MHD Thermal Stagnation Point Flow towards a Stretching Porous Surface

E.M. Arthur* Y.I Seini
Faculty of Mathematical Sciences, University for Development Studies, P. O. Box 1350, Tamale – Ghana

Abstract

This paper presents an investigation of the hydromagnetic stagnation flow of an incompressible viscous, electrically conducting fluid, towards a stretching sheet in the presence of axially in- creasing free stream velocity. The Newton-Raphson shooting method along with the fourth-order Runge-Kutta integration algorithm has been employed to tackle the third order, nonlinear boundary layer equation governing the problem. The results indicate that suction and thermal Grashof number have the same effect on the rate of heat transfer. The magnetic parameter has the effect of increasing the skin friction coefficient whilst the reverse is observed for increasing the velocity ratio parameter.

Keywords: Convective flow; Viscous; MHD; Free stream; Stagnation Point; Suction, Velocity Ratio

1.0 Introduction

The need for cooling has resulted in a major paradigm shift for many chemical engineers, electrical system designers and manufacturers. The fluid mechanical properties of the penultimate product depend mainly on the cooling liquid used and the rate of stretching. In practice, fluids having better electromagnetic properties are normally used as cooling liquid, since imposing a magnetic field on it can control its flow to improve the quality of the expected product. As a result of this, MHD boundary layer flow over a stretching surface has attracted considerable attention in recent times due to its numerous practical applications. Such applications include hot rolling, wire drawing, glass fiber and paper production, drawing of paper films, metal and polymer extrusion.

The problem of steady two-dimensional boundary layer flow due to stretching sheets was initiated by Sakiadis (1971). This triggered many researchers to study heat and mass transfer under various physical conditions. To mention a few: Datta, *et al.* (1985); McLeod and Rajagopal (1987); Gupta and Gupta (1997); Cortell (2008); and Ohwada (2009). Yian et al. (2007) discussed the non-orthogonal stagnation point towards a stretching vertical plate and found that the flow had an inverted boundary layer structure, when the stretching velocity of the surface exceeded the stagnation velocity of the free stream. A review article on the stagnation point solutions have been given by Wang (2007). Kechil and Hashim (2008) studied the boundary-layer equation of flow over a nonlinearly stretching sheet in a magnetic field with chemical reaction.

Fang et al. (2009) presented an analytical investigation of the hydrodynamic boundary layer of slip MHD viscous flow over a stretching sheet and concluded that the wall drag force increases with the magnetic parameter. Ishak et al. (2009) studied numerically a steady two-dimensional MHD stagnation point flow towards a stretching sheet with variable surface temperature. They found that the heat transfer rate at the surface increased with the magnetic parameter when the free stream velocity exceeded the stretching velocity. Recently, Javed et al. (2012) studied heat transfer of a viscous fluid over a non-linear shrinking sheet in the presence of a magnetic field and obtained dual solutions for the exact and numerical solutions in the shrinking sheet problem. Computational dynamics of hydrodynamic of hydromagnetic stagnation flow towards a stretching sheet was studied by Makinde and Charles (2010). They concluded that the cooling rate of a stretching sheet in an electrically conducting fluid, subject to a magnetic field can be controlled and a final product with desired characteristics can be achieved.

Ibrahim and Makinde (2010) investigated the MHD boundary layer flow of chemically reacting fluid with heat and mass transfer past a stretching sheet and concluded that both the magnetic field strength and the uniform heat source have significant impact in controlling the rate of heat and mass transfer in the boundary layer region. Rana and Bhargava (2012) studied the problem of steady laminar boundary fluid flow which resulted from non-linear stretching of a flat surface in a nanofluid. Recently, Seini and Makinde (2013) studied MHD boundary layer flow due to exponential stretching surface with radiation and chemical reaction and reported that the rate of heat transfer at the surface decreases with increasing values of the transverse magnetic field parameter and the radiation parameter.

Seini (2013) investigated the flow over an unsteady stretching surface with chemical reaction and non-uniform heat source and observed that the heat and mass transfer rates and the skin friction coefficient increased as the unsteadiness parameter increases and decreased as the space-dependent and temperature-dependent parameters for heat source/sink increase. Alireza et al. (2013) then presented an analytical solution for MHD stagnation point flow and heat transfer over a permeable stretching sheet with chemical reaction. Only a limited attention has been focused on the combined effects of magnetic field and free stream velocity on the stagnation point towards a stretching surface. Stagnation point flow towards a stretching sheet is quite useful and important from practical point of view. This fact motivated the present study to investigate the combined effect of magnetic field and suction on a stagnation point flow over a stretching surface.

2.0 Mathematical Model

Consider a steady two-dimensional flow of an incompressible and electrically conducting fluid towards the stagnation point on a porous stretching sheet in the presence of magnetic field of strength B_0, applied in the positive y direction as shown in Fig. 1. The tangential velocity u_w, and the free stream velocity u_∞ were assumed to vary proportional to the distance x from the stagnation point so that $u_w(x) = ax$ and $u_\infty(x) = bx$. The induced magnetic field due to motion of the electrically conducting fluid and the pressure gradient are neglected. The tangential temperature is maintained at the prescribed constant value T_w.

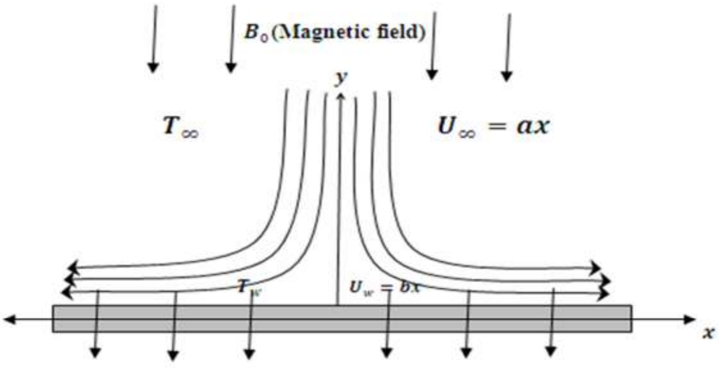

Figure 1: Schematic Diagram of the Problem

The boundary layer equations for a steady incompressible viscous hydrodynamic fluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = a^2 x + \nu \frac{\partial^2 u}{\partial x^2} - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho c_p} u^2 \tag{3}$$

The corresponding boundary conditions are:

$$\text{At } y = 0, u = bx, T = T_w \quad v = -v \tag{4}$$

$$\text{As } y \to \infty, u = ax \quad T \to T_\infty \tag{5}$$

Where u and v are the velocity components in x and y directions respectively, ρ is the density of the liquid, ν is the kinematic viscosity, B_0 is the strength of the applied magnetic field, α is the thermal diffusivity, c_p is the specific heat capacity at constant pressure and σ is the electrical conductivity of the fluid.

Following Nazar et al. (2004), we introduce the following similarity variables and quantities:

$$u = bxf'(\eta), \quad v = -\sqrt{b\nu}\, f(\eta), \quad \eta = y\sqrt{\frac{b}{\nu}}, \quad H = \frac{\sigma B_0^2}{\rho b}, \quad \lambda = \frac{a}{b}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

Substituting (6) into (1), (2), (3), (4) and (5) gives the following nonlinear ordinary differential equations:

$$f''' + ff' - f'^2 - Haf' + \lambda^2 = 0 \tag{7}$$

$$\theta'' + \Pr f\theta' + Ha\Pr G_T f'^2 = 0 \tag{8}$$

Where $\lambda = \frac{a}{b}$ is the velocity ratio parameter, f is the similarity function, and θ is the dimensionless temperature.

The associated boundary conditions are:

$$f(0) = -f_w \qquad f'(0) = 1 \qquad \theta(0) = 1 \qquad f'(\infty) = \lambda \qquad \theta(\infty) = 0 \tag{9}$$

where $Ha = \frac{\sigma B_0^2}{\rho a}$ is the Magnetic parameter, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $G_T = \frac{b^2x^2}{c_p(T_w - T_\infty)}$ is the thermal Grashof number and $f_w = \frac{v}{\sqrt{bv}}$ is the suction/injection parameter. The skin friction coefficient C_f , can be written as

$$C_f = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \bigg/ \frac{\rho U_w^0}{2} \tag{10}$$

Using non-dimensional variables (6), we get

$$\frac{1}{2} C_f \sqrt{Re_x} = f''(0) \tag{11}$$

where Re_x is the local Reynolds number.

3.0 Results and Discussion

a) Numerical Results

The systems of equations (7) and (8) with respect to the boundary condition, equation (9) were solved using the Fourth-Order Runge-Kutta method along with the shooting techniques. For the sake of the validity of our results, we have tabulated a comparison between the present numerical solution and the works of Makinde and Charles (2010), and Nazar (2004), See Table 1. It can be realized that our numerical results are in good agreement with that of Makinde and Charles (2010) and Nazar (2004).

Table 1. Comparison of Skin Friction Coefficient $f''(0)$ for λ when H = 0

λ	Makinde and Charles (2010)	Nazar (2004)	Present Study
0.01	-0.99802	-0.9980	-0.99802
0.02	-0.99578	-0.9958	-0.99578
0.05	-0.98757	-0.9876	-0.98757
0.10	-0.96938	-0.9694	-0.96938
0.20	-0.91810	-0.9181	-0.91810
0.50	-0.66726	-0.6673	-0.66726
2.00	2.01750	2.0176	2.01750
3.00	4.72928	4.7296	4.72928
5.00	11.75199	11.7529	11.7520

The results of varying parameter values on the skin friction coefficient and the local Nusselt number is shown in table 2. It is observed that increasing the velocity ratio parameter decreases both the skin friction coefficient and the rate of heat transfer on the surface. Increases in the magnetic parameter tend to increases both the skin friction coefficient and the Nusselt number due to the presence of the Lorenz force induced by the magnetic field in the flow. Furthermore, both the Prandtl number and the thermal Grashof number do not have any effects on the skin friction coefficient. However, both parameters result in the reduction of the local number. It is however interesting to note that increasing the suction parameter has the effects on reducing not only the skin friction coefficient but also the rate of heat transfer for obvious reasons.

Table 2 Numerical results for varying parameter values

λ	Ha	Pr	G_T	f_W	$-f''(0)$	$-\theta'(0)$
0	1	0.71	0.1	0.1	1.36509	0.07494
0.5	1	0.71	0.1	0.1	1.16742	0.05321
1.0	1	0.71	0.1	0.1	0.63552	-0.07201
1.0	2	0.71	0.1	0.1	1.08676	-0.08463
1.0	3	0.71	0.1	0.1	1.43756	-0.07432
1.0	1	2.00	0.1	0.1	0.63552	- 0.38454
1.0	1	4.00	0.1	0.1	0.63552	-0.86908
1.0	1	7.00	0.1	0.1	0.63552	-1.59590
1.0	1	0.71	1	0.1	0.63552	-1.62013
1.0	1	0.71	2	0.1	0.63552	-3.34026
1.0	1	0.71	3	0.1	0.63552	-5.06039
1.0	1	0.71	0.1	0.5	0.56220	-0.07259
1.0	1	0.71	0.1	1.0	0.48259	-0.07378
1.0	1	0.71	0.1	1.5	0.41590	-0.07545

b) Graphical Results

Velocity and temperature profiles have been plotted to show the behavior of the flow in the boundary layer region. Figures 1 – 4 presents the velocity profiles for varying parameters. In figure 1, it is observed that increasing the velocity ration tends to increase the velocity boundary layer. It is observed that when the velocity ratio is zero, the usual profile at free stream is obtained far away from the surface.

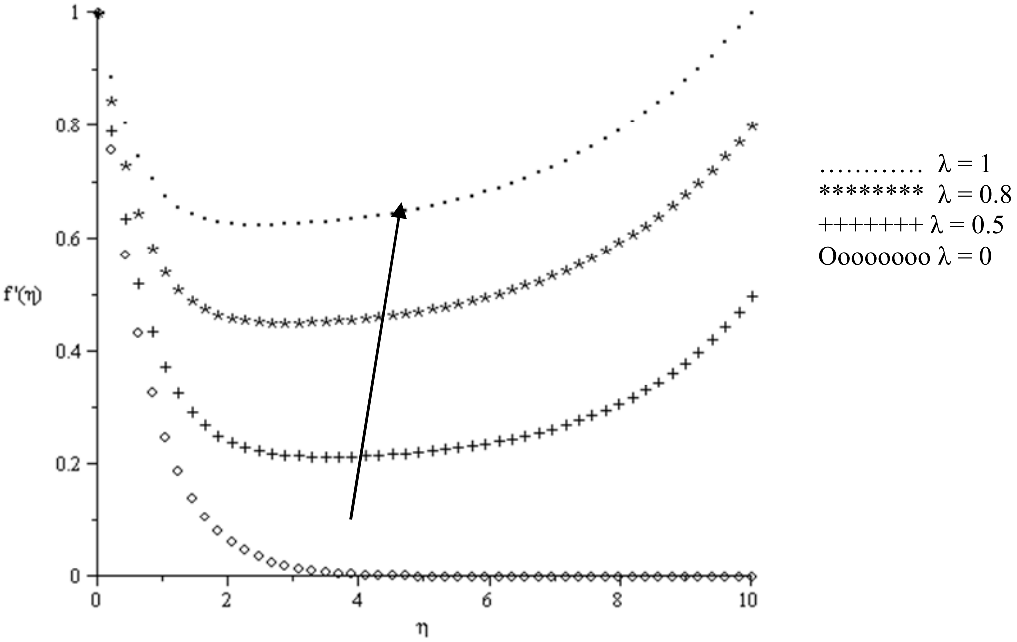

Figure 2. Velocity Profile for Varying Velocity Ratio Parameter.

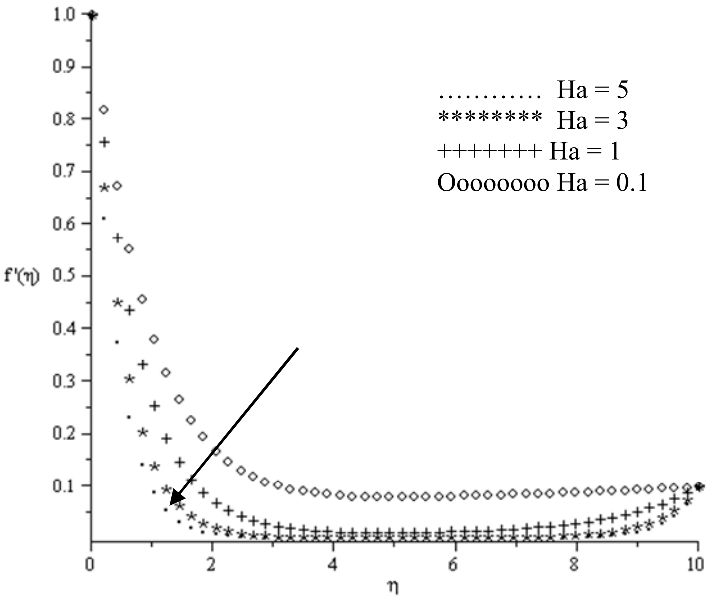

Figure 3. Velocity profiles for varying values of the magnetic parameter

Figure 3 illustrates the velocity profile when the Hartman number is increased. It is observed that increasing the Hartman number reduces the velocity profiles due to the presence of the Lorenz force induced by the magnetic field.

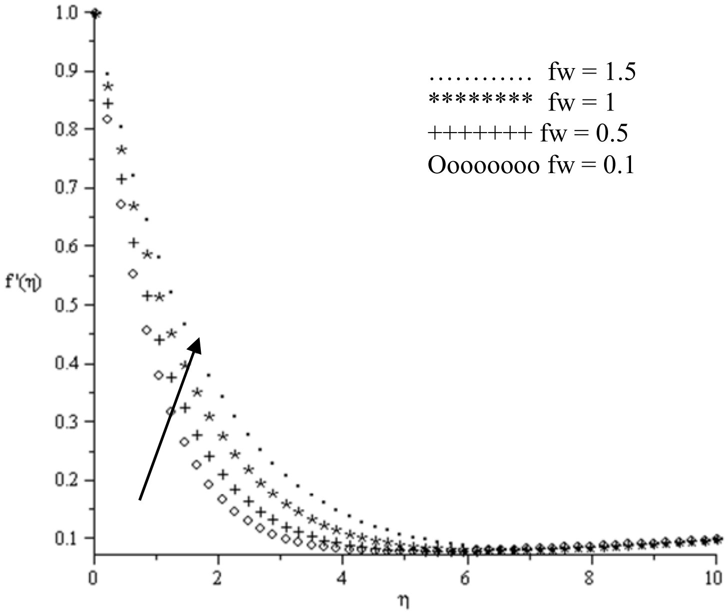

Figure 4. Velocity Profiles for varying suction parameter

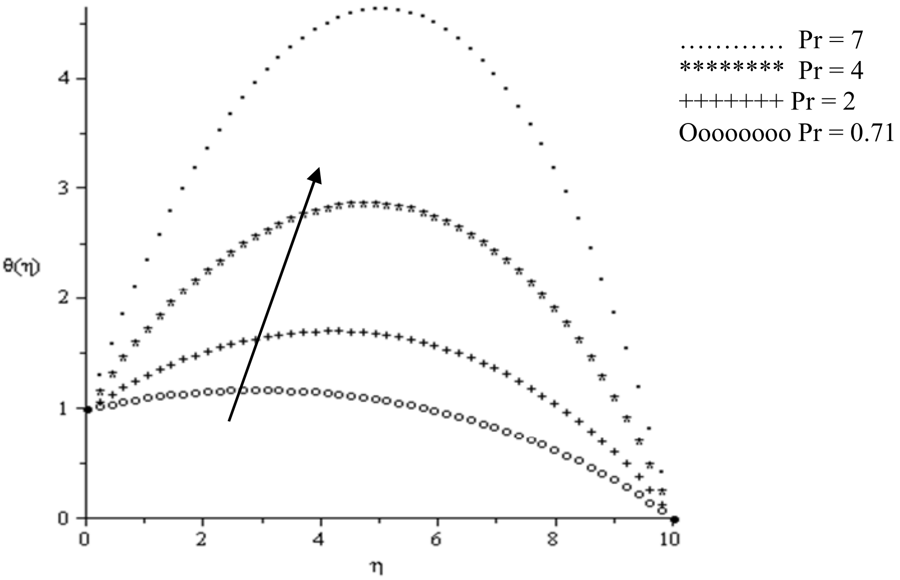

Figure 5. Temperature Profiles for increasing Prandtl number

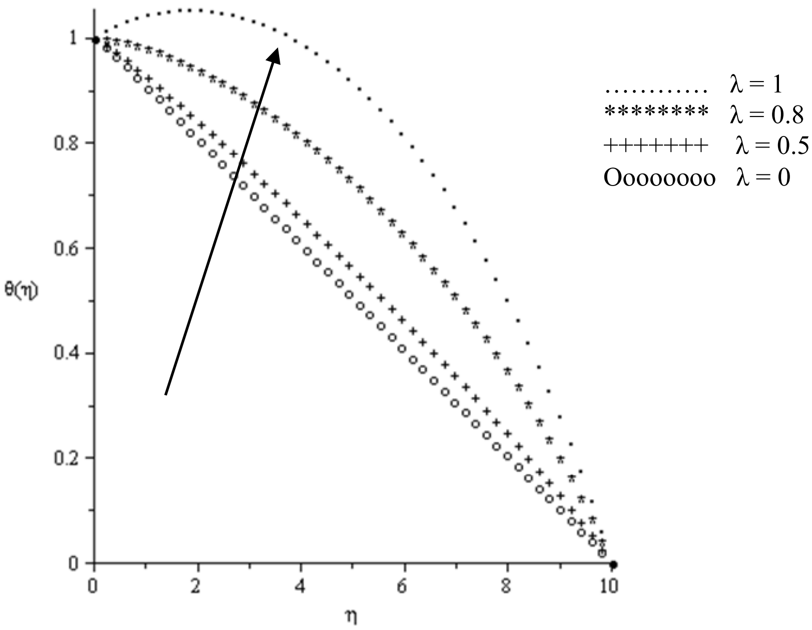

Figure 6. Temperature Profiles for increasing velocity ratio parameter

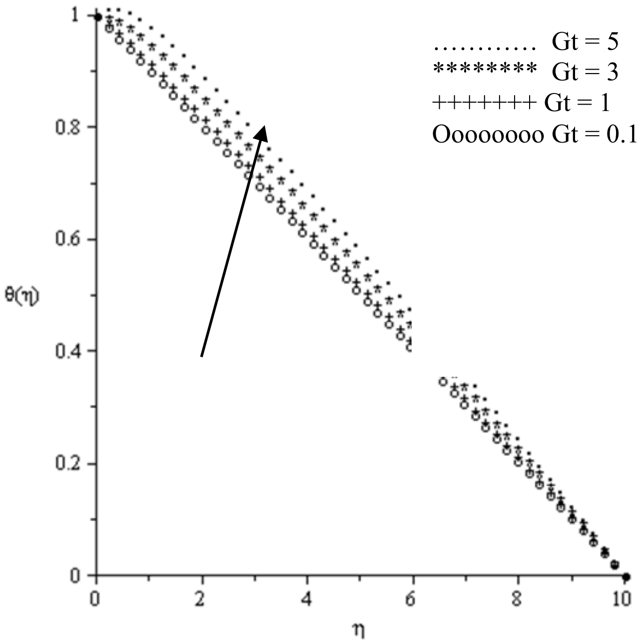

Figure 7. Temperature Profiles for varying thermal Grashof number

Figures 5 – 7 presents the temperature profiles for various parameter variation. In all these figures, it is clear that the temperature profiles increases with increasing values of the Prandtl number, the velocity ratio parameter and the thermal Grashof number.

4.0 Conclusions

The MHD thermal stagnation point flow towards a stretching permeable surface has been investigated. Numerical results have been compared to earlier results published in the literature and a perfect agreement was achieved. Among others, our results reveal that the cooling rate of a porous stretching sheet in an electrically conducting fluid, subject to a magnetic field can be controlled and a final product with desired characteristics can be achieved.

References

Alireza, R., Farzaneh-Gord, M., Varedi, S. R., *Ganji*, D.D. (2013), Analytical Solution for Magnetohydrodynamic Stagnation Point Flow and Heat Transfer over a Permeable Stretching Sheet with Chemical Reaction, Journal of Theoretical and Applied Mechanics 51 (3) 675-686, Warsaw.

Cortell, R. (2008), Effects of dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet, Phys. Lett. A, V. 372, pp.631-636.

Datta, B.K., Roy, P., Gupta, A.S. (1985), Temperature field in the flow over a stretching sheet with uniform heat flux , International Communications in Heat and Mass Transfer, 12, 89-94

Fang, T., Zhang, J., Yao, S. (2009), Slip MHD viscous flow over a stretching sheet – an exact solution, Communications in Nonlinear Science and Numerical Simulation , 14, 3731- 3737

Gupta, P.S., Gupta, A.S. (1977), Heat and mass transfer on a stretching sheet with suction or blowing, Canadian Journal of Chemical Engineering, 55, 744-750

Ibrahim, S.Y., Makinde, O.D. (2010). On MHD boundary layer flow of chemically reacting fluid with heat and mass transfer past a stretching sheet, International Journal of fluid Mechanics, 2(2),123-132.

Ishak, A., Jafar, K., Nazar, R., Pop, I. (2009), MHD stagnation point flow towards a stretching sheet, Physica A, 388, 3377-3383.

Javed, T., Abbas, Z., Sajid, M., Ali, N. (2012), Heat transfer analysis for a hydromagnetic viscous fluid over a nonlinear shrinking sheet, International Journal of Heat and Mass Transfer, doi:10.1016/ j.ijheat mass transfer .2010.12 .025 (in press)

Kechil, S.A., Hashim, I. (2008), Series solution of flow over nonlinearly stretching sheet with chemical reaction and magnetic field, Physics Letters B, 372, 2258-2263

Makinde, O.D., Charles, W.M. (2010). Computational dynamics of hydromagnetic stagnation flow towards a stretching sheet, Appl. Comput. Math., 9(2), 243-251.

McLeod, B., Rajagopal, K.R. (1987), On the non- uniqueness of the flow of a Navier-Stokes fluid due to stretching boundary, Archive for Rational Mechanics and Analysis, 98, 385-493

Nazar, R., Amin, N., Filip, D., Pop, I. (2004), Unsteady boundary layer flow in the region of the stagnation point on a stretching sheet, Int. J. Eng. Sci., V.42, pp.1241-1253

Ohwada, T. (2009), Cole-Hopf transformation as numerical tool for the burgers equation, Appl. Comput. Math. V.8, N.1, pp.107-113.

Rana, P., Bhargava, R. (2012), Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: A numerical study, Communications in Nonlinear Science and Numerical Simulation, 17, 212-226

Sakiadis, B.C. (1971), Boundary layer behavior on continuous solid surfaces: boundary layer equations for two dimensional and axisymmetric flow, American Institute of Chemical Engineers, 61, 26-34

Seini, Y.I. (2013), Flow over unsteady stretching surface with chemical reaction and non-uniform heat source, Journal of Engineering and Manufacturing Technology, JEMT 1 (2013) 24-35

Seini, Y.I., Makinde, O.D., (2013). MHD boundary layer flow due to exponential stretching surface with radiation and chemical reaction, Mathematical Problems in Engineering, http://dx.doi.org/10.1155/2013/163614

Wang, C.Y. (2007), Natural convection on a vertical radially stretching sheet, Math. Anal. App., V. 332, pp. 877-883

Yian, Y.L., Amin, N., Pop, I. (2007), Mixed convection flow near a non-orthogonal stagnation point towards a stretching vertical plate, International Journal of Heat and Mass Transfer, V. 50, N.(23-24), pp.4855-4863.