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Barsoum et al.

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(54) **METHODOLOGY AND METHOD AND APPARATUS FOR SIGNALING WITH CAPACITY OPTIMIZED CONSTELLATIONS**

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H04L 27/18 (2006.01)
H04L 27/20 (2006.01)

(Continued)

(52) **U.S. Cl.**
CPC ***H04L 1/0003*** (2013.01); ***H04B 15/00*** (2013.01); ***H04L 1/0009*** (2013.01); ***H04L 27/3405*** (2013.01)

(58) **Field of Classification Search**
CPC H04L 2025/0342; H04L 27/3488; H04L 27/34; H04L 5/02; H04L 1/0041; H04L 1/0058; H04L 27/18; H04L 5/12; H04L 2001/0098; H04L 1/0009; H04L 1/0042; H04L 27/3405; H04L 1/007; H04L 1/1893; H03M 13/255; H03M 13/356; H03M 13/258; H03M 13/2957; H03M 13/1177; H03M 13/6561; H04N 19/67; H04N 21/2383
USPC 375/259–262, 268, 269, 295, 298, 279, 375/280, 329, 308, 316, 340, 341, 332; 332/103; 329/304
See application file for complete search history.

(56) **References Cited**

U.S. PATENT DOCUMENTS

5,289,501	A	2/1994	Seshadri et al.
5,862,179	A	1/1999	Goldstein et al.

(Continued)

FOREIGN PATENT DOCUMENTS

EP	1578021	A1	9/2005
EP	1971098	A1	9/2008

(Continued)

OTHER PUBLICATIONS

Nuno Sout et al., "Iterative Turbo Multipath Interference Cancellation for WCDMA System with Non-Uniform Modulations," IEEE, pp. 811-815, 2005.*

(Continued)

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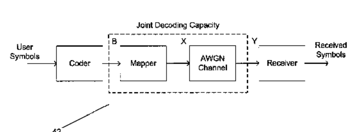
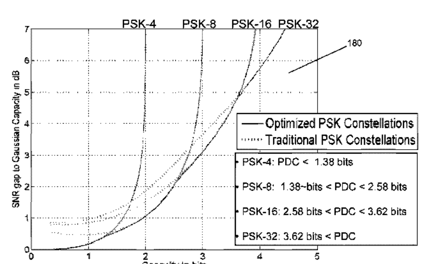
(57) **ABSTRACT**

Design Methodology and Method and Apparatus for Signaling with Capacity Optimized Constellation Abstract Communication systems are described that use geometrically PSK shaped constellations that have increased capacity compared to conventional PSK constellations operating within a similar SNR band. The geometrically shaped PSK constellation is optimized based upon parallel decoding capacity. In many embodiments, a capacity optimized geometrically shaped constellation can be used to replace a conventional constellation as part of a firmware upgrade to transmitters and receivers within a communication system. In a number of embodiments, the geometrically shaped constellation is optimized for an Additive White Gaussian Noise channel or a fading channel. In numerous embodiments, the communication uses adaptive rate encoding and the location of points within the geometrically shaped constellation changes as the code rate changes.

19 Claims, 43 Drawing Sheets





Related U.S. Application Data

continuation of application No. 12/156,989, filed on Jun. 5, 2008, now Pat. No. 7,978,777.

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H04L 27/22	(2006.01)
H04L 1/00	(2006.01)
H04B 15/00	(2006.01)
H04L 27/34	(2006.01)

(56) **References Cited**

U.S. PATENT DOCUMENTS

6,115,415	A	9/2000	Goldstein et al.	
6,157,678	A	12/2000	Wei	
6,603,801	B1	8/2003	Andren et al.	
6,606,355	B1	8/2003	Wei	
6,611,554	B1	8/2003	Chouly et al.	
6,665,831	B1	12/2003	Yoshida et al.	
7,245,666	B1	7/2007	Gardner et al.	
7,376,203	B2	5/2008	Brunel et al.	
7,620,067	B2	11/2009	Niu et al.	
7,978,777	B2	7/2011	Barsoum et al.	
8,265,175	B2	9/2012	Barsoum et al.	
8,270,511	B2	9/2012	Barsoum et al.	
9,191,148	B2	11/2015	Barsoum et al.	
2002/0044597	A1*	4/2002	Shively et al.	375/222
2002/0106010	A1	8/2002	Jones	
2003/0231715	A1	12/2003	Shoemake et al.	
2004/0022179	A1	2/2004	Giannakis et al.	
2004/0066738	A1	4/2004	Stopler	
2004/0161050	A1*	8/2004	Larsson et al.	375/267
2004/0258177	A1	12/2004	Shen et al.	
2005/0180531	A1	8/2005	Wellig et al.	
2005/0207507	A1	9/2005	Mitsutani	
2005/0276343	A1	12/2005	Jones	
2005/0286409	A1	12/2005	Yoon et al.	
2006/0045169	A1	3/2006	Kim	
2006/0155843	A1	7/2006	Glass et al.	
2007/0054614	A1	3/2007	Walker et al.	
2007/0116161	A1	5/2007	Tokoro et al.	
2007/0147530	A1*	6/2007	Li	375/261
2007/0280147	A1	12/2007	Catreux-Erceg et al.	
2008/0200114	A1*	8/2008	Eberlein et al.	455/3.02
2009/0161786	A1	6/2009	Nakagawa et al.	
2010/0195743	A1	8/2010	Barsoum et al.	
2010/0303174	A1	12/2010	Oh et al.	
2011/0228869	A1	9/2011	Barsoum et al.	
2012/0147983	A1	6/2012	Barsoum et al.	
2013/0083862	A1	4/2013	Barsoum et al.	
2013/0170571	A1	7/2013	Barsoum et al.	

FOREIGN PATENT DOCUMENTS

JP	09130438	A	5/1997
JP	2003229835		8/2003
JP	2005269258	A	9/2005
WO	9832257		7/1998
WO	2007074524	A1	7/2007
WO	2008151308	A1	12/2008
WO	2010078472	A1	7/2010

OTHER PUBLICATIONS

Roalds Otnes et al., "Adaptive Data Rate using ARQ and Nonuniform Constellations," Vehicular Technology Conference, pp. 1211-1215, 2001.*

Fei Zesong et al., "Shaping Gain by Non-Uniform QAM Constellation with Binary Turbo Coded Modulation," IEEE, pp. 1863-1867, 2003.*

Magged F. Barsoumet al., "Constellation Design via Capacity maximization," IEEE, pp. 1821-1825, 2007.*

Frank et al., "Optimal Nonuniform Signaling for Gaussian Channels," IEEE, pp. 913-929, 1993.*

Brendan Moor, "Pairwise optimization of modulation constellations for non-uniform sources Modulation," Can. J. Elect. Computer Eng. vol. 34, pp. 167-177, 2009.*

European Supplementary Search Report for Application No. EP 08795885, International Filing Date Jun. 5, 2008, Search Completed Apr. 1, 2014, 8 pgs.

International Search Report for International Application No. PCT/US 09/69881, date completed Apr. 12, 2010, date mailed May 3, 2010, 2 pgs.

International Search Report for International Application No. PCT/US2008/065994, Report completed Oct. 3, 2008, mailed Oct. 22, 2008, 2 pgs.

Written Opinion for International Application No. PCT/US2008/065994, completed Oct. 3, 2008, mailed Oct. 22, 2008, 5 pgs.

Written Opinion of the International Searching Authority for International Application No. PCT/US 09/69881, date completed Apr. 13, 2010, date mailed May 3, 2010, 10 pgs.

U.S. Appl. No. 13/618,630, "Notice of Allowance Received", May 15, 2014, 10 pgs.

Agrell et al., "On the Optimality of the Binary Reflected Gray Code", IEEE Transactions on Information Theory, Dec. 2004, vol. 50, No. 12, pp. 3170-3182.

Betts et al., "Performance of Nonuniform Constellations on the Gaussian Channel", IEEE Transactions on Information Theory, Sep. 1994, vol. 40, No. 5, pp. 1633-1638.

Conway et al., "A Fast Encoding Method for Lattice Codes and Quantizers", IEEE Transactions on Information Theory, Nov. 1983, vol. IT-29, No. 6, pp. 820-824.

De Gaudenzi, Riccardo et al., "Performance Analysis of Turbo-Coded APSK Modulations over Nonlinear Satellite Channels", IEEE Transactions on Wireless Communications, vol. 5, No. 9, Sep. 6, 2006, pp. 2396-2407, XP002457811, ISSN: 1536-1276, DOI: 10.1109/TWC.2006.1687763.

De Gaudenzi, Riccardo et al., "Turbo-Coded APSK Modulations Design for Satellite Broadband Communications", International Journal of Satellite Communications and Networking, vol. 24, No. 4, Jul. 1, 2006, pp. 261-281, XP008134566, ISSN: 1542-0973, DOIL 10.002/SAT.841.

Forney, Jr. et al., "Multidimensional Constellations—Part II: Voronoi Constellations", IEEE Journal on Selected Areas in Communications, Aug. 1989, vol. 7, No. 6, pp. 941-958.

Forney, Jr. et al., "Efficient Modulation for Band-Limited Channels", IEEE Journal on Selected Areas in Communications, Sep. 1984, vol. SAC-2, No. 5, pp. 632-647.

Forney, Jr. et al., "Multidimensional Constellations—Part I: Introduction, Figures of Merit, and Generalized Cross Constellations", IEEE Journal on Selected Areas in Communication, Aug. 1989, vol. 7, No. 6, pp. 877-892.

Foschini et al., "Optimization of Two-Dimensional Signal Constellations in the Presence of Gaussian Noise", IEEE Transactions on Communications, Jan. 1974, vol. Com-22, No. 1, pp. 28-38.

Fragouli et al., "Turbo Codes with Non-Uniform Constellations", IEEE Int. Conf. Commun., Jun. 2001, pp. 70-73.

Hamkins et al., "Asymptotically Dense Spherical Codes—Part II: Laminated Spherical Codes", IEEE Transactions on Information Theory, Nov. 1997, vol. 43, No. 6, pp. 1786-1798.

Hamkins et al., "Asymptotically Dense Spherical Codes—Part I: Wrapped Spherical Codes", IEEE Transactions on Information Theory, Nov. 1997, vol. 43, No. 6, pp. 1774-1785.

Makowski, "On the Optimality of Uniform Pulse Amplitude Modulation", IEEE Transactions on information Theory, Dec. 2006, vol. 52, No. 12, pp. 5546-5549.

Milovanovic et al., "Simple Optimization Method of One-Dimensional M-PAM Constellations for the AWGN Channels", 4th International Conference on Telecommunications in Modern Satellite, Cable and Broadcasting Services, Oct. 13-15, 1999, pp. 263-266.

(56) **References Cited**

OTHER PUBLICATIONS

Muhammad et al., "Joint Optimization of Signal Constellation and Bit Labeling for Bit-Interleaved Coded Modulation with Iterative Decoding", IEEE Communications Letters, Sep. 2005, vol. 9, No. 9, pp. 775-777.

Raphaeli et al., "Constellation Shaping for Pragmatic Turbo-Coded Modulation with High Spectral Efficiency", IEEE Transactions on Communications, Mar. 2004, vol. 52, No. 3, pp. 341-345.

Ruotsalainen, "On the construction of the higher dimensional constellations", ISIT 2000, Lausanne, Switzerland, Jun. 30-Jul. 5, 2002, p. 490.

Sommer, Dirk et al., "Signal Shaping by Non-Uniform QAM for AWGN Channels and Applications Using Turbo Coding", Proc. ITG Conf. Source and Channel Coding, Jan. 31, 2000, pp. 81-86, XP055110228.

Sun et al., "Approaching Capacity by Equiprobable Signaling on the Gaussian Channel", IEEE Transactions on Information Theory, Sep. 1993, vol. 39, No. 5, pp. 1714-1716.

Ungerboeck, "Channel Coding with Multilevel/Phase Signals", IEEE Transactions on Information Theory, Jan. 1982, vol. IT-28, No. 1, pp. 55-67.

* cited by examiner

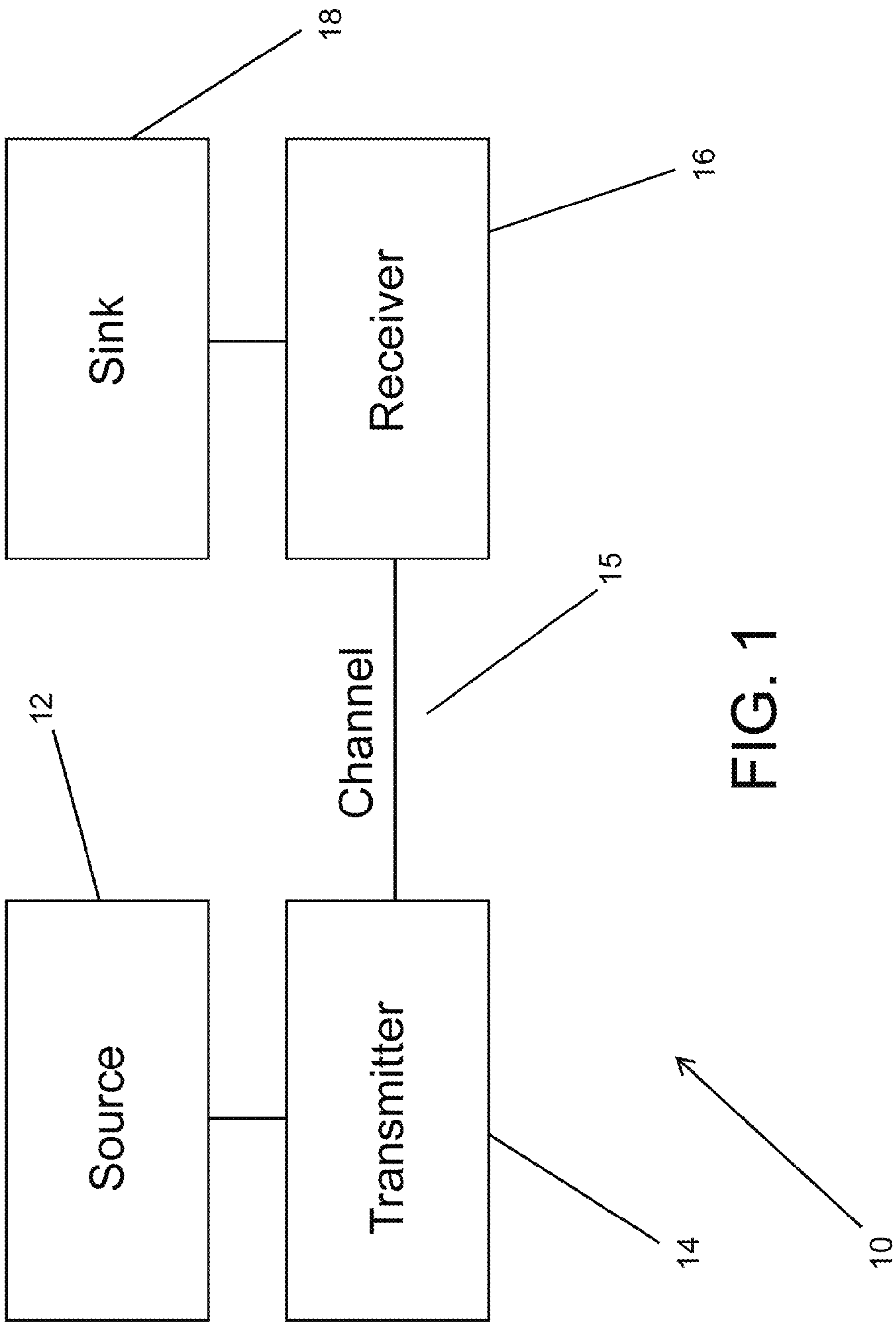


FIG. 1

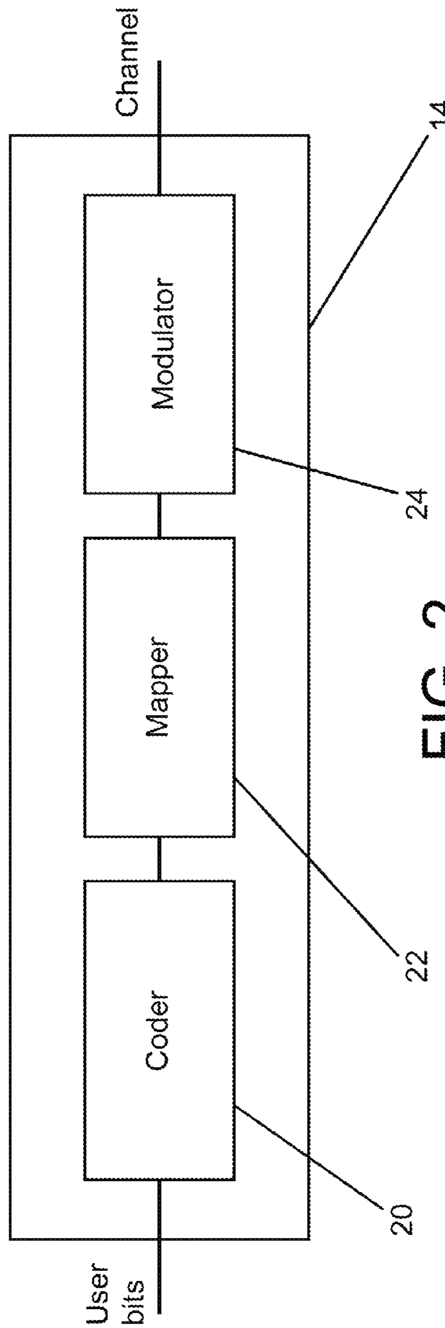

Channel
14
Modulator
24
Mapper
22
Coder
20
User bits

FIG. 2

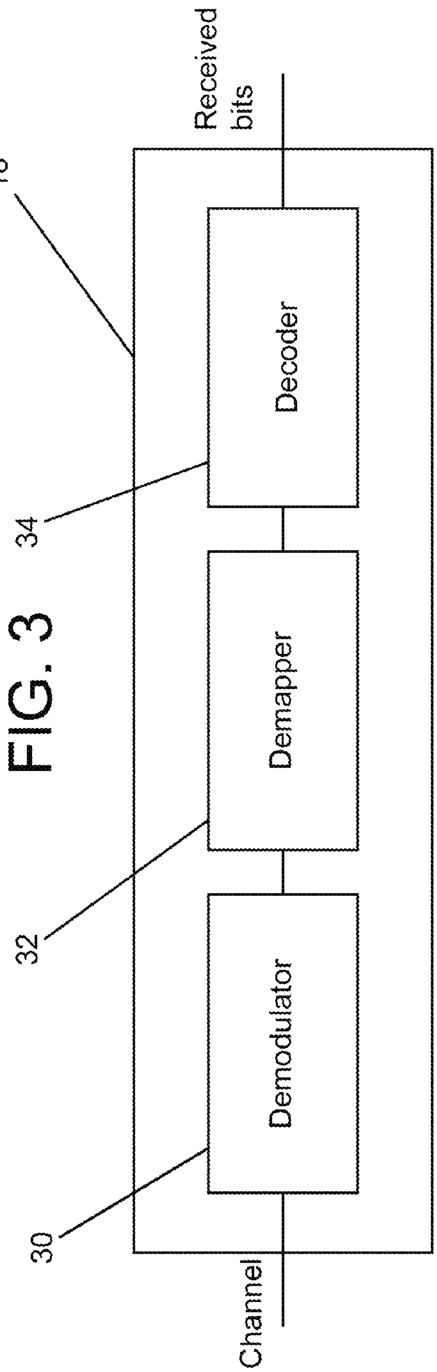

Received bits
16
Decoder
34
Demapper
32
Demodulator
30
Channel

FIG. 3

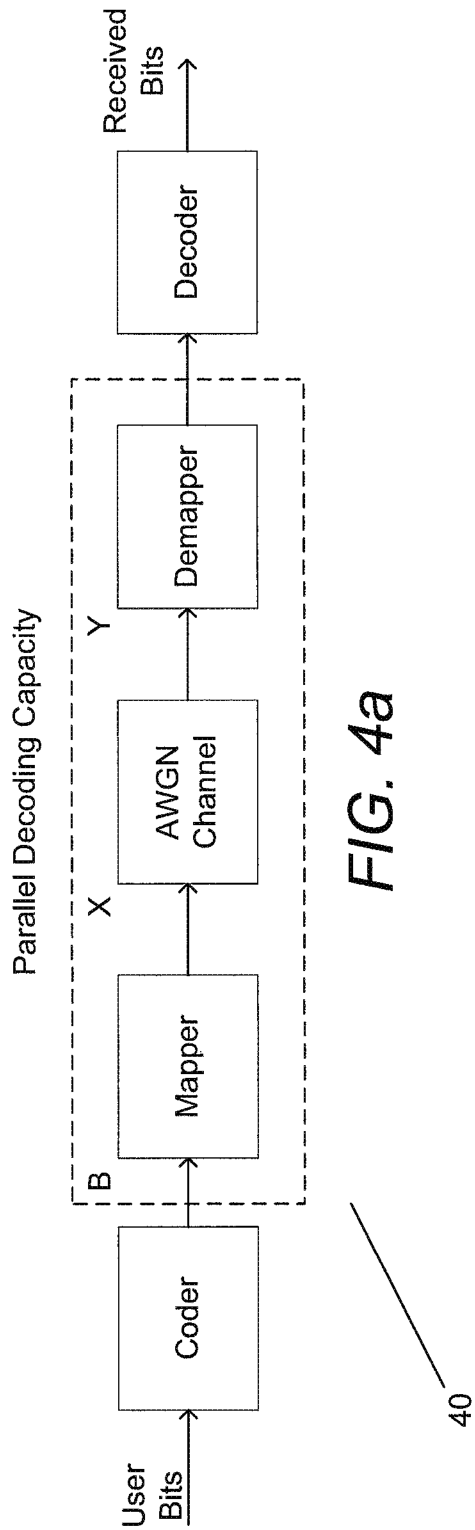

Parallel Decoding Capacity
User Bits
Coder
B
Mapper
X
AWGN Channel
Y
Demapper
Decoder
Received Bits
40

FIG. 4a

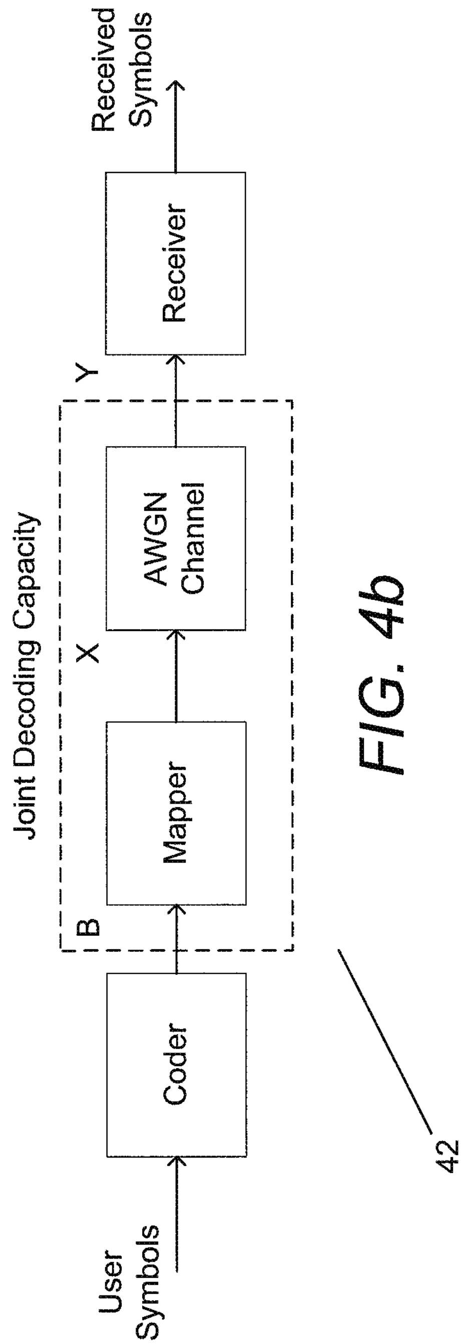

Joint Decoding Capacity
User Symbols
Coder
B
Mapper
X
AWGN Channel
Y
Receiver
Received Symbols
42

FIG. 4b

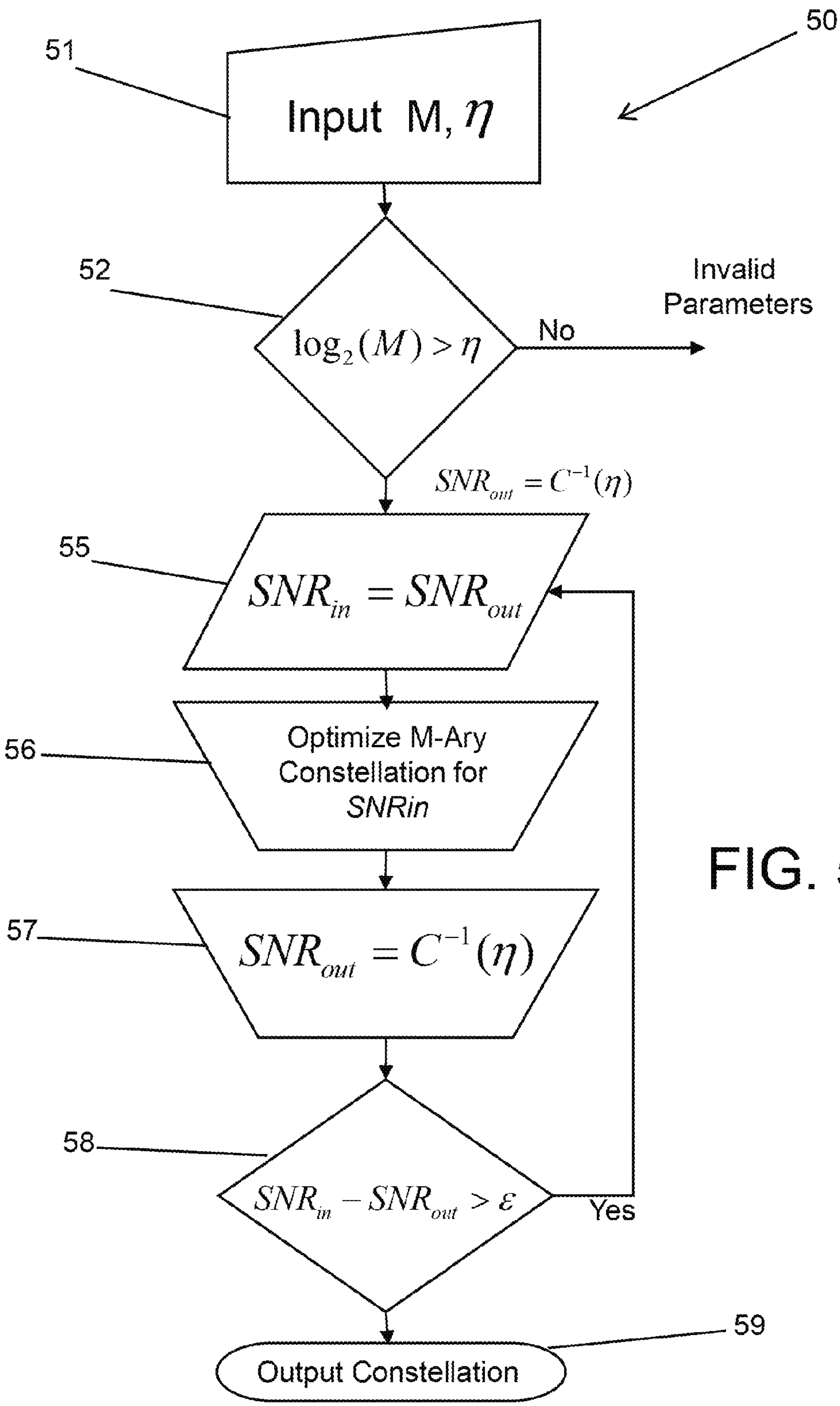

50
51
Input M, η
52
$\log_2(M) > \eta$
No
Invalid Parameters
$SNR_{out} = C^{-1}(\eta)$
55
$SNR_{in} = SNR_{out}$
56
Optimize M-Ary Constellation for *SNRin*
57
$SNR_{out} = C^{-1}(\eta)$
58
$SNR_{in} - SNR_{out} > \varepsilon$
Yes
59
Output Constellation

FIG. 5

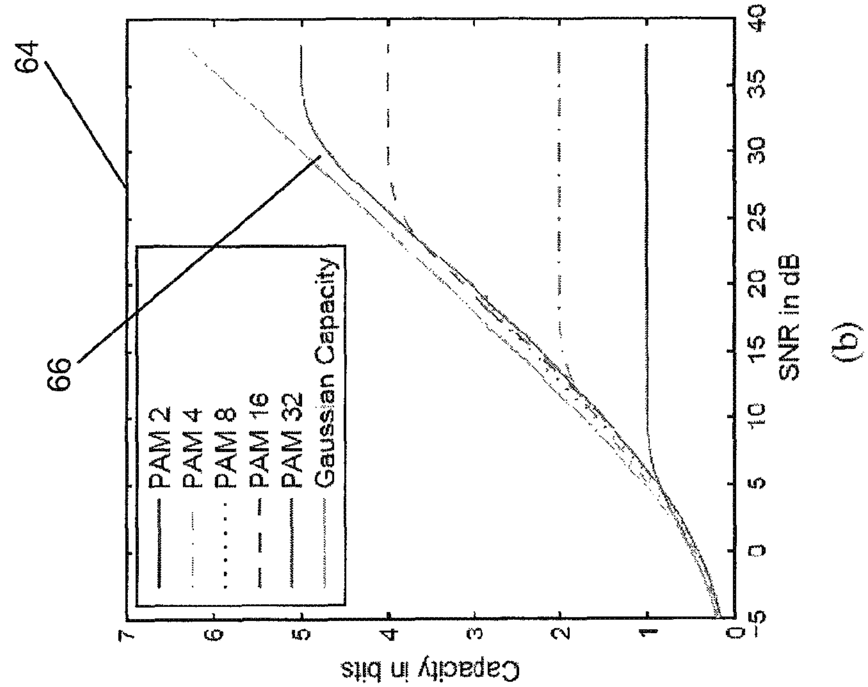

64
66
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(b)

FIG. 6b

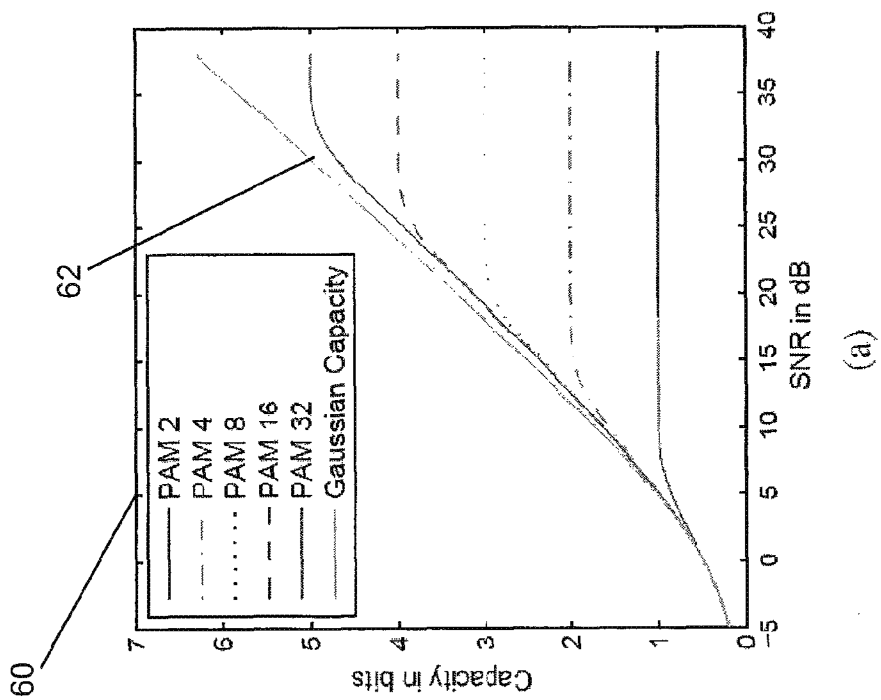

60
62
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(a)

FIG. 6a

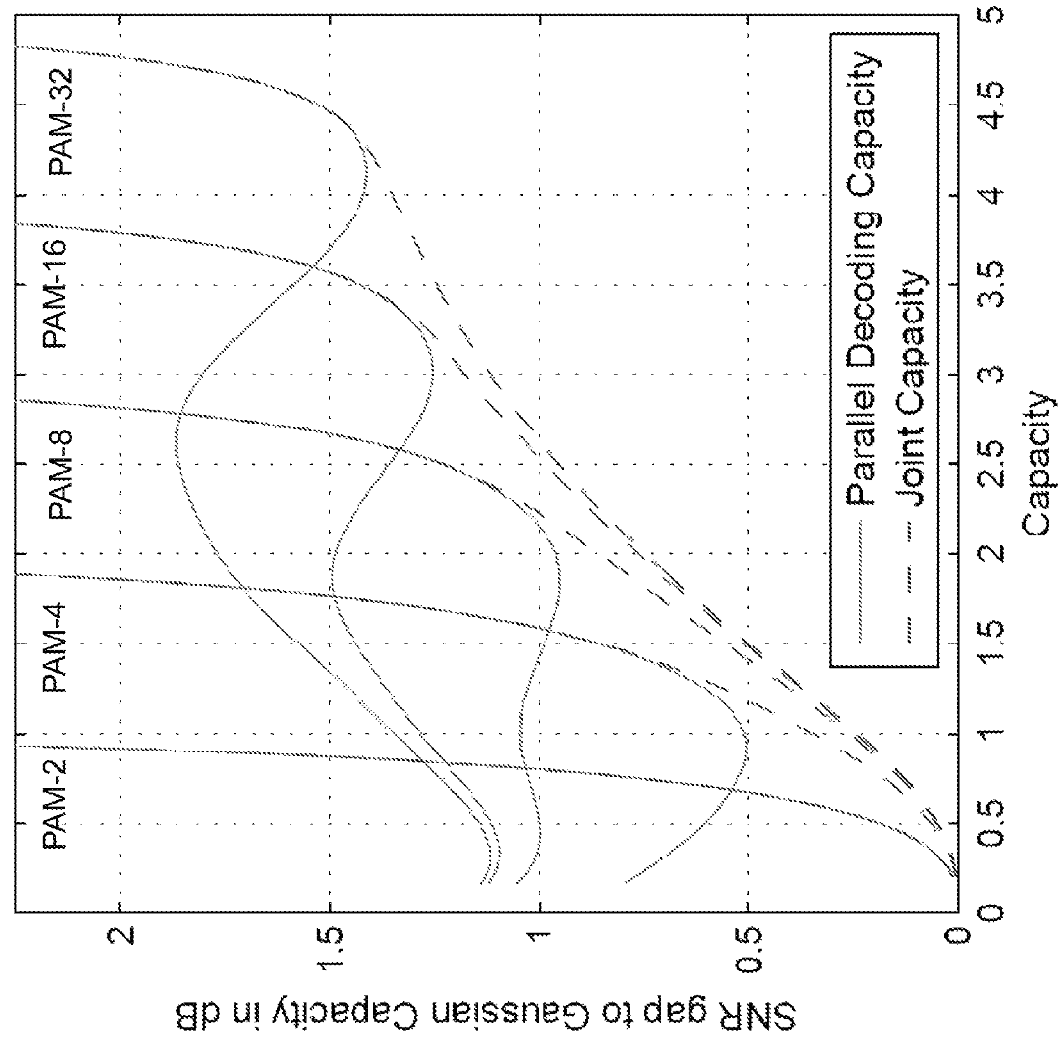

PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
Parallel Decoding Capacity
Joint Capacity
Capacity
SNR gap to Gaussian Capacity in dB
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 7

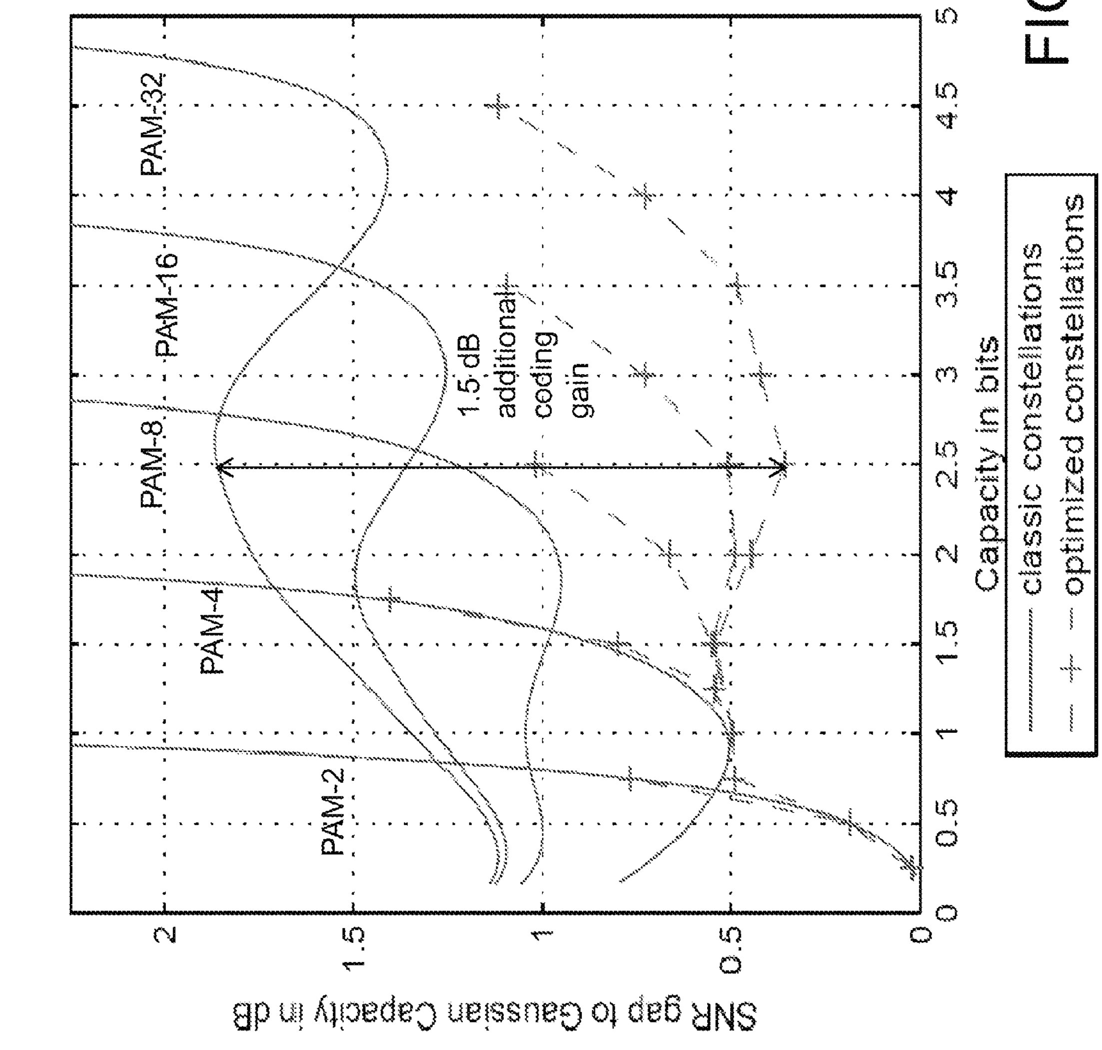

PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
1.5 dB
additional
coding
gain
Capacity in bits
SNR gap to Gaussian Capacity in dB
classic constellations
optimized constellations
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 8a

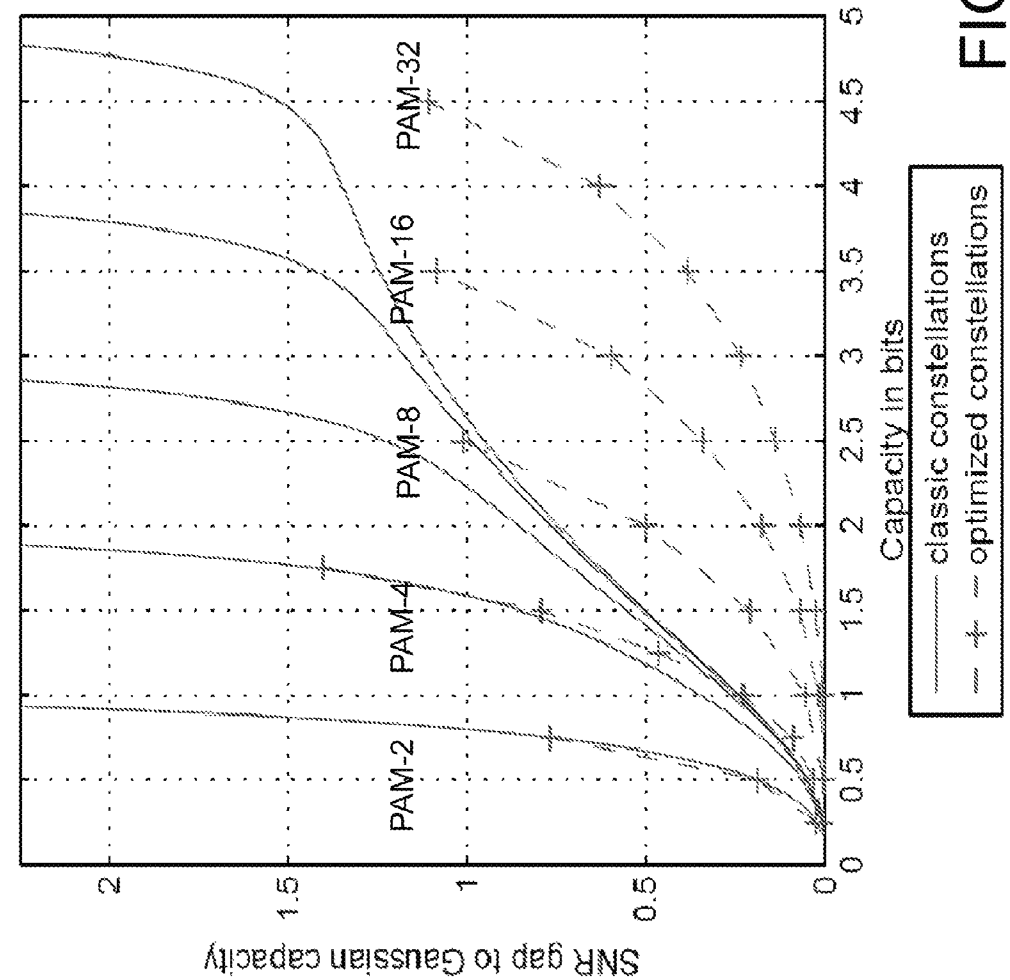
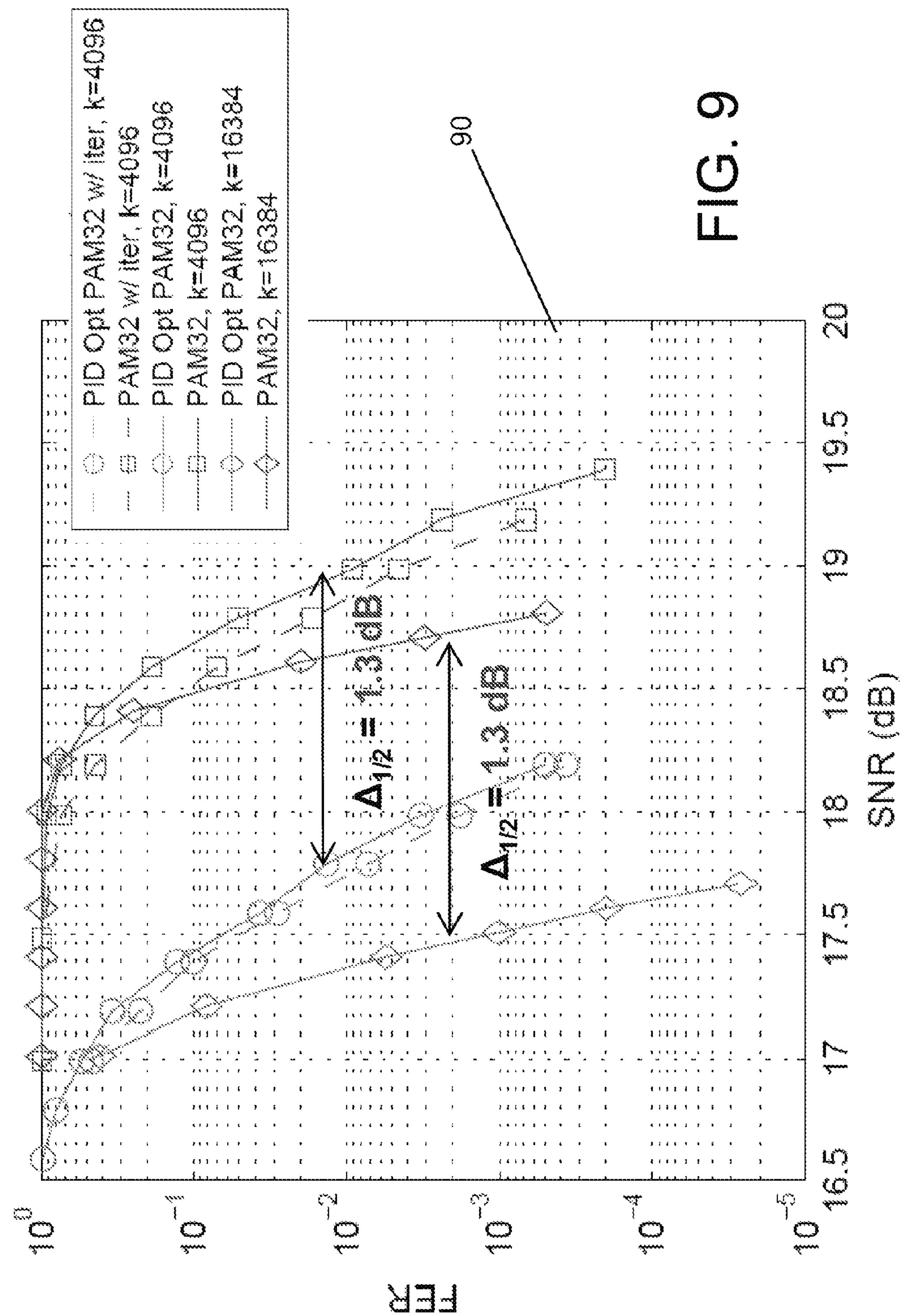

SNR gap to Gaussian capacity
Capacity in bits
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
classic constellations
optimized constellations
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 8b



PID Opt PAM32 w/ iter, k=4096
PAM32 w/ iter, k=4096
PID Opt PAM32, k=4096
PAM32, k=4096
PID Opt PAM32, k=16384
PAM32, k=16384
$\Delta_{1/2}$ = 1.3 dB
$\Delta_{1/2}$ = 1.3 dB
90
FER
SNR (dB)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
16.5
17
17.5
18
18.5
19
19.5
20

FIG. 9

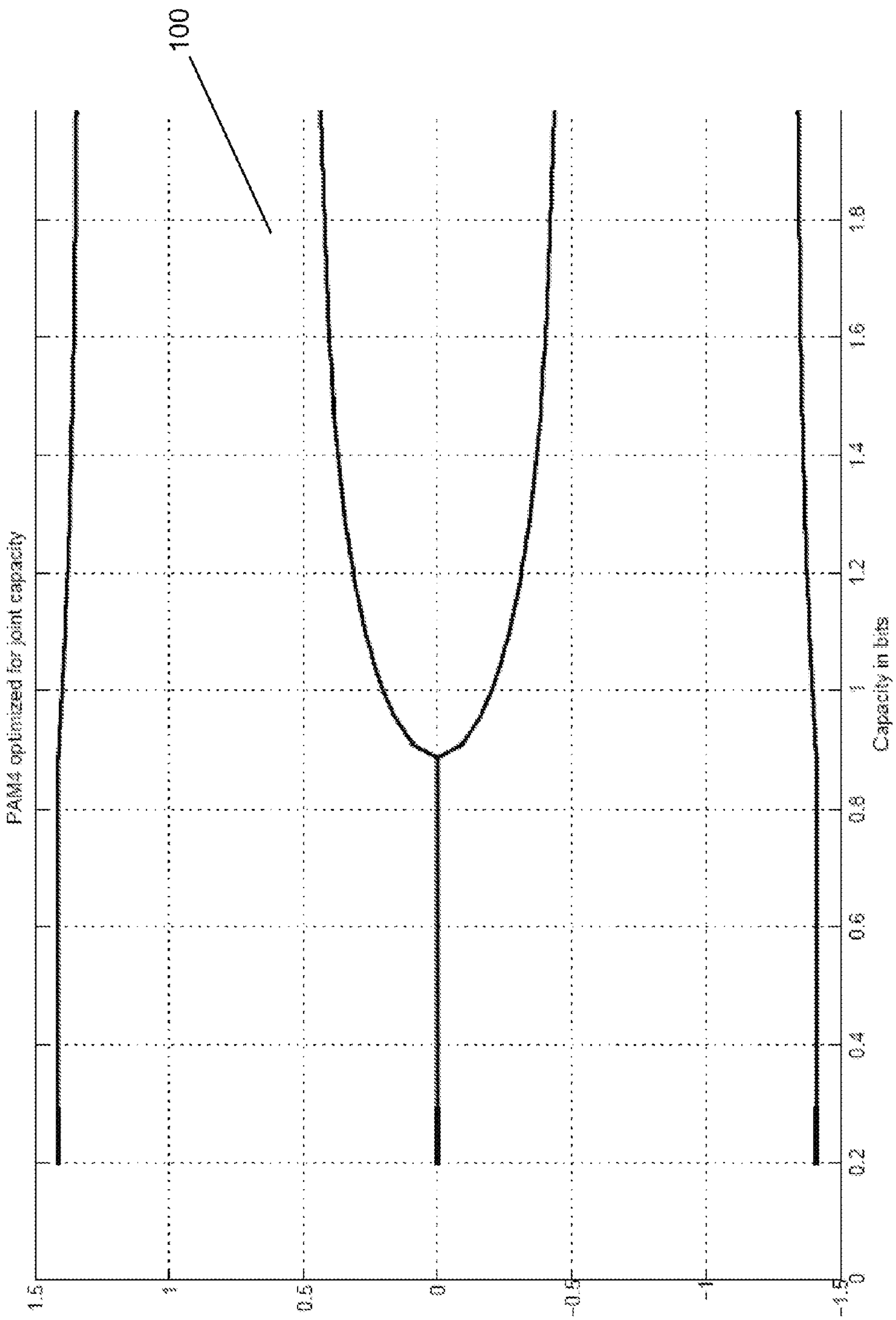

100
PAM4 optimized for joint capacity
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10a

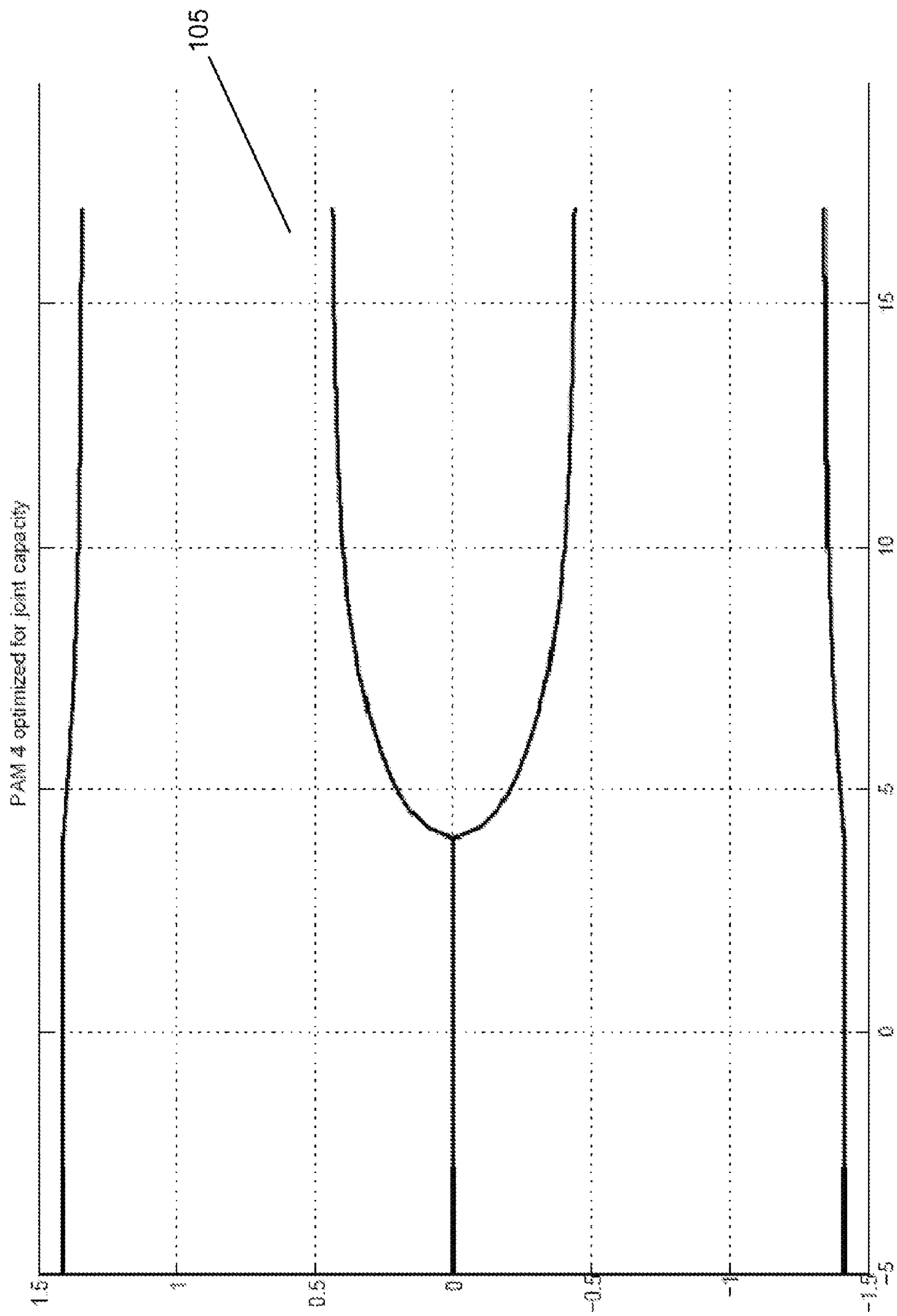

105
PAM 4 optimized for joint capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15

FIG. 10b

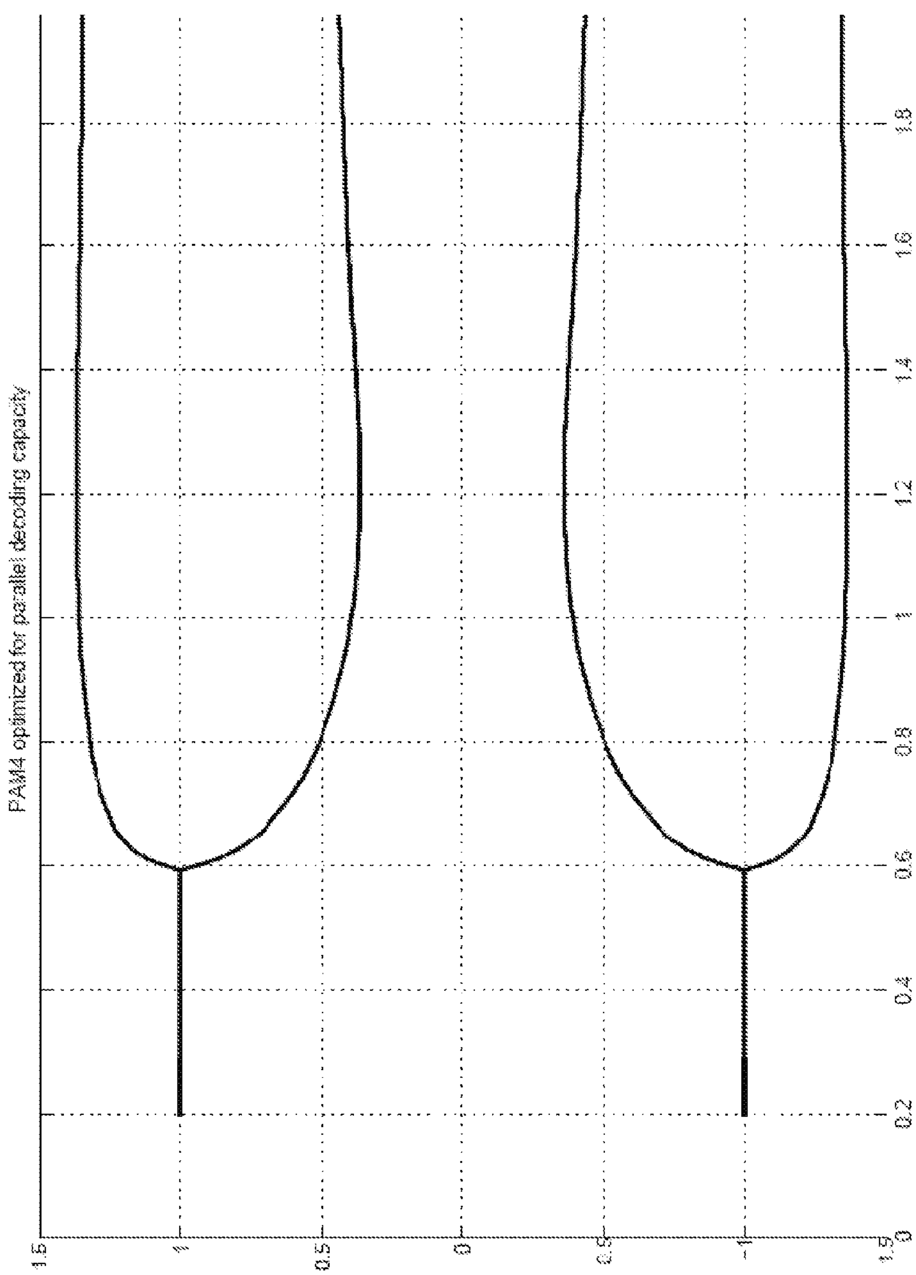

PAM4 optimized for parallel decoding capacity
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10c

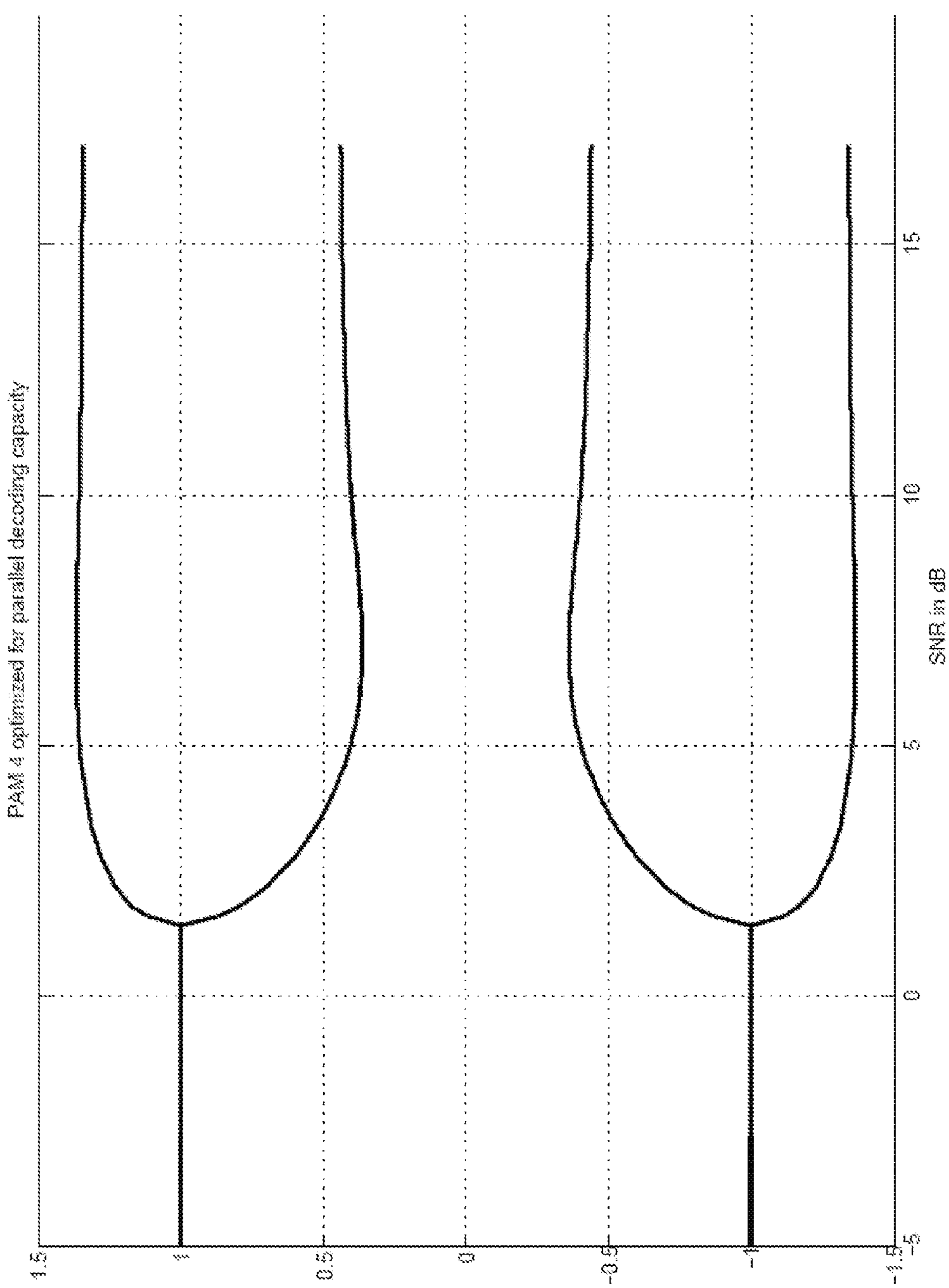


FIG. 10d

PAM-4 constellations optimized for joint capacity at different rates

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.03	2.71	5.00	7.15	9.24
x_0	-1.41	-1.41	-1.40	-1.37	-1.36
x_1	0.00	0.00	-0.20	-0.33	-0.39
x_2	0.00	0.00	0.20	0.33	0.39
x_3	1.41	1.41	1.40	1.37	1.36

FIG. 11a

PAM-4 constellations optimized for parallel decoding capacity at different

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.19	3.11	5.26	7.22	9.25
x_0	-1.00	-1.30	-1.36	-1.37	-1.36
x_1	-1.00	-0.56	-0.39	-0.33	-0.39
x_2	1.00	1.30	1.36	0.33	1.36
x_3	1.00	0.56	0.39	1.37	0.39

FIG. 11b

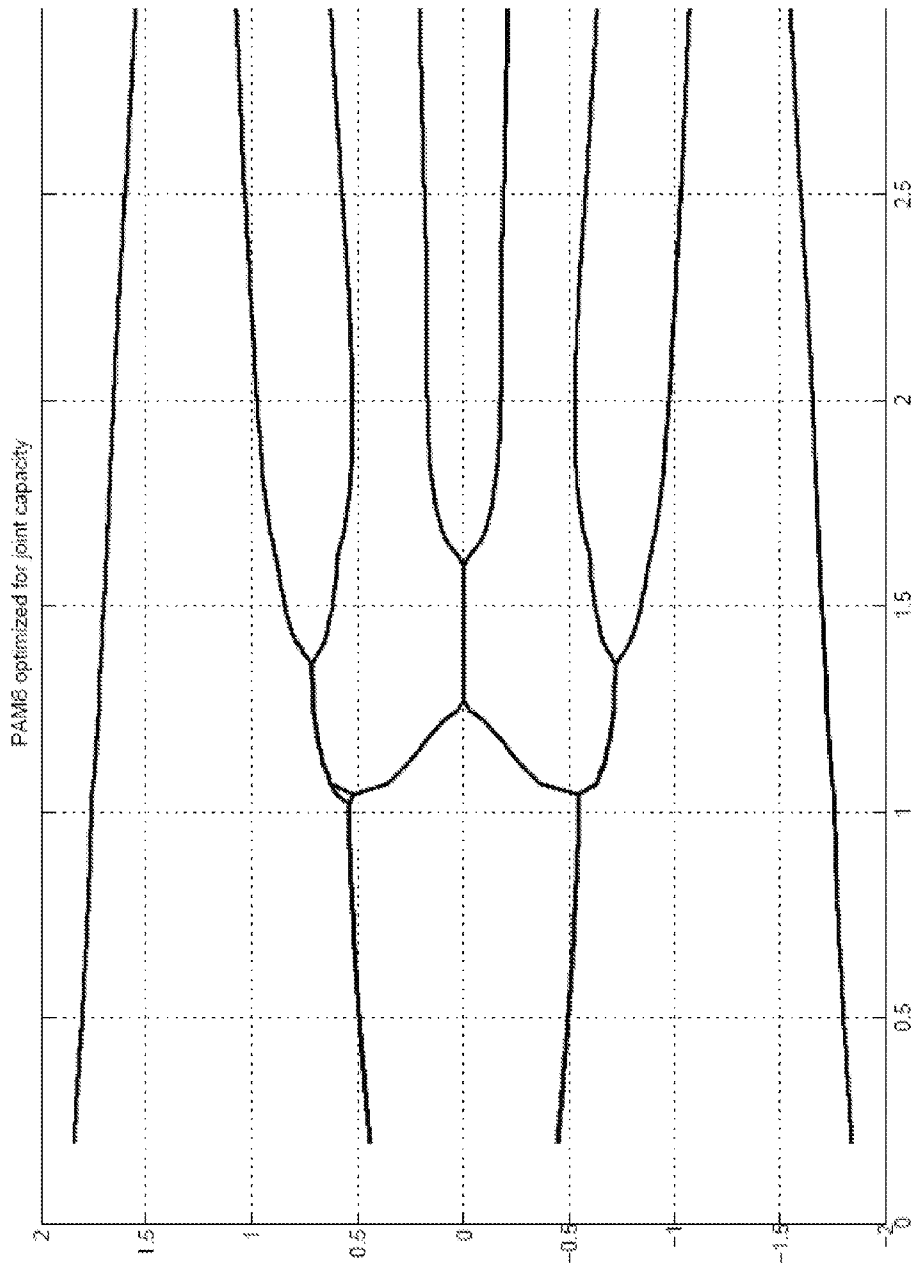

PAM8 optimized for joint capacity
Capacity in bits
0
0.5
1
1.5
2
2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

FIG. 12a

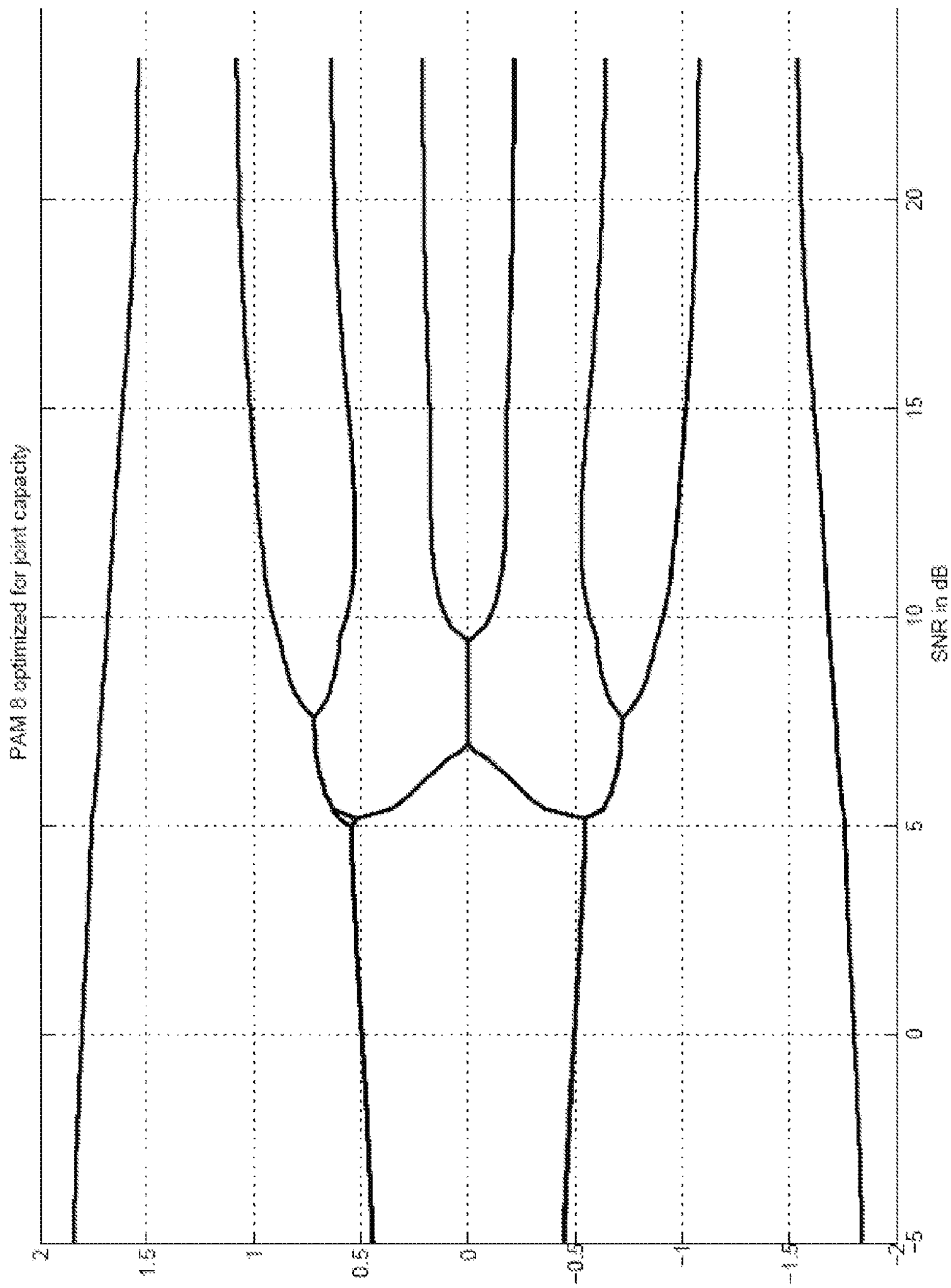

PAM 8 optimized for joint capacity
SNR in dB
-5
0
5
10
15
20
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12b

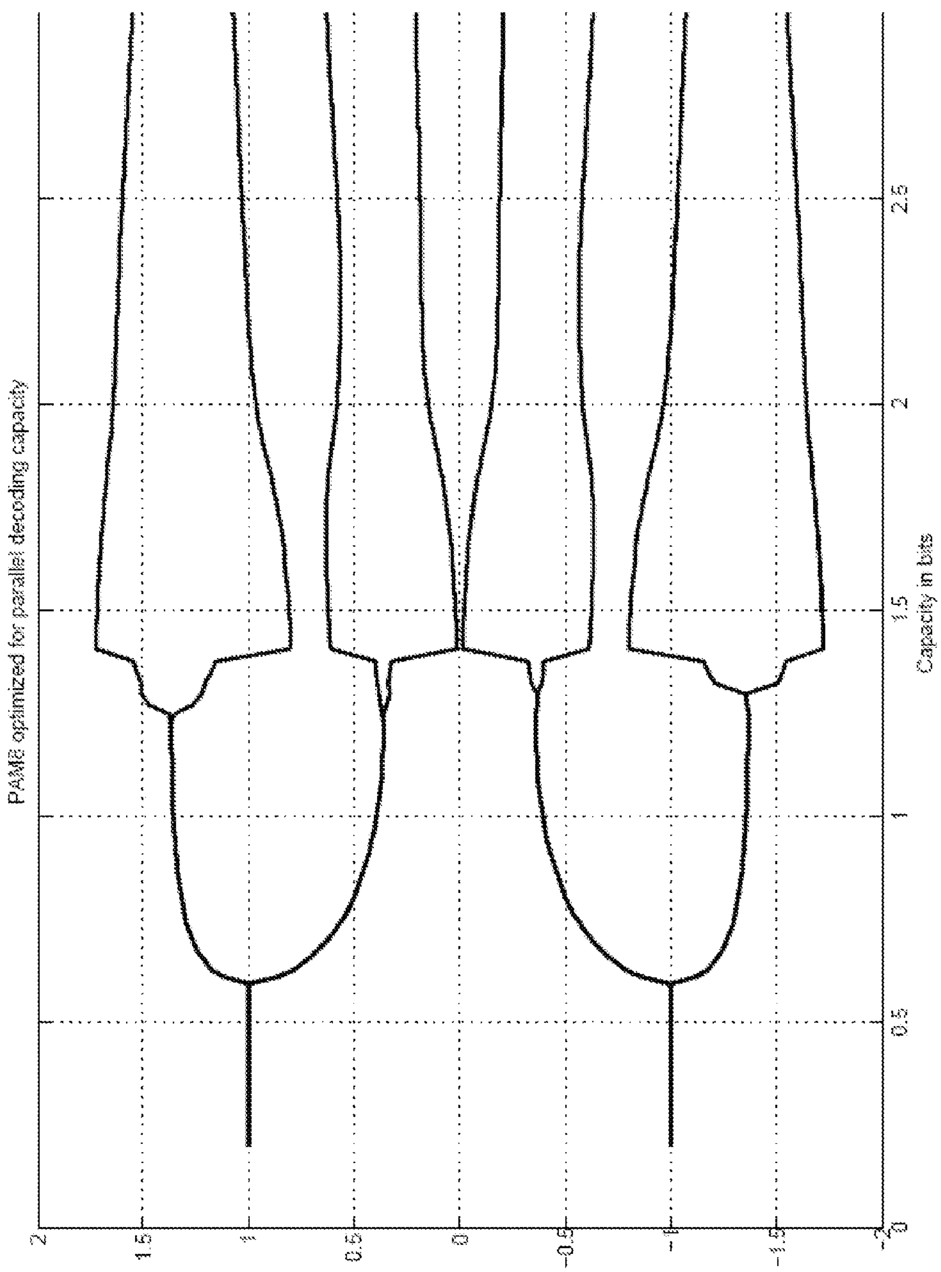

PAM8 optimized for parallel decoding capacity
Capacity in bits
0
0.5
1
1.5
2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12c

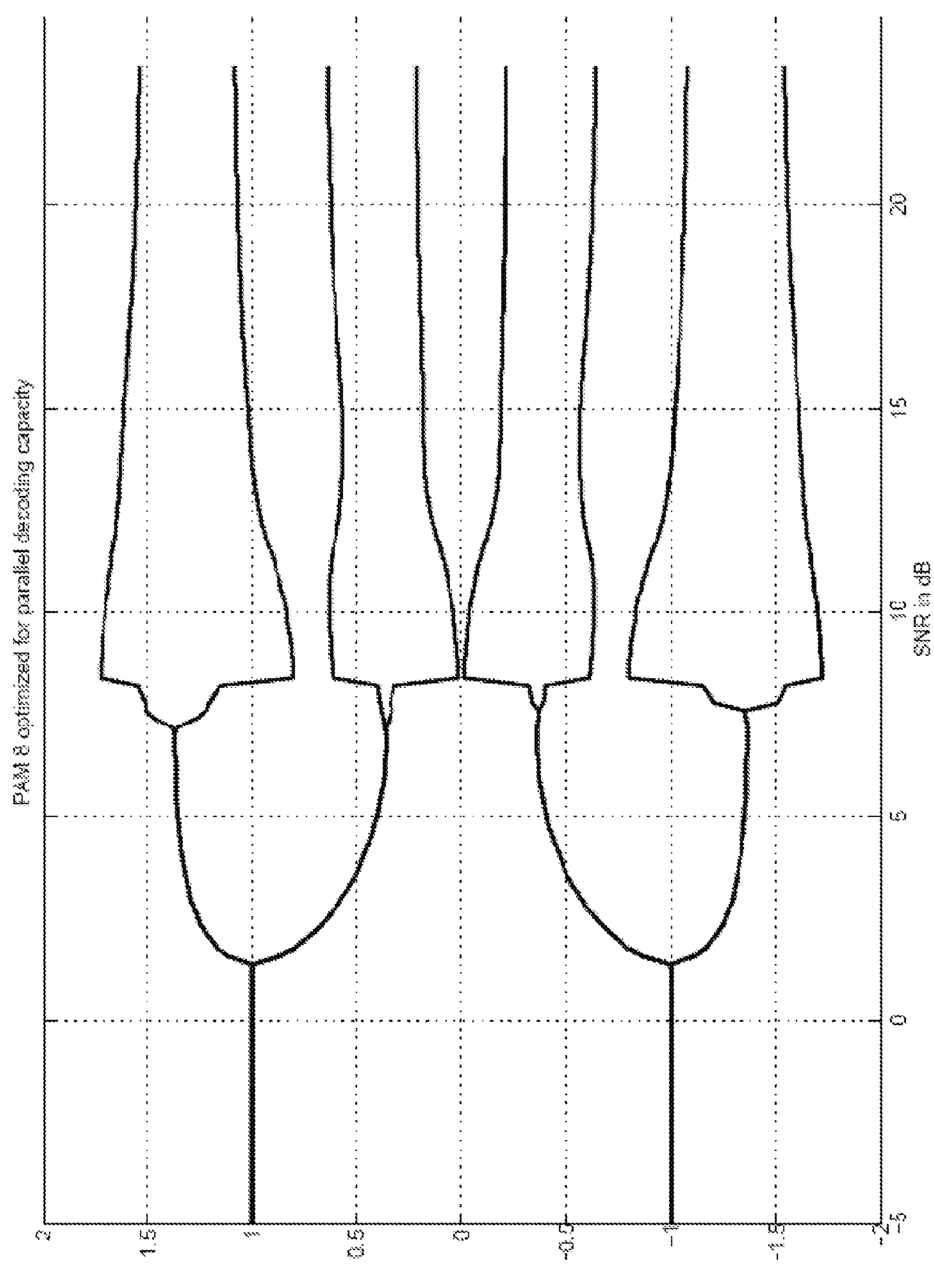

PAM 8 optimized for parallel decoding capacity
SNR in dB
-5
0
5
10
15
20
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12d

PAM-8 constellations optimized for joint capacity at different rates

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.00	4.82	8.66	12.26	15.93
x_0	-1.81	-1.76	-1.70	-1.66	-1.60
x_1	-0.50	-0.55	-0.84	-0.97	-1.03
x_2	-0.50	-0.55	-0.63	-0.53	-0.58
x_3	-0.50	-0.55	-0.00	-0.17	-0.19
x_4	0.50	0.55	0.00	0.17	0.19
x_5	0.50	0.55	0.63	0.53	0.58
x_6	0.50	0.55	0.84	0.97	1.03
x_7	1.81	1.76	1.70	1.66	1.60

FIG. 13a

PAM-8 constellations optimized for parallel decoding capacity at different

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.19	5,27	9.00	12.42	15.93
x_0	-1.00	-1.36	-1.72	-1.64	-1.60
x_1	-1.00	-1.36	-0.81	-0.97	-1.03
x_2	-1.00	-0.39	1.72	1.64	-0.19
x_3	-1.00	-0.39	-0.62	-0.58	-0.58
x_4	1.00	1.36	0.62	0.58	1.60
x_5	1.00	1.36	0.02	0.15	1.03
x_6	1.00	0.39	0.81	0.97	0.19
x_7	1.00	0.39	-0.02	-0.15	0.58

FIG. 13b

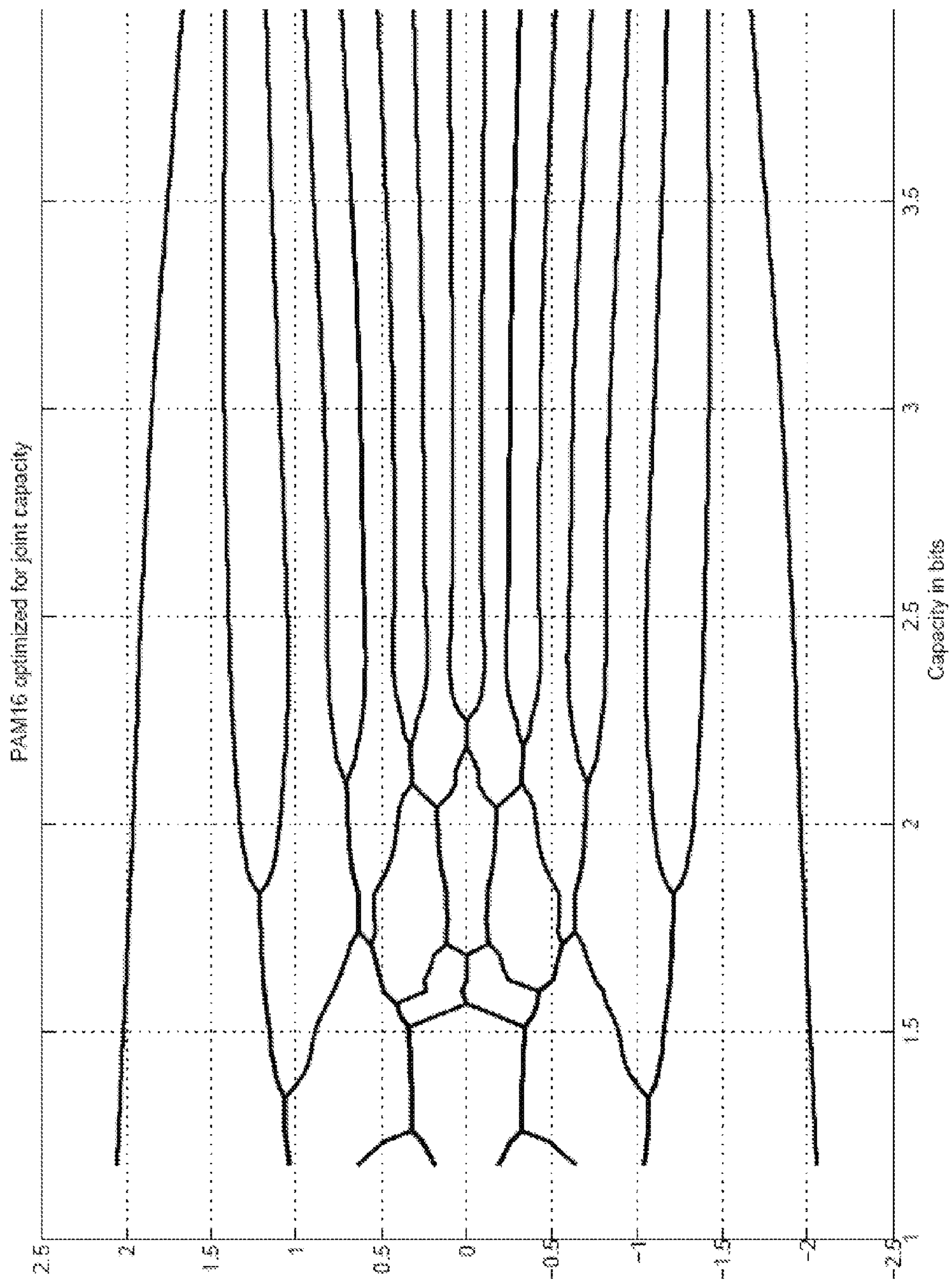

PAM16 optimized for joint capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1
1.5
2
2.5
3
3.5

FIG. 14a

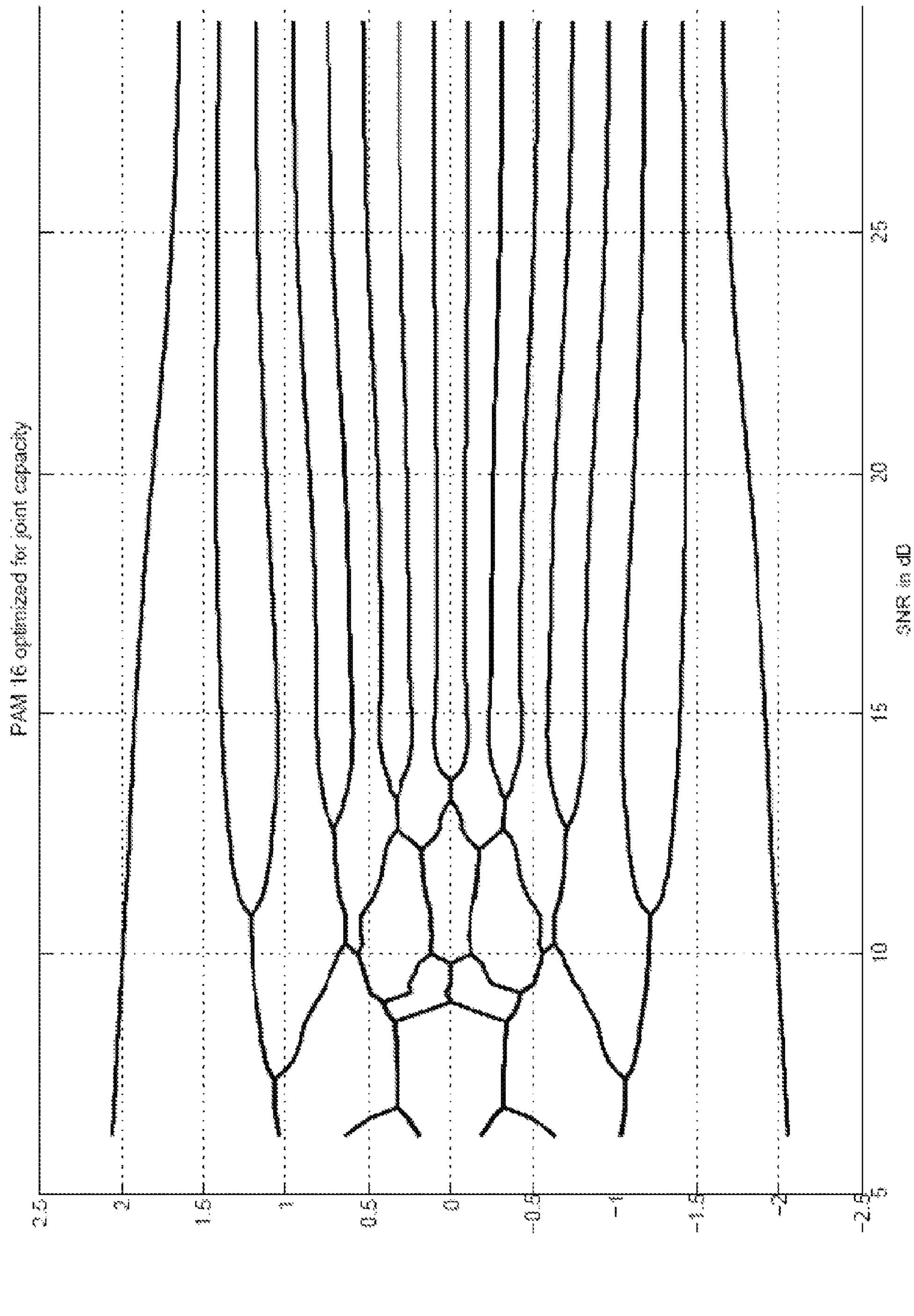

PAM 16 optimized for joint capacity
SNR in dB
5
10
15
20
25
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 14b

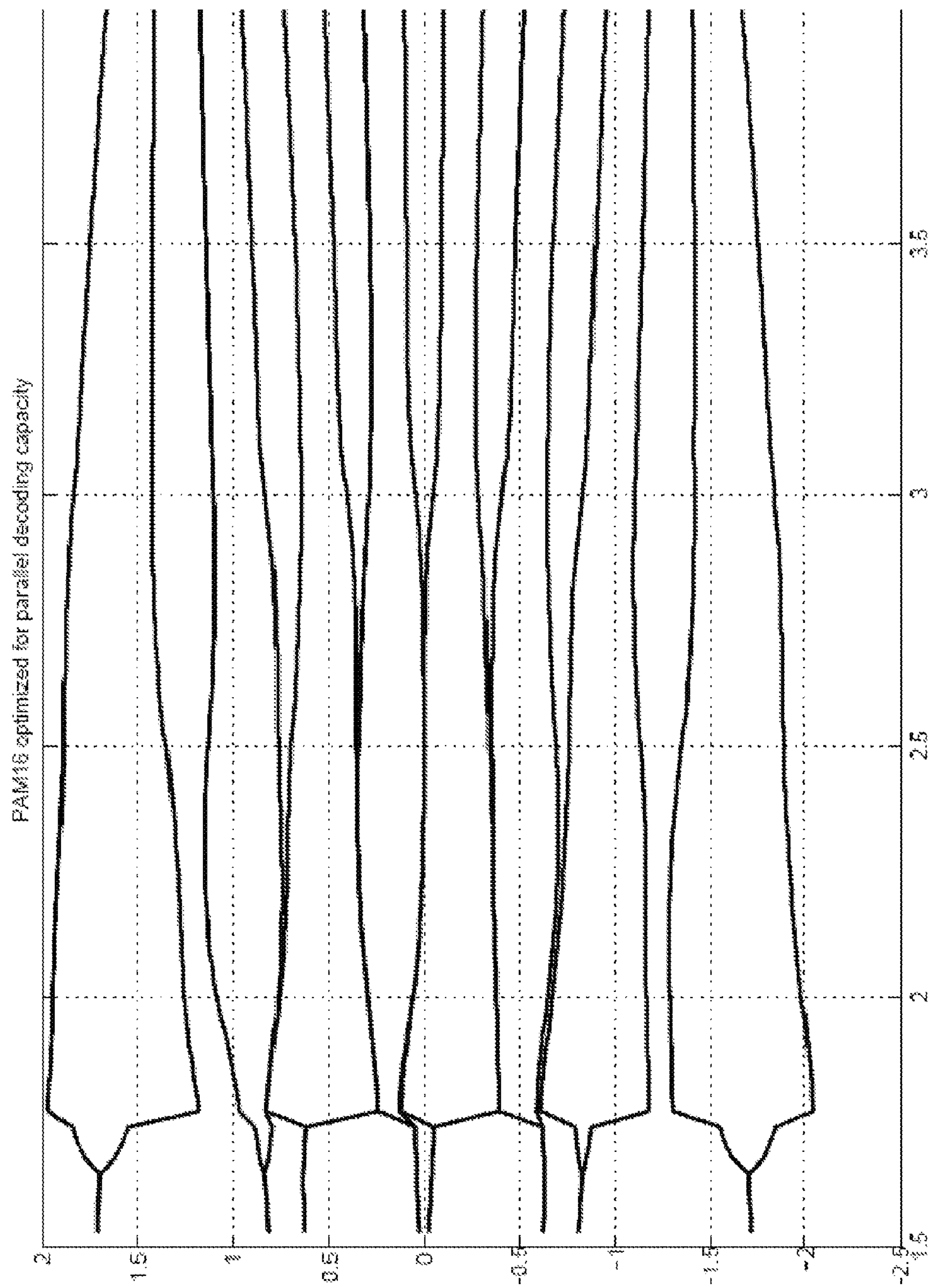

PAM16 optimized for parallel decoding capacity
Capacity in bits
1.5
2
2.5
3
3.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 14c

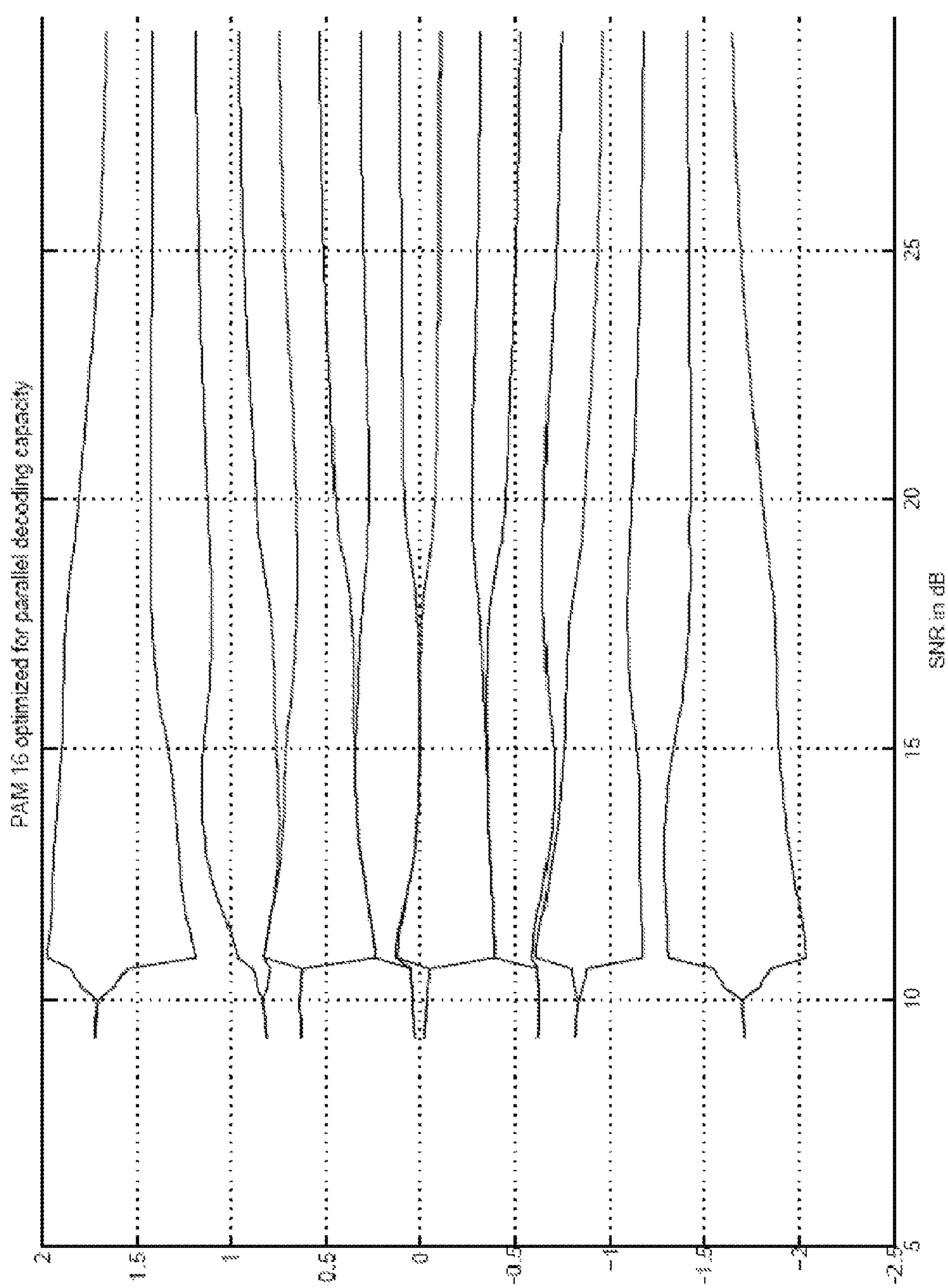

PAM 16 optimized for parallel decoding capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14d

PAM-16 constellations optimized for joint capacity at different rates

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	8.52	11.94	15.25	18.60	22.12
x_0	-2.02	-1.96	-1.91	-1.85	-1.76
x_1	-1.16	-1.33	-1.40	-1.42	-1.42
x_2	-1.16	-1.10	-1.05	-1.10	-1.15
x_3	-0.90	-0.69	-0.82	-0.84	-0.90
x_4	-0.34	-0.69	-0.60	-0.62	-0.68
x_5	-0.34	-0.40	-0.43	-0.43	-0.47
x_6	-0.34	-0.17	-0.24	-0.26	-0.28
x_7	-0.34	-0.17	-0.09	-0.08	-0.09
x_8	0.34	0.17	0.09	0.08	0.09
x_9	0.34	0.17	0.24	0.26	0.28
x_{10}	0.34	0.40	0.43	0.43	0.47
x_{11}	0.34	0.69	0.60	0.62	0.68
x_{12}	0.90	0.69	0.82	0.84	0.90
x_{13}	1.16	1.10	1.05	1.10	1.15
x_{14}	1.16	1.33	1.40	1.42	1.42
x_{15}	2.02	1.96	1.91	1.85	1.76

FIG. 15a

PAM-16 constellations optimized for parallel decoding capacity at different

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	9.00	12.25	15.42	18.72	22.13
x_0	-1.72	-1.98	-1.89	-1.84	-1.75
x_1	-1.72	-1.29	-1.36	-1.42	-1.42
x_2	-0.81	1.94	1.89	1.84	1.75
x_3	-0.81	-1.17	-1.14	-1.11	-1.15
x_4	1.72	-0.38	-0.35	-0.40	-0.47
x_5	1.72	-0.65	-0.70	-0.65	-0.68
x_6	-0.62	-0.38	-0.34	-0.29	-0.28
x_7	-0.62	-0.68	-0.76	-0.83	-0.90
x_8	0.62	1.09	1.13	1.11	1.15
x_9	0.62	0.76	0.76	0.84	0.90
x_{10}	0.02	1.26	1.35	1.42	1.42
x_{11}	0.02	0.76	0.70	0.65	0.68
x_{12}	0.81	0.06	0.00	0.05	0.09
x_{13}	0.81	0.29	0.34	0.29	0.28
x_{14}	-0.02	0.06	0.00	-0.05	-0.09
x_{15}	-0.02	0.29	0.35	0.40	0.47

FIG. 15b

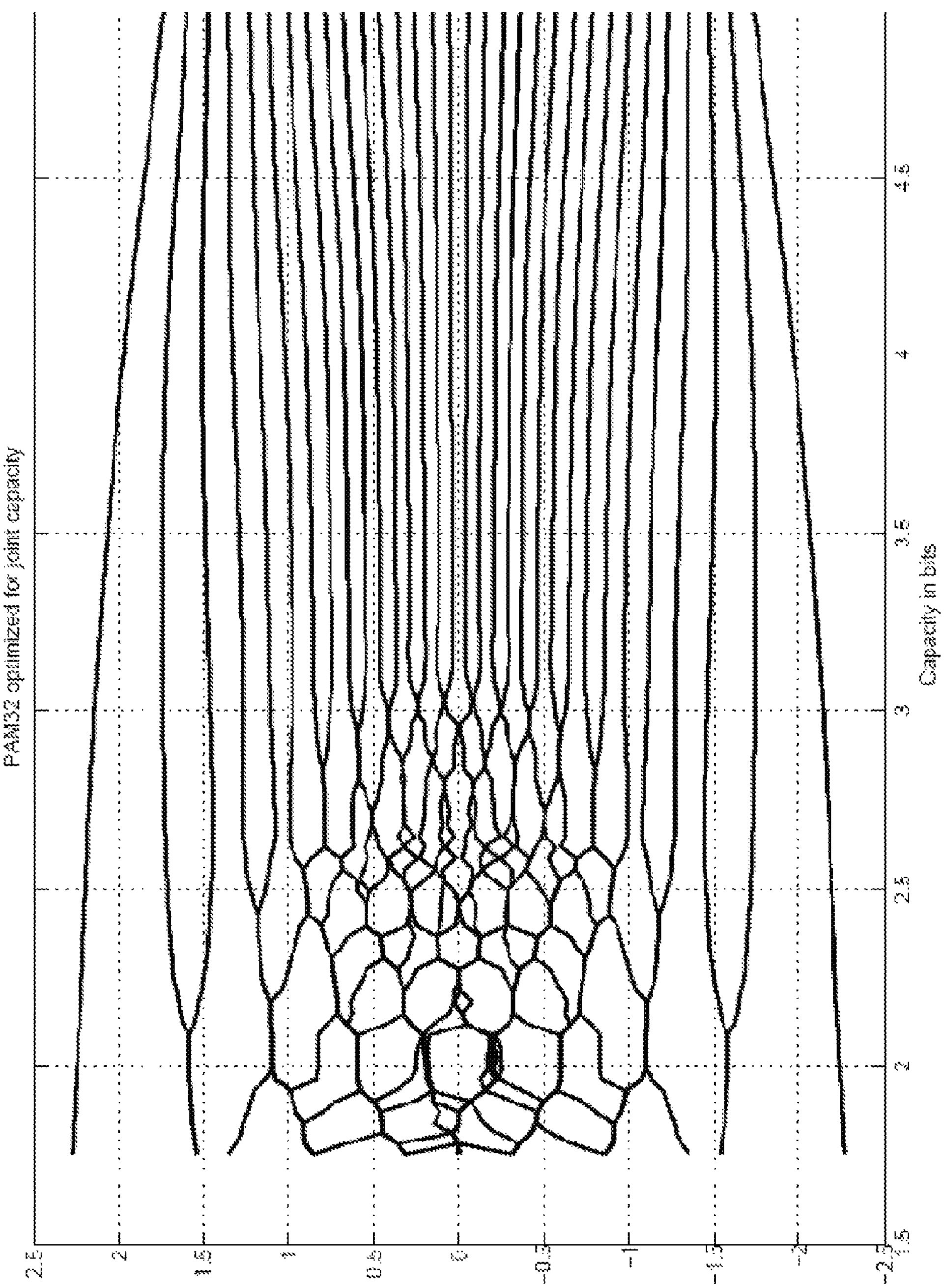


FIG. 16a

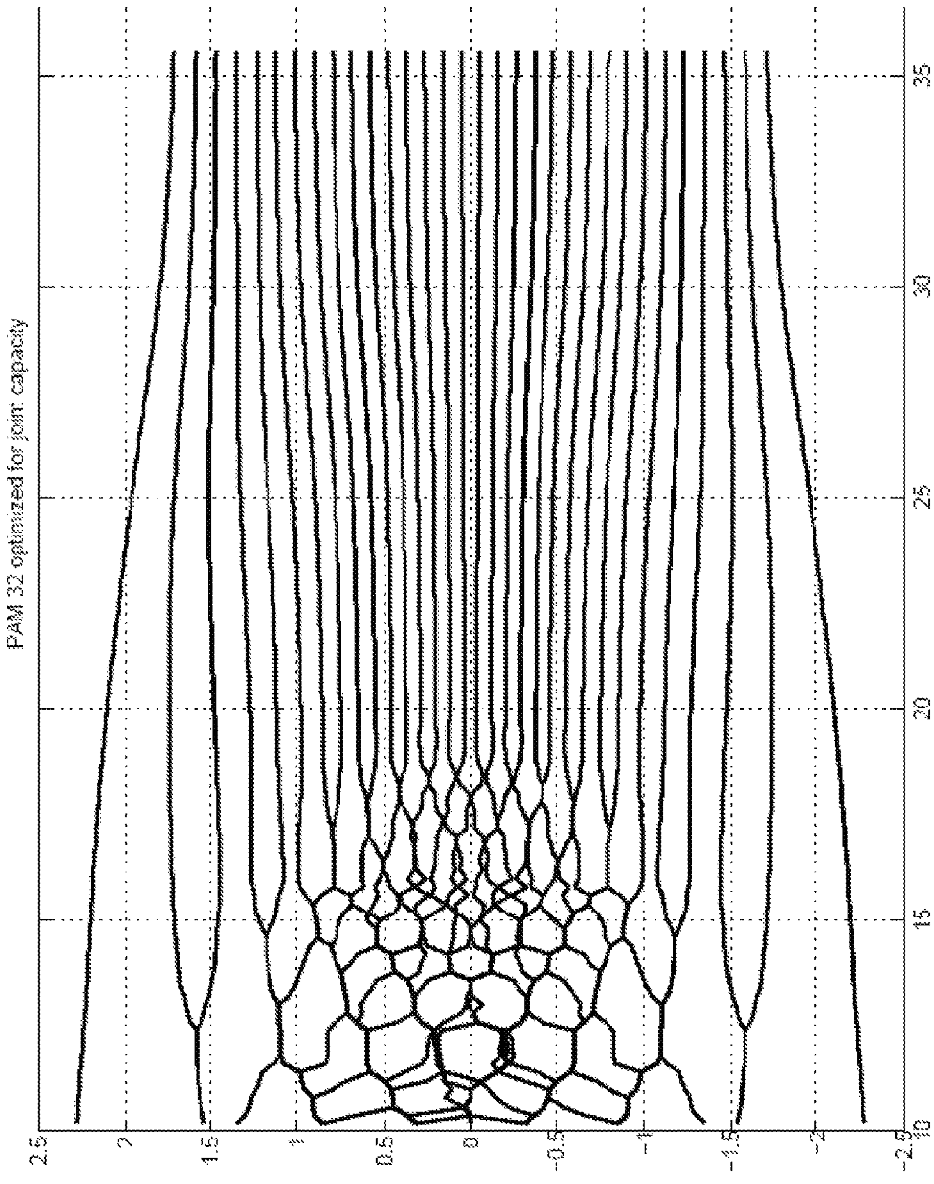

PAM 32 optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
10
15
20
25
30
35

FIG. 16b

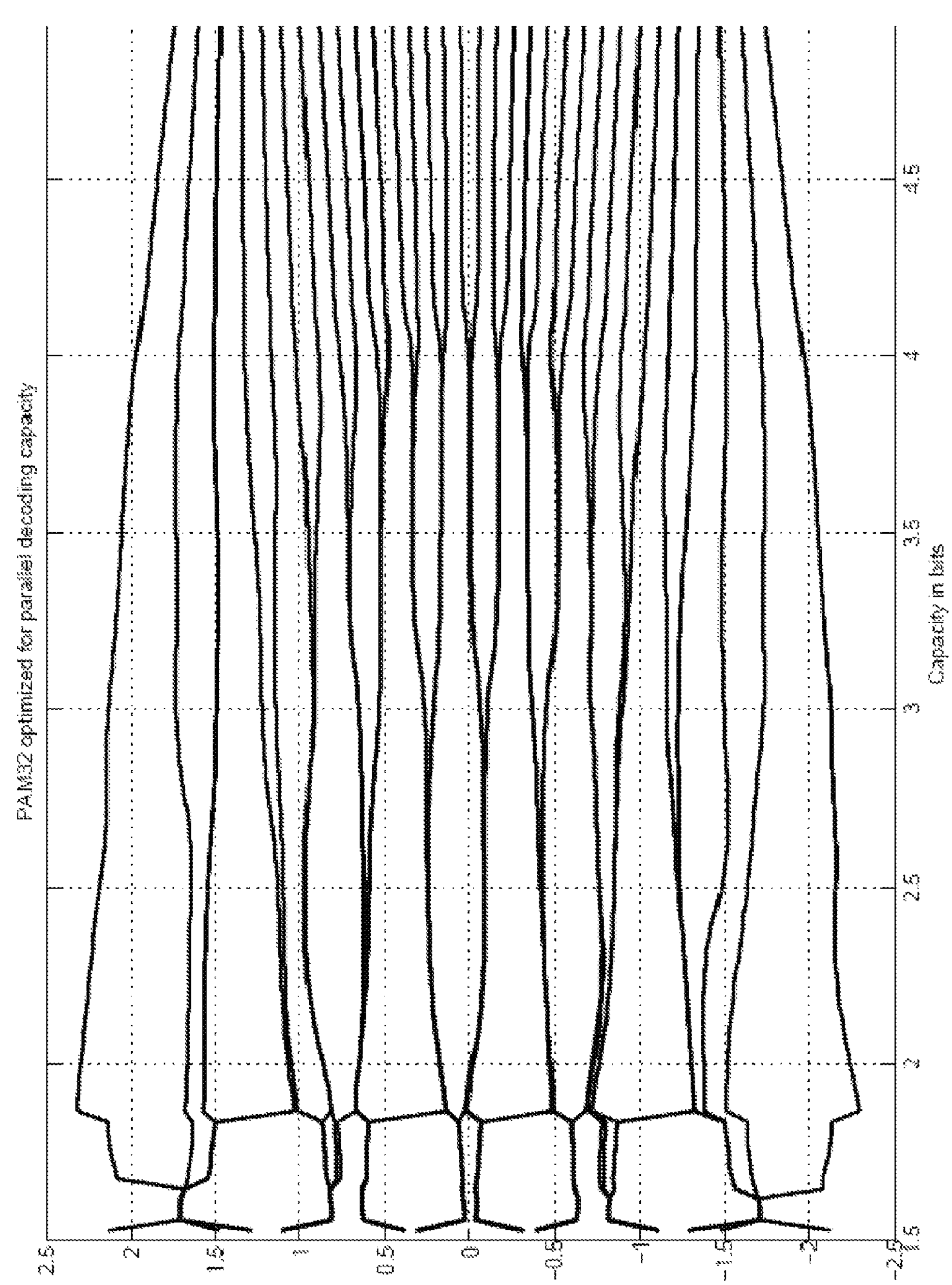

PAM32 optimized for parallel decoding capacity
Capacity in bits
1.5
2
2.5
3
3.5
4
4.5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 16c

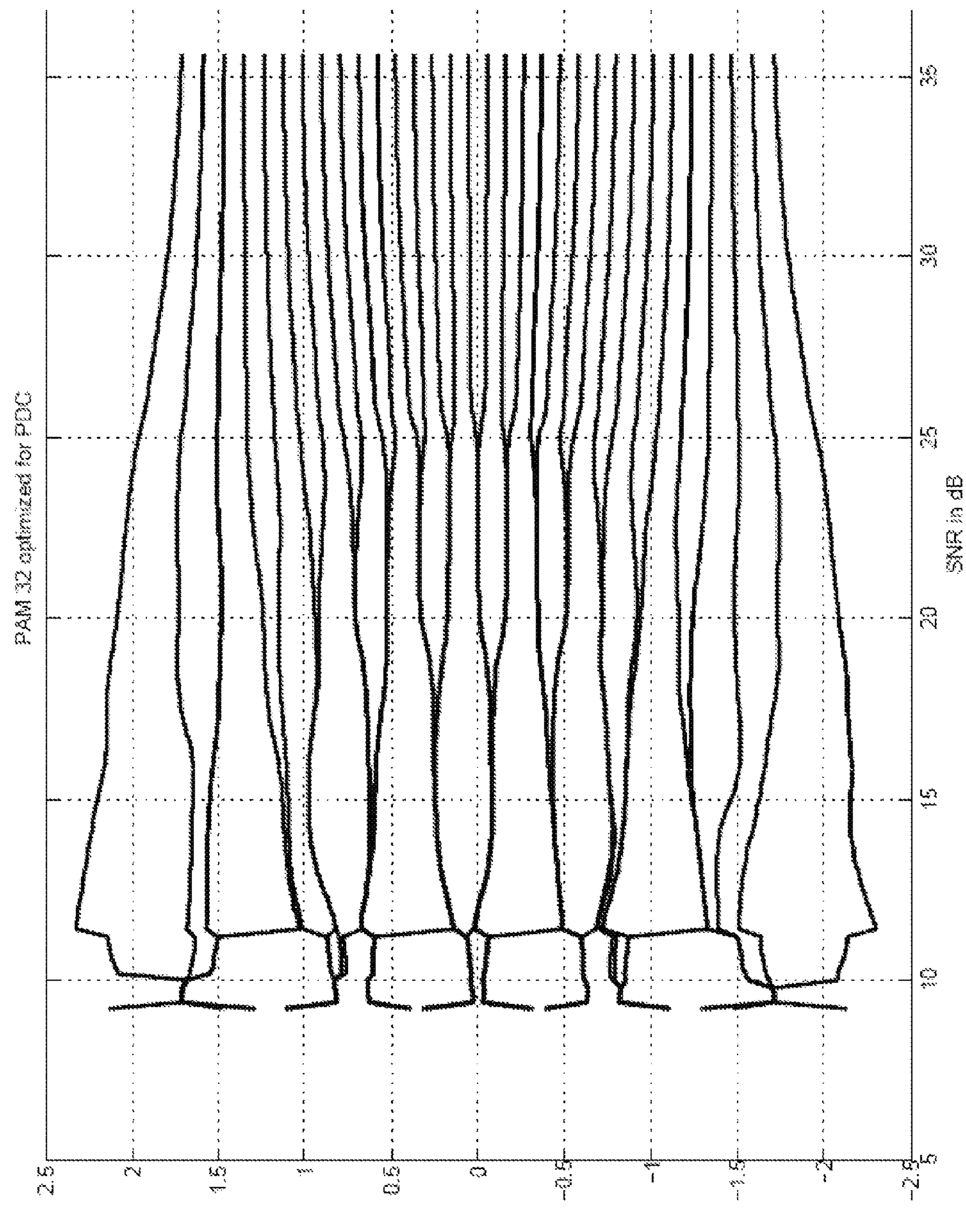

PAM 32 optimized for PDC
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25
30
35

FIG. 16d

PAM-32 constellations optimized for joint capacity at different rates

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	11.83	15.05	18.23	21.42	24.69	28.19
x_0	-2.25	-2.19	-2.14	-2.07	-1.97	-1.85
x_1	-1.58	-1.71	-1.74	-1.74	-1.72	-1.66
x_2	-1.58	-1.46	-1.46	-1.49	-1.51	-1.50
x_3	-1.10	-1.23	-1.27	-1.29	-1.33	-1.35
x_4	-1.10	-1.13	-1.11	-1.13	-1.17	-1.21
x_5	-1.10	-0.90	-0.98	-0.99	-1.02	-1.08
x_6	-0.83	-0.90	-0.85	-0.87	-0.90	-0.95
x_7	-0.60	-0.75	-0.75	-0.76	-0.78	-0.84
x_8	-0.60	-0.58	-0.63	-0.65	-0.67	-0.73
x_9	-0.60	-0.58	-0.57	-0.56	-0.57	-0.62
x_{10}	-0.60	-0.49	-0.42	-0.46	-0.48	-0.52
x_{11}	-0.24	-0.29	-0.40	-0.38	-0.39	-0.42
x_{12}	-0.21	-0.28	-0.24	-0.29	-0.30	-0.32
x_{13}	-0.20	-0.28	-0.24	-0.21	-0.21	-0.23
x_{14}	-0.20	-0.09	-0.09	-0.12	-0.13	-0.14
x_{15}	-0.16	-0.00	-0.07	-0.04	-0.04	-0.05
x_{16}	0.16	0.00	0.07	0.04	0.04	0.05
x_{17}	0.19	0.09	0.09	0.12	0.13	0.14
x_{18}	0.21	0.28	0.24	0.21	0.21	0.23
x_{19}	0.22	0.28	0.24	0.29	0.30	0.32
x_{20}	0.23	0.28	0.41	0.38	0.39	0.42
x_{21}	0.60	0.49	0.42	0.46	0.48	0.52
x_{22}	0.60	0.58	0.57	0.56	0.57	0.62
x_{23}	0.60	0.58	0.62	0.65	0.67	0.73
x_{24}	0.60	0.75	0.75	0.76	0.78	0.84
x_{25}	0.83	0.90	0.85	0.87	0.90	0.95
x_{26}	1.10	0.90	0.98	0.99	1.02	1.08
x_{27}	1.10	1.13	1.11	1.13	1.17	1.21
x_{28}	1.10	1.23	1.27	1.29	1.33	1.35
x_{29}	1.58	1.46	1.46	1.49	1.51	1.50
x_{30}	1.58	1.71	1.74	1.74	1.72	1.66
x_{31}	2.25	2.19	2.14	2.07	1.97	1.85

FIG. 17a

PAM-32 constellations optimized for parallel decoding capacity at different

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	12.21	15.27	18.42	21.52	24.79	28.20
x_0	-2.25	-2.16	-2.14	-2.05	-1.97	-1.85
x_1	-1.52	-1.64	-1.75	-1.74	-1.72	-1.66
x_2	2.30	2.19	-1.31	2.05	1.97	-1.35
x_3	-1.39	-1.48	-1.43	-1.49	-1.51	-1.49
x_4	1.56	1.54	2.14	-0.96	-1.03	1.85
x_5	-1.31	-1.23	1.75	-1.15	-1.17	1.66
x_6	1.67	1.65	-1.07	-0.91	-0.90	-1.21
x_7	-1.31	-1.24	-1.04	-1.28	-1.33	-1.08
x_8	-0.48	-0.43	-0.36	-0.17	-0.17	-0.42
x_9	-0.72	-0.76	-0.36	-0.34	-0.31	-0.52
x_{10}	-0.48	-0.43	-0.62	-0.17	-0.15	-0.73
x_{11}	-0.73	-0.76	-0.62	-0.34	-0.35	-0.62
x_{12}	-0.48	-0.42	-0.29	-0.71	-0.67	-0.33
x_{13}	-0.76	-0.86	-0.29	-0.52	-0.55	-0.23
x_{14}	-0.48	-0.42	-0.77	-0.72	-0.77	-0.84
x_{15}	-0.76	-0.86	-0.77	-0.52	-0.48	-0.96
x_{16}	0.87	0.98	1.07	1.49	1.51	1.21
x_{17}	0.66	0.63	1.04	1.28	1.33	1.08
x_{18}	0.87	0.98	0.77	1.74	1.72	0.84
x_{19}	0.66	0.63	0.77	1.15	1.17	0.96
x_{20}	1.07	1.13	1.31	0.72	0.77	1.35
x_{21}	0.66	0.59	1.43	0.91	0.90	1.49
x_{22}	1.05	1.10	0.62	0.71	0.67	0.73
x_{23}	0.66	0.60	0.62	0.96	1.03	0.62
x_{24}	-0.01	-0.08	0.02	0.00	0.01	0.05
x_{25}	0.17	0.25	0.02	0.17	0.15	0.14
x_{26}	-0.01	-0.08	0.29	0.00	-0.01	0.33
x_{27}	0.17	0.25	0.29	0.17	0.17	0.23
x_{28}	-0.01	-0.08	-0.02	0.52	0.48	-0.05
x_{29}	0.17	0.25	-0.02	0.34	0.35	-0.14
x_{30}	-0.01	-0.08	0.36	0.52	0.55	0.42
x_{31}	0.17	0.25	0.36	0.34	0.31	0.52

FIG. 17b

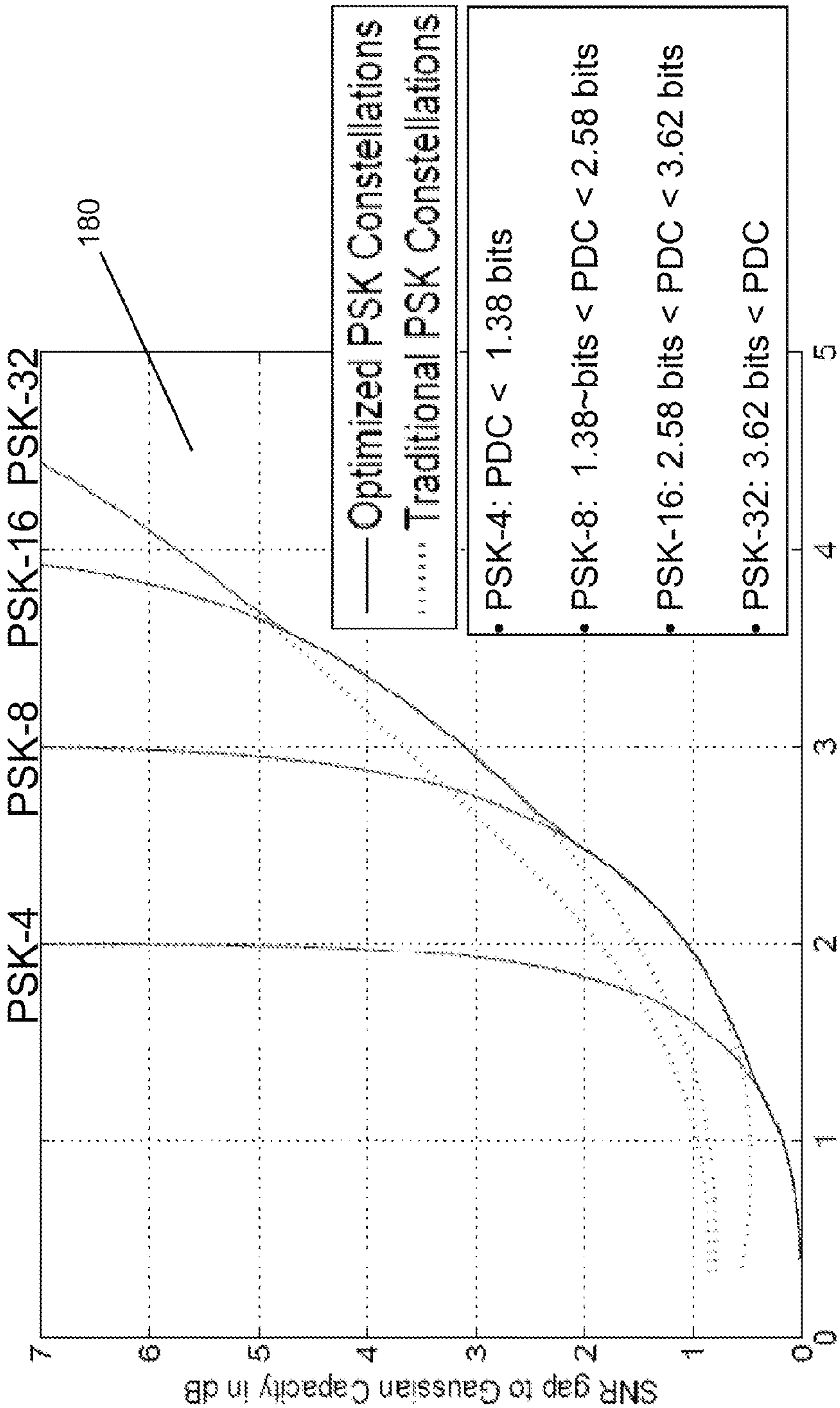

180
PSK-4
PSK-8
PSK-16
PSK-32
Optimized PSK Constellations
Traditional PSK Constellations
PSK-4: PDC < 1.38 bits
PSK-8: 1.38~bits < PDC < 2.58 bits
PSK-16: 2.58 bits < PDC < 3.62 bits
PSK-32: 3.62 bits < PDC
SNR gap to Gaussian Capacity in dB
Capacity in bits
0
1
2
3
4
5
6
7

FIG. 18

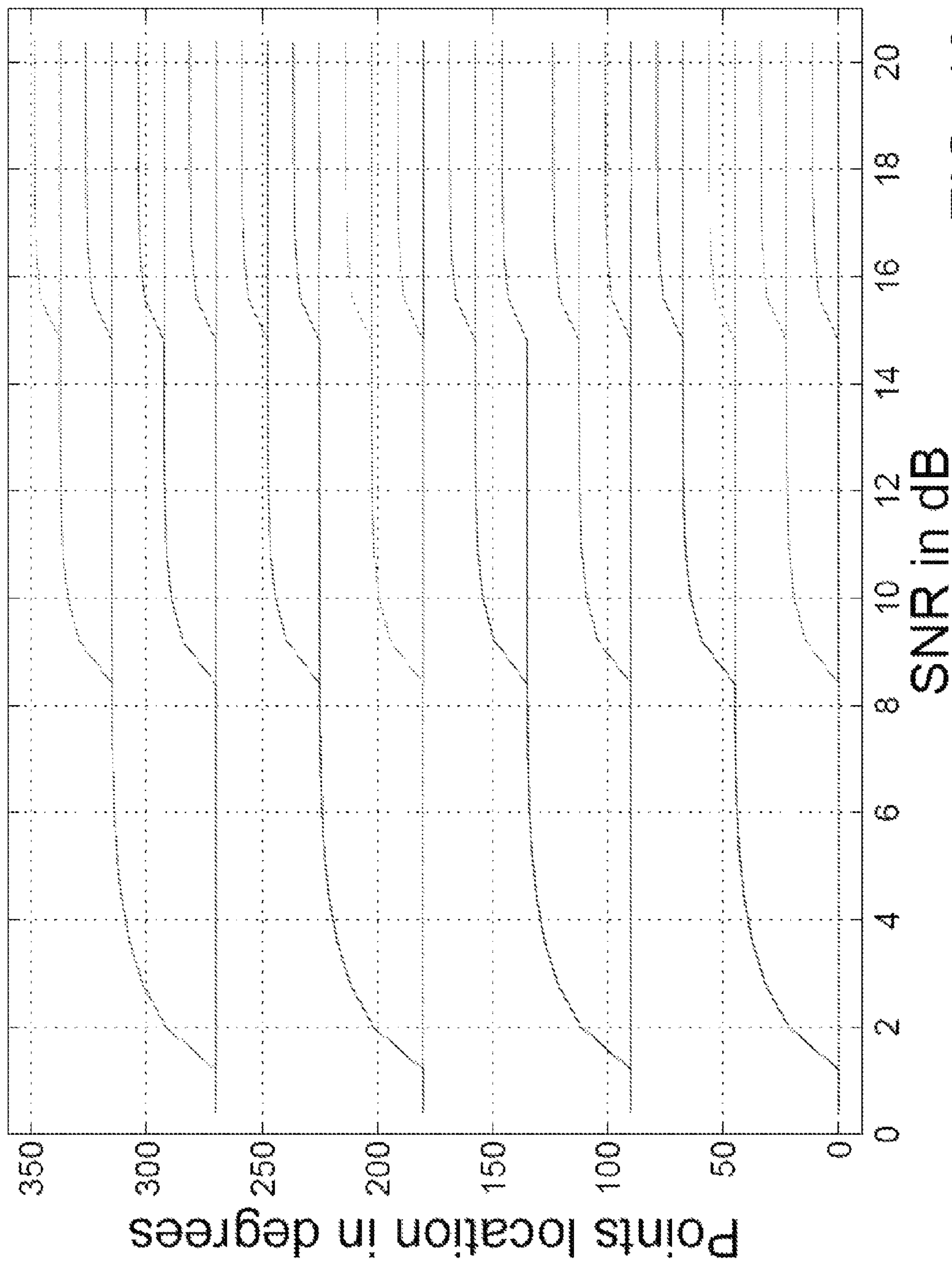

Points location in degrees
0
50
100
150
200
250
300
350
SNR in dB
0
2
4
6
8
10
12
14
16
18
20

FIG. 19

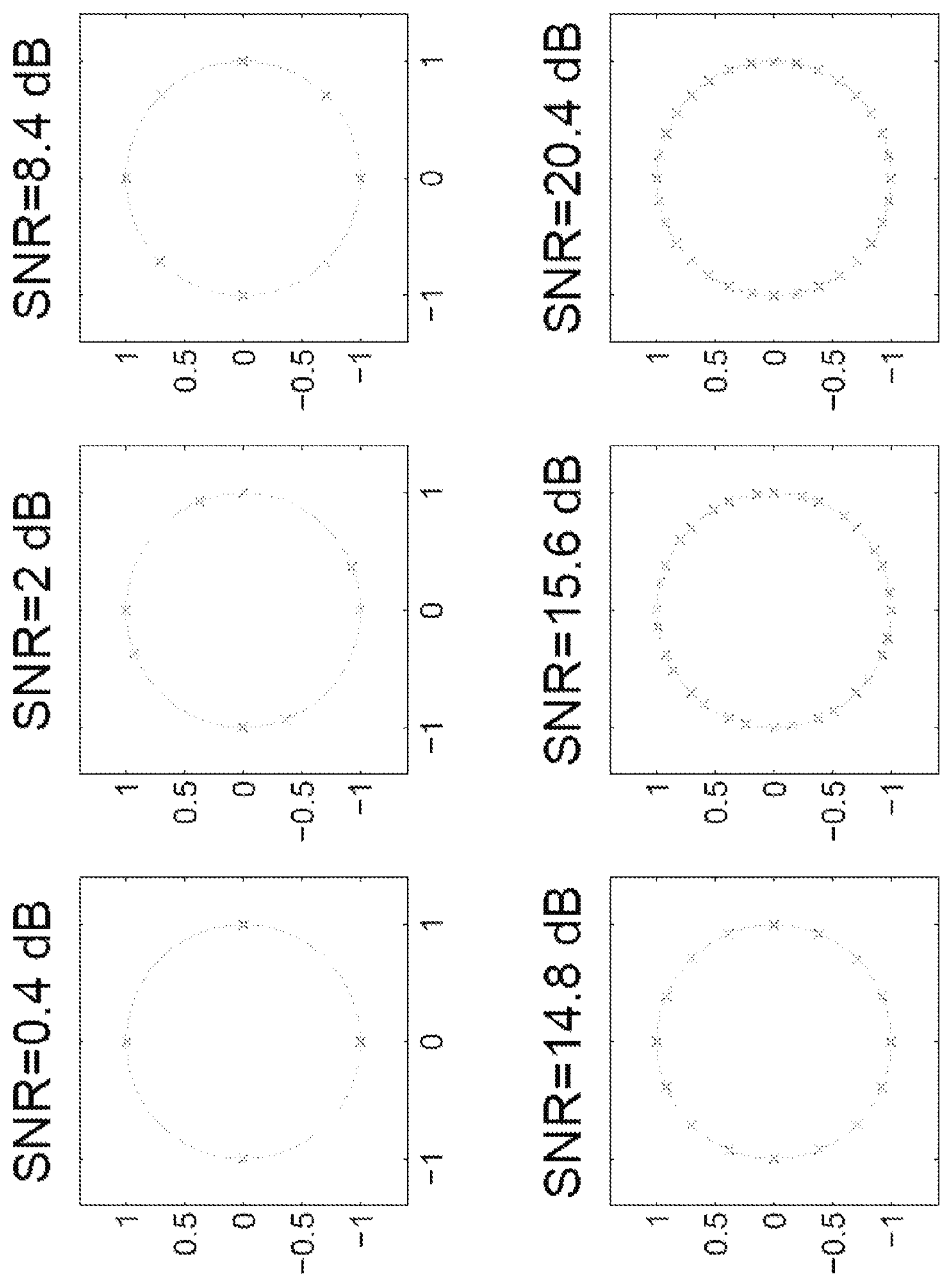


FIG. 20

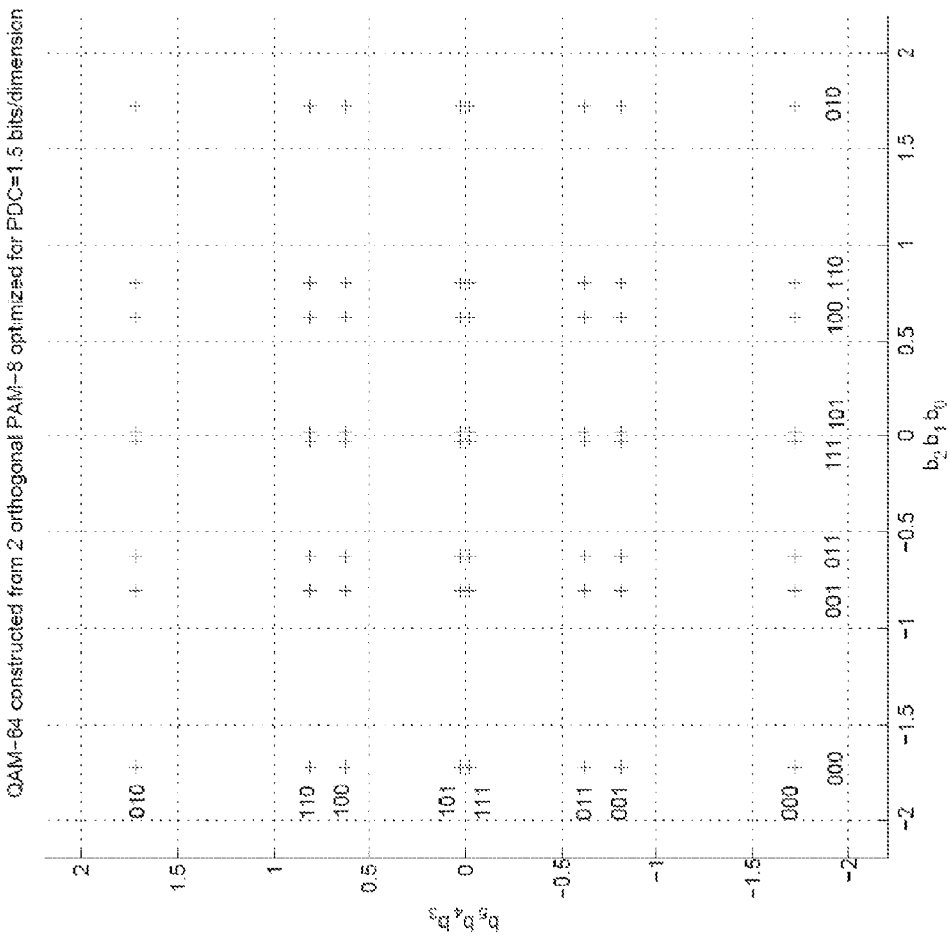

QAM−64 constructed from 2 orthogonal PAM−8 optimized for PDC=1.5 bits/dimension
$b_2 b_1 b_0$
$b_5 b_4 b_3$
000
001 011
111 101
100 110
010
011
001
101
111
110
100
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2

FIG. 21

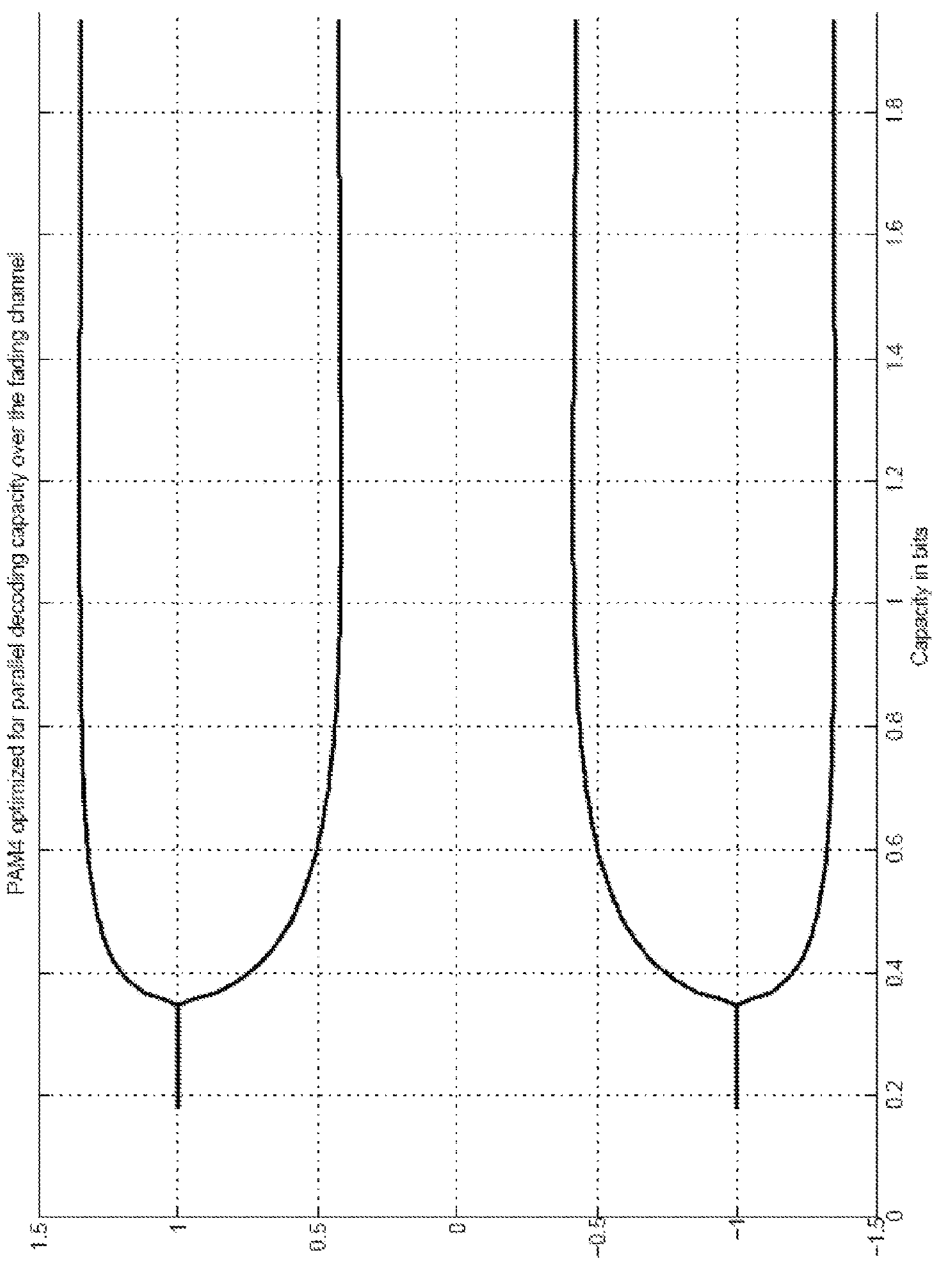

PAM4 optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
1.5
1
0.5
0
−0.5
−1
−1.5

FIG. 22a

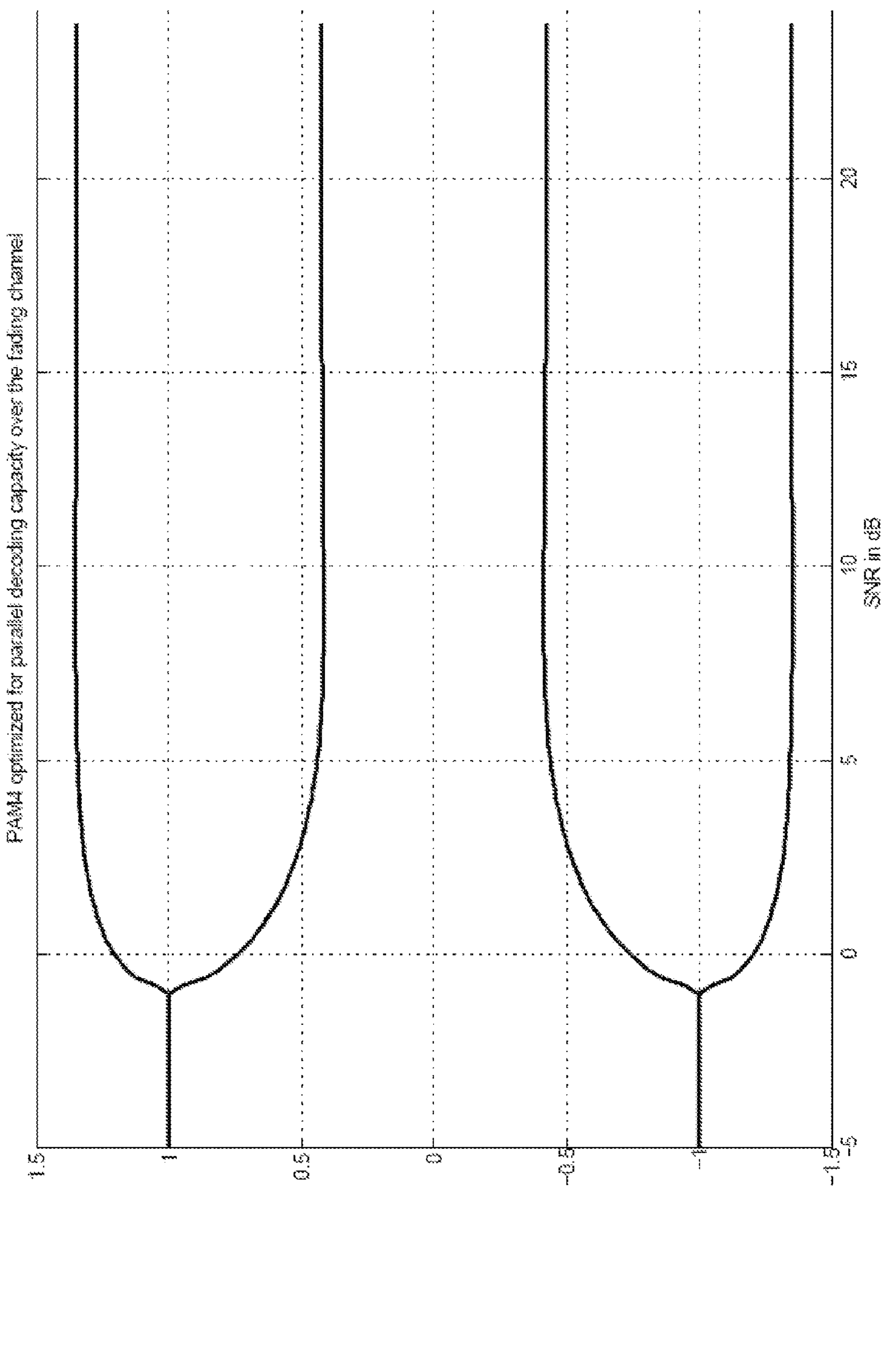

PAM4 optimized for parallel decoding capacity over the fading channel
SNR in dB
-5
0
5
10
15
20
1.5
1
0.5
0
-0.5
-1
-1.5

FIG. 22b

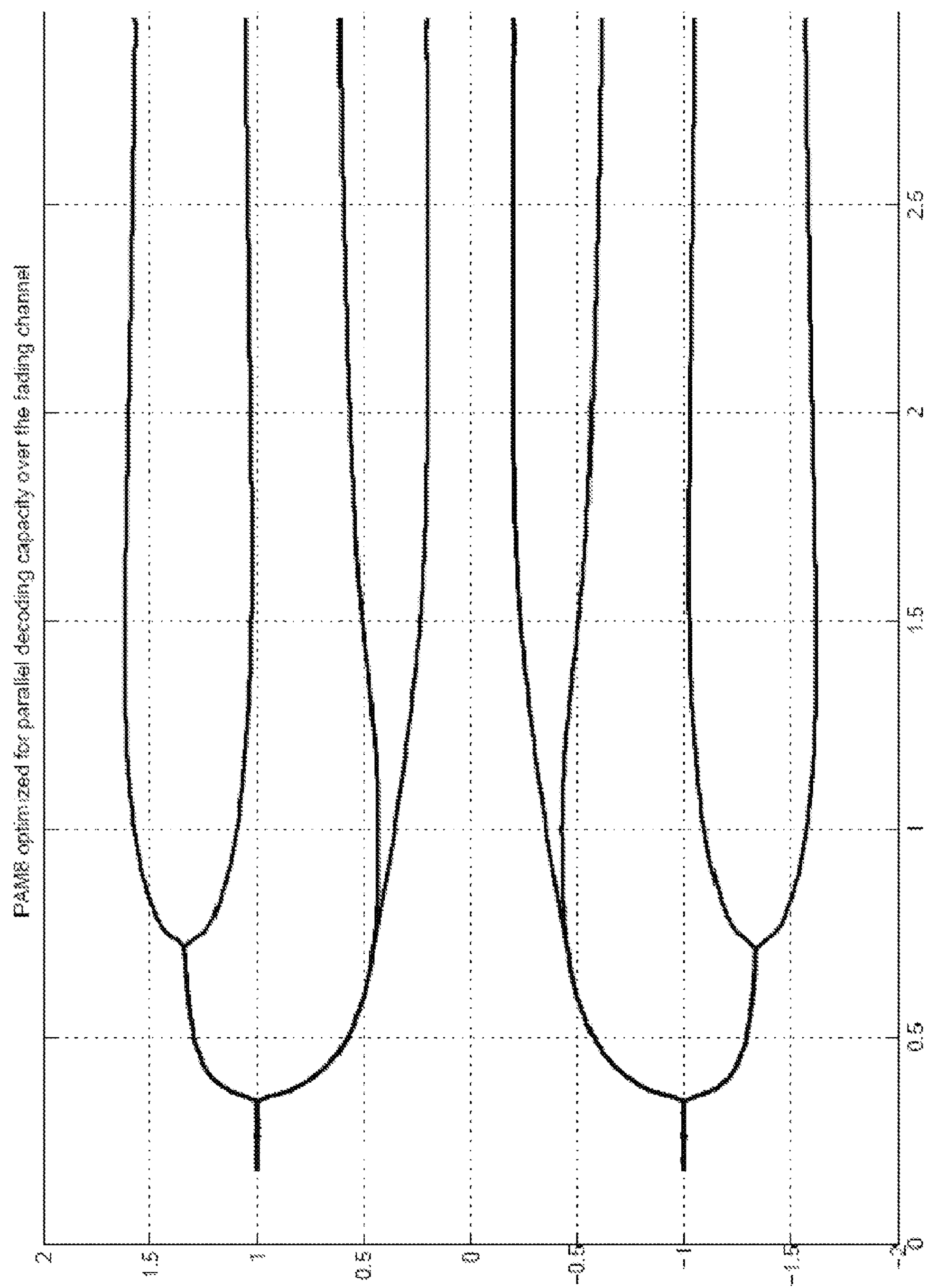

PAM8 optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.5
1
1.5
2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 23a

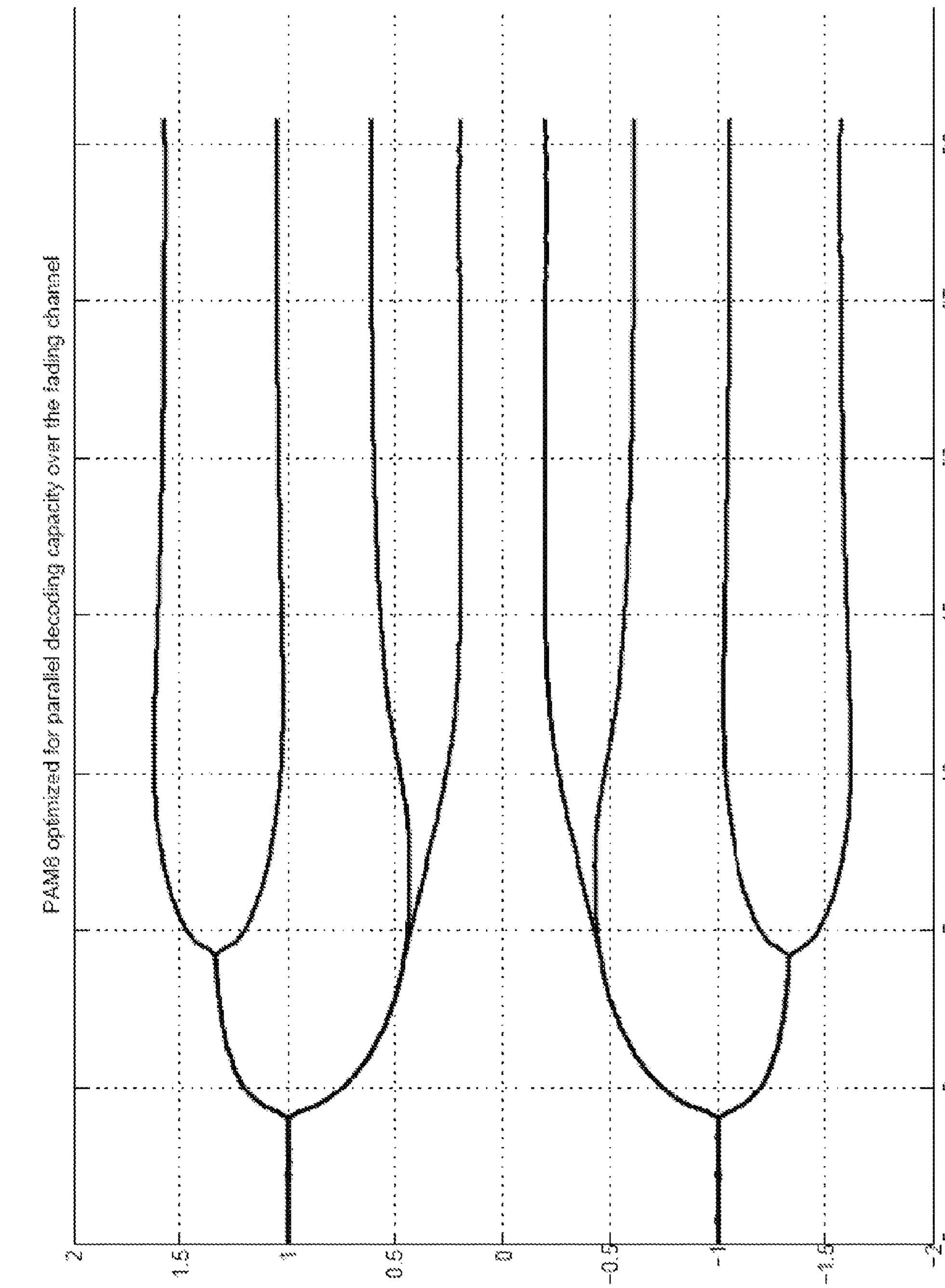

PAM8 optimized for parallel decoding capacity over the fading channel
SNR in dB
-5
0
5
10
15
20
25
30
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 23b

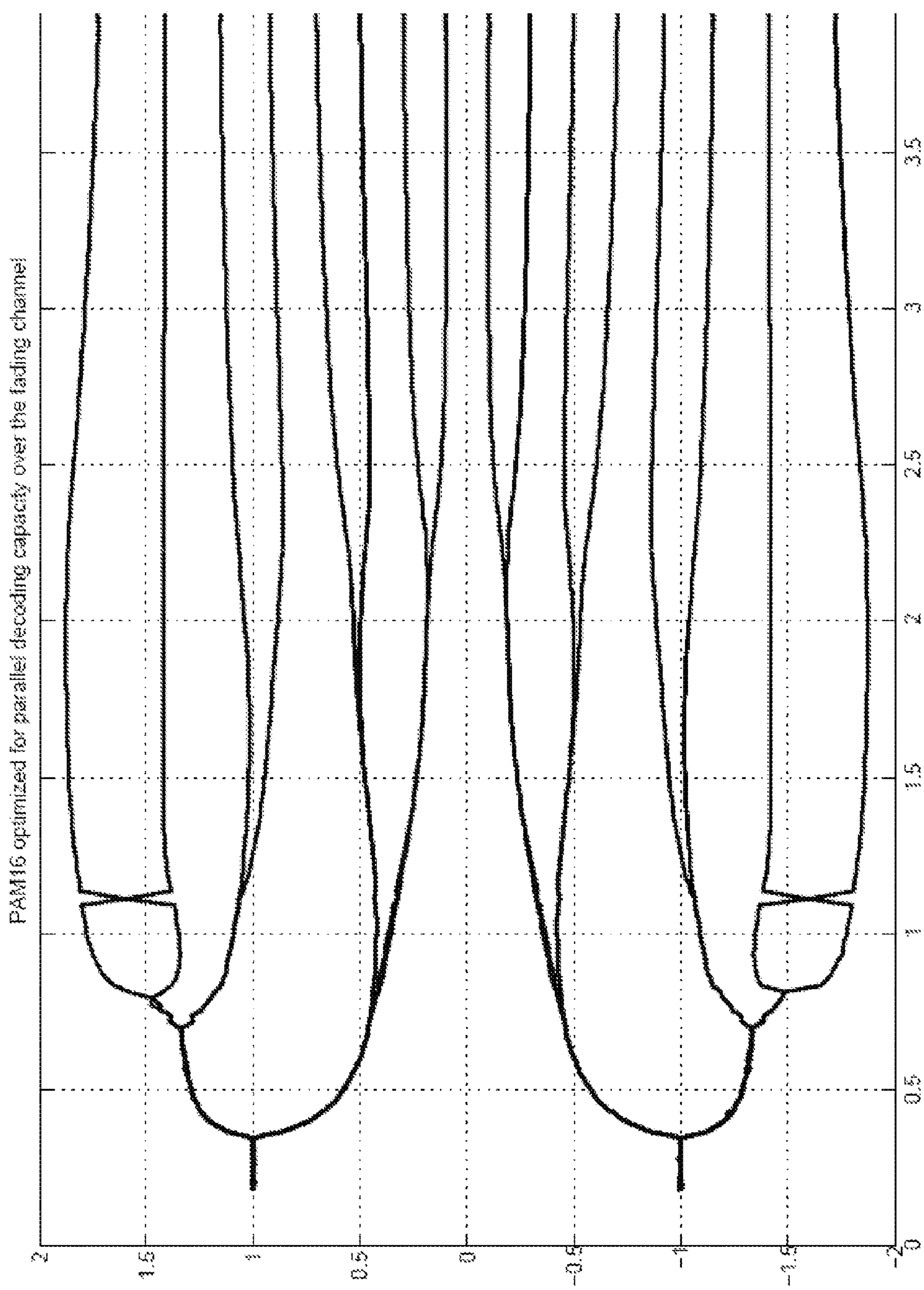

PAM16 optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.5
1
1.5
2
2.5
3
3.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 24a

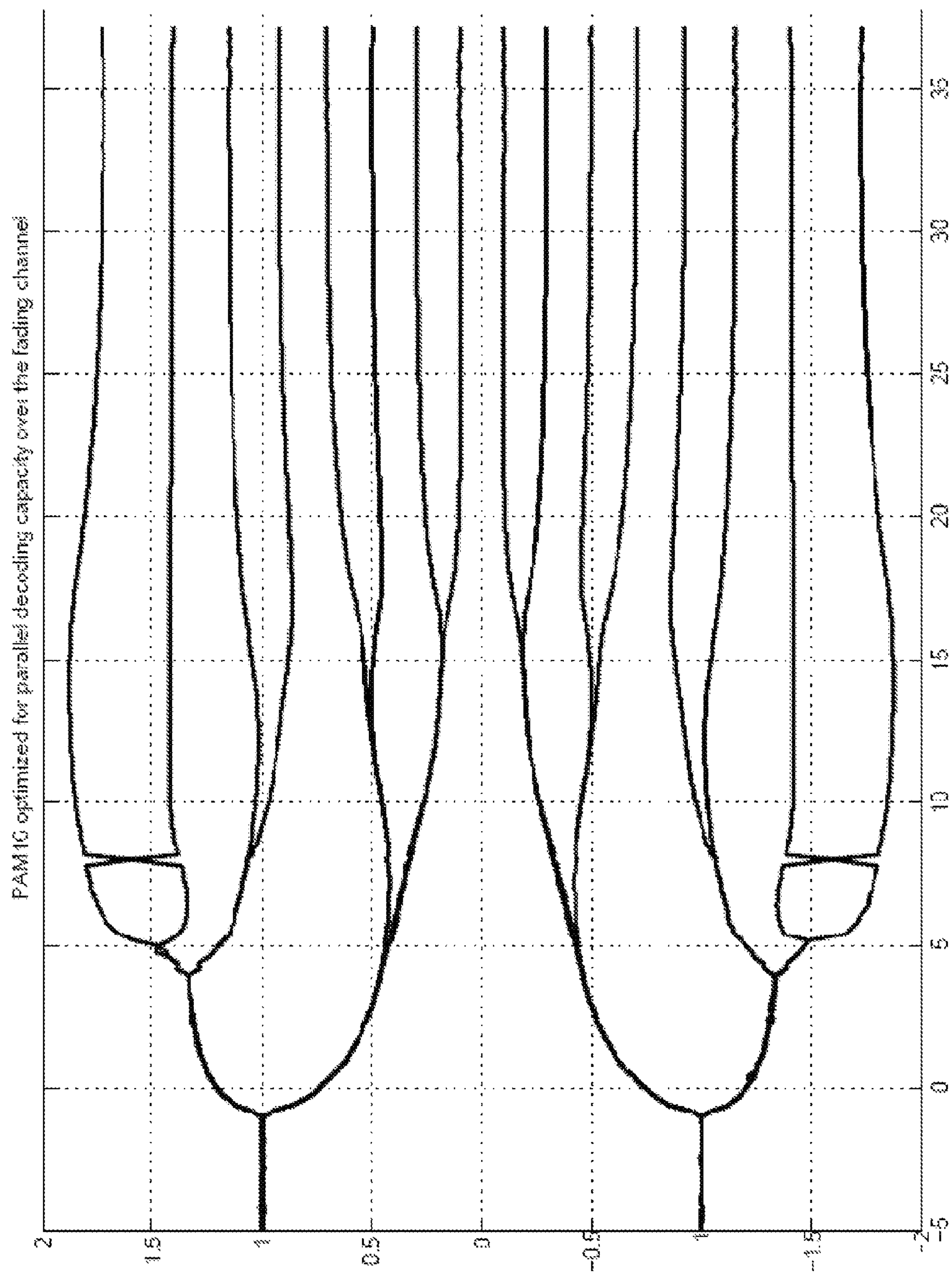

PAM16 optimized for parallel decoding capacity over the fading channel
SNR in dB
-5
0
5
10
15
20
25
30
35
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 24b

METHODOLOGY AND METHOD AND APPARATUS FOR SIGNALING WITH CAPACITY OPTIMIZED CONSTELLATIONS

RELATED APPLICATIONS

This application is a continuation of application Ser. No. 13/618,630 filed Sep. 14, 2012, issued on Sep. 23, 2014 as U.S. Pat. No. 8,842,761, which application is a continuation of application Ser. No. 13/118,921 filed May 31, 2011, issued on Sep. 18, 2012 as U.S. Pat. No. 8,270,511, which application is a continuation of application Ser. No. 12/156,989 filed Jun. 5, 2008, issued on Jul. 12, 2011 as U.S. Pat. No. 7,978,777, which application claimed priority to U.S. Provisional Application 60/933,319 filed Jun. 5, 2007, the disclosures of which are incorporated herein by reference.

STATEMENT OF FEDERALLY SPONSORED RESEARCH

This invention was made with Government support under contract NAS7-03001 awarded by NASA. The Government has certain rights in this invention.

BACKGROUND

The present invention generally relates to bandwidth and/or power efficient digital transmission systems and more specifically to the use of unequally spaced constellations having increased capacity.

The term "constellation" is used to describe the possible symbols that can be transmitted by a typical digital communication system. A receiver attempts to detect the symbols that were transmitted by mapping a received signal to the constellation. The minimum distance (d_{min}) between constellation points is indicative of the capacity of a constellation at high signal-to-noise ratios (SNRs). Therefore, constellations used in many communication systems are designed to maximize d_{min}. Increasing the dimensionality of a constellation allows larger minimum distance for constant constellation energy per dimension. Therefore, a number of multi-dimensional constellations with good minimum distance properties have been designed.

Communication systems have a theoretical maximum capacity, which is known as the Shannon limit. Many communication systems attempt to use codes to increase the capacity of a communication channel. Significant coding gains have been achieved using coding techniques such as turbo codes and LDPC codes. The coding gains achievable using any coding technique are limited by the constellation of the communication system. The Shannon limit can be thought of as being based upon a theoretical constellation known as a Gaussian distribution, which is an infinite constellation where symbols at the center of the constellation are transmitted more frequently than symbols at the edge of the constellation. Practical constellations are finite and transmit symbols with equal likelihoods, and therefore have capacities that are less than the Gaussian capacity. The capacity of a constellation is thought to represent a limit on the gains that can be achieved using coding when using that constellation.

Prior attempts have been made to develop unequally spaced constellations. For example, a system has been proposed that uses unequally spaced constellations that are optimized to minimize the error rate of an uncoded system. Another proposed system uses a constellation with equiprobable but unequally spaced symbols in an attempts to mimic a Gaussian distribution.

Other approaches increases the dimensionality of a constellation or select a new symbol to be transmitted taking into consideration previously transmitted symbols. However, these constellation were still designed based on a minimum distance criteria.

SUMMARY OF THE INVENTION

Systems and methods are described for constructing a modulation such that the constrained capacity between a transmitter and a receiver approaches the Gaussian channel capacity limit first described by Shannon [ref Shannon 1948]. Traditional communications systems employ modulations that leave a significant gap to Shannon Gaussian capacity. The modulations of the present invention reduce, and in some cases, nearly eliminate this gap. The invention does not require specially designed coding mechanisms that tend to transmit some points of a modulation more frequently than others but rather provides a method for locating points (in a one or multiple dimensional space) in order to maximize capacity between the input and output of a bit or symbol mapper and demapper respectively. Practical application of the method allows systems to transmit data at a given rate for less power or to transmit data at a higher rate for the same amount of power.

One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, wherein the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper. In addition, the receiver includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. Furthermore, the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}.

A further embodiment of the invention includes encoding the bits of user information using a coding scheme, mapping the encoded bits of user information to a symbol constellation, wherein the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}, modulating the symbols in accordance with a modulation scheme, transmitting the modulated signal via the communication channel, receiving a modulated signal, demodulating the modulated signal in accordance with the modulation scheme, demapping the demodulated signal using the geometrically shaped signal constellation to produce likelihoods, and decoding the likelihoods to obtain an estimate of the decoded bits.

Another embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

A still further embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

Still another embodiment of the invention includes selecting an appropriate constellation size and a desired SNR, and optimizing the location of the points of the constellation to maximize a capacity measure of the constellation.

A yet further embodiment of the invention includes obtaining a geometrically shaped PAM constellation with a constellation size that is the square root of said given constellation size, where the geometrically shaped PAM constellation has a capacity greater than that of a PAM constellation that maximizes d_{min}, creating an orthogonalized PAM constellation using the geometrically shaped PAM constellation, and combining the geometrically shaped PAM constellation and the orthogonalized PAM constellation to produce a geometrically shaped QAM constellation.

Another further embodiment of the invention includes transmitting information over a channel using a geometrically shaped symbol constellation, and modifying the location of points within the geometrically shaped symbol constellation to change the target user data rate.

BRIEF DESCRIPTION OF DRAWINGS

FIG. **1** is a conceptual illustration of a communication system in accordance with an embodiment of the invention.

FIG. **2** is a conceptual illustration of a transmitter in accordance with an embodiment of the invention.

FIG. **3** is a conceptual illustration of a receiver in accordance with an embodiment of the invention.

FIG. **4***a* is a conceptual illustration of the joint capacity of a channel.

FIG. **4***b* is a conceptual illustration of the parallel decoding capacity of a channel.

FIG. **5** is a flow chart showing a process for obtaining a constellation optimized for capacity for use in a communication system having a fixed code rate and modulation scheme in accordance with an embodiment of the invention.

FIG. **6***a* is a chart showing a comparison of Gaussian capacity and PD capacity for traditional PAM-2,4,8,16,32.

FIG. **6***b* is a chart showing a comparison between Gaussian capacity and joint capacity for traditional PAM-2,4,8,16,32.

FIG. **7** is a chart showing the SNR gap to Gaussian capacity for the PD capacity and joint capacity of traditional PAM-2, 4,8,16,32 constellations.

FIG. **8***a* is a chart comparing the SNR gap to Gaussian capacity of the PD capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

FIG. **8***b* is a chart comparing the SNR gap to Gaussian capacity of the joint capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

FIG. **9** is a chart showing Frame Error Rate performance of traditional and PD capacity optimized PAM-32 constellations in simulations involving several different length LDPC codes.

FIGS. **10***a*-**10***d* are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **11***a* and **11***b* are design tables of PD capacity and joint capacity optimized PAM-4 constellations in accordance with embodiments of the invention.

FIGS. **12***a*-**12***d* are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **13***a* and **13***b* are design tables of PD capacity and joint capacity optimized PAM-8 constellations in accordance with embodiments of the invention.

FIGS. **14***a*-**14***d* are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **15***a* and **15***b* are design tables of PD capacity and joint capacity optimized PAM-16 constellations in accordance with embodiments of the invention.

FIGS. **16***a*-**16***d* are locus plots showing the location of constellation points of a PAM-32 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **17***a* and **17***b* are design tables of PD capacity and joint capacity optimized PAM-32 constellations in accordance with embodiments of the invention.

FIG. **18** is a chart showing the SNR gap to Gaussian capacity for traditional and capacity optimized PSK constellations.

FIG. **19** is a chart showing the location of constellation points of PD capacity optimized PSK-32 constellations.

FIG. **20** is a series of PSK-32 constellations optimized for PD capacity at different SNRs in accordance with embodiments of the invention.

FIG. **21** illustrates a QAM-64 constructed from orthogonal Cartesian product of two PD optimized PAM-8 constellations in accordance with an embodiment of the invention.

FIGS. **22***a* and **22***b* are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **23***a* and **23***b* are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **24***a* and **24***b* are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

DETAILED DESCRIPTION OF THE INVENTION

Turning now to the drawings, communication systems in accordance with embodiments of the invention are described that use signal constellations, which have unequally spaced (i.e. 'geometrically' shaped) points. In several embodiments, the locations of geometrically shaped points are designed to provide a given capacity measure at a reduced signal-to-noise ratio (SNR) compared to the SNR required by.a constellation that maximizes d_{min}. In many embodiments, the constellations are selected to provide increased capacity at a predetermined range of channel signal-to-noise ratios (SNR). Capacity measures that can be used in the selection of the location of constellation points include, but are not limited to, parallel decode (PD) capacity and joint capacity.

In many embodiments, the communication systems utilize capacity approaching codes including, but not limited to, LDPC and Turbo codes. As is discussed further below, direct optimization of the constellation points of a communication system utilizing a capacity approaching channel code, can yield different constellations depending on the SNR for which they are optimized. Therefore, the same constellation is unlikely to achieve the same coding gains applied across all code rates; that is, the same constellation will not enable the best possible performance across all rates. In many instances, a constellation at one code rate can achieve gains that cannot be achieved at another code rate. Processes for selecting capacity optimized constellations to achieve increased coding gains based upon a specific coding rate in accordance with embodiments of the invention are described below. In a number of embodiments, the communication systems can adapt location of points in a constellation in response to channel conditions, changes in code rate and/or to change the target user data rate.

Communication Systems

A communication system in accordance with an embodiment of the invention is shown in FIG. **1**. The communication system **10** includes a source **12** that provides user bits to a transmitter **14**. The transmitter transmits symbols over a channel to a receiver **16** using a predetermined modulation scheme. The receiver uses knowledge of the modulation scheme, to decode the signal received from the transmitter. The decoded bits are provided to a sink device that is connected to the receiver.

A transmitter in accordance with an embodiment of the invention is shown in FIG. **2**. The transmitter **14** includes a coder **20** that receives user bits from a source and encodes the bits in accordance with a predetermined coding scheme. In a number of embodiments, a capacity approaching code such as a turbo code or a LDPC code is used. In other embodiments, other coding schemes can be used to providing a coding gain within the communication system. A mapper **22** is connected to the coder. The mapper maps the bits output by the coder to a symbol within a geometrically distributed signal constellation stored within the mapper. The mapper provides the symbols to a modulator **24**, which modulates the symbols for transmission via the channel.

A receiver in accordance with an embodiment of the invention is illustrated in FIG. **3**. The receiver **16** includes a demodulator **30** that demodulates a signal received via the channel to obtain symbol or bit likelihoods. The demapper uses knowledge of the geometrically shaped symbol constellation used by the transmitter to determine these likelihoods. The demapper **32** provides the likelihoods to a decoder **34** that decodes the encoded bit stream to provide a sequence of received bits to a sink.

Geometrically Shaped Constellations

Transmitters and receivers in accordance with embodiments of the invention utilize geometrically shaped symbol constellations. In several embodiments, a geometrically shaped symbol constellation is used that optimizes the capacity of the constellation. Various geometrically shaped symbol constellations that can be used in accordance with embodiments of the invention, techniques for deriving geometrically shaped symbol constellations are described below.

Selection of a Geometrically Shaped Constellation

Selection of a geometrically shaped constellation for use in a communication system in accordance with an embodiment of the invention can depend upon a variety of factors including whether the code rate is fixed. In many embodiments, a geometrically shaped constellation is used to replace a conventional constellation (i.e. a constellation maximized for d_{min}) in the mapper of transmitters and the demapper of receivers within a communication system. Upgrading a communication system involves selection of a constellation and in many instances the upgrade can be achieved via a simple firmware upgrade. In other embodiments, a geometrically shaped constellation is selected in conjunction with a code rate to meet specific performance requirements, which can for example include such factors as a specified bit rate, a maximum transmit power. Processes for selecting a geometric constellation when upgrading existing communication systems and when designing new communication systems are discussed further below.

Upgrading Existing Communication Systems

A geometrically shaped constellation that provides a capacity, which is greater than the capacity of a constellation maximized for d_{min}, can be used in place of a conventional constellation in a communication system in accordance with embodiments of the invention. In many instances, the substitution of the geometrically shaped constellation can be achieved by a firmware or software upgrade of the transmitters and receivers within the communication system. Not all geometrically shaped constellations have greater capacity than that of a constellation maximized for d_{min}. One approach to selecting a geometrically shaped constellation having a greater capacity than that of a constellation maximized for d_{min} is to optimize the shape of the constellation with respect to a measure of the capacity of the constellation for a given SNR. Capacity measures that can be used in the optimization process can include, but are not limited to, joint capacity or parallel decoding capacity.

Joint Capacity and parallel Decoding Capacity

A constellation can be parameterized by the total number of constellation points, M, and the number of real dimensions, N_{min}. In systems where there are no belief propagation iterations between the decoder and the constellation demapper, the constellation demapper can be thought of as part of the channel. A diagram conceptually illustrating the portions of a communication system that can be considered part of the channel for the purpose of determining PD capacity is shown in FIG. **4***a*. The portions of the communication system that are considered part of the channel are indicated by the ghost line **40**. The capacity of the channel defined as such is the parallel decoding (PD) capacity, given by:

$$C_{PD} = \sum_{i=0}^{l-1} I(X_i; Y)$$

where X_i is the ith bit of the I-bits transmitted symbol, and Y is the received symbol, and I(A;B) denotes the mutual information between random variables A and B.

Expressed another way, the PD capacity of a channel can be viewed in terms of the mutual information between the output bits of the encoder (such as an LDPC encoder) at the transmitter and the likelihoods computed by the demapper at the receiver. The PD capacity is influenced by both the placement of points within the constellation and by the labeling assignments.

With belief propagation iterations between the demapper and the decoder, the demapper can no longer be viewed as part of the channel, and the joint capacity of the constellation becomes the tightest known bound on the system performance. A diagram conceptually illustrating the portions of a communication system that are considered part of the channel for the purpose of determining the joint capacity of a constellation is shown in FIG. **4***b*. The portions of the communication system that are considered part of the channel are indicated by the ghost line **42**. The joint capacity of the channel is given by:

$$C_{JOINT} = I(X; Y)$$

Joint capacity is a description of the achievable capacity between the input of the mapper on the transmit side of the link and the output of the channel (including for example

AWGN and Fading channels). Practical systems must often 'demap' channel observations prior to decoding. In general, the step causes some loss of capacity. In fact it can be proven that $C_G \geq C_{JOINT} \geq C_{PD}$. That is, C_{JOINT} upper bounds the capacity achievable by C_{PD}. The methods of the present invention are motivated by considering the fact that practical limits to a given communication system capacity are limited by C_{JOINT} and C_{PD}. In several embodiments of the invention, geometrically shaped constellations are selected that maximize these measures.

Selecting a Constellation Having an Optimal Capacity

Geometrically shaped constellations in accordance with embodiments of the invention can be designed to optimize capacity measures including, but not limited to PD capacity or joint capacity. A process for selecting the points, and potentially the labeling, of a geometrically shaped constellation for use in a communication system having a fixed code rate in accordance with an embodiment of the invention is shown in FIG. **5**. The process **50** commences with the selection (**52**) of an appropriate constellation size M and a desired capacity per dimension η. In the illustrated embodiment, the process involves a check (**52**) to ensure that the constellation size can support the desired capacity. In the event that the constellation size could support the desired capacity, then the process iteratively optimizes the M-ary constellation for the specified capacity. Optimizing a constellation for a specified capacity often involves an iterative process, because the optimal constellation depends upon the SNR at which the communication system operates. The SNR for the optimal constellation to give a required capacity is not known a priori. Throughout the description of the present invention SNR is defined as the ratio of the average constellation energy per dimension to the average noise energy per dimension. In most cases the capacity can be set to equal the target user bit rate per symbol per dimension. In some cases adding some implementation margin on top of the target user bit rate could result in a practical system that can provide the required user rate at a lower rate. The margin is code dependent. The following procedure could be used to determine the target capacity that includes some margin on top of the user rate. First, the code (e.g. LDPC or Turbo) can be simulated in conjunction with a conventional equally spaced constellation. Second, from the simulation results the actual SNR of operation at the required error rate can be found. Third, the capacity of the conventional constellation at that SNR can be computed. Finally, a geometrically shaped constellation can be optimized for that capacity.

In the illustrated embodiment, the iterative optimization loop involves selecting an initial estimate of the SNR at which the system is likely to operate (i.e. SNR_{in}). In several embodiments the initial estimate is the SNR required using a conventional constellation. In other embodiments, other techniques can be used for selecting the initial SNR. An M-ary constellation is then obtained by optimizing (**56**) the constellation to maximize a selected capacity measure at the initial SNR_{in} estimate. Various techniques for obtaining an optimized constellation for a given SNR estimate are discussed below.

The SNR at which the optimized M-ary constellation provides the desired capacity per dimension η (SNR_{out}) is determined (**57**). A determination (**58**) is made as to whether the SNR_{out} and SNR_{in} have converged. In the illustrated embodiment convergence is indicated by SNR_{out} equaling SNR_{in}. In a number of embodiments, convergence can be determined based upon the difference between SNR_{out} and SNR_{in} being less than a predetermined threshold. When SNR_{out} and SNR_{in} have not converged, the process performs another iteration selecting SNR_{out} as the new SNR_{in} (**55**). When SNR_{out} and SNR_{in} have converged, the capacity measure of the constellation has been optimized. As is explained in more detail below, capacity optimized constellation at low SNRs are geometrically shaped constellations that can achieve significantly higher performance gains (measured as reduction in minimum required SNR) than constellations that maximize d_{min}.

The process illustrated in FIG. **5** can maximize PD capacity or joint capacity of an M-ary constellation for a given SNR. Although the process illustrated in FIG. **5** shows selecting an M-ary constellation optimized for capacity, a similar process could be used that terminates upon generation of an M-ary constellation where the SNR gap to Gaussian capacity at a given capacity is a predetermined margin lower than the SNR gap of a conventional constellation, for example 0.5 db. Alternatively, other processes that identify M-ary constellations having capacity greater than the capacity of a conventional constellation can be used in accordance with embodiments of the invention. A geometrically shaped constellation in accordance with embodiments of the invention can achieve greater capacity than the capacity of a constellation that maximizes d_{min}, without having the optimal capacity for the SNR range within which the communication system operates.

We note that constellations designed to maximize joint capacity may also be particularly well suited to codes with symbols over GF(q), or with multi-stage decoding. Conversely constellations optimized for PD capacity could be better suited to the more common case of codes with symbols over GF(2)

Optimizing the Capacity of an M-Ary Constellation at a Given SNR

Processes for obtaining a capacity optimized constellation often involve determining the optimum location for the points of an M-ary constellation at a given SNR. An optimization process, such as the optimization process **56** shown in FIG. **5**, typically involves unconstrained or constrained non-linear optimization. Possible objective functions to be maximized are the Joint or PD capacity functions. These functions may be targeted to channels including but not limited to Additive White Gaussian Noise (AWGN) or Rayleigh fading channels. The optimization process gives the location of each constellation point identified by its symbol labeling. In the case where the objective is joint capacity, point bit labelings are irrelevant meaning that changing the bit labelings doesn't change the joint capacity as long as the set of point locations remains unchanged.

The optimization process typically finds the constellation that gives the largest PD capacity or joint capacity at a given SNR. The optimization process itself often involves an iterative numerical process that among other things considers several constellations and selects the constellation that gives the highest capacity at a given SNR. In other embodiments, the constellation that requires the least SNR to give a required PD capacity or joint capacity can also be found. This requires running the optimization process iteratively as shown in FIG. **5**.

Optimization constraints on the constellation point locations may include, but are not limited to, lower and upper bounds on point location, peak to average power of the resulting constellation, and zero mean in the resulting constellation. It can be easily shown that a globally optimal constellation will have zero mean (no DC component). Explicit inclusion of a zero mean constraint helps the optimization routine to converge more rapidly. Except for cases where exhaustive search of all combinations of point locations and labelings is possible it will not necessarily always be the case that solutions are provably globally optimal. In cases where exhaustive search is possible, the solution provided by the non-linear optimizer is in fact globally optimal.

The processes described above provide examples of the manner in which a geometrically shaped constellation having an increased capacity relative to a conventional capacity can be obtained for use in a communication system having a fixed code rate and modulation scheme. The actual gains achievable using constellations that are optimized for capacity compared to conventional constellations that maximize d_{min} are considered below.

Gains Achieved by Optimized Geometrically Spaced Constellations

The ultimate theoretical capacity achievable by any communication method is thought to be the Gaussian capacity, C_G which is defined as:

$$C_G = \frac{1}{2}\log_2(1 + SNR)$$

Where signal-to-noise (SNR) is the ratio of expected signal power to expected noise power. The gap that remains between the capacity of a constellation and C_G can be considered a measure of the quality of a given constellation design.

The gap in capacity between a conventional modulation scheme in combination with a theoretically optimal coder can be observed with reference to FIGS. **6***a* and **6***b*. FIG. **6***a* includes a chart **60** showing a comparison between Gaussian capacity and the PD capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}, Gaps **62** exist between the plot of Gaussian capacity and the PD capacity of the various PAM constellations. FIG. **6***b* includes a chart **64** showing a comparison between Gaussian capacity and the joint capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}, Gaps **66** exist between the plot of Gaussian capacity and the joint capacity of the various PAM constellations. These gaps in capacity represent the extent to which conventional PAM constellations fall short of obtaining the ultimate theoretical capacity i.e. the Gaussian capacity.

In order to gain a better view of the differences between the curves shown in FIGS. **6***a* and **6***b* at points close to the Gaussian capacity, the SNR gap to Gaussian capacity for different values of capacity for each constellation are plotted in FIG. **7**. It is interesting to note from the chart **70** in FIG. **7** that (unlike the joint capacity) at the same SNR, the PD capacity does not necessarily increase with the number of constellation points. As is discussed further below, this is not the case with PAM constellations optimized for PD capacity.

FIGS. **8***a* and **8***b* summarize performance of constellations for PAM-4, 8, 16, and 32 optimized for PD capacity and joint capacity (it should be noted that BPSK is the optimal PAM-2 constellation at all code rates). The constellations are optimized for PD capacity and joint capacity for different target user bits per dimension (i.e. code rates). The optimized constellations are different depending on the target user bits per dimension, and also depending on whether they have been designed to maximize the PD capacity or the joint capacity. All the PD optimized PAM constellations are labeled using a gray labeling which is not always the binary reflective gray labeling. It should be noted that not all gray labels achieve the maximum possible PD capacity even given the freedom to place the constellation points anywhere on the real line. FIG. **8***a* shows the SNR gap for each constellation optimized for PD capacity. FIG. **8***b* shows the SNR gap to Gaussian capacity for each constellation optimized for joint capacity. Again, it should be emphasized that each '+' on the plot represents a different constellation.

Referring to FIG. **8***a*, the coding gain achieved using a constellation optimized for PD capacity can be appreciated by comparing the SNR gap at a user bit rate per dimension of 2.5 bits for PAM-32. A user bit rate per dimension of 2.5 bits for a system transmitting 5 bits per symbol constitutes a code rate of 1/2. At that code rate the constellation optimized for PD capacity provides an additional coding gain of approximately 1.5 dB when compared to the conventional PAM-32 constellation.

The SNR gains that can be achieved using constellations that are optimized for PD capacity can be verified through simulation. The results of a simulation conducted using a rate 1/2 LDPC code in conjunction with a conventional PAM-32 constellation and in conjunction with a PAM-32 constellation optimized for PD capacity are illustrated in FIG. **9**. A chart **90** includes plots of Frame Error Rate performance of the different constellations with respect to SNR and using different length codes (i.e. k=4,096 and k=16,384). Irrespective of the code that is used, the constellation optimized for PD capacity achieves a gain of approximately 1.3 dB, which closely approaches the gain predicted from FIG. **8***a*.

Capacity Optimized PAM Constellations

Using the processes outlined above, locus plots of PAM constellations optimized for capacity can be generated that show the location of points within PAM constellations versus SNR. Locus plots of PAM-4, 8, 16, and 32 constellations optimized for PD capacity and joint capacity and corresponding design tables at various typical user bit rates per dimension are illustrated in FIGS. **10***a*-**17***b*. The locus plots and design tables show PAM-4,8,16,32 constellation point locations and labelings from low to high SNR corresponding to a range of low to high spectral efficiency.

In FIG. **10***a*, a locus plot **100** shows the location of the points of PAM-4 constellations optimized for Joint capacity plotted against achieved capacity. A similar locus plot **105** showing the location of the points of Joint capacity optimized PAM-4 constellations plotted against SNR is included in FIG. **10***b*. In FIG. **10***c*. the location of points for PAM-4 optimized for PD capacity is plotted against achievable capacity and in FIG. **10***d* the location of points for PAM-4 for PD capacity is plotted against SNR. At low SNRs, the PD capacity optimized PAM-4 constellations have only 2 unique points, while the Joint optimized constellations have 3. As SNR is increased, each optimization eventually provides 4 unique points. This phenomenon is explicitly described in FIG. **11***a* and FIG. **11***b* where vertical slices of FIGS. **10***ab* and **10***cd* are captured in tables describing some PAM-4 constellations designs of interest. The SNR slices selected represent designs that achieve capacities={0.5, 0.75, 1.0, 1.25, 1.5} bits per symbol (bps). Given that PAM-4 can provide at most $\log_2(4)$=2 bps, these design points represent systems with information code rates R={1/4, 3/8, 1/2, 5/8, 3/4} respectively.

FIGS. **12***ab* and **12***cd* present locus plots of PD capacity and joint capacity optimized PAM-8 constellation points versus achievable capacity and SNR. FIGS. **13***a* and **13***b* provide slices from these plots at SNRs corresponding to achievable capacities η={0.5, 1.0, 1.5, 2.0, 2.5} bps. Each of these slices correspond to systems with code rate R=η bps/$\log_2(8)$, resulting in R={1/6, 1/3, 1/2, 2/3, 5/6}. As an example of the relative performance of the constellations in these tables, consider FIG. **13***b* which shows a PD capacity optimized PAM-8 constellation optimized for SNR=9.00 dB, or 1.5 bps. We next examine the plot provided in FIG. **8***a* and see that the gap of the optimized constellation to the ultimate, Gaussian, capacity (CG) is approximately 0.5 dB. At the same spectral efficiency, the gap of the traditional PAM-8 constellation is approximately 1.0 dB. The advantage of the optimized constellation is 0.5 dB for the same rate (in this case R=1/2). This gain can be obtained by only changing the mapper and demapper in the communication system and leaving all other blocks the same.

Similar information is presented in FIGS. **14***abcd*, and **15***ab* which provide loci plots and design tables for PAM-16 PD capacity and joint capacity optimized constellations. Likewise FIGS. **16***abcd*, **17***ab* provide loci plots and design tables for PAM-32 PD capacity and joint capacity optimized constellations.

Capacity Optimized PSK Constellations

Traditional phase shift keyed (PSK) constellations are already quite optimal. This can be seen in the chart **180** comparing the SNR gaps of tradition PSK with capacity optimized PSK constellations shown in FIG. **18** where the gap between PD capacity and Gaussian capacity is plotted for traditional PSK-4,8,16,32 and for PD capacaity optimized PSK-4,8,16,32.

The locus plot of PD optimized PSK-32 points across SNR is shown in FIG. **19**, which actually characterizes all PSKs with spectral efficiency $\eta \leq 5$. This can be seen in FIG. **20**. Note that at low SNR (0.4 dB) the optimal PSK-32 design is the same as traditional PSK-4, at SNR=8.4 dB optimal PSK-32 is the same as traditional PSK-8, at SNR=14.8 dB optimal PSK-32 is the same as traditional PSK-16, and finally at SNRs greater than 20.4 dB optimized PSK-32 is the same as traditional PSK-32. There are SNRs between these discrete points (for instance SNR=2 and 15. dB) for which optimized PSK-32 provides superior PD capacity when compared to traditional PSK constellations.

We note now that the locus of points for PD optimized PSK-32 in FIG. **19** in conjunction with the gap to Gaussian capacity curve for optimized PSK-32 in FIG. **18** implies a potential design methodology. Specifically, the designer could achieve performance equivalent or better than that enabled by traditional PSK-4,8,16 by using only the optimized PSK-32 in conjunction with a single tuning parameter that controlled where the constellation points should be selected from on the locus of FIG. **19**. Such an approach would couple a highly rate adaptive channel code that could vary its rate, for instance, rate 4/5 to achieve and overall (code plus optimized PSK-32 modulation) spectral efficiency of 4 bits per symbol, down to 1/5 to achieve an overall spectral efficiency of 1 bit per symbol. Such an adaptive modulation and coding system could essentially perform on the optimal continuum represented by the rightmost contour of FIG. **18**.

Adaptive Rate Design

In the previous example spectrally adaptive use of PSK-32 was described. Techniques similar to this can be applied for other capacity optimized constellations across the link between a transmitter and receiver. For instance, in the case where a system implements quality of service it is possible to instruct a transmitter to increase or decrease spectral efficiency on demand. In the context of the current invention a capacity optimized constellation designed precisely for the target spectral efficiency can be loaded into the transmit mapper in conjunction with a code rate selection that meets the end user rate goal. When such a modulation/code rate change occurred a message could propagated to the receiver so that the receiver, in anticipation of the change, could select a demapper/decoder configuration in order to match the new transmit-side configuration.

Conversely, the receiver could implement a quality of performance based optimized constellation/code rate pair control mechanism. Such an approach would include some form of receiver quality measure. This could be the receiver's estimate of SNR or bit error rate. Take for example the case where bit error rate was above some acceptable threshold. In this case, via a backchannel, the receiver could request that the transmitter lower the spectral efficiency of the link by swapping to an alternate capacity optimized constellation/code rate pair in the coder and mapper modules and then signaling the receiver to swap in the complementary pairing in the demapper/decoder modules.

Geometrically Shaped QAM Constellations

Quadrature amplitude modulation (QAM) constellations can be constructed by orthogonalizing PAM constellations into QAM inphase and quadrature components. Constellations constructed in this way can be attractive in many applications because they have low-complexity demappers.

In FIG. **21** we provide an example of a Quadrature Amplitude Modulation constellation constructed from a Pulse Amplitude Modulation constellation. The illustrated embodiment was constructed using a PAM-8 constellation optimized for PD capacity at user bit rate per dimension of 1.5 bits (corresponds to an SNR of 9.0 dB) (see FIG. **13***b*). The label-point pairs in this PAM-8 constellation are {(000, −1.72), (001, −0.81), (010, 1.72), (011, −0.62), (100, 0.62), (101, 0.02), (110, 0.81), (111, −0.02)}. Examination of FIG. **21** shows that the QAM constellation construction is achieved by replicating a complete set of PAM-8 points in the quadrature dimension for each of the 8 PAM-8 points in the in-phase dimension. Labeling is achieved by assigning the PAM-8 labels to the LSB range on the in-phase dimension and to the MSB range on the quadrature dimension. The resulting 8×8 outer product forms a highly structured QAM-64 for which very low-complexity de-mappers can be constructed. Due to the orthogonality of the in-phase and quadrature components the capacity characteristics of the resulting QAM-64 constellation are identical to that of the PAM-8 constellation on a per-dimension basis.

N-Dimensional Constellation Optimization

Rather than designing constellations in 1-D (PAM for instance) and then extending to 2-D (QAM), it is possible to take direct advantage in the optimization step of the additional degree of freedom presented by an extra spatial dimension. In general it is possible to design N-dimensional constellations and associated labelings. The complexity of the optimization step grows exponentially in the number of dimensions as does the complexity of the resulting receiver de-mapper. Such constructions constitute embodiments of the invention and simply require more 'run-time' to produce.

Capacity Optimized Constellations for Fading Channels

Similar processes to those outlined above can be used to design capacity optimized constellations for fading channels in accordance with embodiments of the invention. The processes are essentially the same with the exception that the manner in which capacity is calculated is modified to account for the fading channel. A fading channel can be described using the following equation:

$$Y=a(t).X+N$$

where X is the transmitted signal, N is an additive white Gaussian noise signal and a(t) is the fading distribution, which is a function of time.

In the case of a fading channel, the instantaneous SNR at the receiver changes according to a fading distribution. The fading distribution is Rayleigh and has the property that the average SNR of the system remains the same as in the case of the AWGN channel, $E[X^2]/E[N^2]$. Therefore, the capacity of the fading channel can be computed by taking the expectation of AWGN capacity, at a given average SNR, over the Rayleigh fading distribution of a that drives the distribution of the instantaneous SNR.

Many fading channels follow a Rayleigh distribution. FIGS. **22***a*-**24***b* are locus plots of PAM-4, 8, and 16 constellations that have been optimized for PD capacity on a Rayleigh fading channel. Locus plots versus user bit rate per dimension and versus SNR are provided. Similar processes can be used to obtain capacity optimized constellations that are optimized using other capacity measures, such as joint capacity, and/or using different modulation schemes.

What is claimed is:

1. A digital communication system, comprising:

a transmitter configured to transmit signals to a receiver via a communication channel;

wherein the transmitter, comprises:

a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate using a code;

a mapper configured to map said encoded bits to symbols in a phase shift keyed (PSK) symbol constellation;

a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper;

wherein the PSK symbol constellation is a geometrically spaced symbol constellation in which the phases of the constellation points are optimized for capacity using parallel decode capacity that provides a given capacity at a reduced signal-to-noise ratio compared to a PSK signal constellation that maximizes d_{min}.

2. The communication system of claim **1**, wherein the geometrically spaced symbol constellation is capacity optimized subject to additional constraints.

3. The communication system of claim **1**, wherein the code is a Turbo code.

4. The communication system of claim **1**, wherein the code is a LDPC code.

5. The communication system of claim **1**, wherein the channel is an AWGN channel.

6. The communication system of claim **1**, wherein the channel is a fading channel.

7. The communication system of claim **1**, wherein the constellation points of the geometrically spaced symbol constellation are variable depending on the user rate.

8. The communication system of claim **7**, wherein:

the transmitter is configured to modify the rate of the code in order to change the user bit rate of the communication system; and

the transmitter is configured to move the constellation points of the symbol constellation used by the mapper based upon the code rate of the coder.

9. The communication system of claim **1**, wherein the constellation points of the geometrically spaced symbol constellation are variable depending on the channel.

10. The communication system of claim **9**, wherein the constellation points of the geometrically spaced symbol constellation are variable depending on channel SNR.

11. The communication system of claim **1**, wherein the constellation points of the geometrically spaced symbol constellation are variable depending on the channel.

12. The communication system of claim **11**, wherein the constellations points of the geometrically spaced symbol constellation are variable depending on the channel SNR.

13. A digital communication system, comprising:

a receiver configured to receive signals transmitted via communication channel using a PSK constellation;

wherein the receiver, comprises:

a demodulator configured to demodulate the received signal via the communication channel;

a demapper configured to estimate likelihoods of symbols in a PSK constellation from the demodulated signal;

a decoder that is configured to estimate the likelihood of decoded bits from the likelihoods generated by the demapper based upon a code; and

wherein the PSK symbol constellation is a geometrically spaced symbol constellation in which the phases of the constellation points are optimized for capacity using parallel decode capacity that provides a given capacity at a reduced signal-to-noise ratio compared to a PSK signal constellation that maximizes d_{min}.

14. The communication system of claim **13**, wherein the geometrically spaced symbol constellation is capacity optimized subject to additional constraints.

15. The communication system of claim **13**, wherein the code is a Turbo code.

16. The communication system of claim **13**, wherein the code is a LDPC code.

17. The communication system of claim **13**, wherein the channel is an AWGN channel.

18. The communication system of claim **13**, wherein the channel is a fading channel.

19. The communication system of claim **13**, wherein the constellation points of the geometrically spaced symbol constellation are variable depending on a user rate.

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