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MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion

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Abstract

An analysis is performed to study the effects of thermal radiation on unsteady free convective flow over a moving vertical plate with mass transfer in the presence of magnetic field. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised to T'_w and the concentration level near the plate is also raised linearly with time. The dimensionless governing equations are solved using the Laplace transform technique. The velocity, temperature and concentration are studied for different parameters like the magnetic field parameter, radiation parameter, thermal Grashof number, mass Grashof number and time. It is observed that the velocity decreases with increasing magnetic field parameter or radiation parameter.

Keywords: gray, thermal radiation, magnetic field, vertical plate, mass diffusion.

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List of symbols

a^*	absorption coefficient
A	constant
B_0	external magnetic field
C'	species concentration in the fluid
C'_w	concentration of the plate
C'_∞	concentration in the fluid far away from the plate
C	dimensionless concentration
C_p	specific heat at constant pressure
g	acceleration due to gravity
Gr	thermal Grashof number
Gc	mass Grashof number
k	thermal conductivity of the fluid
M	magnetic field parameter
Pr	Prandtl number
q_r	radiative heat flux in the y-direction
R	radiation parameter
Sc	Schmidt number
T'	temperature of the fluid near the plate
T'_w	temperature of the plate
T'_∞	temperature of the fluid far away from the plate
t'	time
t	dimensionless time
u'	velocity of the fluid in the x'-direction
u_0	velocity of the plate
u	dimensionless velocity
y'	coordinate axis normal to the plate
y	dimensionless coordinate axis normal to the plate

Greek symbols

α	thermal diffusivity
β	volumetric coefficient of thermal expansion
β^*	volumetric coefficient of expansion with concentration
μ	coefficient of viscosity
ν	kinematic viscosity

ρ	density of the fluid
σ	electric conductivity
τ'	skin-friction
τ	dimensionless skin-friction
θ	dimensionless temperature
η	similarity parameter
$erfc$	complementary error function

1 Introduction

Magnetoconvection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals. MHD finds applications in electromagnetic pumps, controlled fusion research, crystal growing, MHD couples and bearings, plasma jets and chemical synthesis.

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion.

Boundary layer flow on moving horizontal surfaces was studied by Sakiadis [1]. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar *et al* [2]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar *et al* [3]. The dimensionless governing equations were solved using Laplace transform technique. Kumari and Nath [4] studied the development of the asymmetric flow of a viscous electrically conducting fluid in the forward stagnation point region of a two-dimensional body and over a stretching surface with an applied magnetic field, when the external stream or the stretching surface was set into impulsive motion from the rest. The governing equation were tackled using implicit finite difference scheme.

England and Emery[5] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar[6] have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar[7]. In all above studies, the stationary vertical plate is considered. Raptis and Perdikis[8] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das *et al*[9] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique.

It is proposed to study thermal radiation effects on flow past an impulsively started infinite vertical plate with uniform temperature and variable mass diffusion in the presence of transverse applied magnetic field. The governing equations are solved by the Laplace-transform technique. The effect of velocity, temperature and concentration for different magnetic field parameter, radiation parameter and time are studied graphically.

2 Mathematical Analysis

Thermal radiation effects on unsteady flow of a viscous incompressible fluid past an impulsively started infinite vertical isothermal plate with variable mass diffusion, in the presence of transverse applied magnetic field is studied. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity u_0 in a fluid, in the presence of thermal radiation. At the same time the plate temperature is raised to T_w and the level of concentration near the plate are raised linearly with time. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}\, u' \quad (1)$$

$$\rho\, C_p\, \frac{\partial T}{\partial t'} = k\, \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\, \frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

with the following initial and boundary conditions:

$$\begin{array}{ll} t' \leq 0: & u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y' \\ t' > 0: & u' = u_0, \quad T' = T'_w, \quad C' = C'_\infty + (C'_w - C'_\infty)\, A\, t' \quad at \quad y' = 0 \\ & u' = 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y' \to \infty \end{array} \quad (4)$$

where $A = \frac{u_0^2}{\nu}$. The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\, \sigma \left(T_\infty^4 - T^4\right) \quad (5)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\, T - 3T_\infty^4 \quad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho\, C_p\, \frac{\partial T}{\partial t'} = k\, \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3\, (T_\infty - T) \tag{7}$$

On introducing the following non-dimensional quantities:

$$\begin{gathered} u = \frac{u'}{u_0},\ t = \frac{t' u_0^2}{\nu},\ y = \frac{y' u_0}{\nu},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \\ Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{u_0^3},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gm = \frac{\nu g \beta^*(C'_w - C'_\infty)}{u_0^3}, \\ Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D},\ M = \frac{\sigma B_0^2 \nu}{\rho u_0^2},\ R = \frac{16a^* \nu^2 \sigma T_\infty^3}{k u_0^2} \end{gathered} \tag{8}$$

in equations (1) to (4), leads to

$$\frac{\partial u}{\partial t} = Gr\ \theta + Gc\ C + \frac{\partial^2 u}{\partial y^2} - M\ u \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\ \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\ \theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\ \frac{\partial^2 C}{\partial y^2} \tag{11}$$

The initial and boundary conditions in dimensionless form are as follows:

$$\begin{array}{lllll} & u = 0, & \theta = 0, & C = 0 & \text{for all} \quad y, t \leq 0 \\ t > 0: & u = 1, & \theta = 1, & C = t & \text{at} \quad y = 0 \\ & u = 0, & \theta \to 0, & C \to 0 & \text{as} \quad y \to \infty \end{array} \tag{12}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydromagnetic flow field in the presence of thermal radiation.

The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned} \theta = \frac{1}{2} \Big[& \exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr} + \sqrt{at}) + \\ & \exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr} - \sqrt{at}) \Big] \end{aligned} \tag{13}$$

$$C = t\left[(1+2\eta^2 Sc)\ erfc(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\ \exp(-\eta^2\ Sc)\right] \tag{14}$$

$$\begin{aligned}
u = \frac{1}{2}\left(1+\frac{Gr}{b(1-Pr)}+\frac{Gc(1+ct)}{c^2(1-Sc)}\right)&\big[\exp(2\ \eta\sqrt{Mt})erfc(\eta+\sqrt{Mt})+ \\
&\exp(-2\ \eta\ \sqrt{M\ t})\ erfc(\eta-\sqrt{Mt})\big]- \\
\frac{Gc\eta\sqrt{t}}{2c(1-Sc)\sqrt{M}}&\big[\exp(-2\ \eta\ \sqrt{Mt})\ erfc(\eta-\sqrt{Mt}) \\
&-\exp(2\ \eta\sqrt{Mt})\ erfc(\eta+\sqrt{Mt})\big]- \\
\frac{Gc}{c^2(1-Sc)}&\ erfc(\eta\ \sqrt{Sc})- \\
\frac{Gr\ \exp(bt)}{2b(1-Pr)}&\big[\exp(2\ \eta\ \sqrt{(M+b)t})\ erfc(\eta+\sqrt{(M+b)t}) \\
&+\exp(-2\eta\sqrt{(M+b)t})\ erfc(\eta-\sqrt{(M+b)t})\big]- \\
\frac{Gc\ \exp(ct)}{2c^2(1-Sc)}&\big[\exp(2\ \eta\ \sqrt{(M+c)t})\ erfc(\eta+\sqrt{(M+c)t})+ \\
&\exp(-2\eta\sqrt{(M+c)t})\ erfc(\eta-\sqrt{(M+c)t})\big]- \\
\frac{Gr}{2b(1-Pr)}&\big[\exp(2\ \eta\sqrt{Rt})erfc(\eta\ \sqrt{Pr}+\sqrt{at})+ \\
&\exp(-2\ \eta\ \sqrt{Rt})\ erfc(\eta\ \sqrt{Pr}-\sqrt{at})\big]+ \\
\frac{Gr\ \exp(bt)}{2b(1-Pr)}&\big[\exp(2\ \eta\ \sqrt{Pr(a+b)t})\ erfc(\eta\sqrt{Pr}+\sqrt{(a+b)t})+ \\
&\exp(-2\eta\sqrt{Pr(a+b)t})\ erfc(\eta\ \sqrt{Pr}-\sqrt{(a+b)t})\big]- \\
\frac{Gc\ t}{c(1-sc)}&\big[(1+2\eta^2 Sc)\ erfc(\eta\sqrt{Sc})-\frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\ \exp(-\eta^2\ Sc)\big]+ \\
\frac{Gc\ \exp(ct)}{2c^2(1-Sc)}&\big[\exp(2\eta\sqrt{ct\ Sc})\ erfc(\eta\sqrt{Sc}+\sqrt{ct})+ \\
&\exp(-2\eta\sqrt{ct\ Sc})\ erfc(\eta\sqrt{Sc}-\sqrt{ct})\big]
\end{aligned} \tag{15}$$

where

$$a = \frac{R}{Pr}, \; b = \frac{M-R}{Pr-1}, \;\; c = \frac{M}{Sc-1} \text{ and } \; \eta = \frac{y}{2\sqrt{t}}.$$

3 Discussion of Results

The numerical values of the velocity, temperature and wall concentration are computed for different parameters like magnetic field parameter, radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number, time and $Pr = 0.71$. The purpose of the calculations given here is to study the effects of the parameters M, R, t, Gr, Gc and Sc upon the nature of the flow and transport.

The temperature profiles are calculated for different values of thermal radiation parameter($R = 2, 5, 7, 10$) from Equation (13) and these are shown in Figure 1. for air ($Pr = 0.71$). The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

The effect of concentration for different time ($t = 0.2, 0.4, 0.6, 0.8, 1$) are shown in Figure 2. In this case, the concentration increases with increasing time t.

Figure 3 represents the effect of concentration profiles at time $t = 1$ for different Schmidt number($Sc = 0.16, 0.3, 0.6, 2.01$). The effect of concentration is important in concentration field. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

The velocity profiles for different values of the radiation parameter($R = 3, 7, 20$), $Gr = Gc = 2, Sc = 2.01, M = 2, Pr = 0.71$ and $t = 0.4$ are shown in Figure 4. It is observed that the velocity increases with decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation.

The velocity profiles for different magnetic field parameter($M = 2, 5, 10), Gr = Gc = 2, R = 12, Sc = 2.01, Pr = 0.71$ and $t = 0.2$ are presented in Figure 5. It is clear that the velocity increases with decreasing magnetic field parameter.

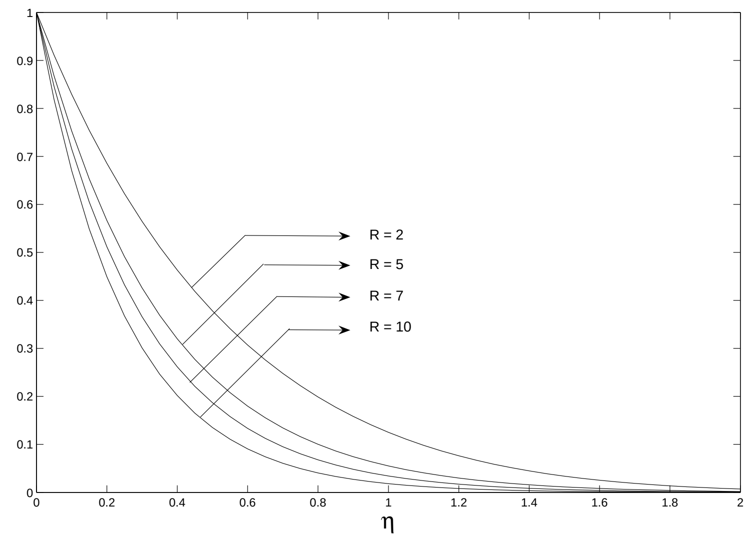

Figure 1: Temperature profiles for different R

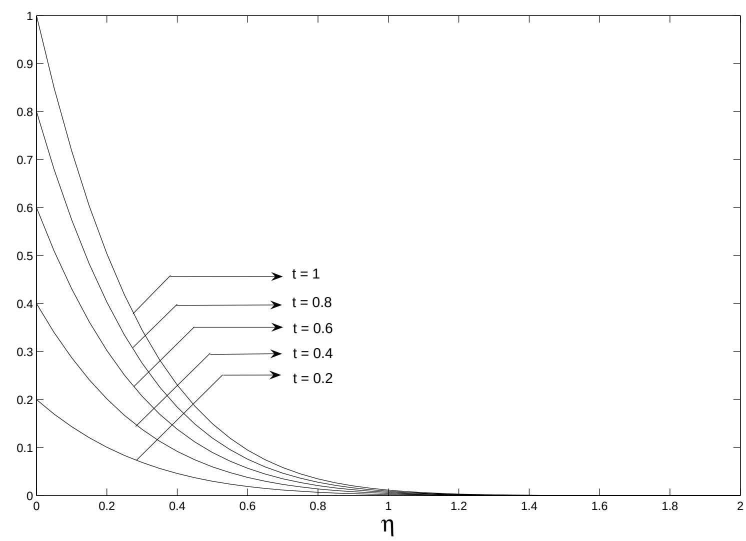

Figure 2: Concentration profiles for different t

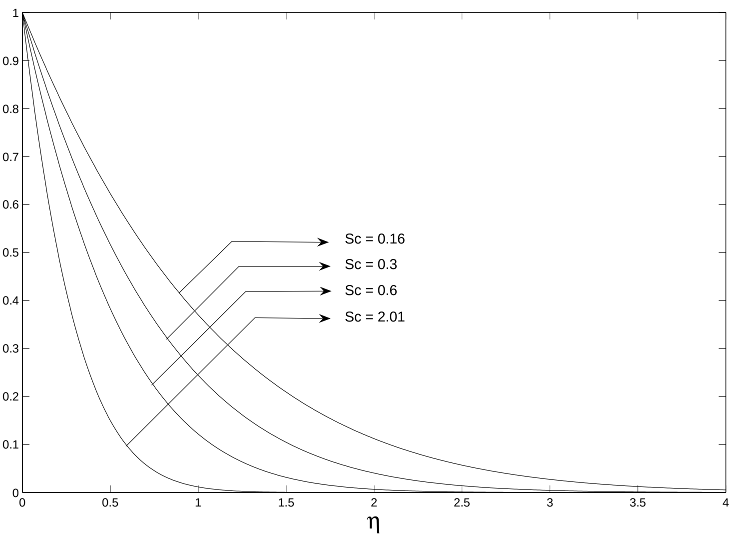

Figure 3: Concentration profiles for different S_c

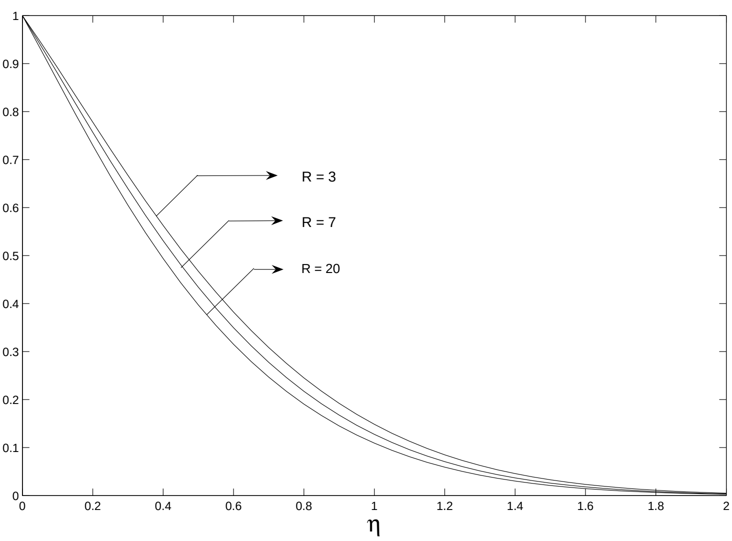

Figure 4: Velocity profiles for different R

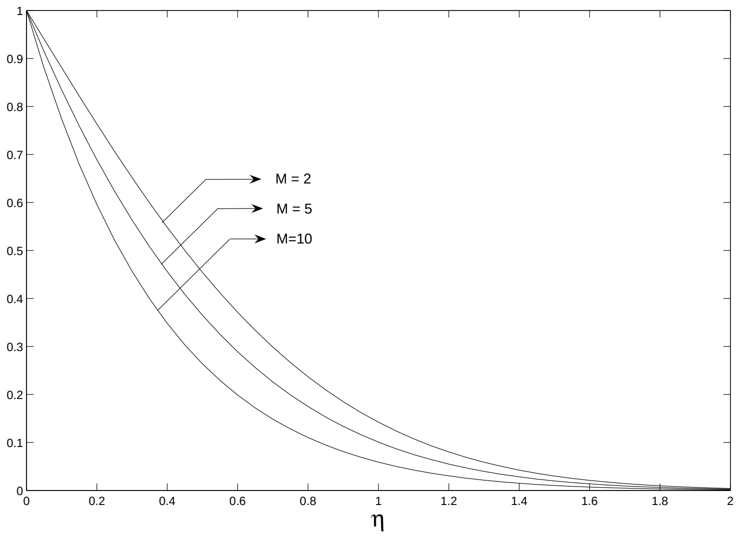

Figure 5: Velocity profiles for different M

4 Concluding Remarks

An analysis is performed to study the thermal radiation effects on flow past an impulsively started infinite vertical plate with uniform temperature and variable mass diffusion in the presence of transverse applied magnetic field. The dimensionless governing equations are solved by the usual Laplace-transform technique. The velocity, temperature and wall concentration are studied for different physical parameters are studied graphically. The conclusions of the study are as follows:

(i) The velocity decreases with increasing Radiation parameter or magnetic field parameter.

(ii) The temperature increases with decreasing values of thermal radiation parameter.

(iii) As time increases, it is found that there is a rise in wall concentration.

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Radijacioni i MHD uticaji na pokretnu izotermsku vertikalnu ploču sa promenljivom difuzijom mase

UDK 537.84

Proučavaju se uticaji termičke radijacije na nestacionarno slobodno konvektivno tečenje preko pokretne vertikalne ploče sa prenosom mase u prisistvu magnetnog polja. Smatra se da je fluid siv, apsorbuje i emituje radijaciju ali da nema raspršivanja. Temperatura ploče raste do T'_w pri čemu se nivo koncentracije takodje podiže linearno sa vremenom. Bezdimenzione jednačine problema se rešavaju metodom Laplasove transformacije. Brzina, temperatura i koncentracija se proučavaju pri raznim vrednostima parametra magnetskog polja, radijacionog parametra, termičkog Grashofovog broja, masenog Grashofovog broja i vremena. Uočava se opadanje brzine pri rastu parametra magnetskog polja ili radijacionog parametra.