a wider class of processes with rather arbitrary changes in pressure as a function of time.

Using the model equation (10), we calculate the transitional process in the SRM combustor with the pressure changing from p_h to $p_0 < p_h$. The change in pressure occurs owing to a linear change in the nozzle throat area.

Let us consider a propellant for which $Q = 0$ and $u_0(p) = u_1 p^\nu$ in the examined pressure range (u_1 and ν are constant coefficients).

The transitional process is described by the equations

$$\frac{W}{RT}\frac{dp}{dt} = u(t)\gamma S - Ap\sigma(t), \quad (11)$$

$$\frac{d^2u}{dt^2} + 2\lambda\frac{du}{dt} + \omega_0^2 u = \omega_0^2 u_0(p(t)) + \frac{u_0^2}{æ}\frac{P}{\nu}\frac{du_0}{dt}, \quad (12)$$

where the first equation is the mass conservation law [6]. Here W is the combustor volume, R is the gas constant of combustion products, T is the temperature of combustion products, A is the flow rate coefficient, γ is the propellant density, S is the burning surface area, and σ is the nozzle throat area. The values of W, T, R, A, ν, k, r, Q, and P are assumed to be constant during the entire process. In addition to Eqs. (11) and (12), we specify the law of variation of the nozzle throat area $\sigma(t)$.

Retaining, for convenience, the previous notations, we pass to the dimensionless variables

$$p := \frac{p}{p_0}, \quad u := \frac{u}{u_1 p_0^\nu}, \quad t := \frac{(u_1 p_0^\nu)^2}{æ}t, \quad \sigma := \frac{\sigma}{\sigma_0}, \quad (13)$$

where σ_0 is the nozzle throat area corresponding to the pressure p_0; it satisfies the relation $u_1\gamma S = Ap_0^{1-\nu}\sigma_0$ determining the steady solution of Eqs. (11) and (12).

In the dimensionless variables, system (11), (12) takes the form

$$\chi\frac{dp}{dt} = u - p\sigma(t), \quad (14)$$

$$\frac{d^2u}{dt^2} + 2p^{2\nu}\lambda\frac{du}{dt} + p^{4\nu}\omega_0^2 u = p^{5\nu}\omega_0^2 + Pp^{3\nu-1}\frac{dp}{dt}, \quad (15)$$

where $\chi = Wæ/[ART\sigma_0(u_1p_0^\nu)^2]$ is the hardware constant of the facility, which is equal to the ratio of the characteristic time of exhaustion from the combustor to the characteristic time of relaxation of the heated propellant layer.

The law of variation of the nozzle throat area has the following form in the dimensionless variables:

$$\sigma(t) = \begin{cases} \sigma_h, & t < 0, \\ \sigma_h - \dfrac{(\sigma_h - 1)t}{\tau_h}, & 0 \leqslant t \leqslant \tau_h, \\ 1, & t > \tau_h \end{cases} \quad (16)$$

($\sigma_h = p_h^{\nu-1}$ and τ_h is the dimensionless duration of nozzle switching).

The results calculated for the transitional processes in combustors with different volumes (different values of χ) are shown in Fig. 3. The parameters of the propellant and of the transitional process were identical in all calculations: $\nu = 0.7$, $k = 2.0$, $r = 0.5$, $P = 11.5$, $Q = 0$, $p_h = 2.0$, and $\tau_h = 10.0$; for the propellant considered, $\omega_0 = 2.83$ and $\lambda = 1.0$ in accordance with Eq. (1). The values of ν, k, r, Q, and P were assumed to be independent of pressure in the calculations.

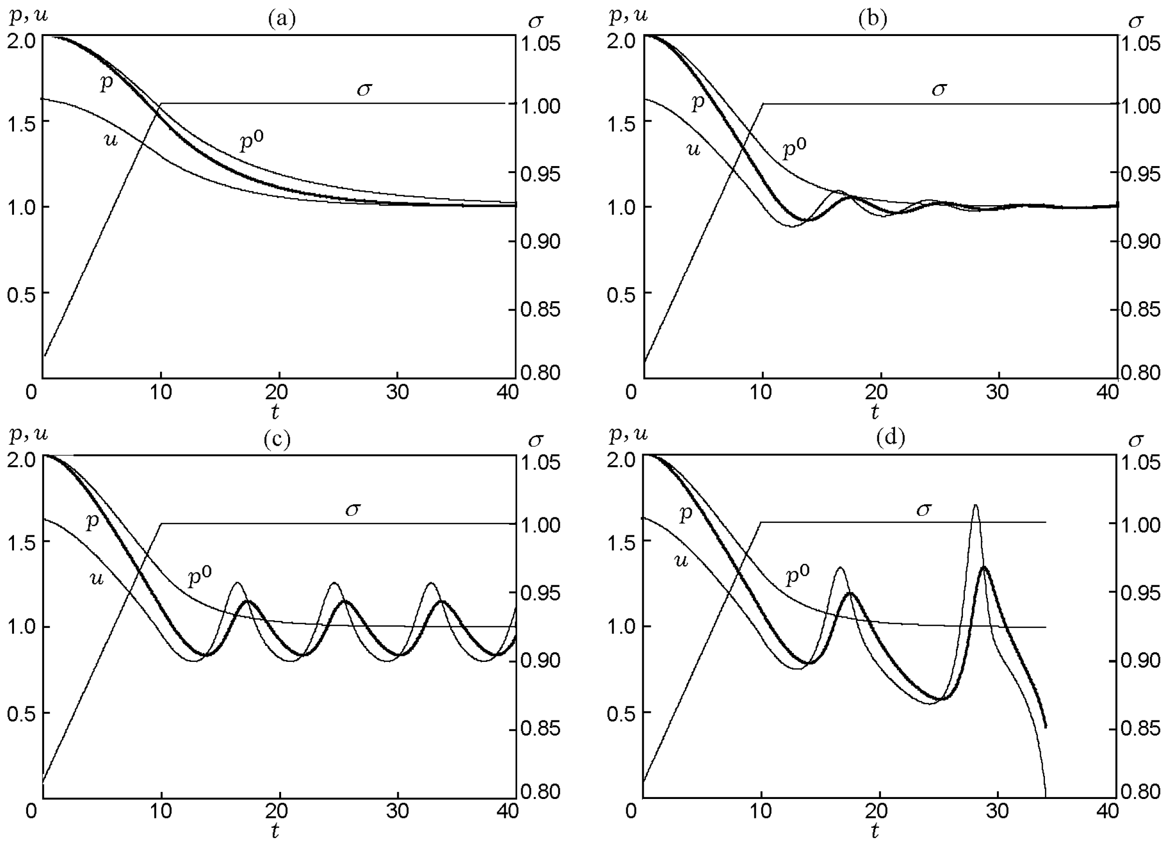

Fig. 3. Dimensionless pressure and burning rate versus the dimensionless time [$\nu = 0.7$, $k = 2.0$, $r = 0.5$, $P = 11.5$, $Q = 0$, $p_h = 2.0$, $\tau_h = 10.0$, and $\chi = 3$ (a), 1.4 (b), 1.3 (c), and 1.275 (d)].

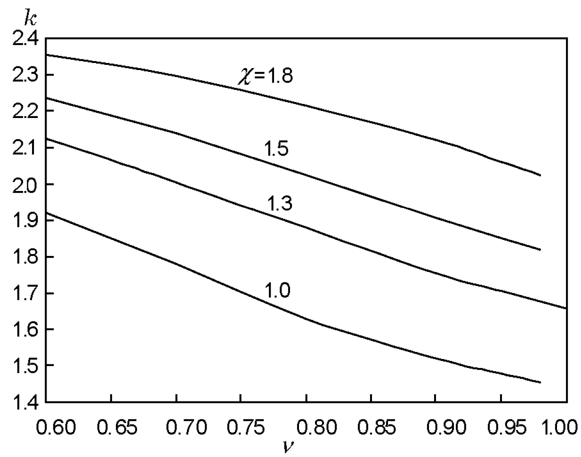

Fig. 4. Boundaries of stability of the transitional process for different values of χ.

For comparison, Fig. 3 shows the time evolution of pressure in the absence of the effects of unsteady combustion $p^0(t)$ with the calculation being performed by the unsteady equation of internal ballistics (14) for the steady burning rate $u = u_0(p)$.

It is seen from Fig. 3 that the transition to a new level of pressure in a large-volume combustor ($\chi = 3$) occurs steadily and monotonically. As χ decreases, other conditions being identical, the process acquires the character of decaying oscillations. At a certain critical value $\chi = \chi_{\rm cr}$ (in the case considered, $\chi_{\rm cr} = 1.3$; Fig. 3c), the process does not reach a new steady regime; it continues in the form of nondecaying nonlinear oscillations with a frequency approximately four times lower than the frequency of the eigenvalue oscillations of the burning rate ω_0, which is observed in experiments during the emergence of low-frequency instability in SRMs [7]. A small decrease in χ with respect to $\chi_{\rm cr}$ leads to amplification of oscillations and, finally, to propellant extinction (Fig. 3d). The smaller the value of χ with respect to $\chi_{\rm cr}$, the smaller the number of periods of pressure oscillations prior to propellant extinction. This agrees well with the experimental data. The experiments show that sometimes propellant extinction can be avoided even at values of χ somewhat smaller than $\chi_{\rm cr}$ if the amplification of oscillations proceeds slowly and the SRM has enough time to finish its operation or switch to a new

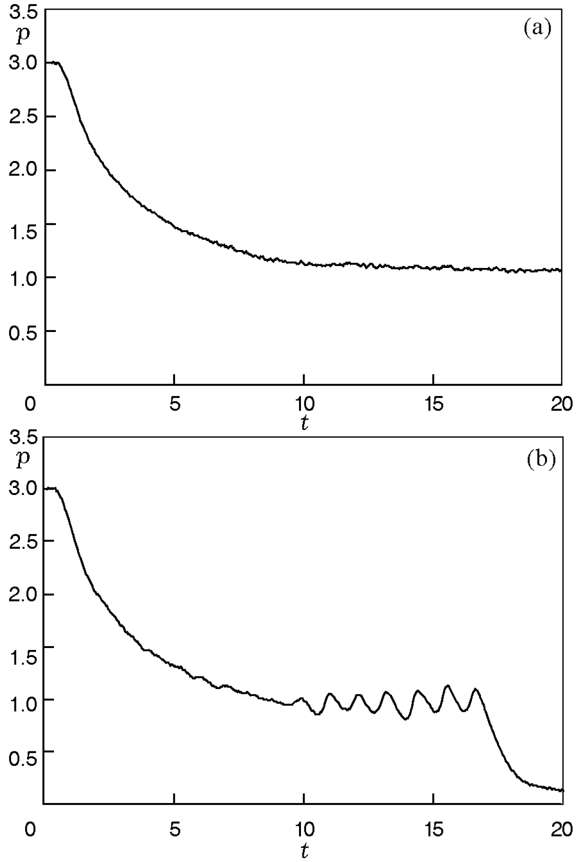

Fig. 5. Experimental dependences of pressure on time for the transitional process in the SRM ($\chi = 1.3$): the data are given in the dimensionless form: (a) stable process ($\nu = 0.79$ and $k = 1.5$); (b) unstable process ($\nu = 0.95$ and $k = 1.76$).

operation mode before the amplitude of pressure oscillations becomes sufficiently high for propellant extinction to occur.

The calculations were performed for $\nu = 0.7$, i.e., for a propellant with a comparatively stable combustion process. As the value of ν increases to 0.9–0.95, the propellant becomes more prone to extinction during the transitional processes; other conditions being identical, extinction can occur at large free volumes and slower decrease in pressure.

The parametric studies of the transitional process for different values of the parameters ν, k, r, P, p_h, and τ_h showed that the proposed model equation for the unsteady burning rate predicts SP extinction; this is an extremely important property of the model because, for instance, the PTUC predicts only a tendency to extinction, but not the extinction proper [8, 9].

STABILITY OF THE TRANSITIONAL PROCESS

A real SP is characterized by the parameters ν and k, which are experimentally determined by burning propellant samples in constant-pressure vessels or special small-size SRMs.

Of particular interest for applications is the analysis of the influence of the propellant parameters, first of all, ν and k, on stability of the transitional process at a fixed combustor volume. The reason is that the SP parameters ν and k may differ in the same SRM from one test to another because of tolerances for the raw material, production tolerances, and specific features of manufacturing of SP charges.

Even at $Q = 0$, the proposed model has two undetermined parameters, which can be found only by comparing the calculated results with experimental data for some unsteady processes in the SRM where unsteady combustion effects were manifested. Moreover, if the values of r and P can be determined in this manner for some particular charge, these values cannot be used for calculating processes with other charges of the same propellant because of the scatter in SP characteristics. For this reason, there arises a problem of determining the parameters r and P for a particular charge prior to its burning with due allowance for possible random deviations of SP characteristics. Let us consider an approximate method of estimating the parameters r and P from the measured parameters ν and k.

Let us use the hypothesis of a unique dependence of the steady burning rate on the burning surface temperature [1, 2], which is commonly used in the theory of unsteady combustion. It is a reasonable assumption because it reflects the commonly observed tendency of increasing of a certain mean burning surface temperature with increasing burning rate.

In this case, the ratio $r/k = \mu/\nu = \alpha$ can be considered as a constant characteristic of the propellant. An analysis of the PTUC transfer function [1, 2] shows that we have $P = A(k, \alpha)\nu$ and $Q = B(k, \alpha)\nu$, where $A(k, \alpha)$ and $B(k, \alpha)$ are independent of ν. In addition, we assume that the functions $A(k, \alpha)$ and $B(k, \alpha)$ depend comparatively weakly on k, so that the values of A and B can be considered as constant characteristics of the propellant in a comparatively narrow range of variations of the parameter k for real charges of the same SRM. Then, in the entire range of variation of the parameters ν and k, we have

$$P = A\nu, \quad Q = B\nu, \quad r = \alpha k. \qquad (17)$$

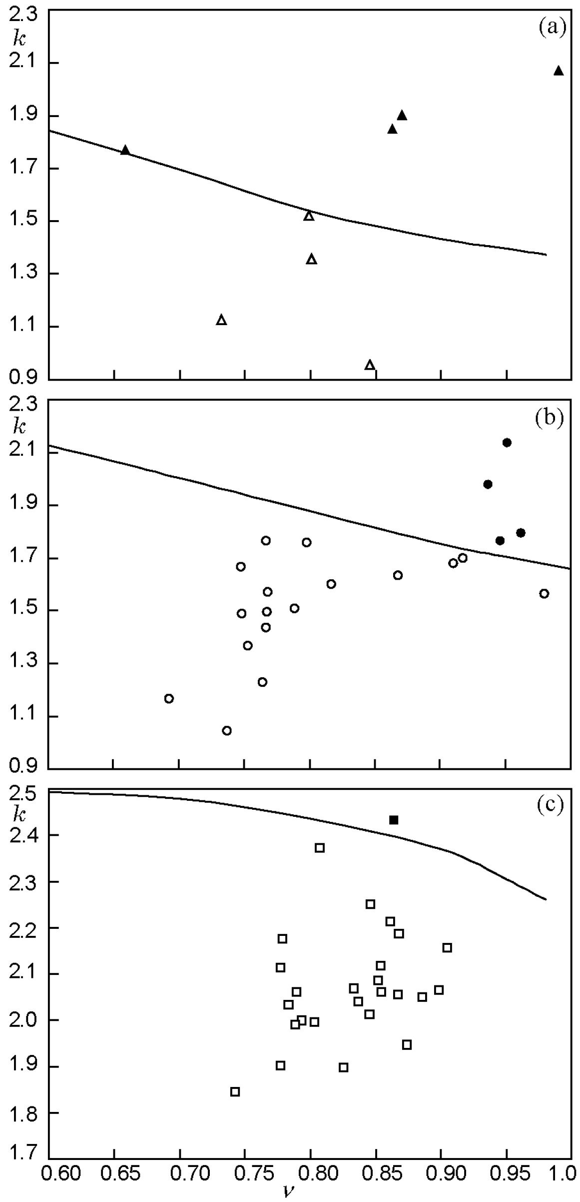

Fig. 6. Comparison of the calculated results with the experimental data on stability of the process in a small-scale SRM: the curves are the calculated boundaries of stability for a given value of the hardware constant χ; the stability and instability domains are located below and above the curves, respectively; the experimental data show the stable SRM operation (open points) and unstable SRM operation (filled points); $\chi = 0.89$ (a), 1.3 (b), and 2.15 (c).

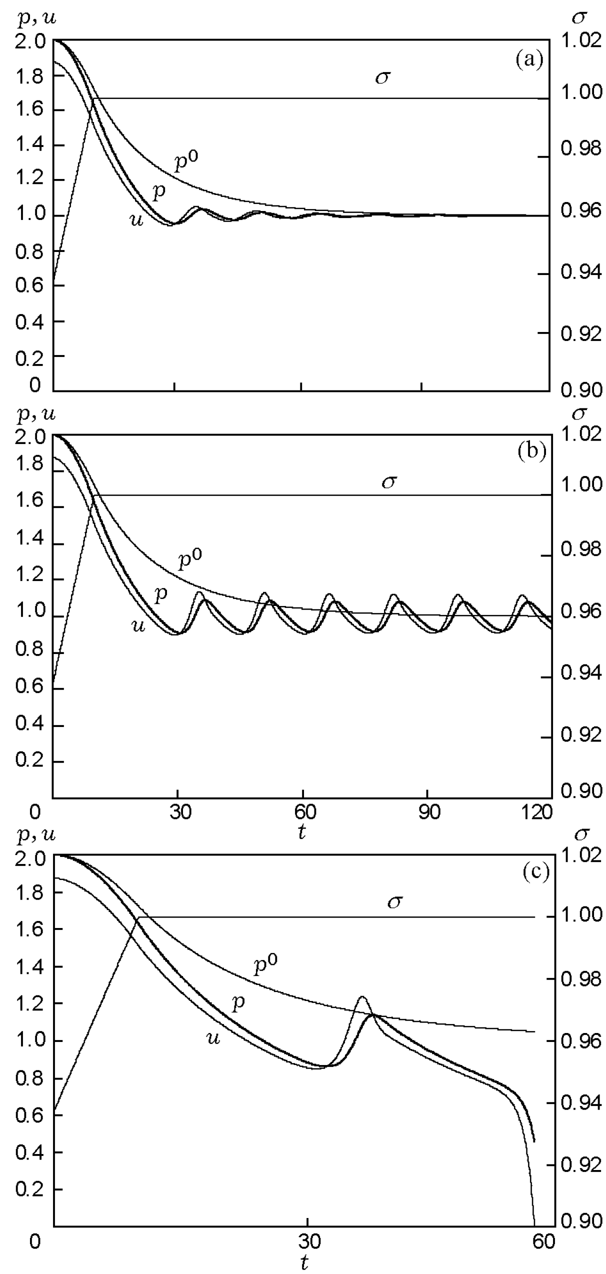

Fig. 7. Calculated dependences of dimensionless pressure and burning rate versus the dimensionless time [$\nu = 0.907$, $\chi = 1.8$, and $k = 2.07$ (a), 2.1 (b), and 2.109 (c)].

Note that relations (17) are not exact dependences: they reflect only the tendency of the behavior of the parameters P, Q, and r with variations of the parameters ν and k for a particular propellant. At the same time, as is shown below, relation (17) yields results consistent with experimental data.

Let us consider the effect of variations of the parameters ν and k for a conventional propellant on stability of the transitional process. For this propellant, we have

$$\alpha = 0.25, \quad A = 16.4, \quad B = 0. \qquad (18)$$

As an example, let us consider the transitional process in an SRM with $p_h = 2.0$ and $\tau_h = 10.0$ for various values of χ. The calculations were performed in the range of the parameters $k = 1.5$–2.4 and $\nu = 0.6$–0.98. The change in the parameter k at fixed values of ν and χ determines the boundary of stability in the coordinates (ν, k) for this value of χ: below this boundary, the process in the SRM reaches a certain new regime (steady or oscillatory one); above this boundary, propellant extinction occurs, and the burning rate in the calculation vanishes. The calculated results are shown in Fig. 4.

Let us compare the results calculated with the use of the proposed model equation with experimental data for a small-size SRM in which a slowly burning composite propellant was used and the transition from one regime to another was ensured by changing the nozzle throat area. Figure 5 shows the experimental dependences $p(t)$ in the dimensionless form for stable (a) and unstable (b) transitional processes corresponding to different values of the parameters ν and k.

Figure 6 shows the experimental data on stability of transitional processes in a small-scale SRM in the coordinates (ν, k) for different values of the hardware constant χ. The SP parameters ν and k were determined before each test; the results are shown as open points if the transitional process was stable and as filled points if the propellant extinguished after the transitional process and SRM operation ceased. The figures also show the theoretical boundaries of stability calculated by the model equation (15) with parameters (17) and (18).

The results obtained show that the calculation by the model equation (12) correctly reproduces the main features observed in experiments for the chosen propellant in a wide range of the main ballistic parameters ν and k, and in a wide range of the hardware constant χ.

Note that the relative error of measurement of the parameter k is estimated as $\pm 5\%$.

Let us estimate the sensitivity of the transitional process to variations of the parameter k with other conditions being identical. Figure 7 shows the calculated results corresponding to different degrees of process stability at $\chi = 1.8$. In these calculations, we used the same SP parameters (17) and (18) and transitional process parameters as in the calculations presented in Fig. 6. For all calculation variants, we used $\nu = 0.907$. For comparison, Fig. 7 also shows the time evolution of pressure in the absence of unsteady combustion effects $p^0(t)$; the calculation is performed by the unsteady equation (14) with the steady burning rate $u = u_0(p)$. Other conditions being identical, the transition from a stable to an unstable process occurs in a comparatively narrow range of the values of the parameter k, and the width of this range is smaller than the measurement accuracy of k. This fact should be taken into account in estimating stability of SRM operation with a particular charge.

CONCLUSIONS

A new model equation is proposed for the unsteady burning rate of a solid propellant. In the range of frequencies of interest for practice, the model is in perfect agreement with the phenomenological theory of unsteady combustion [1, 2], but the model is more convenient for practical calculations because it reduces to an ordinary differential equation of the second order with respect to the burning rate. The model predicts oscillatory modes of combustion and propellant extinction with a decrease in pressure. The calculations performed by this model qualitatively agree with experimental data for a model solid-propellant rocket motor.

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