Micro–level stochastic loss reserving for general insurance

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Abstract

[1] To meet future liabilities general insurance companies will set–up reserves. Predicting future cash–flows is essential in this process. Actuarial loss reserving methods will help them to do this in a sound way. The last decennium a vast literature about stochastic loss reserving for the general insurance business has been developed. Apart from few exceptions, all of these papers are based on data aggregated in run–off triangles. However, such an aggregate data set is a summary of an underlying, much more detailed data base that is available to the insurance company. We refer to this data set at individual claim level as 'micro–level data'. We investigate whether the use of such micro–level claim data can improve the reserving process. A realistic micro–level data set on liability claims (material and injury) from a European insurance company is modeled. Stochastic processes are specified for the various aspects involved in the development of a claim: the time of occurrence, the delay between occurrence and the time of reporting to the company, the occurrence of payments and their size and the final settlement of the claim. These processes are calibrated to the historical individual data of the portfolio and used for the projection of future claims. Through an out–of–sample prediction exercise we show that the micro–level approach provides the actuary with detailed and valuable reserve calculations. A comparison with results from traditional actuarial reserving techniques is included. For our case–study reserve calculations based on the micro–level model are to be preferred; compared to traditional methods, they reflect real outcomes in a more realistic way.

Key words: actuarial science, reserving, general insurance, poisson process, recurrent events, survival analysis, prediction.

1 Introduction

We develop a micro–level stochastic model for the run–off of general insurance (also called 'non–life' or 'property and casualty') claims. Figure 1 illustrates the run–off (or development) process of a general insurance claim. It shows that a claim occurs at a certain point in time (t_1), consequently it is declared to the insurer (t_2) (possibly after a period of delay) and one or several payments follow until the settlement (or closing) of the claim. Depending on the nature of the business and claim, the claim can re–open and payments can follow until the claim finally settles.

At the present moment (say τ) the insurer needs to put reserves aside to fulfill his liabilities in the future. This actuarial exercise will be denoted as 'loss' or 'claims reserving'. Insurers,

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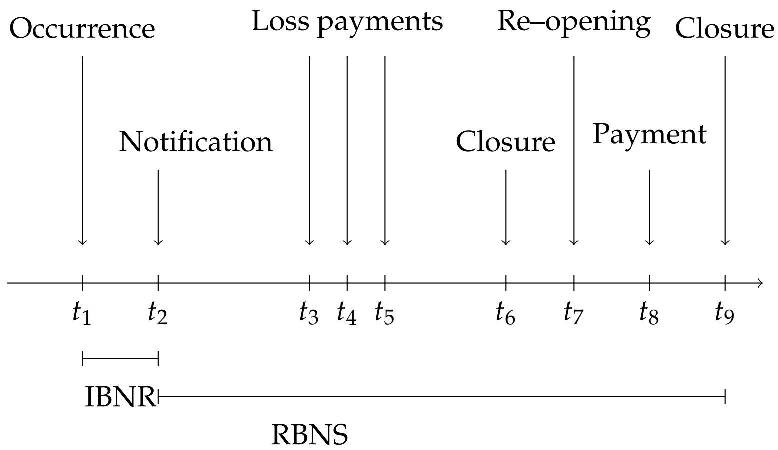
[1]The authors would like to thank Jan–Willem Vulto and Joris van Kempen for supplying and explaining the data. Please note that the original frequency and severity data have been transformed for reasons of confidentiality.

Figure 1: *Development of a general insurance claim*



share holders, regulators and tax authorities are interested in a rigorous picture of the distribution of future payments corresponding with open (i.e. not settled) claims in a loss reserving exercise. General insurers distinguish between RBNS and IBNR reserves. 'RBNS' claims are claims that are Reported to the insurer But Not Settled, whereas 'IBNR' claims Incurred But are Not Reported to the company. For an RBNS claim occurrence and declaration take place before the present moment and settlement occurs afterwards (i.e. $\tau \geq t_2$ and $\tau < t_6$ (or $\tau < t_9$) in Figure 1). An IBNR claim has occurred before the present moment, but its declaration and settlement follow afterwards (i.e. $\tau \in [t_1, t_2)$ in Figure 1). The interval $[t_1, t_2]$ represents the so–called reporting delay. The interval $[t_2, t_6]$ (or $[t_2, t_9]$) is often referred to as the settlement delay. Data bases within general insurance companies typically contain detailed information about the run–off process of historical and current claims. The structure in Figure 1 is generic for the kind of information that is available. In this paper we will use the label 'micro–level' data to denote this sort of data structures.

With the introduction of Solvency 2 (in 2012) and IFRS 4 Phase 2 (in 2013) insurers face major challenges. IFRS 4 Phase 2 will define a new accounting model for insurance contracts, based on market values of liabilities. In the document "Preliminary Views on Insurance Contracts" (May 2007, discussion paper) the IASB ('International Accounting Standards Board')[2] states that an insurer should base the measurement of all its insurance liabilities (for reserving) on 'best estimates' of the contractual cash flows, discounted with current market discount rates. On top of this, a margin that market participants are expected to require for bearing risk should be added to this.

Solvency 2 will lead to a change in the regulatory required solvency capital for insurers. Depending on the type of business, at this moment this capital requirement is a fixed percentage of the mathematical reserve, the risk capital, the premiums or the claims. Under Solvency 2 the so–called Solvency Capital Requirement ('SCR') will be risk–based, and market values of assets and liabilities will be the basis for these calculations.

The measurement of future cash flows and their uncertainty thus becomes more and more important. That also gives rise to the question whether the currently used techniques can be improved. In this paper we will address that question for general insurance. Currently, reserving for general insurance is based on data aggregated in run–off triangles. In a run–off triangle observable variables are summarized per arrival year and development year combination. The term *arrival year* ('AY') or *year of occurrence* is used by general actuaries to indicate the year in

[2] http://www.iasb.org/NR/rdonlyres/08C8BB09-61B7-4BE8-AA39-A1F71F665135/0/InsurancePart1.pdf

which the accident took place. For a claim from AY t its first *development year* will be year t itself, the second development year is $t + 1$ and so on. An example of a run–off triangle is given in Table 3 and 4. A vast literature exists about techniques for claims reserving, largely designed for application to loss triangles. An overview of these techniques is given in England and Verrall (2002), Wüthrich and Merz (2008) or Kaas et al. (2008). These techniques can be applied to run–off triangles containing either 'paid losses' or 'incurred losses' (i.e. the sum of paid losses and case reserves).

The most popular approach is the chain–ladder model, largely because of is practicality. Loosely spoken, the stochastic chain–ladder model applies a Poisson regression model to the observations in a run–off triangle, whereby arrival and development year figure as categorical covariates. However, the use of aggregated data in combination with the chain–ladder approach gives rise to several issues. A whole literature on itself has evolved to solve these issues, which are (in random order):

(1) Different results between projections based on paid losses or incurred losses, addressed by Quarg and Mack (2008), Postuma et al. (2008) and Halliwell (2009).

(2) Lack of robustness and the treatment of outliers, see Verdonck et al. (2009).

(3) The existence of the chain–ladder bias, see Halliwell (2007) and Taylor (2003).

(4) Instability in ultimate claims for recent arrival years, see Bornhuetter and Ferguson (1972).

(5) Modeling negative or zero cells in a stochastic setting, see Kunkler (2004).

(6) The inclusion of calendar year effects, see Verbeek (1972) and Zehnwirth (1994).

(7) The possibly different treatment of small and large claims, see Wuthrich and Alai (2009).

(8) The need for including a tail factor, see for example Mack (1999).

(9) Over parametrization of the chain–ladder method, see Wright (1990) and Renshaw (1994).

(10) Separate assessment of IBNR and RBNS claims, see Schnieper (1991) and Liu and Verrall (2009).

(11) The realism of the Poisson distribution underlying the chain–ladder method.

(12) When using aggregate data, lots of useful information about the claims data remains unused, as noted by England and Verrall (2002) and Taylor et al. (2008).

Without going into detail, we conclude that the references above present useful additions to or comments on the chain–ladder method, but these additions cannot all be applied simultaneously. More importantly, the existence of these issues and the substantial literature about it indicate that the use of aggregate data in combination with the chain–ladder technique (or similar techniques) is not always adequate for capturing the complexities of stochastic reserving for general insurance.

England and Verrall (2002) and Taylor et al. (2008) questioned the use of aggregate loss data when the underlying extensive micro–level data base is available as well. With aggregate data, lots of useful information about the claims data remain unused. Covariate information from policy, policy holder or the past development process cannot be used in the traditional stochastic model, since each cell of the run–off triangle is an aggregate figure. Quoting England and Verrall (2002) (page 507) *"[. . .] it has to be borne in mind that traditional techniques were developed before the advent of desktop computers, using methods which could be evaluated using pencil and paper. With the continuing increase in computer power, it has to be questioned whether it would not be better to examine individual claims rather than use aggregate data."*.

As a result of the observations mentioned above, a small stream of literature has emerged about stochastic loss reserving on an individual claim level. Arjas (1989), Norberg (1993) and Norberg (1999) formulated a mathematical framework for the development of individual claims. Using ideas from martingale theory and point processes, these authors present a probabilistic, rather than statistical, framework for individual claims reserving. Haastrup and Arjas (1996) continue the work by Norberg and present a first detailed implementation of a micro–level stochastic model for loss reserving. They use non–parametric Bayesian techniques which may complicate the accessibility of the paper. Furthermore, their case study is based on a small data set with fixed claim amounts. Recently, Larsen (2007) revisited the work of Norberg, Haastrup and Arjas with a small case–study. Zhao et al. (2009) and Zhao and Zhou (2010) present a model for individual claims development using (semi–parametric) techniques from survival analysis and copula methods. However, a case study is lacking in their work.

In this paper a micro-level stochastic model is developed to quantify the reserve and its uncertainty for a realistic general liability insurance portfolio. Stochastic processes for the occurrence time, the reporting delay, the development process and the payments are fit to the historical individual data of the portfolio and used for projection of future claims and its (estimation and process) uncertainty. Both the Incurred But Not Reported (IBNR) reserve as well as the Reported But Not Settled (RBNS) reserve are quantified and the results are compared with those of traditional actuarial techniques.

We investigate whether the quality of reserves and their uncertainty can be improved by using more detailed claims data instead of the classical run–off triangles. Indeed, a micro–level approach allows much closer modeling of the claims process. Lots of the above mentioned issues will not exist when using a micro–level approach, because of the availability of lots of data and the potential flexibility in modeling the future claims process. For example, covariate information (e.g. deductibles, policy limits, calendar year) can be included in the projection of the cash flows when claims are modeled at an individual level. The use of lots of (individual) data avoids robustness problems and over parametrization. Also the problems with negative or zero cells and setting the tail factor are circumvented, and small and large claims can be handled simultaneously. Furthermore, individual claim modeling can provide a natural solution for the dilemma within the traditional literature whether to use triangles with paid claims or incurred claims. This dilemma is important because practicing actuaries put high value to their companies' expert opinion which is expressed by setting an initial case reserve. Incurred payments are the sum of paid losses and these case reserves. Using micro–level data we use the initial case reserve as a covariate in the projection process of future cash flows.

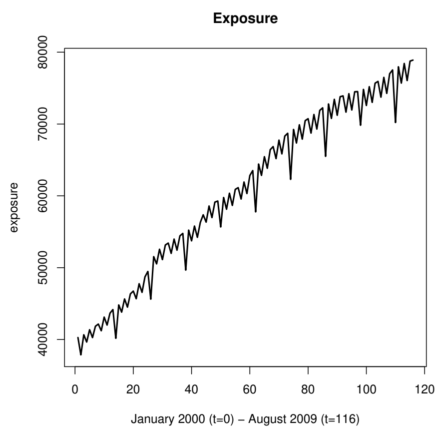
The remainder of the paper is organized as follows. First, the data set is introduced in Section 2. In Section 3 the statistical model is described. Results from estimating all components of the model are in Section 4. Section 5 presents the prediction routine and in Section 6 we give results and a comparison with traditional actuarial techniques. Section 7 concludes.

2 Data

The data set used in this paper contains information about a general liability insurance portfolio (for private individuals) of a European insurance company. The available data consist of the exposure per month from January 2000 till August 2009, as well as a claim file that provides a record of each claim filed with the insurer from January 1997 till August 2009. Note that we are missing exposure information for the period January 1997 till December 1999, but the impact of this lack on our reserve calculations will be negligible.

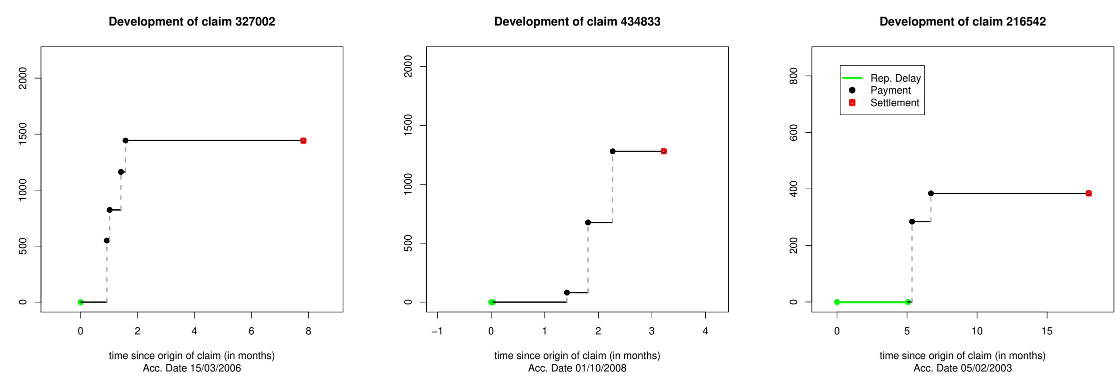
Exposure The exposure is not the number of policies, but the 'earned' exposure. That implies that two policies which are both only insured for half of the period are counted as 1. Figure 2 shows the exposure per month. Note that the downward spikes correspond to the month February.

Figure 2: *Available exposure per month from January 2000 till August 2009.*



Random development processes The claim file consists of 1,525,376 records corresponding with 491,912 claims. Figure 3 shows the development of three claims, taken at random from our data set. It shows the timing of events as well as the cost of the corresponding payments (if any). These are indicated as jumps in the figure. Starting point of the development process is the accident date. This is indicated with a sub-title in each of the plots and corresponds with the point $x = 0$. The x-axis is in months since the accident date. The y-axis represents the cumulative amount paid for the claim.

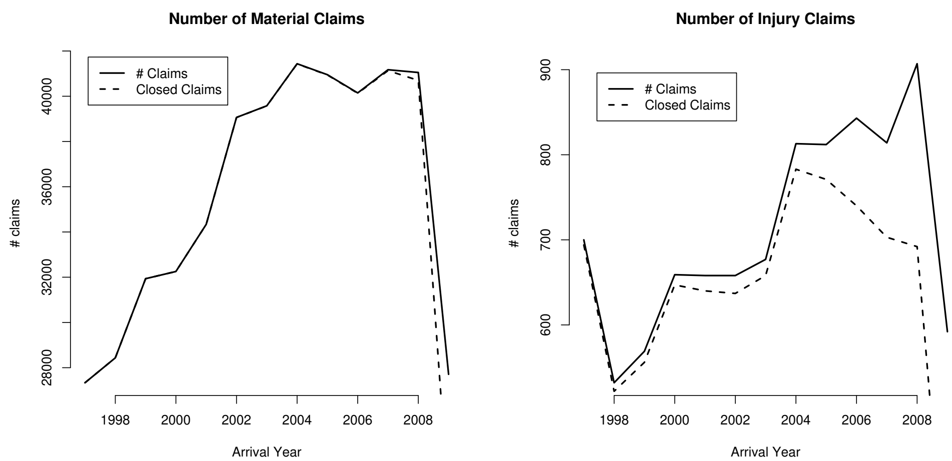
Figure 3: *Development of 3 random claims from the data set.*



Type and number of claims In this general liability portfolio, we have to deal with two types of claims: material damage ('material') and bodily injury ('injury').
Figure 4 shows the number of claims per arrival year, and whether they are closed or still open (at the end of August 2009). The development pattern and loss distributions of these claim types are usually very different. In practice they are therefore treated separately in separate run–off triangles. Following this approach, we will treat them separately too.

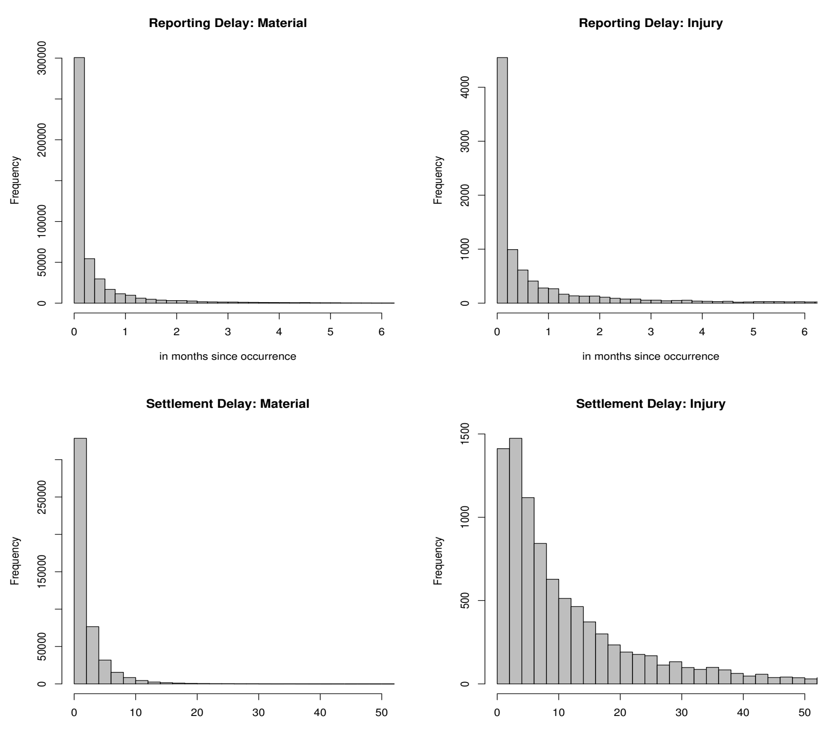
Reporting and settlement delay Important drivers of the IBNR and RBNS reserves are the reporting delays and settlement delays. Figure 5 shows the reporting delays for material and injury claims. The reporting delay is the time that passes between the occurrence date of the accident and the date it was reported to the insurance company. It is measured in months since occurrence of the claim. Of course, the reporting delay is only available for claims that have been reported to the insurer at the present moment. Figure 5 shows the settlement delay separately for injury and material claims. The settlement delay is the time elapsed between the

Figure 4: *Number of open and closed claims of type material (left) and injury (right).*



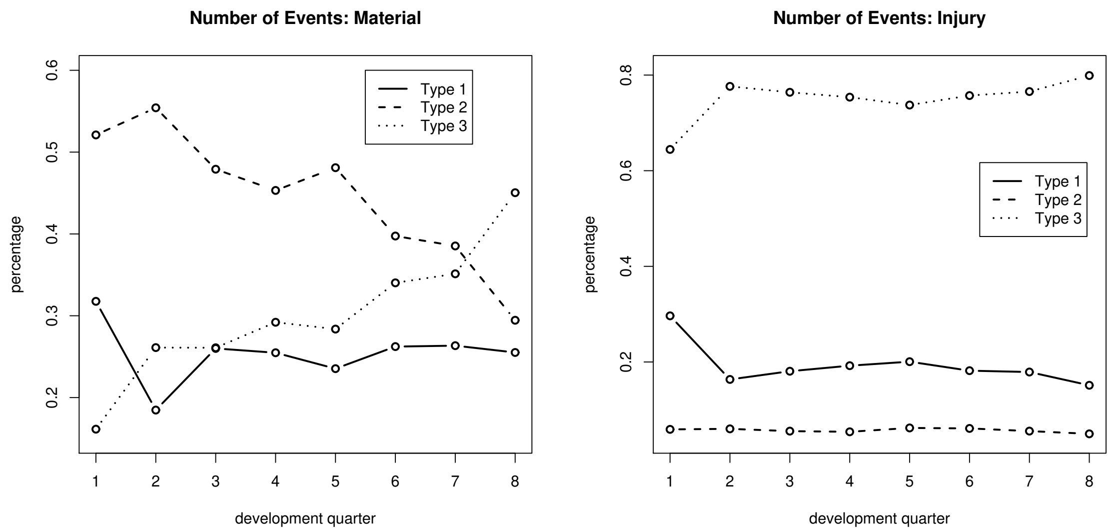
reporting date of the claim and the date of final settlement by the company. It is measured in months and only available for closed claims. These figures show that the observed reporting delays are of similar length for material and injury losses. However, the settlement delay is very different. The settlement delay is far more skewed to the right for the injury claims than for the material claims.

Figure 5: *Upper: reporting delay for material (left) and injury (right) claims. Lower: settlement delay for material (left) and injury (right) claims.*



Events in the development In this paper we will distinguish three types of events which can occur during the development of a claim. "Type 1" events imply settlement of the claim without a payment. With a "type 2" event we will refer to a payment with settlement at the same time. Intermediate payments (without settlement) are "type 3" events. Figure 6 gives the relative frequency of the different types of events over development quarters. With micro–level data the first development quarter is the period of 3 months following the reporting date of the claim, the second quarter is the period of 3 months following the first development quarter, et cetera. In the last development quarter shown in the graph we collect the remainder development. The graph shows that the proportions of each event type are stable over the development quarters for injury claims. For material claims, the proportion of event type 2 decreases for later development quarters, while the proportion of event type 3 increases.

Figure 6: *Relative frequency of events over development quarters.*



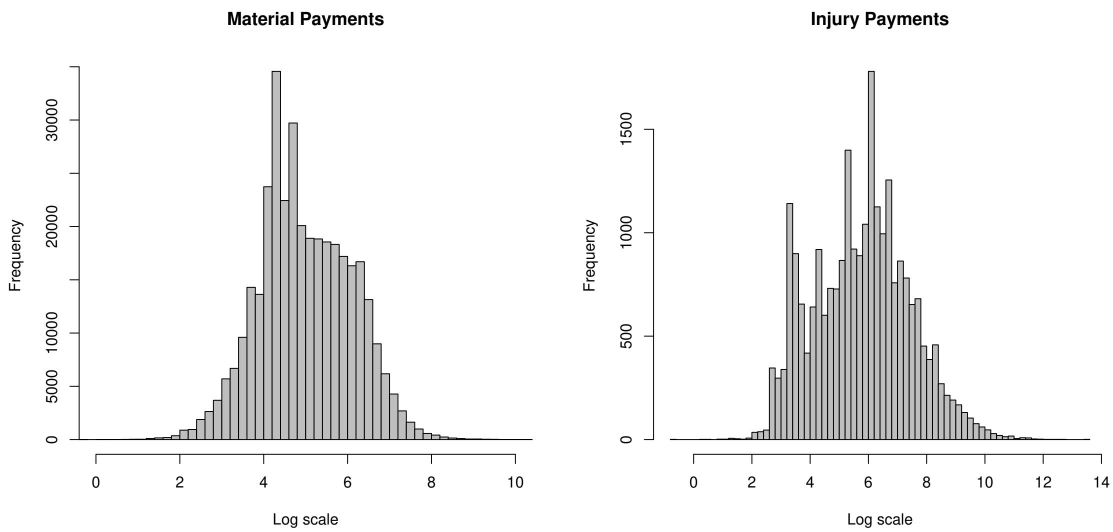
Payments Events of type 2 and type 3 come with a payment. The distribution of these payments differs materially for the different types of claims. Figure 7 shows the distribution of the payments, separately for material and injury claims. The payments are discounted to 1-1-1997 with the Dutch consumer price inflation, to exclude the impact of inflation on the distribution of the payments. The figures suggest that a lognormal distribution would probably be reasonable for describing the distribution of the payments. This will be discussed further in Section 4. Table 1 gives characteristics of the observed payments for both material and injury losses.

Table 1: *Characteristics observed payments.*

	Mean	Median	Min.	Max.	1%	5%	25%	75%	95%	99%
Material	277	129	8×10^{-4}	198,931	12	25	69	334	890	1,768
Injury	1,395	361	0.4875	779,398	16	25	89	967	4,927	16,664

Initial case estimates As noted in Section 1, often the problem arises that the projection based on paid losses is far different than the projection based on incurred losses. This problem is addressed recently by Quarg and Mack (2008), Postuma et al. (2008) and Halliwell (2009), who

Figure 7: *Distribution of payments for material (right) and injury (left) claims.*



simultaneously model paid and incurred losses. Disadvantage of those methods is that models based on incurred losses can be instable because the methods for setting the case reserves are often changed (for example, as a result of adequacy test results or profit policy of the company). Reserving models that are directly based on these case reserves (as part of the incurred losses) can therefore be instable. However, the case reserves may have added value as an explaining variable when projecting future payments. We have defined different categories of initial case reserves (separately for material claims and injury claims) that can be used as explanatory variables. Table 2 shows the number of claims, the average settlement delay (in months) and the average cumulative paid amount for these categories. The table clearly shows the differences in settlement delay and cumulative payments for the different initial reserve categories. Therefore, it might be worthwhile to include these categories as explanatory variables in the prediction routine.

Table 2: *Initial reserve categories.*

Material				Injury			
Initial Case Reserve	# claims	Average settl. delay (months)	Average Cum. payments	Initial Case Reserve	# claims	Average settl. delay (months)	Average Cum. payments
$\leq 10,000$	465,015	1.87	252	$\leq 1,000$	3,709	9.87	2,570
$> 10,000$	385	10.88	7,950	(1,000 -15,000]	5,165	15.17	3,872
				$> 15,000$	360	35.2	33,840

3 The statistical model

By a claim i is understood a combination of an occurrence time T_i, a reporting delay U_i and a development process $\mathbf{X}_i$. Hereby $\mathbf{X}_i$ is short for $(E_i(v), P_i(v))_{v \in [0,V_i]}$. $E_i(v_{ij}) := E_{ij}$ is the type of the jth event in the development of claim i. This event occurs at time v_{ij}, expressed in time units after notification of the claim. V_i is the total waiting time from notification to settlement for claim i. If the event includes a payment, the corresponding severity is given by $P_i(v_{ij}) := P_{ij}$.

The different types of events are specified in Section 2. The development process $\mathbf{X}_i$ is a jump process. It is modeled here with two separate building blocks: the timing and type of events and their corresponding severities. The complete description of a claim is given by:

$$(T_i, U_i, \mathbf{X}_i) \text{ with } \mathbf{X}_i := (E_i(v), P_i(v))_{v \in [0, V_i]}. \tag{1}$$

Assume that outstanding liabilities are to be predicted at calendar time τ. We distinguish IBNR, RBNS and settled claims.

- for an IBNR claim: $T_i + U_i > \tau$ and $T_i < \tau$;
- for an RBNS claim: $T_i + U_i \leq \tau$ and the development of the claim is censored at $(\tau - T_i - U_i)$, i.e. only $(E_i(v), P_i(v))_{v \in [0, \tau - T_i - U_i]}$ is observed;
- for a settled claim: $T_i + U_i \leq \tau$ and $(E_i(v), P_i(v))_{v \in [0, V_i]}$ is observed.

3.1 Position dependent marked Poisson process

Following the approach in Arjas (1989) and Norberg (1993) we treat the claims process as a Position Dependent Marked Poisson Process (PDMPP), see Karr (1991). In this application, a point is an occurrence time and the associated mark is the combined reporting delay and development of the claim. We denote the intensity measure of this Poisson process with λ and the associated mark distribution with $(P_{Z|t})_{t \geq 0}$. In the claims development framework the distribution $P_{Z|t}$ is given by the distribution $P_{U|t}$ of the reporting delay, given occurrence time t, and the distribution $P_{X|t,u}$ of the development, given occurrence time t and reporting delay u. The complete development process then is a Poisson process on claim space $\mathcal{C} = [0, \infty) \times [0, \infty) \times \chi$ with intensity measure:

$$\lambda(dt) \times P_{U|t}(du) \times P_{X|t,u}(dx) \;\text{ with } (t, u, x) \in \mathcal{C}. \tag{2}$$

The reported claims (which are not necessarily settled) belong to the set:

$$\mathcal{C}^r = \{(t, u, x) \in \mathcal{C} | t + u \leq \tau\}, \tag{3}$$

whereas the IBNR claims belong to:

$$\mathcal{C}^i = \{(t, u, x) \in \mathcal{C} | t \leq \tau,\ t + u > \tau\}. \tag{4}$$

Since both sets are disjoint, both processes are independent (see Karr (1991)). The process of reported claims is a Poisson process on $\mathcal{C}$ with measure

$$\begin{aligned} & \lambda(dt) \times P_{U|t}(du) \times P_{X|t,u}(dx) \times 1_{[(t,u,x) \in \mathcal{C}^r]} \\ = \; & \underbrace{\lambda(dt) P_{U|t}(\tau - t) 1_{(t \in [0,\tau])}}_{(a)} \times \underbrace{\frac{P_{U|t}(du) 1_{(u \leq \tau - t)}}{P_{U|t}(\tau - t)}}_{(b)} \times \underbrace{P_{X|t,u}(dx)}_{(c)}. \end{aligned} \tag{5}$$

Part (a) is the occurrence measure. The mark of this claim is composed by a reporting delay, given the occurrence time (its conditional distribution is given by (b)), and the conditional distribution (c) of the development, given the occurrence time and reporting delay. Similarly, the process of IBNR claims is a Poisson process with measure:

$$\underbrace{\lambda(dt)\left(1 - P_{U|t}(\tau - t)\right) 1_{(t \in [0,\tau])}}_{(a)} \times \underbrace{\frac{P_{U|t}(du) 1_{u > \tau - t}}{1 - P_{U|t}(\tau - t)}}_{(b)} \times \underbrace{P_{X|t,u}(dx)}_{(c)}, \tag{6}$$

where similar components can be identified as in (5).

3.2 The likelihood

The approach followed in this paper is parametric. Therefore, we will optimize the likelihood expression for observed data over the unknown parameters used in this expression. The observed part of the claims process consists of the development up to time τ of claims reported before τ. We denote these observed claims as follows:

$$(T_i^o, U_i^o, X_i^o)_{i\geq 1}, \tag{7}$$

where the development of claim i is censored $\tau - T_i^o - U_i^o$ time units after notification. The likelihood of the observed claim development process can be written as (see Cook and Lawless (2007)):

$$\begin{aligned}\Lambda(obs) &\propto \left\{\prod_{i\geq 1} \lambda(T_i^o) P_{U|t}(\tau - T_i^o)\right\} \exp\left(-\int_0^\tau w(t)\lambda(t)P_{U|t}(\tau-t)dt\right) \\ &\times \left\{\prod_{i\geq 1} \frac{P_{U|t}(dU_i^o)}{P_{U|t}(\tau - T_i^o)}\right\} \times \prod_{i\geq 1} P_{X|t,u}^{\tau - T_i^o - U_i^o}(dX_i^o).\end{aligned} \tag{8}$$

The superscript in the last term of this likelihood indicates the censoring of the development of this claim $\tau - T_i^o - U_i^o$ time units after notification. The function $w(t)$ gives the exposure at time t.

For the reporting delay and the development process we will use techniques from survival analysis. The reporting delay is a one–time single type event that can be modeled using standard distributions from survival analysis. For the development process the statistical framework of recurrent events will be used. Cook and Lawless (2007) provide a recent overview of statistical techniques for the analysis of recurrent events. These techniques primarily address the modeling of an event intensity (or hazard rate).

As mentioned in (1) for each claim i its development process consists of

$$\mathbf{X}_i = (E_i(v), P_i(v))_{v\in[0,V_i]}. \tag{9}$$

Hereby $E_i(v_{ij}) := E_{ij}$ is the type of the jth event in the development of claim i, occurring at time v_{ij}. V_i is the total waiting time from notification to settlement for claim i. If the event includes a payment, the corresponding severity is given by $P_i(v_{ij}) := P_{ij}$. To model the occurrence of the different events a hazard rate is specified for each type. The hazard rates h_{se}, h_{sep} and h_p correspond to type 1 (settlement without payment), type 2 (settlement with a payment at the same time) and type 3 (payment without settlement) events, respectively.

Events of type 2 and 3 come with a payment. We denote the density of a severity payment with P_p. Using this notation the likelihood of the development process of claim i is given by:

$$\begin{aligned}\left\{\prod_{j=1}^{N_i}\left(h_{se}^{\delta_{ij1}}(V_{ij}) \times h_{sep}^{\delta_{ij2}}(V_{ij}) \times h_p^{\delta_{ij3}}(V_{ij})\right)\right\} &\times \exp\left(-\int_0^{\tau_i}(h_{se}(u) + h_{sep}(u) + h_p(u))du\right) \\ &\times \prod_j P_p(dV_{ij}).\end{aligned} \tag{10}$$

Here δ_{ijk} is an indicator variable that is 1 if the jth event in the development of claim i is of type k. N_i is the total number of events, registered in the observation period for claim i. This observation period is $[0, \tau_i]$ with $\tau_i = \min(\tau - T_i - U_i, V_i)$.

Combining (8) and (10) gives the likelihood for the observed data:

$$\begin{aligned}\Lambda(obs) &\propto \left\{\prod_{i\geq 1}\lambda(T_i^o)P_{U|t}(\tau-T_i^o)\right\}\exp\left(-\int_0^\tau w(t)\lambda(t)P_{U|t}(\tau-t)dt\right)\\ &\times \left\{\prod_{i\geq 1}\frac{P_{U|t}(dU_i^o)}{P_{U|t}(\tau-T_i^o)}\right\}\times\prod_{i\geq 1}\left\{\prod_{j=1}^{N_i}\left(h_{se}^{\delta_{ij1}}(V_{ij})\times h_{sep}^{\delta_{ij2}}(V_{ij})\times h_p^{\delta_{ij3}}(V_{ij})\right)\right\}\\ &\times \exp\left(-\int_0^{\tau_i}(h_{se}(u)+h_{sep}(u)+h_p(u))du\right)\times\prod_{i\geq 1}\prod_j P_p(dV_{ij}).\end{aligned}\tag{11}$$

3.3 Distributional assumptions

We discuss the likelihood in (11) in more detail. Distributional assumptions for the various building blocks, being the reporting delay, the occurrence times –given the reporting delay distribution– and the development process, are presented. At each stage it is possible to include covariate information such as the initial case reserve classes. Our final choices and estimation results will be covered in Section 4.

Reporting delay The notification of the claim is a one–time single type event that can be modeled using standard distributions from survival analysis (such as the Exponential, Weibull or Gompertz distribution). Figure 5 indicates that for a large part of the claims the claim will be reported in the first few days after the occurrence. Therefore we use a mixture of one particular standard distribution with one or more degenerate distributions for notification during the first few days. For example, for a mixture of a survival distribution f_U with n degenerate components the density is given by:

$$\sum_{k=0}^{n-1}p_k I_{\{k\}}(u)+\left(1-\sum_{k=0}^{n-1}p_k\right)f_{U|U>n-1}(u),\tag{12}$$

where $I_{\{k\}}=1$ for the kth day after occurrence time t and $I_{\{k\}}=0$ otherwise.

Occurrence process When optimizing the likelihood for the occurrence process the reporting delay distribution and its parameters (as obtained in the previous step) are used. The likelihood

$$L \propto \left\{\prod_{i\geq 1}\lambda(T_i^o)P_{U|t}(\tau-T_i^o)\right\}\exp\left(-\int_0^\tau w(t)\lambda(t)P_{U|t}(\tau-t)dt\right),\tag{13}$$

needs to be optimized over $\lambda(t)$. We use a piecewise constant specification for the occurrence rate:

$$\lambda(t) = \begin{cases}\lambda_1 & 0\leq t<d_1\\ \lambda_2 & d_1\leq t<d_2\\ \vdots & \\ \lambda_m & d_{m-1}\leq t<d_m,\end{cases}\tag{14}$$

with intervals such that $\tau\in[d_{m-1},d_m)$ and $w(t):=w_l$ for $d_{l-1}\leq t<d_l$.

Let the indicator variable $\delta_1(l,t_i)$ be 1 if $d_{l-1}\leq t_i<d_l$, with t_i the occurrence time of claim i. The number of claims in interval $[d_{l-1},d_l)$ can be expressed as:

$$N_{oc}(l) := \sum_i \delta_1(l,t_i).\tag{15}$$

The likelihood corresponding with the occurrence times is given by

$$
\begin{aligned}
L & \propto & \lambda_1^{N_{oc}(1)}\lambda_2^{N_{oc}(2)}\ldots\lambda_m^{N_{oc}(m)}\prod_{i\geq 1} P_{U|t}(\tau-t_i) \\
& \times & \exp\left(-\lambda_1 w_1\int_0^{d_1} P_{U|t}(\tau-t)dt\right)\exp\left(-\lambda_2 w_2\int_{d_1}^{d_2} P_{U|t}(\tau-t)dt\right) \\
& \times & \ldots\exp\left(-\lambda_m w_m\int_{d_{m-1}}^{d_m} P_{U|t}(\tau-t)dt\right).
\end{aligned}
\tag{16}
$$

Optimizing over λ_l (with $l = 1,\ldots,m$) leads to:

$$
\hat{\lambda}_l = \frac{N_{oc}(l)}{w_l\int_{d_{l-1}}^{d_l} P_{U|t}(\tau-t)dt}. \tag{17}
$$

Development process A piecewise constant specification is used for the hazard rates. This implies:

$$
h_{\{se,sep,p\}}(t) = \begin{cases} h_{\{se,sep,p\};1} \text{ for } 0 \leq t < a_1 \\ h_{\{se,sep,p\};2} \text{ for } a_1 \leq t < a_2 \\ \quad\vdots \\ h_{\{se,sep,p\};d} \text{ for } a_{d-1} \leq t < a_d. \end{cases} \tag{18}
$$

This piecewise specification can be integrated in a straightforward way in likelihood specification (11), although the resulting expression is complex in notation. The optimization of the likelihood expression can be done analytically (which results in very elegant and compact expressions) or numerically. It might be worthwhile to specify a separate hazard rate for 'first events' in the development and 'later events'. This will be investigated in Section 4.

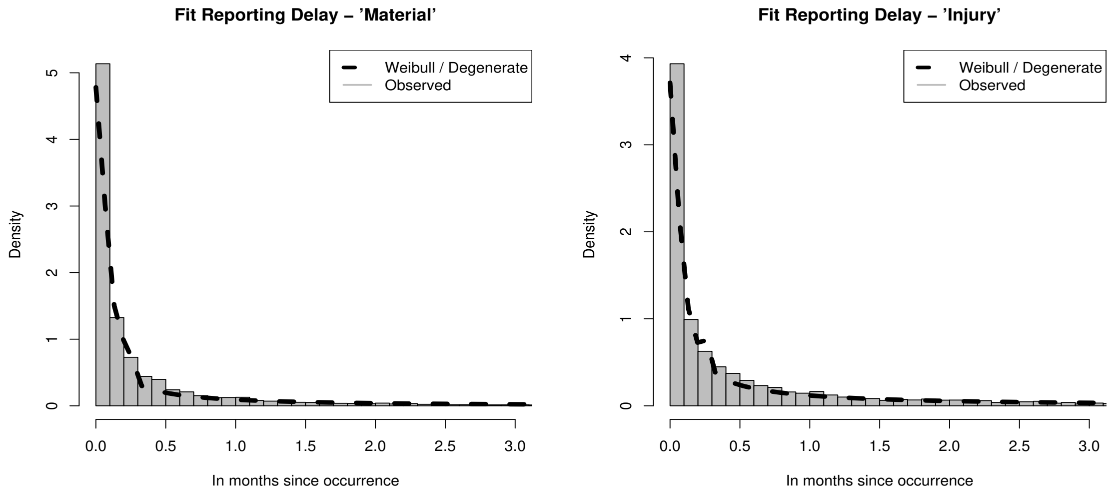
Payments Events of type 2 and type 3 come with a payment. Section 2 showed that the observed distribution of the payments has similarities with a lognormal distribution, but there might be more flexible distributions that fit the historical payment data better. Therefore, next to the lognormal distribution, we experimented with a generalized beta of the second kind (GB2), Burr and Gamma distribution. Covariate information such as the initial reserve category and the development year is taken into account.

4 Estimation results

The outcomes of calibrating these distributions to the historical data are presented. Given the very different characteristics of material and injury claims, the processes described in Section 3 are fitted (and projected) separately for both types of claims. This is in line with actuarial practice, where usually separate run–off triangles are constructed for material and injury claims. Optimization of all likelihood specifications was done with the Proc NLMixed routine in SAS.

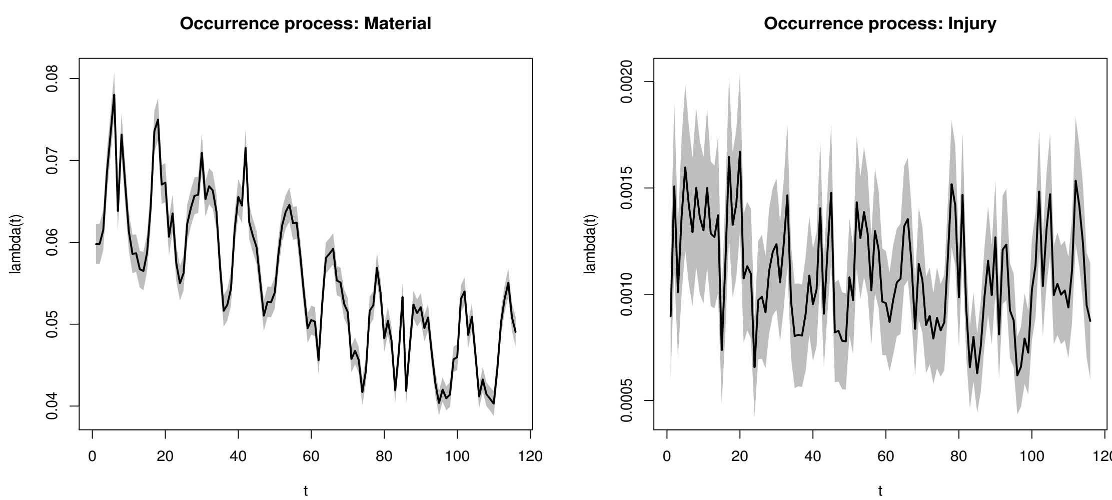
Reporting delay We will use a mixture of a Weibull distribution and 9 degenerate components corresponding with settlement after $0,\ldots,8$ days. Figure 8 illustrates the fit of this mixture of distributions to the actually observed reporting delays.

Figure 8: *Reporting delay for material (left) and injury (right) claims plus degenerate components and truncated Weibull distribution.*



Occurrence process Given the above specified distribution for the reporting delay, the likelihood (16) for the occurrence times can be optimized. Monthly intervals are used for this, ranging from January 2000 till August 2009. Point estimates and a corresponding 95% confidence interval are shown in Figure 9.

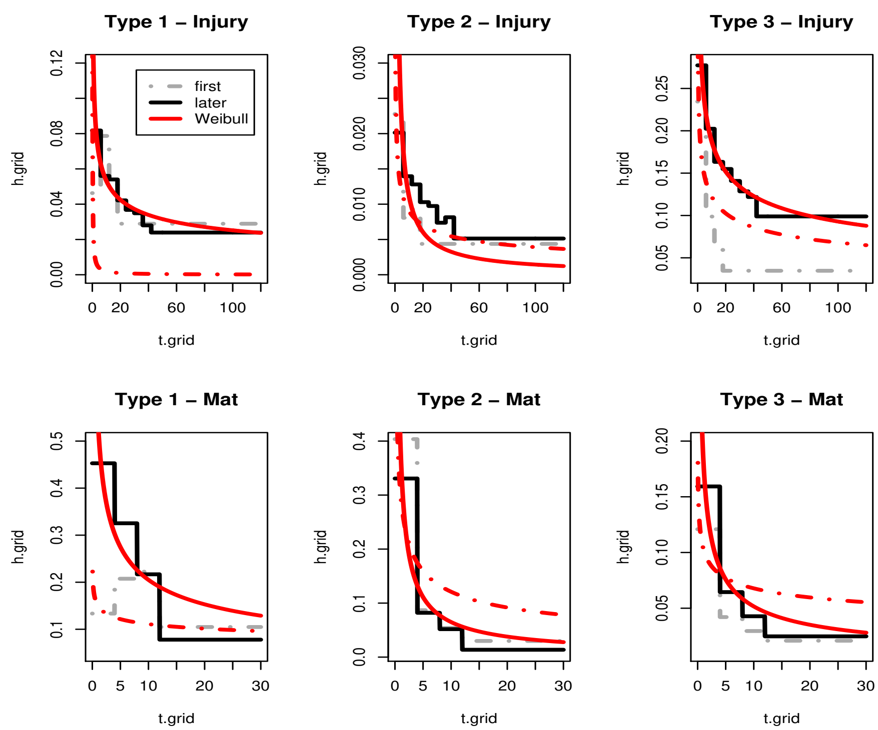
Figure 9: *Estimates of piecewise specification for $\lambda(t)$: (left) material and (right) injury claims.*



Development process For the different events that may occur during the development of a claim, the use of a constant, Weibull as well as a piecewise constant hazard rate was investigated. In the piecewise constant hazard rate specification for the development of material claims, the hazard rate was assumed to be constant on four month intervals: $[0-4)$ months, $[4-8)$ months, ..., $[8-12)$ months and ≥ 12 months. For injury claims, the hazard rate was assumed constant on intervals of six months: $[0-6)$ months, $[6-12)$ months, ..., $[36-42)$

months and $\geq$ 42 months. Figure 10 shows estimates for Weibull and piecewise constant hazard rates. All models are estimated separately for 'first events' and 'later events'.

Figure 10: *Estimates for Weibull and piecewise constant hazard rates: (upper) injury claims and (lower) material claims.*



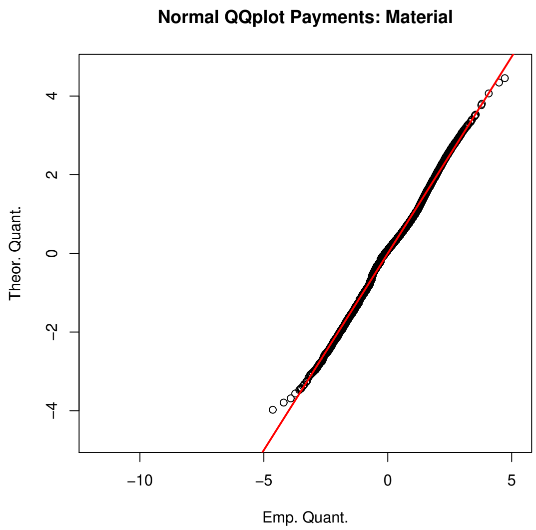
The piecewise constant specification reflects the actual data. The figure shows that the Weibull distribution is reasonably close to the piecewise constant specification. In the rest of this paper we will use the piecewise constant specification. Because the Weibull distribution is a good alternative, we explain how to use both specifications in the prediction routine (see Section 5).

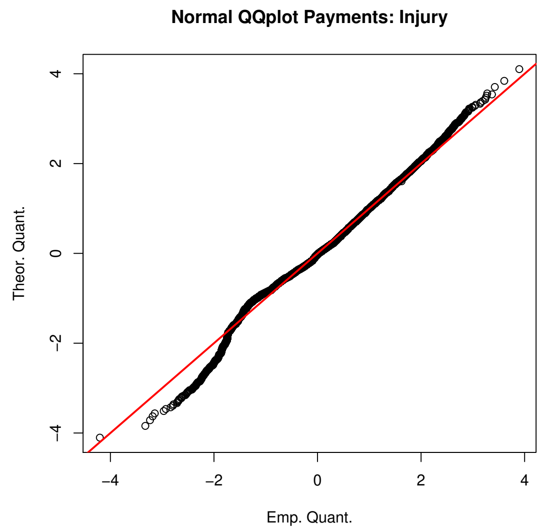
Payments Several distributions have been fitted to the historical payments (which were discounted to 1-1-1997 with Dutch price inflation). We examined the fit of the Burr, gamma and lognormal distribution, combined with covariate information. Distributions for the payments are truncated at the coverage limit of 2.5 million euro per claim. A comparison based on BIC showed that the lognormal distribution achieves a better fit than the Burr and gamma distributions. When including the initial reserve category as covariate or both the initial reserve category and the development year, the fit further improves. Given these results, the lognormal distribution with the initial reserve category and the development year as covariates will be used in the prediction. The covariate information is included in both the mean (μ_i) and standard deviation (σ_i) of the lognormal distribution for observation i:

$$\begin{aligned} \mu_i &= \sum_r \sum_s \mu_{r,s} I_{\mathrm{DY}_i=s} I_{i\in r} \\ \sigma_i &= \sum_r \sum_s \sigma_{r,s} I_{\mathrm{DY}_i=s} I_{i\in r}. \end{aligned} \tag{19}$$

Hereby r is the initial reserve category and DY_i is the development year corresponding with observation i. $I_{\mathrm{DY}_i=s}$ and $I_{i\in r}$ are indicator variables denoting whether observation i corresponds with DY s and reserve category r. Figure 11 shows corresponding qqplots.

Figure 11: *Normal qqplots corresponding with the fit of log(payments) including initial reserve and development year as covariate information.*





5 Predicting future cash–flows

5.1 Prediction routine

To predict the outstanding liabilities with respect to this portfolio of liability claims, we distinguish between IBNR and RBNS claims. The following step by step approach allows to obtain random draws from the distribution of both IBNR and RBNS claims.

Predicting IBNR claims As noted in Section 3, an IBNR claim occurred already but has not yet been reported to the insurer. Therefore, $T_i + U_i > \tau$ and $T_i < \tau$ with T_i the occurrence time of the claim and U_i its reporting delay. The T_is are missing data: they are determined in the development process but unknown to the actuary at time τ. The prediction process for the IBNR claims requires the following steps:

(a) **Simulate the number of IBNR claims in $[0, \tau]$ and their corresponding occurrence times.** According to the discussion in Section 3 the IBNR claims are governed by a Poisson process with non–homogeneous intensity or occurrence rate:

$$w(t)\lambda(t)(1 - P_{U|t}(\tau - t)), \tag{20}$$

where $\lambda(t)$ is piecewise constant according to specification (14). The following property follows from the definition of non–homogeneous Poisson processes:

$$N_{\text{IBNR}}(l) \sim \text{Poisson}\left(\lambda_l w_l \int_{d_{l-1}}^{d_l} (1 - P_{U|t}(\tau - t))dt\right), \tag{21}$$

where $N_{\text{IBNR}}(l)$ is the number of IBNR claims in time interval $[d_{l-1}, d_l)$. Note that the integral expression has already been evaluated (numerically) in the fitting procedure. Given the simulated number of IBNR claims $n_{\text{IBNR}}(l)$ for each interval $[d_{l-1}, d_l)$, the occurrence times of the claims are uniformly distributed in $[d_{l-1}, d_l)$.

(b) **Simulate the reporting delay for each IBNR claim**
Given the simulated occurrence time t_i of an IBNR claim, its reporting delay is simulated by inverting the distribution:

$$P(U \leq u | U > \tau - t_i) = \frac{P(\tau - t_i < U \leq u)}{1 - P(U \leq \tau - t_i)}. \tag{22}$$

In case of our assumed mixture of a Weibull distribution and 9 degenerate distributions this expression has to be evaluated numerically.

(c) **Simulate the initial reserve category**
For each IBNR claim an initial reserve category has to be simulated for use in the development process. Given m initial reserve categories, the probability density for initial reserve category c is:

$$f(c) = \begin{cases} p_c \text{ for } c = 1, 2, \ldots, m-1 \\ 1 - \sum_{k=1}^{m-1} p_k \text{ for } c = m. \end{cases} \tag{23}$$

The probabilities used in (23) are the empirically observed percentages of policies in a particular initial reserve category.

(d) **Simulate the payment process for each IBNR claims**
This step is common with the procedure for RBNS claims and will be explained in the next paragraph.

Predicting RBNS claims Given the RBNS claims and the simulated IBNR claims, the process proceeds as below.

(e) **Simulate the next event's exact time**
In case of RBNS claims, the time of censoring c_i of claim i is known. For IBNR claims this censoring time $c_i := 0$. The next event – at time $v_{i,next}$ – can take place at any time $v_{i,\text{next}} > c_i$. To simulate its exact time we need to invert: (with p randomly drawn from a $\text{Unif}(0,1)$ distribution)

$$\begin{aligned} P(V < v_{i,\text{next}} | V > c_i) &= p \\ &\Updownarrow \\ \frac{P(c_i < V \leq v_{i,\text{next}})}{1 - P(V \leq c_i)} &= p. \end{aligned} \tag{24}$$

From the relation between a hazard rate and cdf, we know

$$P(V \leq v_{i,\text{next}}) = 1 - \exp\left(-\int_0^{v_{i,\text{next}}} \sum_e h_e(t) dt\right), \tag{25}$$

with $e \in \{se, sep, p\}$. For instance with a Weibull specification for the hazard rates this equation will be inverted numerically. With a piecewise constant specification for the hazard rates numerical routines can be used as well. However, closed–form expressions are available. Step (e) should then be replaced by $(e1) - (e2)$:

(e1) **Simulate the next event's time interval**
In case of RBNS claims, the time of censoring c_i of claim i belongs to a certain interval $[a_{k-1}, a_k)$. The next event – at time $v_{i,next} > c_i$ – can take place in any interval from

$[a_{k-1}, a_k)$ on. The probability that $v_{i,next}$ belongs to a certain interval $[a_{k-1}, a_k)$ is given by:

$$P(a_{k-1} \leq V < a_k | V > c_i) = \begin{cases} \frac{P(c_i < V < a_k)}{1 - P(V \leq c_i)} & \text{if } c_i \in [a_{k-1}, a_k) \\ \frac{P(a_{k-1} \leq V < a_k)}{1 - P(V \leq c_i)} & \text{if } c_i \notin [a_{k-1}, a_k). \end{cases} \tag{26}$$

Using the notation introduced above the involved probabilities can be expressed as (for instance):

$$\begin{aligned} & \frac{P(c_i < V < a_k)}{1 - P(V < c_i)} = \frac{P(V < a_k) - P(V \leq c_i)}{1 - P(V \leq c_i)} \\ = & \frac{1 - \exp\left\{-\int_0^{a_k} \sum_e h_e(t)dt\right\} - 1 + \exp\left\{-\int_0^{c_i} \sum_e h_2(t)dt\right\}}{\exp\left\{-\int_0^{c_i} \sum_e h_e(t)dt\right\}}, \\ = & \frac{\exp\left\{-\sum_e \sum_{l=1}^d h_{el}\left[(a_l - a_{l-1})\delta_2(l,c_i) + (c_i - a_{l-1})\delta_1(l,c_i)\right]\right\} - \exp\left\{-\sum_e \sum_{l=1}^d h_{el}\left[(a_l - a_{l-1})\delta_2(l,a_k) + (a_k - a_{l-1})\delta_1(l,a_k)\right]\right\}}{\exp\left\{-\sum_e \sum_{l=1}^d h_{el}\left[(a_l - a_{l-1})\delta_2(l,c_i) + (c_i - a_{l-1})\delta_1(l,c_i)\right]\right\}}, \end{aligned} \tag{27}$$

with $e \in \{se, sep, p\}$, $\delta_2(l,t)$ is 1 if $t > a_l$ and 0 otherwise and $\delta_1(l,t)$ is 1 if $a_{l-1} \leq t < a_l$ and 0 otherwise.

(e2) **Simulate the exact time of the next event**
Given the time interval of the next event, $[a_{k-1}, a_k)$, we simulate its exact time by inverting the following equation for $v_{i,\text{next}}$

$$\begin{aligned} P(V < v_{i,\text{next}} | c_i < V < a_k) &= p \ \text{ if } c_i \in [a_{k-1}, a_k); \\ P(V < v_{i,\text{next}} | a_{k-1} \leq V < a_k) &= p \ \text{ otherwise,} \end{aligned} \tag{28}$$

where p is randomly drawn from a Unif$(0,1)$ distribution. For instance, for $P(V < v_{i,\text{next}} | a_{k-1} \leq V < a_k) = p$ this inverting operation goes as follows:

$$\begin{aligned} & P(V < v_{i,\text{next}}) = pP(a_{k-1} \leq V < a_k) + P(V < a_{k-1}) \\ \Updownarrow & \\ & 1 - \exp\left\{-\int_0^{v_{i,\text{next}}} \sum_e h_e(t)dt\right\} = pP(a_{k-1} \leq V < a_k) + P(V < a_{k-1}) \\ \Updownarrow & \\ & -\log\left[1 - pP(a_{k-1} \leq V < a_k) - P(V < a_{k-1})\right] = \sum_e \sum_{l=1}^{k-1} h_{el}(a_l - a_{l-1}) + \sum_e h_{ek}(v_{i,\text{next}} - a_{k-1}) \\ \Updownarrow & \\ & v_{i,next} = \frac{-\log\left[1 - pP(a_{k-1} \leq V < a_k) - P(V < a_{k-1})\right] - \sum_e \sum_{l=1}^{k-1} h_{el}(a_l - a_{l-1})}{\sum_e h_{ek}} + a_{k-1}, \end{aligned} \tag{29}$$

with $e \in \{se, sep, p\}$.

(f) **Simulate the event type** Given the exact time of the next event, its type is simulated using the following argument

$$\begin{aligned} \lim_{\triangle v \to 0} P(E = e | v \leq V < v + \triangle v) &= \lim_{\triangle v \to 0} \frac{\frac{P(v \leq V < v + \triangle v \cap E = e)}{\triangle v}}{\frac{P(v \leq V < v + \triangle v)}{\triangle v}} \\ &= \frac{h_e(v)}{\sum_e h_e(v)}, \end{aligned} \tag{30}$$

where $e \in \{se, sep, p\}$.

(g) **Simulate the corresponding payment** Given the covariate information for claim i, the payment can be drawn from the appropriate lognormal distribution. Note that the cumulative payment cannot exceed the coverage limit of 2.5 million per claim.

(h) **Stop or continue** Depending on the simulated event type in step (f), the prediction stops (in case of settlement) or continues.

In the next section, this prediction process will be applied separately for the material claims and the injury claims.

5.2 Comment on parameter uncertainty

With respect to the uncertainty of predictions a distinction has to be made between process uncertainty and estimation or parameter uncertainty (see England and Verrall (2002)). The process uncertainty will be taken care of by sampling from the distributions proposed in Section 3. To include parameter uncertainty the bootstrap technique or concepts from Bayesian statistics can be used. While a formal Bayesian approach is very elegant, it generally leads to significantly more complexity, which is not contributing to the accessibility and transparency of the techniques towards practicing actuaries. Applying a bootstrap procedure would be possible, but is very computer intensive, since our sample size is very large and several stochastic processes are used. To avoid computational problems when dealing with parameter uncertainty, we will use the asymptotic normal distribution of our maximum likelihood estimators. At each iteration of the prediction routine we sample each parameter from its corresponding asymptotic normal distribution. Note that –due to our large sample size– confidence intervals are narrow. This is in contrast with run–off triangles where sample sizes are typically very small and parameter uncertainty is an important point of concern.

6 Numerical results

The prediction process described in Section 5 is applied separately for the material and injury claims. In this Section results obtained with the micro–level reserving model are shown. Our results are compared with those from traditional techniques based on aggregate data. We show results for an out–of–sample exercise, so that the estimated reserves can be compared with actual payments. This out–of–sample test is done by estimating the reserves per 1-1-2005. The data set that is available at 1-1-2005 can be summarized using run-off triangles, displaying data from arrival years 1999 – 2004. Table 3 (material) and 4 (injury) show the run–off triangles that are the basis for this out–of–sample exercise. The lower triangle is known up to 3 cells. The actual observations are given in bold. Of course, these were not known at 1-1-2005 so cannot be used as input for calibration of the models.

Output from the micro–level model The distribution of the reserve per 1-1-2005 is determined for the individual (micro–level) model proposed in this paper. We will first look at the output that becomes available when using the micro–level model. Figure 12 shows results for injury payments done in calendar year 2006, based on 10,000 simulations. In Table 4 this is the diagonal going from 412, 268, . . . , up to 97. The first row in Figure 12 shows (from left to right): the number of IBNR claims reported in 2006, the total amount of payments done in this calendar year and the total number of events occurring in 2006. The IBNR claims are claims that occurred before 1-1-2005, but were reported to the insurer during calendar year 2006. The total amount paid in 2006 is the sum of payments for RBNS claims and IBNR claims, which are separately available from the micro–model. In the second row of plots we take a closer look at

Table 3: *Run–off triangle material claims (displayed in thousands), arrival years 1997-2004.*

Arrival	Development Year							
Year	1	2	3	4	5	6	7	8
1997	4,380	972	82	9	36	27	34	11
1998	4,334	976	56	35	76	24	0.572	**17**
1999	5,225	1,218	59	108	108	12	**0.390**	**0**
2000	5,366	1,119	161	14	6	**4**	**0.36**	**10**
2001	5,535	1,620	118	119	**13**	**3**	**0.350**	**2**
2002	6,539	1,547	67	**65**	**17**	**5**	**9**	**8.8**
2003	6,535	1,601	**90**	**21**	**31**	**7**	**1.7**	
2004	7,109	**1,347**	**99**	**76**	**20**	**13**		

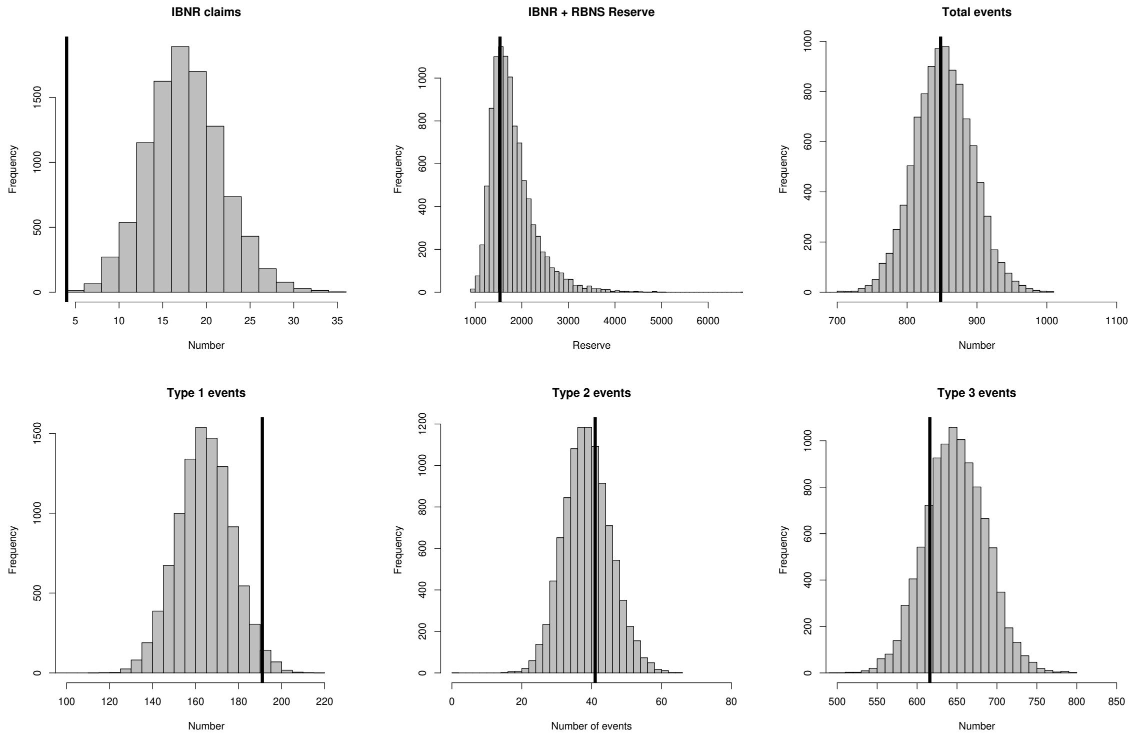
Table 4: *Run–off triangle injury claims (displayed in thousands), arrival years 1997-2004.*

Arrival	Development Year							
Year	1	2	3	4	5	6	7	8
1997	308	635	366	530	549	137	132	339
1998	257	481	312	336	269	56	179	**78**
1999	292	589	410	273	254	287	**132**	**97**
2000	316	601	439	498	407	**371**	**247**	**275**
2001	465	846	566	566	**446**	**375**	**147**	**240**
2002	314	615	540	**449**	**133**	**131**	**332**	**1,082**
2003	304	802	**617**	**268**	**223**	**216**	**173**	
2004	333	**864**	**412**	**245**	**273**	**100**		

the events registered in 2006 by splitting into type 1–type 3 events. In each of the plots the black solid line indicates what was actually observed. This figure shows that the predictive distributions from the micro–level model are realistic, given the actual observations. Only the actual number of IBNR claims is far in the tail of the distribution. However, note that this relates to a relatively low number of IBNR claims.

Comparing reserves The results from the micro–level model are now compared with results from two standard actuarial models developed for aggregate data. To the data in Tables 3 and 4, a stochastic chain–ladder model is applied which is based on the overdispersed Poisson distribution and the lognormal distribution, respectively. With Y_{ij} denoting cell (i, j) from a run–off triangle, corresponding with arrival year i and development year j, the model specifications for overdispersed Poisson ((31)) and lognormal ((32)) are given below. Both aggregate models

Figure 12: *Out–of–sample exercise per 1-1-2005, injury claims. Results are for calendar year 2006, based on 10,000 simulations from the micro–level model. The black solid line indicates actually observed quantities.*



are implemented in a Bayesian framework [3]

$$\begin{aligned} Y_{ij} &= \phi M_{ij} \\ M_{ij} &\sim \text{Poi}(\mu_{ij}/\phi) \\ \mu_{ij} &= \alpha_i + \beta_j; \end{aligned} \tag{31}$$

$$\begin{aligned} \log(Y_{ij}) &= \mu_{ij} + \epsilon_{ij} \\ \mu_{ij} &= \alpha_i + \beta_j \\ \epsilon_{ij} &\sim N(0, \sigma^2). \end{aligned} \tag{32}$$

Figure 13 shows the reserves (in thousands euro) for material claims, as obtained with the different methods. The results are shown for calendar years 2005 (top left) - 2007 (bottom left), and for 2008 (top right), 2009 and the total reserve (bottom right). The total reserve predicts the complete lower triangle (all bold numbers + three missing cells in Tables 3 and 4). In each row of 3 plots we show (in this order) the results from the micro–level model, the aggregate overdispersed Poisson model and the aggregate lognormal. The solid black line in each plot indicates what has really been observed. In the plots of the total reserve the dashed line is the sum of all observed payments in the lower triangle. This is –up to three unknown cells– the total reserve. Corresponding numerical results are in Table 5.

In Figure 13 we use the same scale for plots showing reserves obtained with the micro–level and the overdispersed Poisson model. However, for the lognormal model a different scale on

[3]The implementation of the overdispersed Poisson is in fact empirically Bayesian. ϕ is estimated beforehand and held fixed. We use vague normal priors for the regression parameters in both models and a gamma prior for σ^{-1} in the lognormal model.

the x–axis is necessary because of the long right tail of the frequency histogram obtained for this model. These unrealistically high reserves (see Table 5) are a disadvantage of the lognormal model for the portfolio of material claims. Concerning the Poisson model for aggregate data, we conclude from Figure 13 that the overdispersed Poisson model overstates the reserve; the actually observed amount is always in the left tail of the histogram. For instance, in the plots with the total reserve, the median of the simulations from overdispersed Poisson is at 2,785,000 euro, the median of the simulations from the micro–level model is 2,054,430 euro, whereas the total amount registered for the lower triangle is 1,861,000 euro. Recall that the latter is the total reserve up to the three unknown cells in Triangle 3. The best estimates (see the 'Mean' or 'Median' columns) obtained with the micro–level model are realistic and closer to the true realizations than the best estimates from aggregate techniques.

Figure 14 shows the distributions of the reserve (in thousands of euro) for the different methods for injury claims. Once again the actual payments are indicated with a solid black line. The results of the lognormal model are now presented on the same scale as the other two models. Corresponding numerical results are in Table 6. All models do well for calendar year 2005, the individual model does the best job for calendar years 2006 and 2007. For these calendar years the actual amount paid is –again– in the very left tail of the distributions obtained with aggregate techniques. The overdispersed Poisson and the lognormal model perform better in calendar years 2008 and 2009. Note however that calendar years 2008 and 2009 were extraordinary years, when looking at injury payments. In 2009 the two highest claims of the whole data set settled with a payment in 2009. The highest (the 779,383 euro payment shown in Table 4) is extremely far from all other payments in the data set. The observed outcome from calendar year 2009 should be considered as a very pessimistic scenario. Indeed, this realized outcome is in the very right tail of the distribution obtained with the individual model. The year 2008 was less extreme, but had an unusual number of very large claims (of the 15 highest claims in the data set, 4 of them occurred in 2008).

Note Although we only present the results obtained for the out–of–sample test that calculates reserves per 1-1-2005, we also calculated reserves per 1-1-2006/2007/2008/2009. Our conclusions for these tests were similar to those reported above. Full details are available on the home page of the first author.

Table 5: *Out–of–sample exercise per 1-1-2005: numerical results for material claims (in thousands), 6,000 simulations for the micro–level model..*

Method	Observed	Year	Mean	Median	Min.	Max.	5%	25%	75%	90%	95%	99.5%
Micro–level	1,537	CY 2005	1,404	1,342	1,093	5,574	1,204	1,272	1,449	1,627	1,783	3,143
	139	06	307	248	76	2,738	138	191	346	498	630	1,779
	123	07	246	183	30	2,740	72	123	286	444	618	1,688
	39	08	146	98	7	2,426	30	61	164	283	402	1,225
	23	09	52	26	0	2,216	4	12	53	104	167	639
	> 1,861	Total	2,208	2,054	1,374	7,875	1,622	1,831	2,401	2,871	3,305	5,074
Aggregate ODP	1,537	CY 2005	2,000	1,989	1,194	3,028	1,591	1,834	2,166	2,321	2,431	2,674
	139	06	324	309	44	774	177	265	376	442	486	597
	123	07	214	199	0	619	88	155	265	332	354	464
	39	08	144	133	0	553	44	88	177	243	265	354
	23	09	66	66	0	376	0	22	88	133	155	243
	> 1,861	Total	2,803	2,785	1,613	4,354	2,232	2,564	3,028	3,271	3,426	3,846
Aggregate LogN.	1,537	CY 2005	5,340	2,253	70	587,500	497	1,146	4,896	10,790	17,985	77,671
	139	06	699	410	32	164,200	135	254	710	1,231	1,818	6,522
	123	07	380	228	8	23,720	67	137	403	734	1,110	3,731
	39	08	326	167	2	48,850	41	93	317	627	998	4,053
	23	09	163	71	1	33,660	14	36	146	304	499	2,051
	> 1,861	Total	7,071	3,645	201	645,500	1,110	2,135	6,936	13,692	21,931	84,712

Table 6: *Out–of–sample exercise per 1-1-2005: numerical results for injury claims (in thousands), 10,000 simulations for the micro–level model.*

Method	Observed	Year	Mean	Median	Min.	Max.	5%	25%	75%	90%	95%	99.5%
Micro–level	2,957	CY 2005	2,548	2,453	1,569	6,587	1,951	2,212	2,764	3,154	3,499	4,567
	1,532	06	1,798	1,699	909	6,790	1,246	1,477	2,001	2,393	2,703	3,752
	1,020	07	1,254	1,159	453	4,945	774	968	1,420	1,778	2,088	3,125
	1,060	08	884	776	267	4,381	458	613	1,024	1,393	1,694	2,743
	1,354	09	390	313	63	3,745	149	226	448	678	908	1,875
	$> 7,923$	Total	7,386	7,209	4,209	14,850	5,666	6,489	8,092	9,035	9,721	11,725
Aggregate ODP	2,957	CY 2005	2,798	2,774	1,727	8,247	2,259	2,553	2,994	3,233	3,380	4,298
	1,532	06	2,134	2,112	1,065	6,723	1,670	1,929	2,314	2,498	2,627	3,472
	1,020	07	1,721	1,708	845	6,172	1,286	1,525	1,892	2,076	2,186	3,049
	1,060	08	1,286	1,249	551	5,933	882	1,102	1,433	1,616	1,727	2,627
	1,354	09	759	735	220	4,114	478	625	863	992	1,084	1,543
	$> 7,923$	Total	9,639	9,478	5,474	40,670	7,660	8,688	10,360	11,200	11,770	17,360
Aggregate LogN.	2,957	CY 2005	2,948	2,882	1,175	6,729	2,181	2,570	3,254	3,648	3,944	4,944
	1,532	06	2,251	2,196	957	6,898	1,623	1,940	2,500	2,825	3,050	3,934
	1,020	07	1,817	1,759	567	5,313	1,244	1,526	2,040	2,355	2,583	3,426
	1,060	08	1,377	1,315	374	5,768	864	1,110	1,571	1,861	2,087	2,944
	1,354	09	815	768	195	4,054	472	632	941	1,151	1,313	1,867
	$> 7,923$	Total	10,277	10,040	4,459	26,010	7,661	8,954	11,310	12,680	13,730	17,590

7 Conclusions

The measurement of future cash flows and their uncertainty becomes more and more important, also for general insurance portfolios. Currently, reserving for general insurance is based on data aggregated in run–off triangles. A vast literature on techniques for claims reserving exists, largely designed for application to loss triangles. The most popular approach is the chain–ladder approach, because of is practicality. However, the use of aggregate data in combination with the chain–ladder approach gives rise to several issues, implying that the use of aggregate data in combination with the chain–ladder technique (or similar techniques) is not fully adequate for capturing the complexities of stochastic reserving for general insurance.

In this paper micro–level stochastic modeling is used to quantify the reserve and its uncertainty for a realistic general liability insurance portfolio. Stochastic processes for the occurrence times, the reporting delay, the development process and the payments are fit to the historical individual data of the portfolio and used for projection of future claims and its (estimation and process) uncertainty. A micro–level approach allows much closer modeling of the claims process. Lots of issues mentioned in our discussion of the chain–ladder approach will not exist when using a micro–level approach, because of the availability of lots of data and the potential flexibility in modeling the future claims process.

The paper shows that micro–level stochastic modeling is feasible for real life portfolios with over a million data records, and that it gives the flexibility to model the future payments realistically, not restricted by limitations that exist when using aggregate data. The prediction results of the individual (micro–level) model are compared with models applied to aggregate data, being an overdispersed Poisson and a lognormal model. We present our results through an out–of–sample exercise, so that the estimated reserves can be compared with actual payments. Conclusion of the out–of–sample test is that –for the case–study under consideration– traditional techniques tend to overestimate the real payments. Predictive distributions obtained with the micro–model reflect reality in a more realistic way: 'regular' outcomes are close to the median of the predictive distribution whereas pessimistic outcomes are in the very right tail. As such, reserve calculations based on the micro-level model are to be preferred; they reflect real outcomes in a more realistic way.

The results obtained in this paper make it worthwhile to further investigate the use of micro–level data for reserving purposes. Several directions for future research can be mentioned. One could try to refine the performance of the individual model with respect to very pessimistic scenarios by using a combination of e.g. a lognormal distribution for losses below and a generalized Pareto distribution for losses above a certain threshold. Analyzing the performance of both the micro–level model and techniques for aggregate data on simulated data sets will bring more insight in their performance. In that respect it is our intention to collect and study new case–studies.

Figure 13: *Out–of–sample results per 1-1-2005, material claims. Top left to bottom left: calendar years 2005-2006-2007. Top right to bottom right: calendar years 2008-2009-total reserve. In each graph the black vertical line indicates the amount that was actually paid in that calendar year. The dashed black line in the graph of the total reserve corresponds with the sum of all payments done in calendar years 2005 up to 2009. Up to three missing cells (see Table 3) this is the total reserve.*

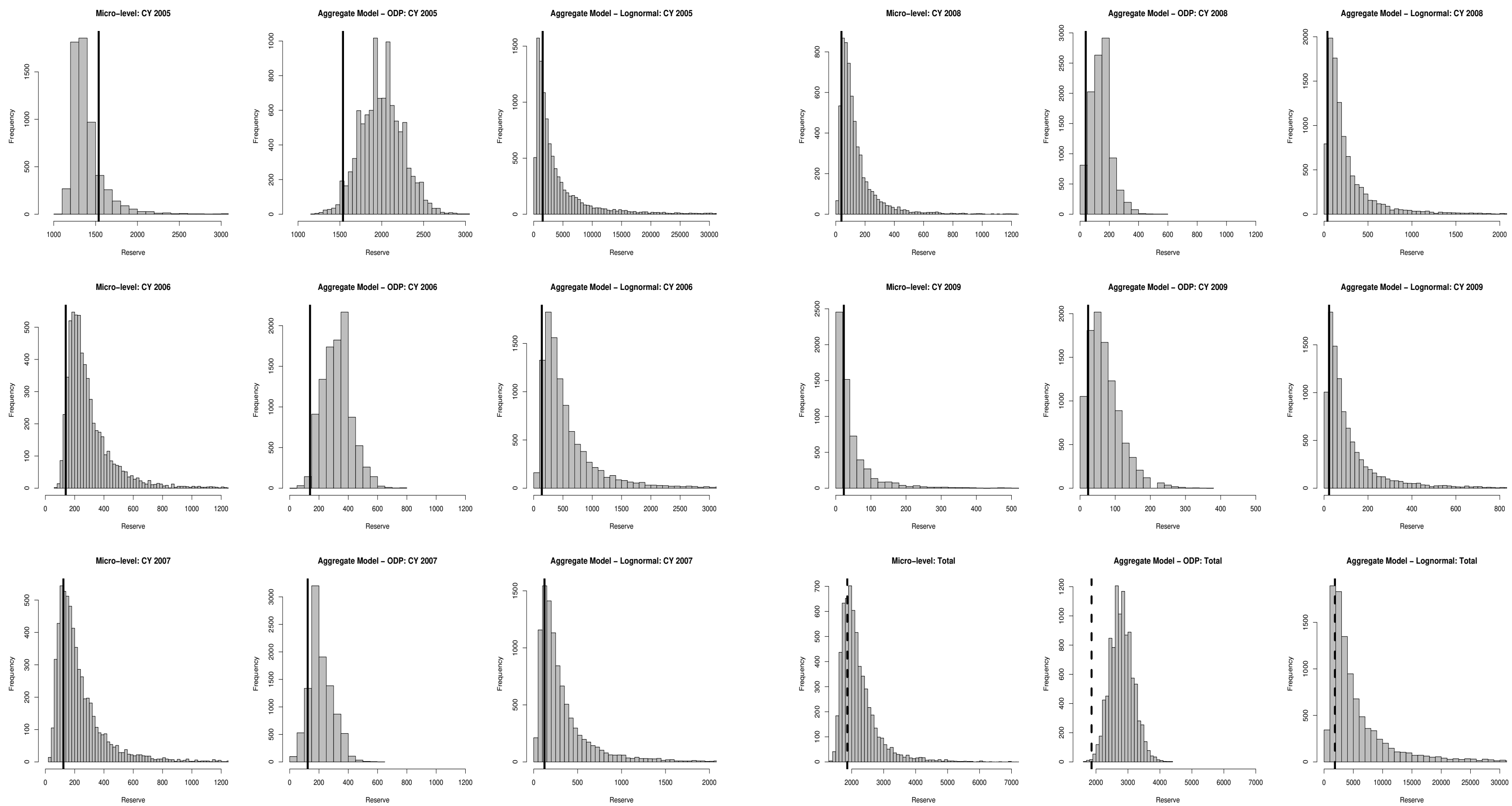
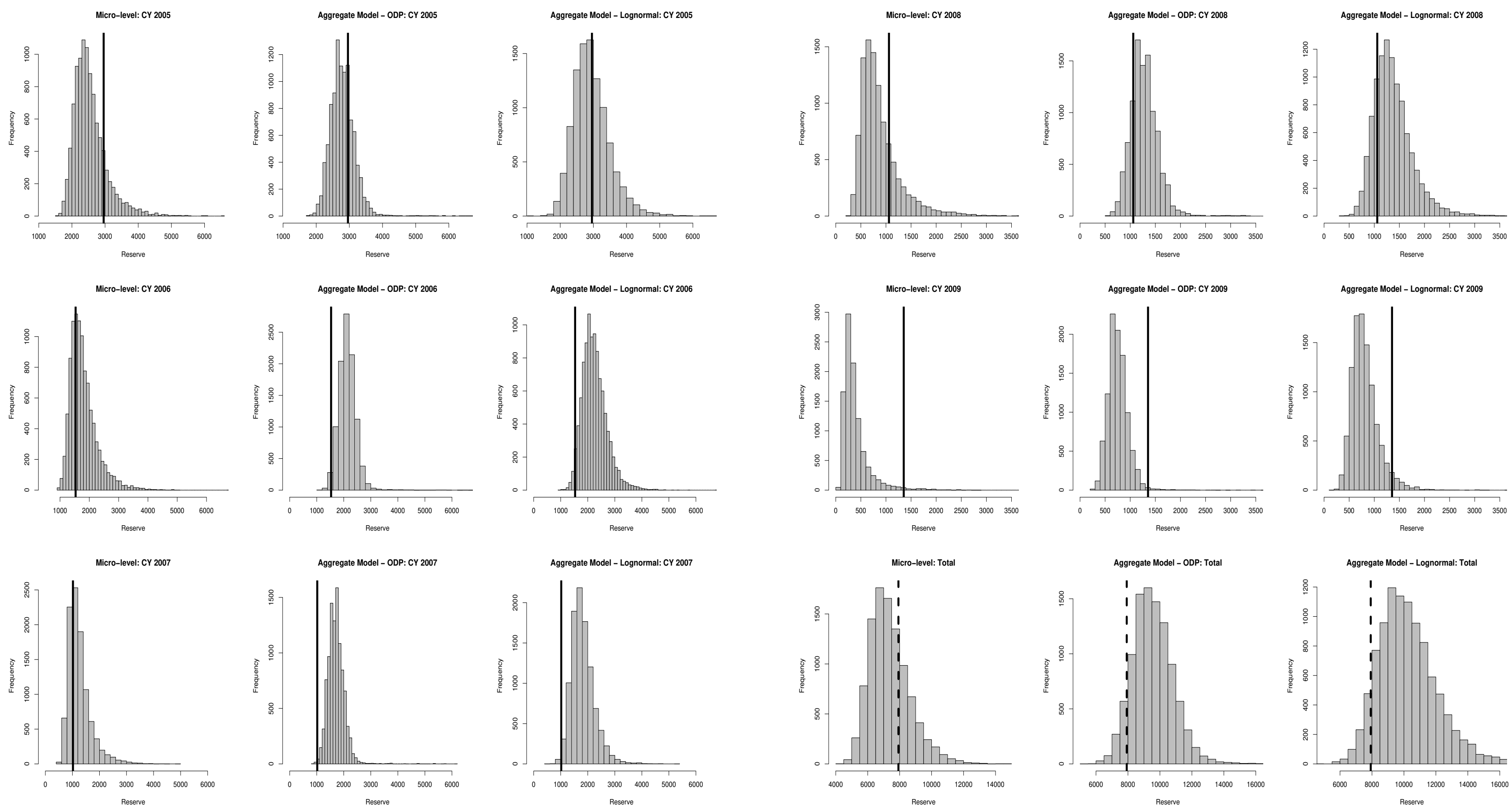


Figure 14: *Out–of–sample results per 1-1-2005, injury claims. Top left to bottom left: calendar years 2005-2006-2007. Top right to bottom right: calendar years 2008-2009-total reserve. In each graph the black vertical line indicates the amount that was actually paid in that calendar year. The dashed black line in the graph of the total reserve corresponds with the sum of all payments done in calendar years 2005 up to 2009. Up to three missing cells (see Table 4) this is the total reserve.*



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