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Metropolitan Spatial Structure and its Determinants: A Case-study of Tokyo

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Summary. In this paper, to describe accurately the spatial structure of the Tokyo metropolitan area, the cubic-spline density function approach is applied and modified. It is shown that, during the 1975–85 decade, there was a substantial suburbanisation in the whole study area, while its three directional regions displayed three different spatial patterns. In identifying the determinants of the spatial dynamics, a varying-parameter model is proposed to confirm that increasing income and decreasing transportation cost would cause suburbanisation. The spatial differences in the directional region are also related to their different values of income and transportation cost.

Introduction

There have been many studies in the empirical literature investigating urban spatial structure. However, most of the works have concentrated on the spatial structure of a single city, or an *intra-urban* system. As the modern urban system becomes increasingly large and complex it becomes important to look at the urban space in a more global view, say from the perspective of the *inter-urban* system. The methods and techniques developed from studying the intra-urban structure seem to need to be improved and reconsidered if they are to be applied to the situation of the inter-urban structure. The term 'metropolitan' used here implies a broad urban space which may include the intra-urban system and also the inter-urban system as well. The purpose of this paper is to present a case-study of the Tokyo metropolitan area to show some advanced methods in investigating the metropolitan spatial structure and its determinants.

The paper is organised as follows. In the second section, we shall briefly review several earlier works concerning metropolitan spatial structure, and then introduce the approaches used. In the third section, the method of cubic-spline density function is applied and modified to describe accurately the spatial distribution of population and its variations over time in the Tokyo metropolitan area. From the estimated density functions we can see a substantial suburbanisation in the area during the 1975–85 period, and three different spatial patterns in three directional regions within the area. To explain the estimation results, we propose a varying-parameter model in the fourth section to identify the determinants of population density dynamics. It is confirmed that increasing income and decreasing transportation cost would cause suburbanisation: the difference in income and transportation cost in the directional regions is

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used to explain the spatial differences. Finally, the fifth section gives the conclusion and suggests some further directions to extend the present work.

Brief Review of Previous, Related Studies

In the spatial structure of a metropolitan area (or an inter-urban system), experience often shows that there exist some central places which are densely inhabited. In some cases, these centres are spatially located in a central part of the area, forming the so-called monocentric pattern. Sometimes, they may appear spatially deconcentrated and distributed far away from each other, in a way characteristic of multicentricity. Relatively few theoretical works in the field of urban economics deal with this type of situation. White (1976), Miyao (1981, ch. 13) and Fujita and Ogawa (1982) have analysed some monocentric and multicentric patterns in different frameworks. Their main conclusion is that the determination of the spatial patterns depends upon the relative cost of the interaction between business firms and the transportation of residents. Recently, Zheng (1990) presented a model of an inter-urban system in which, besides monocentric and multicentric cases, a separate pattern is derived, in which cities are spatially split by agricultural land. In addition to the relative cost, Zheng shows that the determination of inter-urban structure will also be due to levels of urban residential income. That is, when cities are concentrated on the central part, or monocentricity appears, high incomes are needed for the urban residents to pay the high rent of land (or housing). In contrast, the multicentric pattern requires a relatively lower level of income, and the separate pattern the lowest income level of the three.

Turning to the empirical works of urban spatial structure, we note that they contain the following two main aspects: descriptions of the spatial structure, and an identification of the determinants of it and its dynamics over time. In describing urban spatial structure, most of the works use the population density function approach. Among them, the most frequently applied function form is the negative exponential, such as in Clark (1951), Muth (1969), Mills (1972, ch. 3) and Macauley (1985). Besides this, there are also other density function forms, such as general normal function (Newling, 1969), gamma function (Aynvary, 1969) and lognormal function (Parr *et al.*, 1988), which have been used for estimation. (See McDonald and Bowman (1976) and McDonald (1989) for a detailed survey.) However, in reality the metropolitan spatial structure always displays various patterns, sometimes monocentricity and sometimes multicentricity, which seems much more complicated than any single density function form can effectively represent. It seems that we also need a generalised density function form that has greater flexibility to describe the density–distance relationship of various patterns. To meet such a demand, recently Anderson (1982, 1985) has developed an approach of cubic-spline density function which uses a number of cubic-degree polynomials to represent the density–distance relationship. Anderson's works are carried out in the intra-urban situation and seem to be required to be extended to the metropolitan context. Also, the cubic-spline approach should be accompanied by an identification of spatial determinants, which is obviously lacking in the works of Anderson (1982, 1985) and Skaburskis (1989).

In identifying the determinants of urban spatial structure, we have the works of Muth (1969), Mills (1972, ch. 3), Johnson and Kau (1980), Alperovich (1983) and Lahiri and Numrich (1983), among others, which have shown that the determinants involve income, transportation cost, city size, city age and so forth. Most of these works applied a varying-parameter model in which the explanatory variables are introduced to represent the parameters in the negative exponential density function. However, the use of negative exponential function form may give rise to questions regarding the reliability of estimation. Given that the appropriateness of the exponential density form has been questioned (see Mills, 1970; Kau and Lee, 1976; McDonald and Bowman, 1976), one may ask whether the determinants can be efficiently identified by using the exponential density function as the regression basis. In addition, all of these works were concerned with the intra-urban system. In this sense, the confirmation of the determinants should be carried out on a more general basis and in the context of the inter-urban system.

In summary, the foregoing review introduces the cubic-spline estimation approach and a varying-parameter model, which will be used in the following two sections to study the spatial structure and its determinants of the Tokyo metropolitan area.

Metropolitan Spatial Structure and the Dynamics

The objective of this section is to apply the technique of cubic-spline estimation to describe accurately the spatial structure of the Tokyo metropolitan area and the spatial dynamics during the past period of 1975–85.

Cubic-spline Density Functions

The cubic-spline density function approach is to estimate the density–distance relationship of a distance interval by using a few piecewise, continuous cubic polynomials. In econometrics, the cubic-spline technique is also called switching regression, and belongs to the varying-parameter models (see Maddala, 1977, ch. 17; Judge *et al.*, 1980, ch. 10). It has been applied by Anderson (1982, 1985) to study urban density–distance patterns. In this section, we shall follow Anderson's procedure to estimate the density functions of the Tokyo metropolitan area, and give some modification to his approach so as to get more accurate estimation results.

The approach of cubic-spline density functions can be stated as follows. Suppose there exists a centre in the metropolitan area and any location within the study area can be expressed by its distance to the centre. Cubic-spline estimation at first divides the distance interval $[X_0, X_B]$ in question into several segments of *equal length*. The generalised density function, according to Suits *et al.* (1978), can be written as

$$\begin{aligned} D_t = a + b(X_t - X_0) + c(X_t - X_0)^2 \\ + d_1(X_t - X_0)^3 + \sum_{i=1}^{n-1} (d_{i+1} \\ - d_i)(X_t - X_i)^3 Y_i + u_t \qquad (1) \end{aligned}$$

where D_t is the density in census tract t; X_t is the distance between tract t and the centre; X_i $(X_i < X_{i+1},\ i = 1, 2, \ldots, n-1)$ are the knots dividing $[X_0, X_B]$ into n segments; a, b, c and d_i are the parameters to be estimated; u_t is a normally distributed disturbance term with zero mean and constant variance; and Y_i is a dummy variable such that

$$\begin{aligned} Y_i &= 1 \quad \text{if } X_t \geq X_i \\ Y_i &= 0 \quad \text{otherwise.} \end{aligned} \qquad (2)$$

It can be easily confirmed that the density function defined, and its first and second derivatives, are continuous at all the knots X_i $(i = 1, 2, \ldots, n-1)$. So the density function is smooth along $[X_0, X_B]$. From (1), it also becomes clear that the cubic-spline estimation is just a multiple regression about $n + 3$ parameters, a, b, c, d_1 and $(d_{i+1} - d_i)$, which can be carried out by using any standard regression procedure.

There is, however, a problem remaining to be solved before the estimation. That is, how to determine the optimal number of segments n in (1). In Anderson (1985) it was suggested that the number could be decided by using minimum standard error of regression, and an attempt was made to do this. However, it failed to be pointed out that the determination of n should also be according to how the statistical significance of the coefficients estimated would be affected. In fact, in Anderson's works some coefficients have relatively large estimated errors (say table 1a in Anderson, 1985), which may not make one confident of his estimated results. In the case of studying a metropolitan space by using the cubic-spline density function, it seems more important especially to assure the significance of every estimated coefficient, because a tiny inaccuracy in the coefficients may possibly give rise to wider swings in the function at points far removed from the centre so that the description of the spatial structure becomes unreliable. For these reasons, in estimating the density functions of the Tokyo metropolitan area, we will examine a few cases of different segments for each regression and choose to accept the one that yields highly significant estimates and a small standard error of regression as well.

Outline of the Tokyo Metropolitan Area

The Tokyo metropolitan area to be studied here is defined as the inter-urban space within a range of 50 km from Tokyo Station as the area centre (Figure 1). It contains more than 150 cities, towns, villages and districts (or *shi*, *machi*, *mura* and *ku* in Japanese) which administratively belong to Tokyo Metropolis, Kanagawa Prefecture, Chiba Prefecture, Saitama Prefecture and Ibaraki Prefecture. The study area is broader than that used in

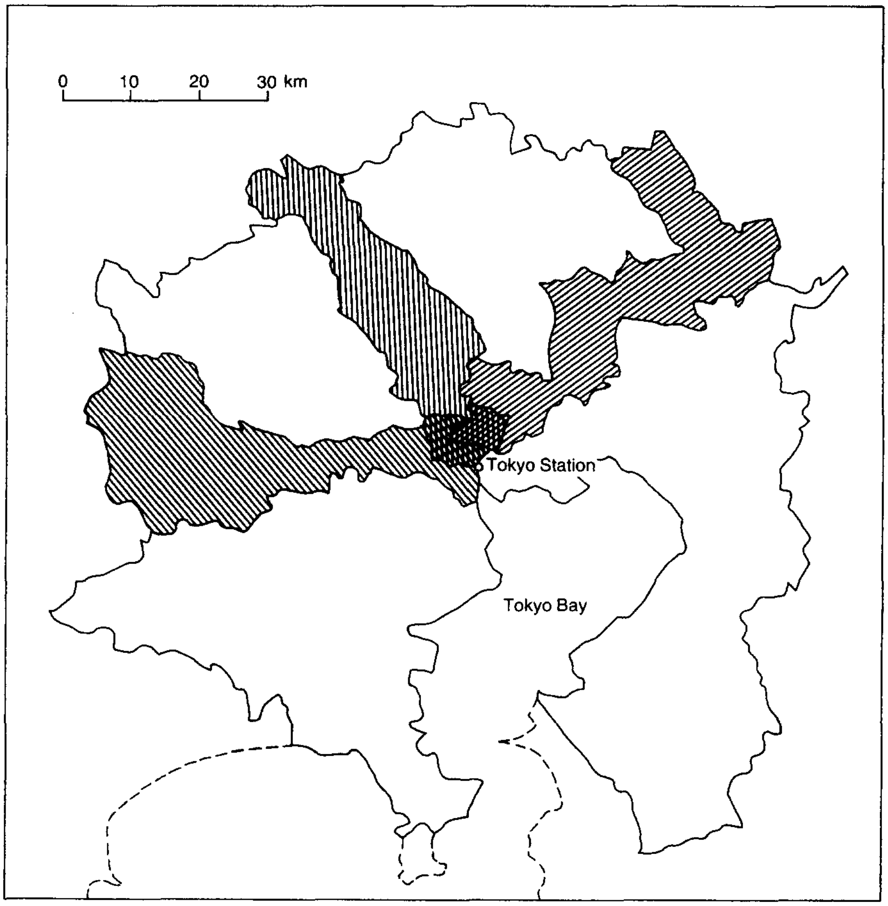

Figure 1. Tokyo metropolitan area: (▧) Chuo Line Region; (▥) Takasaki Line Region; (▨) Joban Line Region.

the previous studies of Tokyo—those by Glickman (1979, ch. 5) and Kawashima (1986), for example, which can be considered as concerning the intra-urban spatial structure.

The data concerning population density in the area are available at the levels of cities, towns, villages and districts from *Population Census of Japan* (Japan Statistics Bureau, 1975, 1985). And the distance of each administrative unit (as the census tract) to the area centre is measured as a straight-line distance between the location of its local government and Tokyo Station. As for the explanation of density patterns, we also gather data concerning income and transportation cost from *Chiiki Keizai Soran* (*Handbook of Regional Economy*) (Toyo Keizai, 1980, 1988). Here, while income is directly represented by per capita income data, transportation cost, because the data are lacking, is replaced by a proxy, car registration rate, as has been used by Muth (1969) and Alperovich (1983). These data are available only at the levels of cities (*shi*) and districts (*ku*) in the area.

Within the Tokyo metropolitan area, the spatial structure may not necessarily display a single pattern in all directions from Tokyo Station, and we therefore further define three railway-line regions along three main railway lines in the study area. The three regions, all spreading from Tokyo Station, are named respectively as Chuo Line Region, which spreads to the west of Tokyo Station along the Chuo railways line, Takasaki Line Region, spreading to the north-west along the Takasaki line, and Joban Line Region, to the north-east along the Joban line (Figure 1). The names of localities in the three regions are listed in Appendix 1.

Results of Estimation

In the estimation of urban density function by using census tract data, it has been pointed out by Frankena (1978) that a bias in coefficients would result from heteroscedasticity in the data of population density. To correct this bias, the weighted least squares method (WLS) is always used instead of ordinary least squares (OLS). In particular, as the residuals of density data seem negatively related to the area of census tract, the weight used in estimation can be defined as the square root of census tract area (see Frankena, 1978 and Anderson, 1985).

Using the WLS procedure, we first estimate the overall population density function of the Tokyo metropolitan area through the cubic-spline regression (equation (1)). Table 1 gives the estimation results for 1975 and 1985 with three cases of segments in each year.

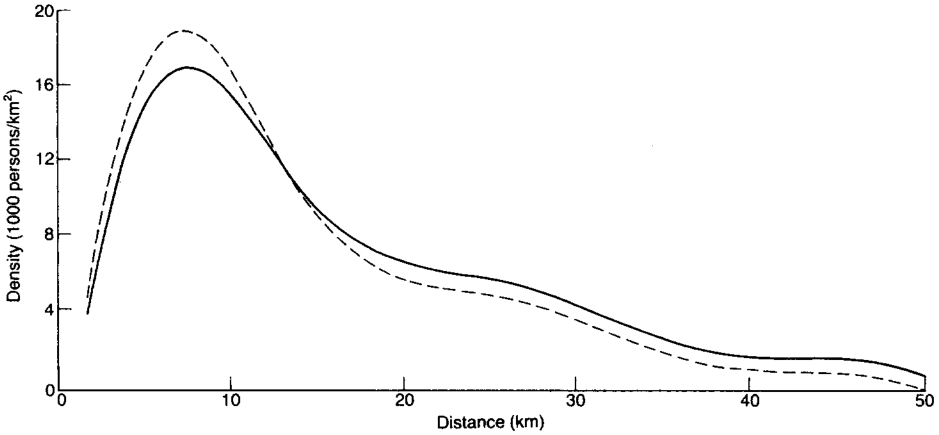

Figure 2. The overall density functions: (– – –) 1975; (——) 1985.

Table 1. Estimation results of the overall density function

	Segments (1975, $N = 154$)			Segments (1985, $N = 154$)		
Parameter	3	4	5	3	4	5
a	10385.86	4688.37	5127.35	8518.44	3835.62	4755.05
	(4.51)***	(1.83)**	(1.80)**	(3.84)***	(1.53)*	(1.71)**
b	2446.60	5932.27	5938.69	2380.14	5242.12	4810.55
	(3.67)***	(6.04)***	(4.33)***	(3.70)***	(5.44)***	(3.62)***
c	−280.47	−742.79	−796.43	−251.36	−632.83	−605.12
	(−5.01)***	(−6.92)***	(−4.23)***	(−4.65)***	(−6.02)***	(−3.32)***
d_1	7.28	24.17	28.80	6.34	20.33	20.94
	(5.23)***	(6.96)***	(3.75)***	(4.72)***	(5.98)***	(2.83)***
$d_2 - d_1$	−9.27	−28.65	−27.94	−7.86	−23.93	−18.88
	(−5.20)***	(−6.77)***	(−2.96)***	(−4.57)***	(−5.78)***	(−2.07)**
$d_3 - d_2$	3.80	6.17	−4.50	3.02	5.22	−5.75
	(3.45)***	(4.02)***	(−1.35)*	(2.84)***	(3.47)***	(−1.78)**
$d_4 - d_3$	—	−3.93	7.16	—	−3.86	7.46
		(−1.77)**	(2.93)***		(−1.76)**	(3.13)***
$d_5 - d_4$	—	—	−10.18	—	—	−11.16
			(−2.49)***			(−2.79)***
SE	13796.5	13031.6	13155.1	13381.0	12861.0	12863.2
$\bar{R}^2$	0.75	0.78	0.77	0.75	0.76	0.76

* Significant at 10 per cent level; ** significant at 5 per cent level; *** significant at 1 per cent level.

Note: N is the number of observations. Figures in parentheses are associated t values. $\bar{R}^2$ is only for reference as it is indefinable, strictly.

Table 2. Estimation results of density function for Chuo Line Region

	Segments (1975, $N = 22$)			Segments (1985, $N = 22$)		
Parameter	2	3	4	2	3	4
a	8864.58	4262.40	2891.00	7468.27	3391.64	2305.63
	(3.15)***	(1.66)**	(1.00)	(3.04)***	(1.53)*	(0.91)
b	2684.88	5297.87	6544.19	2546.69	4862.68	5869.76
	(3.56)***	(5.62)***	(4.94)***	(3.86)***	(5.95)***	(5.03)***
c	−196.95	−497.66	−713.84	−178.80	−445.52	−622.85
	(−3.97)***	(−5.48)***	(−4.31)***	(−4.12)***	(−5.65)***	(−4.27)***
d_1	3.43	12.24	21.62	3.06	10.87	18.65
	(3.74)***	(5.01)***	(3.72)***	(3.81)***	(5.12)***	(3.65)***
$d_2 - d_1$	−4.63	−14.86	−23.38	−4.06	−13.18	−19.98
	(−3.02)***	(−4.31)***	(−3.03)***	(−3.03)***	(−4.40)***	(−2.94)***
$d_3 - d_2$	—	5.07	1.82	—	4.58	1.27
		(1.86)**	(0.49)		(1.93)**	(0.38)
$d_4 - d_3$	—	—	0.68	—	—	1.03
			(0.13)			(0.22)
SE	12980.9	10118.6	10419.7	11359.4	8780.5	9179.2
$\bar{R}^2$	0.83	0.90	0.89	0.83	0.90	0.89

Note: Key and notes as in Table 1.

Since all the cases of segments in each year give highly significant coefficients, we choose the case that has the minimum standard error of regression (SE), i.e. the case of four segments in each year, as the result. Illustrating the two accepted density functions in Figure 2, we can see the overall spatial structure of the Tokyo metropolitan area and the changes during the past decade.

From Figure 2, it is seen that the maximum height of the density function is not at the centre but at about 8 km distant from it. Beyond that, density declines with distance in a pattern that seems unable to be best represented by any single function form. This result differs from other previous works concerning Tokyo (Mills and Ohta, 1976 and Glickman, 1979, ch. 5), which applied specific function forms, such as the exponential, in estimation. It also supports the hypothesis that business activities are located exclusively in the metropolitan centre while residents live outside. Another important point is that, by comparing the density functions of the two years, we find that, during the period, while the density near the centre has declined with the peak moving out a little, all densities at the places beyond 14 km from the centre have risen. This indicates the substantial suburbanisation in the study area during the 1975–85 decade.

The spatial structure of a metropolitan area will not necessarily have only one pattern in all directions from the centre. We now estimate the three directional density functions for the three railway-line regions defined. The results of estimation for the three regions are listed in Tables 2, 3 and 4 respectively.

Here, which cases of segments would be acceptable is judged from the significance of coefficients, and also from the SE. From Table 2 we can see that in both 1975 and 1985, the case of four segments yields a few estimates of lower statistical significance while the case of two segments has a larger SE, so the case of three segments seems to be the most acceptable of the three. In Table 3, although the case of four segments has a smaller SE than the case of three segments, it has one insignificant coefficient; hence the case of three segments is accepted. Similarly, from Table 4 we find the case of three segments to be more acceptable than the other two cases. In short, we have accepted all the cases of three segments for the three railway-line regions, and their density functions are illustrated in Figures 3, 4 and 5 respectively.

It is clear from these figures that the

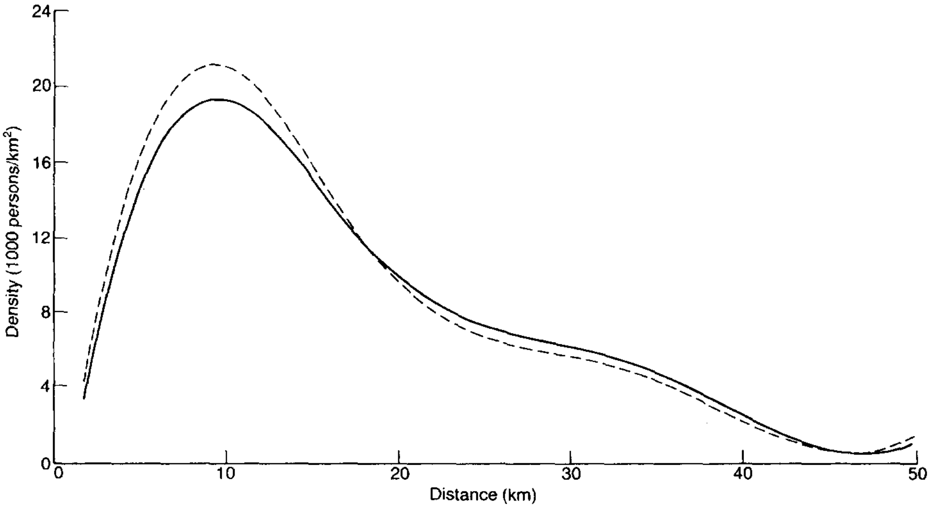

Figure 3. Density functions of Chuo Line Region: (– – –) 1975; (——) 1985.

Table 3. Estimation results of density function for Takasaki Line Region

	Segments (1975, $N = 20$)			Segments (1985, $N = 20$)		
Parameter	2	3	4	2	3	4
a	16865.10	9161.72	4945.49	13982.64	7487.59	4174.00
	(4.11)***	(2.55)***	(1.59)*	(4.08)***	(2.53)***	(1.55)*
b	555.82	4958.38	8328.60	865.89	4586.54	7260.36
	(0.54)	(3.62)***	(5.67)***	(1.01)	(4.06)***	(5.72)***
c	−103.84	−589.92	−1108.02	−111.80	−523.43	−937.69
	(−1.48)*	(−4.34)***	(−6.17)***	(−1.91)**	(−4.66)***	(−6.05)***
d_1	2.36	16.50	37.65	2.41	14.42	31.43
	(1.72)**	(4.38)***	(6.03)***	(2.10)**	(4.63)***	(5.83)***
$d_2 - d_1$	−4.52	−22.67	−44.93	−4.49	−19.45	−36.92
	(−1.77)**	(−4.16)***	(−5.55)***	(−2.10)**	(−4.33)***	(−5.28)***
$d_3 - d_2$	—	11.97	9.11	—	9.37	6.65
		(2.43)***	(2.04)**		(2.30)**	(1.72)**
$d_4 - d_3$	—	—	−4.79	—	—	−3.74
			(−0.55)			(−0.49)
SE	20455.3	14803.3	11621.9	17073.7	12221.3	10041.8
$\bar{R}^2$	0.56	0.78	0.87	0.60	0.81	0.87

Note: Key and notes as in Table 1.

Table 4. Estimation results of density function for Joban Line Region

	Segments (1975, $N = 16$)			Segments (1985, $N = 16$)		
Parameter	2	3	4	2	3	4
a	21886.56	20806.01	20239.58	18932.94	18061.28	17713.37
	(18.81)***	(19.74)***	(18.63)***	(22.22)***	(25.62)***	(23.59)***
b	−1678.54	−902.55	−204.61	−1122.88	−487.87	−51.19
	(−5.44)***	(−2.22)**	(−0.35)	(−4.97)***	(−1.79)**	(−0.13)
c	43.65	−54.09	−178.13	20.55	−60.03	−138.83
	(1.93)**	(−1.22)	(−2.15)**	(1.24)	(−2.04)**	(−2.42)***
d_1	−0.34	2.76	8.28	−0.05	2.52	6.08
	(−0.71)	(2.09)**	(2.56)***	(−0.14)	(2.85)***	(2.72)***
$d_2 - d_1$	−0.32	−4.85	−11.27	−0.43	−4.09	−7.83
	(−0.36)	(−2.53)***	(−2.55)***	(−0.65)	(−3.19)***	(−2.56)***
$d_3 - d_2$	—	3.56	3.16	—	2.84	1.48
		(2.12)**	(1.25)		(2.53)***	(0.85)
$d_4 - d_3$	—	—	−0.07	—	—	1.26
			(−0.02)			(0.49)
SE	5992.0	4931.5	4747.6	4386.8	3299.2	3282.3
$\bar{R}^2$	0.96	0.98	0.98	0.98	0.99	0.99

Note: Key and notes as in Table 1.

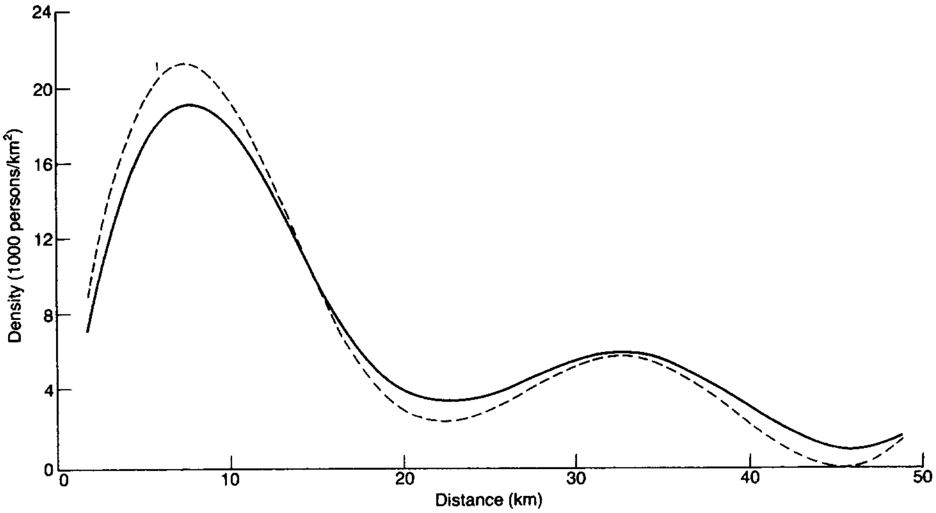

Figure 4. Density functions of Takasaki Line Region: (---) 1975; (——) 1985.

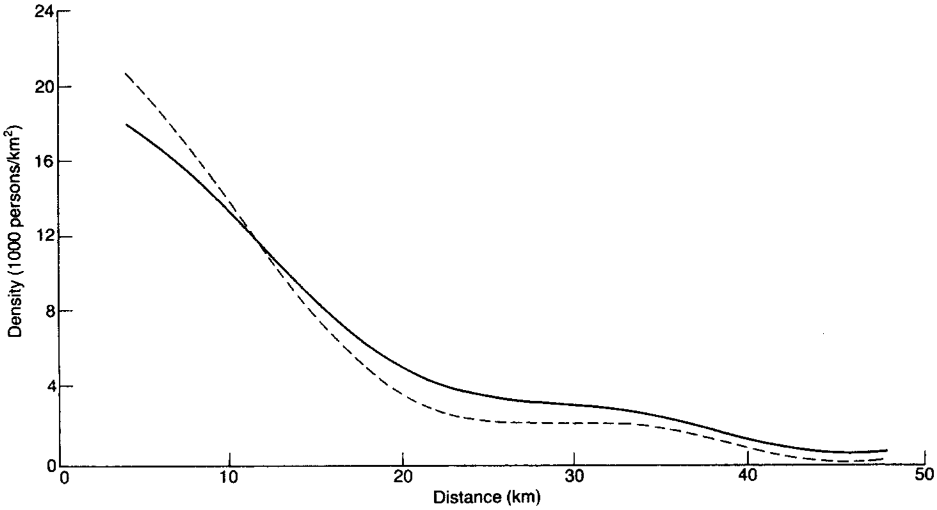

Figure 5. Density functions of Joban Line Region: (---) 1975; (——) 1985.

three railway-line regions have exhibited three different spatial patterns during the period 1975–85. Among them, Figure 3, the Chuo Line Region, presents a similar pattern to that of the overall density function (Figure 2), in which density reaches a maximum value at about 8 km from Tokyo Station (near Nakano Station) and beyond it density declines as the distance increases. By Figure 4, we can see that the density function of the Takasaki Line Region has two local maximum values at about 8 km (near Ikebukuro Station) and 32 km (between Omiya Station and Ageo Station) respectively from the centre. It makes one imagine that there exists a large peripheral city along the Takasaki railway line. In Figure 5, the Joban Line Region gives a density function that tends to have the maximum value at the centre and declines with distance along the Joban line. However, there seems to be one common feature appearing in the three figures. That is, by comparing the density functions of 1975 and 1985 we see that all the density near the area centre has declined while that far away from the centre has increased. This implies the suburbanisation in all directions of the area during the past decade.

From the estimated cubic-spline density functions of the Tokyo metropolitan area, we have seen a global phenomenon of suburbanisation in the 1975–85 period. Meanwhile, we also found that along three main railway lines spreading from the area centre in three different directions, the spatial structure displays three different patterns.

Determinants of Metropolitan Spatial Structure

In this section, first, we shall develop a varying-parameter econometric model to identify the factors that caused the spatial dynamics of the Tokyo metropolitan area during the past decade. We shall then discuss the possible reasons for the three different spatial patterns of the three directional regions, based on the estimated results.

A Varying-parameter Model

To identify the determinants of metropolitan space, we find that the varying-parameter models developed by Johnson and Kau (1980), Alperovich (1983) and Lahiri and Numrich (1983) are very useful. However, since, as suggested in the second section, their works are based on the questionable exponential density function form, it is necessary to rebuild the model on a more general ground, and in the metropolitan context.

By definition, the varying-parameter model is a regression model such that the parameters to be estimated are also regarded as functions of other independent variables (Judge *et al.*, 1980, ch. 10). In the case of the exponential density function, the parameters that represent the density at the city centre and the density gradient are supposed to be varying with income, transportation cost, city size and the like. However, this kind of setting seems impossible for the cubic-spline density function, because it is not clear what role each parameter in the spline function (equation (1)) is playing in forming the smooth density curve. So, we have to look at the relationship in the original data or the estimated results of the cubic-spline density functions.

In observing the two estimated overall density functions of the study area shown in Figure 2, it may, naturally, come to one's mind that during the past decade the *density variation* seems related to the distance from the area centre. In other words, as the distance increases from the area centre to the border, the sign of density variation is changing from minus to plus. In this sense, we may assume that the density variation rate in census tract t, denoted by $\Delta D_t/D_t$, can be expressed as a function of the distance from the tract to the area centre. In particular, and as suggested intuitively by the scatter diagram of density variation rate versus distance, we tend to specify the function to be log-linear, or

$$\frac{\Delta D_t}{D_t} = \alpha + \beta \log X_t + v_t \qquad (3)$$

where v_t is a normally distributed error term with zero mean and constant variance, and α and β are parameters to be estimated. Here, since α implies the density variation rate at or very near the centre (in the log-linear form X_t cannot be zero) and β the relation between the density variation rate and the distance from the centre, we expect α and β to have negative and positive signs respectively.

Furthermore, to explain the variation of metropolitan density we suppose α and β to be functions of the variation rates of some key variables at each locality during the same period. As indicated by the theoretical result of Zheng (1990), we consider the personal income and transportation cost as the key variables. Here, while personal income can be expressed by the per capita income data, transportation cost is represented only by the car registration data. Specifically, we propose to express the functions in the linear form as follows:

$$\alpha = a_1 \frac{\Delta W_t}{W_t} + b_1 \frac{\Delta T_t}{T_t} \tag{4}$$

$$\beta = a_2 \frac{\Delta W_t}{W_t} + b_2 \frac{\Delta T_t}{T_t} \tag{5}$$

where $\Delta W_t / W_t$ is the variation rate of personal income and $\Delta T_t / T_t$ is the variation rate of car registration in census tract t, and a_i and b_i $(i = 1, 2)$ are the parameters to be estimated. Suppose personal income increases or transportation cost decreases rapidly; the density at or near the metropolitan centre will decrease greatly, so a_1 and b_1 can be expected to have negative signs. At the same time, the density near the border will increase greatly so that the density variation rate will change sharply as the distance increases from the centre to the boundary; so the signs of a_2 and b_2 should be positive. Substituting equations (4) and (5) into equation (3), we have

$$\frac{\Delta D_t}{D_t} = a_1 \frac{\Delta W_t}{W_t} + b_1 \frac{\Delta T_t}{T_t} + a_2 \frac{\Delta W_t}{W_t} \log X_t + b_2 \frac{\Delta T_t}{T_t} \log X_t + v_t \tag{6}$$

which apparently can be estimated through OLS.

Results of the Estimation

As the income and car registration data are only available at the levels of cities and districts, the number of observations is reduced to 109. The variation rates of density, income and car registration are calculated by dividing the difference between the values for 1985 and 1975 by the value for 1975.

First of all, we estimate the regression equation (3) by OLS and obtain

$$\frac{\Delta D_t}{D_t} = \underset{(-6.33)}{-0.42} + \underset{(8.87)}{0.19} \log X_t \tag{7}$$

$$\bar{R}^2 = 0.42$$

where figures in parentheses are associated t values and $\bar{R}^2$ is adjusted R^2. It is clear that the estimated coefficients have just the expected signs, with high significance. The value of $\bar{R}^2$ is not low in the sense of cross-sectional data. This result has verified the hypothesis described by equation (3), that the density variation rate is a function increasing with distance from the centre, and confirmed once again the suburbanisation in the area during the 1975–85 period.

To explain the variation of metropolitan spatial structure in terms of personal income and transportation cost, we also estimate the varying-parameter regression (6) by OLS procedure and have the following result:

$$\frac{\Delta D_t}{D_t} = \underset{(-3.47)}{-0.26} \frac{\Delta W_t}{W_t} \underset{(-1.65)}{-0.26} \frac{\Delta T_t}{T_t} + \underset{(3.21)}{0.10} \frac{\Delta W_t}{W_t} \log X_t + \underset{(2.32)}{0.13} \frac{\Delta T_t}{T_t} \log X_t \tag{8}$$

$$\bar{R}^2 = 0.47$$

All the estimated coefficients in equation (8) are highly significant and have just the expected signs. The value of $\bar{R}^2$ is a little larger than that in equation (7), indicating the regression has been improved by the varying-parameter model. By calculation, the correlation coefficient of $\Delta W_t / W_t$ and $\Delta T_t / T_t$ is 0.10. Noting that the signs of a_1, b_1, a_2 and b_2 are verified, we can say that the greater the income rises and/or the transportation cost declines, the more the density near the centre declines, and the more the density near the border increases.

Table 5. Income and transportation costs of the three regions

	Average income (1000 yen/person)		Car registration (cars/100 persons)	
	1975	1985	1975	1985
Chuo ($N = 19$)	653	1403	15.2	20.8
Takasaki ($N = 19$)	587	1233	12.7	19.2
Joban ($N = 12$)	547	1164	8.9	16.4

Note: N is the number of observations.

This estimated result implies that income and transportation cost constitue the determinants for the metropolitan spatial structure, which seems comparable with the previous work concerning the intra-urban space by Muth (1969), Johnson and Kau (1980), Alperovich (1983) and others.

Explanation for Directional Spatial Patterns

The estimated result (equation (8)) implies that personal income and transportation cost are involved in the determinants for the spatial structure of the Tokyo metropolitan area during the period 1975–85. It seems reasonable that personal income and transportation cost will also affect the directional spatial patterns within the study area. In fact, if we calculate the average levels of personal income and transportation cost (represented by the reciprocal of car registration rate) of the three railway-line regions for the two years of 1975 and 1985, we have Table 5 from the available data at the levels of cities and districts. It shows that, during the 1975–85 period, the Chuo Line Region had the highest income level and lowest transportation cost, the Joban Line Region had the lowest income and the highest transportation cost, and the Takasaki Line Region had the medium levels between them. That is, different directional spatial patterns seem related to different values of income and transportation cost.

Now let us look further into the spatial structures of the three directional regions. Consider the three regions as the linear inter-urban systems with Tokyo Station as their centre. By taking the 4000 persons/km^2 standard, which is an index of the Densely Inhabited District (DID) in Japan, as an urbanised area indicator, we can see from Figure 3 that along the Chuo line, most places of the region have been urbanised, and the peripheral city seems to have developed so that it is spatially integrated with the central city. In the Takasaki Line Region, in Figure 4, the peripheral city seems to exist at about 32 km from the centre and can be considered as spatially connected with the central city. As for the Joban Line Region, Figure 5 shows that there has been no peripheral city formed along the Joban line in the metropolitan area, or the peripheral city may exist beyond the area, that is, spatially split far away from the central city. Intuitively, the spatial patterns of the three railway-line regions have some correspondence to the monocentricity, multicentricity and the separate pattern displayed in the theoretical inter-urban model by Zheng (1990).

To give more evidence for the foregoing arguments, we estimate the daytime population density functions to see the urban concentration in the three directional regions. By using the approach of cubic-spline estimation described in the third section, we have obtained the daytime density functions of the three regions for 1985 (listed in Appendix 2) and illustrated them in Figure 6.

As the daytime population density can be considered to be expressing the spatial

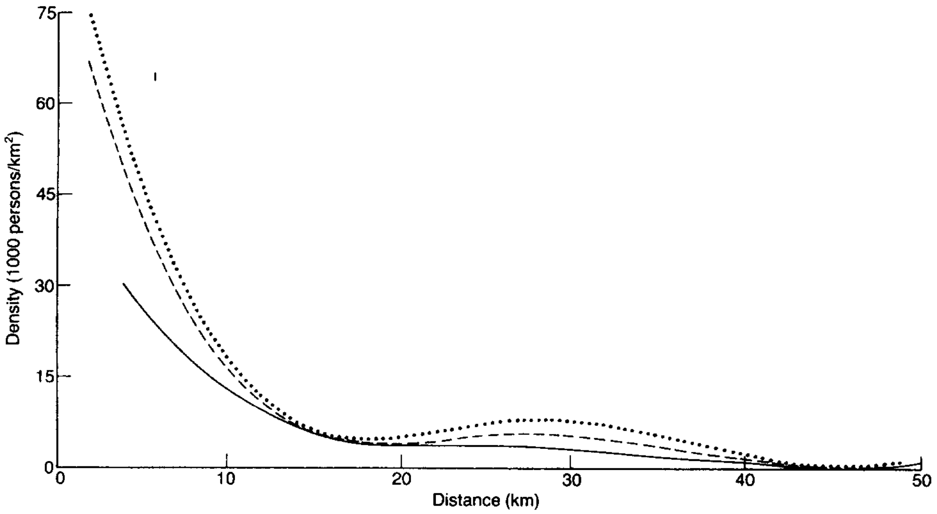

Figure 6. Daytime density functions of 1985: (····) Chuo Line Region; (– – –) Takasaki Line Region; (——) Joban Line Region.

distribution of urban business activities that attract people in the daytime, we may use it to represent the urban concentration to some extent. From Figure 6, the centre and the subcentre in the Chuo Line Region have attracted many people around them, so the central and peripheral cities have developed so much that they are spatially integrated. In the Takasaki Line Region, the density around the centre and subcentre seems a little less than in the Chuo Line Region; hence the central and peripheral cities would be only spatially connected. The daytime density in the Joban Line Region shows that only one centre exists along the region, and thus that no peripheral city has developed along the Joban line within the study area.

In these arguments, we have seen that the spatial structures of the three directional regions differ from each other in their urban concentrations, which seem to be related to the levels of income and the transportation conditions in each directional region.

Concluding Remarks

We have applied the approach of the cubic-spline density function to explore the spatial structure of the Tokyo metropolitan area. During the estimation, in determining the optimal number of segments that make up the whole distance interval, we have taken into account the statistical significance of each estimated coefficient as well as the SE so that the estimated cubic-spline density functions become more statistically reliable. The estimated overall density functions have shown a substantial suburbanisation in the study area during the period 1975–85. By estimating the directional density functions of three railway-line regions, we found that they display three different spatial patterns. To identify the determinants of the spatial dynamics of the Tokyo metropolitan area, a varying-parameter model is proposed. In this model, we have confirmed the hypothesis that an increase in income and a reduction in transportation cost would result in suburbanisation. And the spatial difference in the three directional regions is also related to their different values of income and transportation cost.

The present work is only a first step towards studying the metropolitan spatial structure empirically. In the future, we want to build a more comprehensive theoretical model of the urban system, and apply it empirically to investigate the Tokyo metropolitan area. Also, we need to apply the approaches used here to study comparatively other large metropolitan areas, say Shanghai and Seoul in the East Asian region, to see the similarities and differences in different urban areas.

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Appendix 1. Names of the Census Tracts of the Three Directional Regions

Although the names of the three regions are the same as in Kawashima (1986), the constituent census tracts are rather different. In Table A1 are the names of the census tracts and their distances (km) from Tokyo Station.

Table A1.

Region	Name	Distance (km)	Name	Distance (km)
Chuo Line Region	Chiyoda-ku	1.7	Minato-ku	4.5
	Shinjuku-ku	5.3	Toshima-ku	6.5
	Shibuya-ku	6.8	Nakano-ku	9.8
	Suginami-ku	13.0	Mitaka-shi	18.4
	Koganei-shi	24.1	Fuchu-shi	25.9
	Kokubunji-shi	27.8	Kunitachi-shi	29.1
	Tachikawa-shi	31.7	Hino-shi	33.2
	Akishima-shi	36.4	Hachioji-shi	39.3
	Fussa-shi	40.0	Akigawa-shi	40.1
	Hamura-machi	42.0	Ome-shi	45.5
	Itsukaichi-machi	48.8	Hinode-machi	49.8
Takasaki Line Region	Chiyoda-ku	1.7	Taito-ku	3.8
	Bunkyo-ku	4.5	Shinjuku-ku	5.3
	Toshima-ku	6.5	Arakawa-ku	6.7
	Kita-ku	10.5	Itabashi-ku	12.5
	Kawaguchi-shi	14.6	Toda-shi	16.6
	Hatogaya-shi	17.0	Warabi-shi	17.7
	Urawa-shi	23.0	Yono-shi	25.7
	Omiya-shi	27.8	Ageo-shi	36.4
	Okegawa-shi	39.9	Kitamoto-shi	43.4
	Konosu-shi	46.8	Yoshimi-machi	48.7
Joban Line Region	Taito-ku	3.8	Bunkyo-ku	4.5
	Sumida-ku	6.1	Arakawa-ku	6.7
	Adachi-ku	11.2	Katsushika-ku	11.3
	Matsudo-shi	23.3	Nagareyama-shi	23.3
	Kashiwa-shi	27.8	Abiko-shi	31.4
	Toride-shi	36.4	Fujishiro-machi	41.8
	Ryugasaki-shi	45.9	Kukizaki-machi	46.8
	Ushiku-machi	47.8	Yatabe-machi	47.8

Appendix 2. Estimated Results of Daytime Density Functions

The daytime density functions of the three directional regions in 1985 are estimated as follows. (Figures in parentheses are associated t values and $\bar{R}^2$ is adjusted R^2.)

Chuo Line Region

$$\begin{aligned} D_t = {} & \underset{(21.93)}{75848.03} - \underset{(-11.85)}{11025.84}(X_t - X_0) + \underset{(9.00)}{550.69}(X_t - X_0)^2 \\ & \underset{(-7.74)}{-8.75}(X_t - X_0)^3 + \underset{(5.90)}{11.14}(X_t - X_1)^3 Y_1 \\ & \bar{R}^2 = 0.95 \end{aligned} \tag{9}$$

$$\begin{aligned} & X_0 = 1.7, \quad X_1 = 24.05, \\ & Y_1 = 1, \quad \text{if } X_t \geq X_1 \\ & Y_1 = 0, \quad \text{otherwise.} \end{aligned} \tag{10}$$

Takasaki Line Region

$$\begin{aligned} D_t = {} & \underset{(11.14)}{67095.05} - \underset{(-6.34)}{9546.10}(X_t - X_0) + \underset{(4.56)}{469.53}(X_t - X_0)^2 \\ & \underset{(-3.70)}{-7.45}(X_t - X_0)^3 + \underset{(2.44)}{9.17}(X_t - X_1)^3 Y_1 \\ & \bar{R}^2 = 0.82 \end{aligned} \tag{11}$$

$$\begin{aligned} & X_0 = 1.7, \quad X_1 = 23.5, \\ & Y_1 = 1, \quad \text{if } X_t \geq X_1 \\ & Y_1 = 0, \quad \text{otherwise.} \end{aligned} \tag{12}$$

Joban Line Region

$$\begin{aligned} D_t = {} & \underset{(26.74)}{30094.34} - \underset{(-13.10)}{3908.17}(X_t - X_0) + \underset{(8.73)}{191.33}(X_t - X_0)^2 \\ & \underset{(-6.82)}{-3.15}(X_t - X_0)^3 + \underset{(4.35)}{3.76}(X_t - X_1)^3 Y_1 \\ & \bar{R}^2 = 0.97 \end{aligned} \tag{13}$$

$$\begin{aligned} & X_0 = 3.8, \quad X_1 = 22.0, \\ & Y_1 = 1, \quad \text{if } X_t \geq X_1 \\ & Y_1 = 0, \quad \text{otherwise.} \end{aligned} \tag{14}$$