

Erratum to: Methods of harmonic synthesis for global geopotential models and their first-, second- and third-order gradients

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The expression for the coefficient $A_m^{(rr)}$ given in the third row of the first column of Table 5 should be corrected to include the factor \bar{C}_{nm} as follows:

$$A_m^{(rr)} = \sum_{n=m}^N \left(\frac{a}{r}\right)^n (n+1)(n+2) \bar{C}_{nm} \bar{P}_n^m.$$

Similarly, the expression for the coefficient $A_m^{(rrr)}$ given in the fourth row of the first column of Table 6 should be corrected to include the factor \bar{C}_{nm} as follows:

$$A_m^{(rrr)} = \sum_{n=m}^N \left(\frac{a}{r}\right)^n (n+1)(n+2)(n+3) \bar{C}_{nm} \bar{P}_n^m.$$

The right-hand sides of the formulas appearing in Table 10 should be corrected to include the factor $(1/r)$ as follows:

$$a_1 = \frac{1}{r} \sum_{m=1}^N m \left(A_m^{(1)} c_{m-1} + B_m^{(1)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_2 = \frac{1}{r} \sum_{m=1}^N m \left(B_m^{(1)} c_{m-1} - A_m^{(1)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_3 = \frac{1}{r} \sum_{m=0}^N \left(A_m^{(2)} c_m + B_m^{(2)} s_m \right) \cos^m \phi$$

$$a_4 = -\frac{1}{r} \sum_{m=0}^N \left(A_m^{(3)} c_m + B_m^{(3)} s_m \right) \cos^m \phi.$$

The right-hand sides of the formulas appearing in Table 11 should be corrected to include the factor $(1/r)^2$. The expression of the coefficient a_{11} contains further misprints. Hence, the formulas of Table 11 should read:

$$a_{11} = \frac{1}{r^2} \sum_{m=2}^N m(m-1) \left(A_m^{(1)} c_{m-2} + B_m^{(1)} s_{m-2} \right) \cos^{m-2} \phi$$

$$a_{12} = \frac{1}{r^2} \sum_{m=2}^N m(m-1) \left(B_m^{(1)} c_{m-2} - A_m^{(1)} s_{m-2} \right) \cos^{m-2} \phi$$

$$a_{13} = \frac{1}{r^2} \sum_{m=1}^N m \left(A_m^{(2)} c_{m-1} + B_m^{(2)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_{14} = -\frac{1}{r^2} \sum_{m=1}^N m \left(A_m^{(3)} c_{m-1} + B_m^{(3)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_{22} = -a_{11}$$

$$a_{23} = \frac{1}{r^2} \sum_{m=1}^N m \left(B_m^{(2)} c_{m-1} - A_m^{(2)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_{24} = -\frac{1}{r^2} \sum_{m=1}^N m \left(B_m^{(3)} c_{m-1} - A_m^{(3)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_{33} = \frac{1}{r^2} \sum_{m=0}^N \left(A_m^{(4)} c_m + B_m^{(4)} s_m \right) \cos^m \phi$$

$$a_{34} = -\frac{1}{r^2} \sum_{m=0}^N \left(A_m^{(5)} c_m + B_m^{(5)} s_m \right) \cos^m \phi$$

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$$a_{44} = \frac{1}{r^2} \sum_{m=0}^N \left(A_m^{(6)} c_m + B_m^{(6)} s_m \right) \cos^m \phi.$$

The right-hand sides of the formulas appearing in Table 12 should be corrected by including the factor $(1/r)^3$. The expressions of the coefficients a_{111} , a_{143} , a_{222} and a_{444} contain further misprints. Hence, the formulas of Table 12 should read:

$$a_{111} = \frac{1}{r^3} \sum_{m=3}^N m(m-1)(m-2) \times \left(A_m^{(1)} c_{m-3} + B_m^{(1)} s_{m-3} \right) \cos^{m-3} \phi$$

$$a_{112} = -a_{222}$$

$$a_{113} = \frac{1}{r^3} \sum_{m=2}^N m(m-1) \left(A_m^{(2)} c_{m-2} + B_m^{(2)} s_{m-2} \right) \cos^{m-2} \phi$$

$$a_{114} = -\frac{1}{r^3} \sum_{m=2}^N m(m-1) \left(A_m^{(3)} c_{m-2} + B_m^{(3)} s_{m-2} \right) \cos^{m-2} \phi$$

$$a_{123} = \frac{1}{r^3} \sum_{m=2}^N m(m-1) \left(B_m^{(2)} c_{m-2} - A_m^{(2)} s_{m-2} \right) \cos^{m-2} \phi$$

$$a_{124} = -\frac{1}{r^3} \sum_{m=2}^N m(m-1) \left(B_m^{(3)} c_{m-2} - A_m^{(3)} s_{m-2} \right) \cos^{m-2} \phi$$

$$a_{143} = -\frac{1}{r^3} \sum_{m=1}^N m \left(A_m^{(5)} c_{m-1} + B_m^{(5)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_{221} = -a_{111}$$

$$a_{222} = -\frac{1}{r^3} \sum_{m=3}^N m(m-1)(m-2) \times \left(B_m^{(1)} c_{m-3} - A_m^{(1)} s_{m-3} \right) \cos^{m-3} \phi$$

$$a_{223} = -a_{113}$$

$$a_{224} = -a_{114}$$

$$a_{243} = -\frac{1}{r^3} \sum_{m=1}^N m \left(B_m^{(5)} c_{m-1} - A_m^{(5)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_{331} = \frac{1}{r^3} \sum_{m=1}^N m \left(A_m^{(4)} c_{m-1} + B_m^{(4)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_{332} = \frac{1}{r^3} \sum_{m=1}^N m \left(B_m^{(4)} c_{m-1} - A_m^{(4)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_{333} = \frac{1}{r^3} \sum_{m=0}^N \left(A_m^{(7)} c_m + B_m^{(7)} s_m \right) \cos^m \phi$$

$$a_{334} = -\frac{1}{r^3} \sum_{m=0}^N \left(A_m^{(8)} c_m + B_m^{(8)} s_m \right) \cos^m \phi$$

$$a_{441} = \frac{1}{r^3} \sum_{m=1}^N m \left(A_m^{(6)} c_{m-1} + B_m^{(6)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_{442} = \frac{1}{r^3} \sum_{m=1}^N m \left(B_m^{(6)} c_{m-1} - A_m^{(6)} s_{m-1} \right) \cos^{m-1} \phi$$

$$a_{443} = \frac{1}{r^3} \sum_{m=0}^N \left(A_m^{(9)} c_m + B_m^{(9)} s_m \right) \cos^m \phi$$

$$a_{444} = -\frac{1}{r^3} \sum_{m=0}^N \left(A_m^{(10)} c_m + B_m^{(10)} s_m \right) \cos^m \phi.$$

The expression for the coefficient $A_m^{(2)}$ in Table 13 should read:

$$A_m^{(2)} = \sum_{n=m}^N \rho_n \bar{C}_{nm} d\bar{H}_n^m / du.$$

There are missing terms in the expression for the function \bar{W}_n^m in Table 14. The formula should read:

$$\begin{aligned} \bar{W}_n^m &= (n+m+1)(n+m+2)(n+m+3)\bar{H}_n^m \\ &\quad + 3u(n+m+2)(n+m+3)d\bar{H}_n^m / du \\ &\quad + 3u^2 d^2 \bar{H}_n^m / du^2 \\ &\quad + u^3 d^3 \bar{H}_n^m / du^3. \end{aligned}$$

The letter l in Eqs. (76) and (77) should be replaced by the letter r , so that the two equations read:

$$\begin{aligned} V_{nm} &= \frac{(-1)^n}{(n-m)!} \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right)^m \left(\frac{\partial}{\partial z} \right)^{(n-m)} \left(\frac{1}{r} \right), \\ V_{nn} &= (-1)^n \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right)^n \left(\frac{1}{r} \right), \end{aligned}$$

respectively.

A + sign should be replaced by a - sign in Eq. (94), in the following way:

$$\begin{aligned} \bar{S}_{nm}^{(\alpha\beta\gamma)} &= \frac{(-1)^{\beta/2}}{(-2)^{\alpha+\beta}} \sum_{p=0}^{\alpha} \sum_{q=0}^{\beta} \left[\epsilon^+ A^+ \bar{S}_{n-\eta, m+\sigma} \right. \\ &\quad \left. - (1 - \delta_{0m}) \epsilon^- A^- \bar{S}_{n-\eta, -m+\sigma} \right]. \end{aligned}$$

Finally, the term 3η appearing in the upper limits of the summations in Eqs. (101), (102) and (103) and related text should be replaced by η :

$$\frac{\partial^\eta V}{\partial x^\alpha \partial y^\beta \partial z^\gamma} = \frac{GM}{a^{1+\eta}} \sum_{m=0}^{N+\eta} \left(A_m^{(\alpha\beta\gamma)} \cos m\lambda + B_m^{(\alpha\beta\gamma)} \sin m\lambda \right),$$

$$A_m^{(\alpha\beta\gamma)} = \sum_{n=m}^{N+\eta} \bar{C}_{nm}^{(\alpha\beta\gamma)} \bar{E}_{nm},$$

$$B_m^{(\alpha\beta\gamma)} = \sum_{n=m}^{N+\eta} \bar{S}_{nm}^{(\alpha\beta\gamma)} \bar{E}_{nm}.$$

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