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Metrics of Positive Scalar Curvature on Spheres and the Gromov-Lawson Conjecture

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1. Introduction

In [2] and [9] an ingenious procedure is given to construct a Riemannian metric of positive scalar curvature on a manifold obtained by surgery from one which already has such a metric. With some improvements of [1] this may be summarized as follows.

1.1. **Theorem.** *Let (M, g) be a compact n-dimensional Riemannian manifold with positive scalar curvature and let W be a cobordism from M to M′ such that W admits a handle decomposition on M with no handles of index greater than $n-2$ (i.e. there exists a Morse function on W which is minimal on M and critical points have indices $\leqq n-2$). Then there exists a metric of positive scalar curvature on W which extends g and is product on a collar of $M \cup M'$.*

This construction applied to the cobordism with one handle of index 1 between the disjoint sum $M \cup N$ and the connected sum $M \# N$ shows immediately that the connected sum is well defined for manifolds of positive scalar curvature. Furthermore, this operation gives an abelian group structure in the set π_n^c of concordance classes of positive scalar curvature metrics on S^n, with the zero class represented by the standard metric g_{can} [1]. We say that two metrics g_0, g_1 of positive scalar curvature on M are concordant if there exists a metric g of positive scalar curvature on $M \times [0,1]$ such that $g|M \times \{i\} = g_i$, $i=0,1$, and g is product near $M \times \partial[0,1]$. Our aim is to show how this group, or its subgroup $\tilde{\pi}_n^c$ of classes of those metrics which are boundary restrictions of metrics of positive scalar curvature on compact spin manifolds, is related to some questions concerning positive scalar curvature.

If X^n is a 2-connected closed manifold, B a smooth n-ball in X, then by Morse-Smale theory and Theorem 1.1 there is a metric of positive scalar curvature on $X - \operatorname{Int} B$ which is product near $S^{n-1} = \partial(X - \operatorname{Int} B)$. This metric induces a metric of positive scalar curvatures on S^{n-1}. Our basic observation is that the concordance class $\delta(X)$ of the induced metric depends only on the spin cobordism class of X and $\delta(X) = 0$ if and only if any 1-connected manifold spin cobordant to X admits a metric of positive scalar curvature. The resulting map $\delta: \Omega_n^{\mathrm{Spin}} \to \pi_n^c$ fits into a long exact sequence relating Ω_*^{Spin} with cobordism groups Ω_*^{PSC} of Riemannian spin manifolds with positive scalar curvature:

$$\ldots \to \Omega_n^{\mathrm{PSC}} \to \Omega_n^{\mathrm{Spin}} \xrightarrow{\delta} \pi_{n-1}^c \xrightarrow{i} \Omega_{n-1}^{\mathrm{PSC}} \to \ldots.$$

The homomorphism i sends any metric on S^n to its cobordism class, so $\ker i = \tilde{\pi}_n^c$ and we see that the geometric part of Ω_n^{PSC} is identified with $\pi_n^c/\tilde{\pi}_n^c$.

In Sect. 3 we study the action of the group $\mathrm{Diff}_+ S^n$ of orientation preserving diffeomorphisms of S^n on π_n^c given by $f[g]=[f^*g]$, where f^*g denotes the metric induced by f from g. This is in fact an action of $\pi_0 \mathrm{Diff}_+ S^n$ and we show that it factorizes through an action of the group $\mathbb{Z}_2$ which is free for $n=8k$ or $8k+1$, $k=1,2,\ldots$, and trivial in all other dimensions.

Finally, we discuss the Gromov-Lawson conjecture about existence of positive scalar curvature metrics on 1-connected spin manifolds. It states that a 1-connected spin manifold M admits a positive scalar curvature metric if and only if the generalized $\hat{A}$-genus (with values in $\widetilde{KO}_*$) of M vanishes. This is proved in [2] mod torsion. The torsion part of the conjecture reduces to the following.

Conjecture. *If x is an element of order 2 in $\tilde{\pi}_n^c$, then x contains a representative isometric to the standard metric on S^n.*

We show that metrics isometric to the standard one are characterized by having a symmetric representative in the concordance class, where symmetry means that the metric is invariant under an involution fixing a sphere of codimension 1.

2. The Obstruction to Positive Scalar Curvature

We start with some notation and definitions. Any compact manifold with non-empty boundary admits a metric of positive sectional curvature, so working with such manifolds usually requires the condition that metrics are product on a collar of the boundary to be kept. Thus when we say that a metric on a manifold with boundary has positive scalar curvature we shall understand that the metric is product near the boundary. We denote by $\mathrm{PSC}(M)$ the set of such metrics on M.

We say that two closed n-dimensional spin manifolds with metrics of positive scalar curvature (M_1, g_1), (M_2, g_2) are cobordant if there exist a compact spin manifold (W, g) of positive scalar curvature and a diffeomorphism $f: M_1 \cup (-M_2) \to \partial W$ such that both spin structure and the metric given on $M_1 \cup (-M_2)$ are equal to the spin structure and the metric induced by f from W (hence the same for orientation; $-M$ denotes M with the reversed orientation). The set of cobordism classes Ω_n^{PSC} has an abelian group structure given by disjoint sum (or, equivalently, by connected sum).

We have the forgetful homomorphism $\pi_n^c \to \Omega_n^{\mathrm{PSC}}$ and the subgroup $\tilde{\pi}_n^c$ of π_n^c is defined as its kernel. Moreover, the natural action of $\pi_0 \mathrm{Diff}_+ S^n$ on π_n^c defined by $[f][g]=[f^*g]$, preserves the cobordism class of (S^n, g).

Let N be a closed spin manifold of dimension $n>4$. There exists a 2-connected manifold M^* spin cobordant to N. Let M be the manifold obtained from M^* by deleting the interior of a smoothly embedded disc $D^n \subset M^*$. Since M^* was 2-connected, Theorem 1.1 gives a metric g of positive scalar curvature on M. Denote by $\delta(N) \in \pi^c_{n-1}$ the concordance class of the metric $g|\partial D^n$, or, more precisely, the concordance class of the metric $f^*(g|\partial M)$, where $f: S^{n-1} \to \partial M$ is the diffeomorphism resulting from the embedding $D^n \to M^*$.

2.1. **Proposition.** *$\delta(N)$ depends only on the spin cobordism class of N.*

Proof. If M_1^*, M_2^* are 2-connected spin manifolds which are spin cobordant to N, then there exists a 2-connected cobordism between them. Equivalently, $M_1^* \#(-M_2^*)$ is cobordant to S^n by a cobordism with no handles of index greater then $n-2$. When we remove in this cobordism two copies of $\mathrm{Int}(D^n) \times I$ such that one interval $\{0\} \times I$ joins a point of S^n with a point in M_1^* and the other interval joins another point of S^n with a point in M_2^*, we get a relative cobordism of $(M_1 \#(-M_2), \partial M_1 \cup \partial M_2)$ with $(S^{n-1} \times I, S^{n-1} \times \partial I)$. Applying 1.1 to this cobordism and the metric $g_1 \# g_2$ on $M_1 \#(-M_2)$ [where g_1, g_2 are as in the definition of $\delta(N)$] we get a concordance between $g_1|S^{n-1}$ and $g_2|S^{n-1}$.

If $\delta(N)=0$ and N is itself 2-connected, then the metric constructed on $N-\mathrm{Int}\, D^n$ extends to a positive scalar curvature metric on N. In fact, the following shows that for 1-connected N, $\delta(N)$ is the obstruction to find a metric of positive scalar curvature on N and $\tilde{\pi}^c_n$ is the obstruction group.

2.2. **Theorem.** *If $x \in \Omega^{\mathrm{Spin}}_n$, $n>4$, then $\delta(x)=0$ if and only if any 1-connected manifold in the class x admits a metric of positive scalar curvature.*

Proof. Any 1-connected representative of x is obtained from a 2-connected manifold by a sequence of surgeries of index $n-2$ (because one can kill π_2 by surgeries of index 3). Since on any 2-connected manifold in x we have a positive scalar curvature metric when $\delta(x)=0$, we have such a metric on any 1-connected manifold representing x.

For the reverse consider a 2-connected manifold M^* in the class x, $g_1 \in \mathrm{PSC}(M)$ as in the definition of δ, and $g_2^* \in \mathrm{PSC}(M^*)$. Then g_2^* may be deformed to a metric which is standard "torpedo" metric in a disc in M, resulting in a metric $g_2 \in \mathrm{PSC}(M)$ which is standard on the boundary. Now one uses the same argument as in 2.1 with $M_1 = M_2 = M$ to get a concordance from $\delta(x)$ to $g_2|S^{n-1} = g_{\mathrm{can}}$.

The resulting map $\Omega^{\mathrm{Spin}}_n \to \pi^c_{n-1}$ is a homomorphism of groups and fits to the following exact sequence.

2.3. **Corollary.** *The sequence of group homomorphisms*

$$\dots \xrightarrow{\delta} \pi^c_n \to \Omega^{\mathrm{PSC}}_n \to \Omega^{\mathrm{Spin}}_n \xrightarrow{\delta} \pi^c_{n-1} \to \dots \to \Omega^{\mathrm{Spin}}_4$$

is exact (where unlabeled maps are forgetful homomorphisms).

Remarks. 1. Since $\mathrm{Im}\,\delta = \tilde{\pi}^c_{n-1}$, we have for $n>4$ a short exact sequence

$$0 \to \pi^c_n/\tilde{\pi}^c_n \to \Omega^{\mathrm{PSC}}_n \to \Omega^{\mathrm{Spin}}_n \to \tilde{\pi}^c_{n-1} \to 0 .$$

Thus $\pi^c_*/\tilde{\pi}^c_*$ is the geometric part of Ω^{PSC}_*. In particular, Ω^{PSC}_n is countable, since π^c_n is [every class in π^c_n is an open set in the separable space of metrics $\mathrm{PSC}(S^n)$]. Note that geometric cobordism groups may be very large, as for example the continuous variation of characteristic classes of t-structures [5, 10] produces uncountably many cobordism classes of complete manifolds of finite volume and bounded curvature.

2. Results of this section can be carried over to the case of manifolds (and cobordisms) with fixed fundamental group π. To do this we have to replace the disc in the above constructions by a "thickening" $D(\pi)$ of π (i.e. a manifold with the fundamental group isomorphic to π and of homotopy type of a 2-complex), the sphere by $\partial D(\pi)$ and define appropriately the juxtaposition in $\mathrm{PSC}(D(\pi))$. For instance, if $\pi = Z$, as $D(\pi)$ we may take $D^{n-1} \times S^1$ and the juxtaposition in $\mathrm{PSC}(S^{n-2} \times S^1)$ is given by "round" connected sum along $\{\text{point}\} \times S^1 \subset \partial D(\pi)$.

3. The Action of $\mathrm{Diff}_+ S^n$ on π_n^c

Let $\alpha: \Omega_n^{\mathrm{Spin}} \to \widetilde{KO}(S^n)$ denote the generalized $\hat{A}$-genus [6]. Then $\ker\delta \subset \ker\alpha$, thus the map $a = \alpha \circ \delta^{-1}: \tilde{\pi}_*^c \to KO_{*+1}(pt)$ is a well defined homomorphism.

Remark. There is a natural ring structure in $\tilde{\pi}_*^c$ such that a is a ring homomorphism. The Gromov-Lawson conjecture says that a is a monomorphism, in particular $a \otimes Q$ is an isomorphism [2].

Consider also the natural homomorphism

$$t: \pi_0 \,\mathrm{Diff}_+ S^n \to \tilde{\pi}_n^c : [f] \mapsto [f^* g_{\mathrm{can}}]$$

(note that $f^* g_{\mathrm{can}}$ bounds). These homomorphisms yield the following commutative diagram

3.1.

$$\begin{array}{ccc} \pi_0\,\mathrm{Diff}_+ S^n & \xrightarrow{\ t\ } & \tilde{\pi}_n^c \\ \downarrow \cong & \nearrow \delta & \downarrow a \\ \Theta^{n+1} & \xrightarrow{\ \alpha\ } & \widetilde{KO}(S^{n+1}) \end{array}$$

where Θ^k is the group of (concordance classes of) homotopy k-spheres and the left vertical arrow is the isomorphism given by $f \to \Sigma_f = D^{n+1} \cup_f D^{n+1}$, $n > 4$.

3.2. **Lemma.** *For any $x \in \pi_n^c$ and $f \in \pi_0 \,\mathrm{Diff}_+ S^n$ we have*

$$fx = x + t(f).$$

Proof. Let D_0, D_0' be smoothly embedded n-discs in S^n, $D_0' \subset \mathrm{Int}\, D_0$. In any class x there is a metric of positive scalar curvature such that $g|D_0$ is the standard torpedo metric, and any f can be represented by a diffeomorphism which is the identity on the complement of D_0'. This gives the required decomposition of f^*g as the connected sum of g and $t(f) = f^* g_{\mathrm{can}}$.

3.3. **Lemma.** $t(f) = 0$ *if and only if Σ_f admits a metric of positive scalar curvature.*

Proof. The condition $t(f) = 0$ means that $f^* g_{\mathrm{can}}$ is concordant to g_{can}. Such a concordance $(S^n \times I, h)$ gives a metric of positive scalar curvature on Σ_f, since we may give the lower disc the standard metric, extend it by h on a collar in the upper disc and finish with the standard metric on the remaining part of the upper disc. If Σ_f has a metric of positive scalar curvature, then $t(f)=\delta\Sigma_f=0$ because the composition $\Omega_{n+1}^{\mathrm{PSC}}\to\Omega_{n+1}^{\mathrm{Spin}}\to\pi_n^c$ is zero.

Let $I_x\subset\pi_0\,\mathrm{Diff}_+\,S^n$ be the isotropy subgroup of $x\in\pi_n^c$, I_0 the isotropy subgroup of $[g_{\mathrm{can}}]$.

3.4. **Lemma.** *$I_x=I_0=\ker t$. The action of $\pi_0\,\mathrm{Diff}_+\,S^n$ factorizes through a free action of $\pi_0\,\mathrm{Diff}_+\,S^n/I_0$.*

Proof. It follows from Lemma 3.2 and the definition of t.

To describe properties of the action, we shall need the following characterization of homotopy spheres which admit metrics of positive scalar curvature.

3.5. **Proposition.** *Let $n\geqq 4$. For a homotopy n-sphere Σ, the following conditions are equivalent:*

(i) *Σ is the boundary of a spin manifold,*
(ii) *Σ admits a metric of positive scalar curvature,*
(iii) *$\alpha(\Sigma)=0$.*

Proof. From (i) we know that Σ is the boundary of a manifold X such that $\pi_1(X,\Sigma)=\pi_2(X,\Sigma)=0$ and by 1.1 we can find on Σ a metric with positive scalar curvature. To see that (ii) implies (iii) apply the Lichnerowicz-Hitchin theorem [4]. The implication (iii)→(i) follows from the fact that the image of the framed cobordism in Ω_*^{Spin} is detected by the torsion part of α (cf. [8, Chap. XI]). In particular, for any homotopy sphere Σ, $\alpha(\Sigma)=0$ if and only if Σ is a spin boundary.

3.6. **Theorem.** *The action of $\pi_0\,\mathrm{Diff}_+\,S^n$ on π_n^c factorizes through a free action of $\pi_0\,\mathrm{Diff}_+\,S^n/I_0$, which is isomorphic to Z_2 if $n=8k$ or $8k+1$, $k=1,2,\ldots$, and is trivial in all other dimensions.*

Proof. It is known ([6], see also [4, 4.3]) that for $n=8k$ and $8k+1$, $k\geqq 1$, the homomorphism $\alpha:\Theta^{n+1}\to\widetilde{KO}(S^{n+1})=Z_2$ is onto. In the remaining dimensions α is trivial on Θ^{n+1}. If $n\leqq 3$, then $\pi_0\,\mathrm{Diff}_+\,S^n=0$. For $n\geqq 4$ we have $I_0=\ker(\alpha|\Theta^{n+1})$ by 3.5, thus $\pi_0\,\mathrm{Diff}_+\,S^n/I_0\cong\Theta^{n+1}/\ker\alpha$ gives the required isomorphism.

4. Symmetric Metrics on S^n and the Gromov-Lawson Conjecture

In [2] Gromov and Lawson proved that if M is a 1-connected spin manifold and $\alpha(M)=0$, then for some k, k-fold connected sum $kM=M\#M\#\ldots\#M$ (k times) admits a metric of positive scalar curvature. They conjectured that $k=1$ is good enough (cf. [3] for a general form and discussion of the conjecture). Later Miyazaki [7] has shown that one may take $k=4$. Suppose that we know that the following is true.

4.1. *For any 1-connected spin manifold M, $M\#M$ admits a metric of positive scalar curvature if and only if $M\#\Sigma$ admits such metric, where Σ is a homotopy sphere.*

Consider a 1-connected manifold with $\alpha(M)=0$. Then $4M$ admits a metric of positive scalar curvature and 4.1 implies that $2M\#\Sigma$ does. Since $\alpha(\Sigma)=2\alpha(M)+\alpha(\Sigma)=\alpha(2M\#\Sigma)=0$, we see by 3.5 that Σ has a positive scalar curvature metric.

So the homotopy sphere inverse to Σ, and therefore $2M$ admit positive scalar curvature metrics. Repeating the argument we get a metric of positive scalar curvature on M. When we pass by δ to $\tilde{\pi}_n^c$ one sees immediately that the following implies the torsion part of the Gromov-Lawson conjecture for 1-connected manifolds.

4.2. **Conjecture.** *Any element of order 2 in $\tilde{\pi}_n^c$ has a representative isometric to the standard metric of S^n.*

In other words, if $[g]\in\pi_n^c$ and $g\# g$ is concordant to the standard metric g_{can}, then g should be concordant to f^*g_{can} for some $f\in \operatorname{Diff} S^n$. The following proposition says that up to concordance the property "isometric to the standard one" is equivalent to symmetry, thus it shows that Z_2-symmetries of positive scalar curvature metrics on S^n are reflected as the torsion of $\tilde{\pi}_n^c$.

Let $S^k=\{x\in R^{k+1}: \|x\|=1\}$, $D_\pm^k=\{x\in S^k: \pm x_{k+1}\geqq 0\}$. We say that a metric $g\in \operatorname{PSC}(S^n)$ is symmetric if $g=f^*g$ for an orientation reversing smooth involution f of S^n such that $f|S^{n-1}=\text{id}$.

4.3. **Proposition.** *A metric $g\in\operatorname{PSC}(S^n)$ is concordant to a symmetric metric if and only if g is concordant to φ^*g_{can} for some $\varphi\in\operatorname{Diff}_+S^n$.*

Proof. Consider first the example $g=\phi^*g_{\text{can}}$. Deform ϕ to a diffeomorphism ϕ_1 equal to Id on D_-^n and on a collar of S^{n-1} and define f as $T\phi_1$ on D_+^n and $\phi_1^{-1}T$ on D_-^n, where T is the linear involution $(x_1,\dots,x_{n+1})\to(x_1,\dots,x_n,-x_{n+1})$. The metric $g_1=\phi_1^*g_{\text{can}}$ is symmetric under f.

Suppose now $g=f^*g$. At the expence of changing g in its concordance class we may replace f by another diffeomorphism which is equal to T in a neighbourhood of S^{n-1}. This is done by conjugating f with a diffeomorphism ϕ isotopic to the identity. Namely, let $\alpha_x:(-\varepsilon,\varepsilon)\to S^n$ be the geodesic through $x\in S^{n-1}$ such that $f\alpha_x(t)=\alpha_x(-t)$. By uniqueness of collars there is a smooth isotopy of id_{S^n} which is the identity on S^{n-1}, to a diffeomorphism ϕ such that $\phi(x,t)=\alpha_x(t)$ on $S^{n-1}\times(-\varepsilon,\varepsilon)$. Then in $S^{n-1}\times(-\varepsilon,\varepsilon)$ we have $\phi^{-1}f\phi(x,t)=\phi^{-1}f\alpha_x(t)=\phi^{-1}\alpha_x(-t)=(x,-t)$.

Let $f_1=\phi^{-1}f\phi$. Obviously $f_1^2=\text{id}$ and for $g_1=\phi^*g$ we have $f_1^*g_1=g_1$. Now rotate D_+^n with the metric $g_1|D_+^n$ around S^{n-1} in D_-^{n+1} in the following way. Decompose D_-^{n+1} as the family of discs $D(x_0)$ with the common boundary S^{n-1}, parametrized by the angle x_0 between $D(x_0)$ and $D(0)=D_+^n$, $x_0\in[0,\pi]$. Let $h: R\to R$ be a smooth function with the following properties: (i) $h(t)=\sin(t)$ for $|t|\leqq 1$ and $h(t)=t/|t|$ for $|t|\geqq 2$; (ii) $h'(t)\geqq 0$, $t\cdot h''(t)\leqq 0$, and $h(-t)=-h(t)$ for any t; (iii) $|t\cdot h(t)|\leqq C\cdot|h''(t)|$ for some constant C. Define the rotated metric as

$$\hat{g}=\hat{g}_\varepsilon=\{\varepsilon h(\varepsilon^{-1}r(x))\}^2dx_0^2+g_1,$$

where $x\in D_+^n$, $r(x)$ is the distance from x to S^{n-1} and ε is a small positive number. Each hyperplane $D(x_0)$ is totally geodesic in D_-^{n+1}, in particular the mean curvature of the boundary is zero. To see that the scalar curvature of $\hat{g}$ is positive we can use the formula (cf. [3, Proposition 7.33]):

$$k(\hat{g})=k(g_1)-2f^{-1}\Delta f$$

with $f=\varepsilon h(\varepsilon^{-1}r(x))$. This formula implies that:

(i) if $r(x)\geqq 2\varepsilon$ then $k(\hat{g})=k(g_1)>0$;

(ii) if $r(x)\leqq 2\varepsilon$ and $\Delta r(x)\leqq 0$ then

$$k(\hat{g})-k(g_1)=1/2\{-h''(t)\varepsilon^{-2}\|\Delta r(x)\|^2-\varepsilon^{-1}h'(t)\Delta r(x)\}/h(t)\geqq 0$$

[here $t=\varepsilon^{-1}r(x)$];

(iii) if $r(x)\leqq 2\varepsilon$ and $\Delta r(x)\geqq 0$ then

$$\begin{aligned} k(\hat{g})-k(g_1)&=1/2\{-h''(t)\varepsilon^{-2}-t\cdot h'(t)\Delta r(x)/r(x)\}/h(t)\\ &\geqq -h''(t)\{\varepsilon^{-2}-C\Delta r(x)/r(x)\}/h(t), \end{aligned}$$

which is positive for small ε since $\Delta r(x)/r(x)$ is bounded.

We have a metric of positive scalar curvature on D^{n+1}_- with zero mean curvature and positive scalar curvature on $S^n=\partial D^{n+1}_-$. Any such metric yields a metric of positive scalar curvature which near S^n is the product of the given metric of S^n by an interval. This is done by a construction due to Gromov and Lawson extended to this case by Almeida (Theorem 4.1 in his thesis). On the boundary sphere we have got the metric equal to g_1 on D^n_+ and to T^*g_1 on D^n_-, hence it is of the form ϕ^*g_1, if ϕ is defined as the identity on D^n_+ and f_1T on D^n_-. The required concordance is given by the trick of making a positive scalar curvature metric standard along a small n-sphere in $\operatorname{Int}D^{n+1}$, cf. [2].

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