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# Micromechanics for Particulate Reinforced Composites

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# MICROMECHANICS FOR PARTICULATE REINFORCED COMPOSITES

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## SUMMARY

A set of micromechanics equations for the analysis of particulate reinforced composites is developed using the mechanics of materials approach. Simplified equations are used to compute homogenized or equivalent thermal and mechanical properties of particulate reinforced composites in terms of the properties of the constituent materials. The microstress equations are also presented here to decompose the applied stresses on the overall composite to the microstresses in the constituent materials. The properties of a "generic" particulate composite as well as those of a particle reinforced metal matrix composite are predicted and compared with other theories as well as some experimental data. The micromechanics predictions are in excellent agreement with the measured values.

## SYMBOLS

- C heat capacity
  - E normal modulus
  - G shear modulus
  - K thermal conductivity
  - $V_f$  Volume fraction of particles
  - ~ quantities with tilde refer to particle cell
  - $\alpha$  coefficient of thermal expansion
  - $\epsilon$  strain
  - $\nu$  Poisson's ratio
  - $\nu$  density
  - $\sigma$  stress
- Subscripts

- b binder
- p particles
- pc particulate composite

## INTRODUCTION

Recently, there has been growing interest in the use of particulate reinforced composites in many widely varying applications. The high costs and technical difficulties involved with the fabrication of fiber reinforced composites sometimes limit their use in many applications. Particulate reinforced composites can be thought as a viable alternative. They can be used as either dual or multi-phase materials with the advantage of monolithic materials in that they are easily processed to near net shape and have the improved stiffness, strength and fracture toughness that is characteristic of continuous fiber reinforced composite materials. On the other hand, such dual or multi-phase materials can also be used as matrix materials in a continuous fiber reinforced composite. In the case of aerospace applications, they are potential candidate materials where shock or impact properties are important. Particle reinforced metal matrix composites have shown great potential as candidate materials for a variety of automotive applications. Typically, these materials are aluminum matrix reinforced with SiC or TiC particles. The resulting material has good specific modulus and strength which makes it suitable for disk brake rotors, connecting rods, cylinder liners and other high temperature applications. Concrete can also be thought of as one of the oldest material in this category of multi-phase particle reinforced materials that has been used in a wide variety of civil engineering applications. In-situ reinforced ceramics can also be thought of as particle reinforced composites that have higher fracture toughness compared to monolithic materials and are easier to process in near net-shape.

The characterization of these materials is fundamental to their reliable use. One of the method is to determine the bounds on effective mechanical and thermal properties of particulate composites. The bounds provide a range, as well as maximum error range, in predicting homogenized effective properties. In certain situations, the upper and lower bounds of the effective properties can be far apart and the technique is good only for linear properties. Notable among the works on these bounding techniques are Hashin and Shtrikman (ref. 1) - which assume the constituents to be isotropic and homogeneous, Torquato (ref. 2); Milton (ref. 3) and Davis (ref. 4). Davis (ref. 4) proposed "third-order" bounds. It is seen that third-order bounds that assume the particles to be spherical and arranged in one of the several possible arrangements are much sharper or closer together. Numerical analysis techniques such as finite element analysis have also been used to predict the effective thermal and mechanical properties of the particulate composites as well as certain non-linear properties such as strength, residual (fabrication) stresses, stress-strain relations and creep, etc. (ref. 5). Constituent level numerical analysis techniques are time consuming to set-up, analyze as well as interpret the results. Such techniques are difficult to use at a micro- or nano-scale on a frequent or routine basis to characterize various compositions or different particulate composites. However, on a macro or property averaged scale (element level), they are well suited for component thermal/structural analyses or for occasional verification of the results predicted by some approximate methods.

Another class of methods that can be used to characterize particulate reinforced composites in based on micromechanics. Composite micromechanics equations are simplified equations that are based upon the mechanics of materials approach. They predict effective composite thermal and mechanical properties and easily account for environmental factors such as moisture or temperature. It is also possible to predict, with the help of simple microstress equations, the state of stress in each constituent in an average sense (explained later) due to a variety of applied loading conditions. Since these equations are in closed-form and do not require any numerical integration, they are computationally efficient and yet able to accurately capture the physics and mechanics of the problem. However, it should be noted that the micromechanics approach represents properties or stresses in an averaged sense. While not a mathematically rigorous technique, it does ensure force equilibrium in all directions. A mathematically rigorous solution could be

obtained by solving the three-dimensional elasticity solution. That solution is very tedious. Micromechanics equations sufficiently include the mechanics (as compared with detailed three-dimensional finite element analyses) and the physics of the problem (predictions compared with the measured values).

NASA Lewis Research Center has been involved in the development of computer codes based on simplified micromechanics equations, for over two decades to simulate the continuous fiber reinforced composite materials behavior primarily for aerospace use. The intention of these codes was to include theories and analysis techniques that range in scale from constituent materials to global structural analyses in one integrated code. Several computer codes have been developed as a part of this research activity to describe/simulate the behavior of a range of composite material systems such as polymer matrix composites, metal and inter-metallic reinforced composites, as well as ceramic matrix composites. Different issues are important to each of the above class of materials, and those issues have been addressed in each of the separate computer codes. For analyzing polymer matrix composites, a perfect bond has been assumed to exist between the matrix and the fiber. The constituent properties are degraded for hygral/thermal effects. The resulting micromechanics and macromechanics equations along with a material degradation model are programmed into a computer code ICAN (Integrated Composite Analyzer, ref. 6). The code can carry out a comprehensive linear analysis of continuous fiber reinforced polymer matrix composites. For the analysis of metal matrix composites, a third phase called the "interphase" is also taken into account. The interphase can be thought of as a "reaction-zone" that forms as a result of reaction between the fiber and matrix as these composites are processed at very high temperature, or it can be a fiber coating or a compliant layer to prevent such a reaction. A material non-linear behavior model along with appropriate micromechanics and macromechanics equations have been programmed into a computer code - METCAN (Metal Matrix Composites Analyzer, ref. 7). In the case of ceramic matrix composites, one of the main functions of the reinforcement is to enhance the fracture toughness and avert the catastrophic failure of the brittle matrices. A weak interphase provides a mechanism to increase the fracture toughness. Hence, the interfacial issues in these composites as well as matrix microcracking and the resulting stress redistribution are important in analyzing this class of materials. This has been achieved by using a novel fiber sub-structuring technique. All these are programmed in a computer code called CEMCAN (Ceramic Matrix Composites Analyzer, ref. 8).

The objective of this work is to describe the micromechanics equations for particulate reinforced composites and to predict the effective thermal and mechanical properties as well as the constituent micro-stresses resulting from applied thermal and mechanical loadings. The resulting particulate composite can also be used as a matrix material. These equations can be programmed as a module and combined with any of the computer codes mentioned above. The material degradation models that exist in those codes can be effectively used to characterize these materials. The application of the micromechanics equations developed in this paper is demonstrated by predicting the thermo-mechanical properties of a generic composite as well as those of a particulate reinforced metal matrix composite. The predictions are compared with bounding techniques and some available data for the particle reinforced metal matrix composite. The comparison is excellent and the overall results demonstrate that the micromechanics equations can be used effectively to characterize this class of materials.

## PARTICULATE COMPOSITE MICROMECHANICS

The development of micromechanics equations for the particulate composites follows along the same lines as those for the continuous fiber reinforced composites. In the case of particulate reinforced composites, the particles in various shapes and sizes are dispersed uniformly in the binder material. Similar to the case of continuous fiber reinforced composites, where the fibers are assumed to be arranged in a regular array pattern like square or hexagon, the particulate material is assumed to be dispersed uniformly as spherical particles with a diameter that is the average value of the range of diameters in a cubic lattice. The distance between the neighboring particles is computed from the overall particle volume fraction. Hence, one can think of a representative volume element (RVE) or a "unit cell" for the particulate reinforced composite as shown in figure 1. For the ease of defining certain micro-regions and to compute micro-

stresses, the spherical particle in the unit cell has been replaced by a cubic particle of the same volume so as to maintain the same volume fraction. The unit cell in this case becomes a three-dimensional entity unlike the case of continuous fiber reinforced composites, where the unit cell is two-dimensional as the fiber extends through the full length of the unit cell.

The following assumptions have been made or are implicit in the current micromechanics of particulate composites:

1. The composite is composed of two phases - particles and binder (matrix).
2. Each phase of the composite can be described by the continuum mechanics. Hence, the input parameters are moduli, Poisson's ratios, thermal expansion coefficients and thermal conductivities of the individual phases.
3. The micromechanics is characterized by average values of composite properties and average constituent stresses over a certain region.
4. The interface between the particle and the binder has been assumed to be a perfect bond.
5. The properties of individual phases are assumed to be isotropic.
6. This approach is not mathematically rigorous, i.e. it does not ensure the continuity of stresses, displacement or satisfaction of compatibility equations of elasticity. To be mathematically rigorous, one should solve the three-dimensional elasticity problem. However, such a solution becomes extremely complicated beyond the equivalent properties of the unit cell. The current approach, represented by a set of simplified equations, satisfies the force equilibrium in all directions, and is able to capture the physics and the mechanics involved in the problem. This fact has been previously verified by the authors for continuous fiber reinforced composites and the comparison between the micromechanics predictions and detailed numerical analysis techniques, such as three-dimensional finite element analysis, have shown good agreement.

Unit Cell.—The derivation of the equivalent composite properties using the micromechanics equations starts with the identification of a representative volume element (RVE) or a unit cell. It is assumed that the particles are arranged in a cubic array pattern in the binder or matrix. The average particle size diameter is assumed to be  $d$ . The resulting unit cell is shown in figure 1. The spherical particle within the cubic unit cell is converted to a cubic particle such that the particle volume ratio is maintained the same in both cases. The principal assumptions involved in the mechanics of materials approach are (a) The strain in all constituents is the same if the composite is loaded in the longitudinal (along the fiber) direction and (b) In the transverse direction, all the constituents are subjected to the same stress. The unit cell of figure 1 (b) is further decomposed into two sub-cells referred to as (a) matrix cell - consists purely of binder or matrix material and (b) particle cell - consists of both particle and matrix or binder material, as shown in figure 2. The dimensions of these two sub-cells are also shown in figure 2. These two sub-cells of the overall RVE facilitate the representation of non-uniformity in the local stress distribution.

Micromechanics.—Composite micromechanics theory refers to the collection of physical principles, mathematical models, assumptions and approximations employed to relate the behavior of a simple composite unit like ply/lamina to the behavior of its individual constituents. The primary objective of the composite micromechanics is to determine the equivalent mechanical and thermal properties of a composite unit in terms of the properties of the constituent materials. The mechanical properties of interest are normal and shear moduli and various Poisson's ratios. The thermal properties include the coefficients of thermal expansion, thermal conductivities and heat capacities. The micromechanics of some of these properties for a particulate reinforced composite are described next. The actual derivation of two properties - normal moduli ( $E_{pc}$ ) and coefficient of thermal expansion ( $\alpha_{pc}$ ) for the particulate composite are shown in the Appendix. The derivation of other composite properties as well as microstress equations follows exactly along the same lines as described in the Appendix. The following is a brief list of the various equations.

The mass density of the particulate composite is given by a rule of mixture type of equation:

$$\rho_{pc} = V_f \cdot \rho_p + (1 - V_f) \cdot \rho_b \quad (1)$$

where  $V_f$  is the volume fraction of the particles, subscripts p, c and b stand for particulate composite, particle and the binder respectively.

The normal modulus of the particulate composite is given by

$$E_{pc} = \frac{V_f^{0.67} E_b}{1 - V_f^{0.33} \cdot \left(1 - \frac{E_b}{E_p}\right)} + (1 - V_f^{0.67}) E_b \quad (2)$$

The shear modulus of the particulate composite is also given by a similar expression

$$G_{pc} = \frac{V_f^{0.67} G_b}{1 - V_f^{0.33} \cdot \left(1 - \frac{G_b}{G_p}\right)} + (1 - V_f^{0.67}) G_b \quad (3)$$

Since, the constituents were assumed to have isotropic properties, the resulting particulate composite will also have isotropic properties. Therefore, the Poisson's ratio can be computed from the usual relationship-

$$\nu_{pc} = \frac{E_{pc} - 2 G_{pc}}{2 G_{pc}} \quad (4)$$

Similarly the thermal properties of the particulate composite are computed by analogous expressions. The coefficient of thermal expansion of the particulate composite is given by

$$\alpha_{pc} = \tilde{\alpha} V_f^{0.67} \frac{\tilde{E}_p}{E_{pc}} + \alpha_b \frac{E_b}{E_{pc}} - \alpha_b V_f^{0.67} \frac{E_b}{E_{pc}} \quad (5)$$

where

$$\tilde{\alpha} = \alpha_b - V_f^{0.33} (\alpha_b - \alpha_p) \quad (6)$$

and

$$\tilde{E}_p = \frac{E_b}{1 - V_f^{0.33} \left(1 - \frac{E_b}{E_p}\right)} \quad (7)$$

Similarly, the thermal conductivity of the particulate composite is given by

$$K_{pc} = \frac{V_f^{0.67} K_b}{1 - V_f^{0.33} \left(1 - \frac{K_b}{K_p}\right)} + (1 - V_f^{0.67}) K_b \quad (8)$$

and the heat capacity of the particulate composite is given by

$$C_{pc} = \frac{1}{\rho_{pc}} (\rho_b \cdot (1 - V_f) C_b + \rho_p V_f C_p) \quad (9)$$

where  $\alpha$ ,  $K$  and  $C$  stand for coefficient of thermal expansion, thermal conductivity and heat capacity respectively.

**Microstress Equations.**—The derivation of microstress equations is similar to the derivation of micro-mechanics equations. A complete derivation of the microstress equations will not be provided here for conciseness.

**Matrix cell microstresses due to applied normal stresses.**—The normal stress in the matrix (binder) cell results from applied normal stress in the loading direction as well as due to restraining effects because of mismatch in the Poisson's ratios. For example, the 1-1 stress in the matrix cell is given by

$$\sigma_{b11} = \frac{E_b}{E_{pc}} \cdot \sigma_{\ell 11} + (v_b - v_{pc}) \frac{E_b}{E_{pc}} \sigma_{\ell 22} + (v_b - v_{pc}) \frac{E_b}{E_{pc}} \sigma_{\ell 33} \quad (10)$$

similarly, other normal stresses in the matrix (binder) cell are given by,

$$\sigma_{b22} = \frac{E_b}{E_{pc}} \cdot \sigma_{\ell 22} + (v_b - v_{pc}) \frac{E_b}{E_{pc}} \sigma_{\ell 11} + (v_b - v_{pc}) \frac{E_b}{E_{pc}} \sigma_{\ell 33} \quad (11)$$

and

$$\sigma_{b33} = \frac{E_b}{E_{pc}} \cdot \sigma_{\ell 33} + (v_b - v_{pc}) \frac{E_b}{E_{pc}} \sigma_{\ell 11} + (v_b - v_{pc}) \frac{E_b}{E_{pc}} \sigma_{\ell 22} \quad (12)$$

where  $\sigma_{\ell 11}$ ,  $\sigma_{\ell 22}$  and  $\sigma_{\ell 33}$  are the applied stresses on the unit cell of the particle composite. These equations also assume that the particle and the binder have isotropic properties.

**Matrix cell microstresses due to applied shear stresses.**—The moisture, thermal or Poisson's ratio mismatch have no effect on the shear stresses. They are simply given by,

$$\sigma_{b12} = \left( \frac{G_b}{G_{pc}} \right) \sigma_{\ell 12} \quad (13)$$

and similarly,

$$\sigma_{b13} = \left( \frac{G_b}{G_{pc}} \right) \sigma_{\ell 13} \quad (14)$$

$$\sigma_{b23} = \left( \frac{G_b}{G_{pc}} \right) \sigma_{\ell 23} \quad (15)$$



### Matrix Cell Microstresses Due to Applied Thermal Load

Thermal stresses in the constituent occur because of the mismatch in the coefficients of thermal expansion as well as the restraining effects that occur due to the mismatch in the Poisson's ratios of the constituents.

The 1-1 thermal stress in the matrix (binder) cell is given by

$$\sigma_{b11} = (\alpha_{pc} - \alpha_b)\Delta T E_b + (v_{pc} - v_b)(\alpha_{pc} - \alpha_b)\Delta T E_b + (v_{pc} - v_b)(\alpha_{pc} - \alpha_b)\Delta T E_b \quad (16)$$

because of the isotropy conditions,  $\sigma_{b22}$  and  $\sigma_{b33}$  microstresses due to thermal load are the same as  $\sigma_{b11}$  microstress.

### Microstresses in the Particle Cell Due to Applied Normal Stresses

The microstress equation for the particle cell are similar to those of the binder cell and are given by,

$$\sigma_{p11} = \frac{\tilde{E}_{p11}}{E_{pc}} \sigma_{\ell11} + (\tilde{v}_{21} - v_{pc}) \frac{\tilde{E}_{p11}}{E_{pc}} \cdot \sigma_{\ell22} + (\tilde{v}_{31} - v_{pc}) \frac{\tilde{E}_{p11}}{E_{pc}} \sigma_{\ell33} \quad (17)$$

and microstresses in the other directions are given by similar expressions.

### Particle Cell Shear Stresses

These expressions are similar to the shear stress expressions in the matrix (binder) cell. Temperature, moisture or the Poisson's ratio mismatch do not effect the shear stresses.

$$\begin{aligned} \tilde{\sigma}_{p12} &= \left( \frac{\tilde{G}_{p12}}{G_{pc}} \right) \sigma_{\ell12} \\ \tilde{\sigma}_{p13} &= \left( \frac{\tilde{G}_{p13}}{G_{pc}} \right) \sigma_{\ell13} \\ \tilde{\sigma}_{p23} &= \left( \frac{\tilde{G}_{p23}}{G_{pc}} \right) \sigma_{\ell23} \end{aligned} \quad (18)$$

the quantities with tilde refer to those for the particle cell.

## Particle Cell Thermal Stresses

The thermal stresses in the particle arise due to the mismatch of the coefficients of thermal expansion and the restraining effects arising from the Poisson's ratio mismatch. The thermal stress in the particle cell is given by,

$$\tilde{\sigma}_{p11} = (\alpha_{pc} - \tilde{\alpha}_{11})\Delta T \tilde{E}_{p11} + (v_{pc} - \tilde{v}_{p21}) \cdot (\alpha_{pc} - \tilde{\alpha}_{p22})\Delta T \tilde{E}_{p11} + (v_{pc} - \tilde{v}_{p31}) \cdot (\alpha_{pc} - \tilde{\alpha}_{p33})\Delta T \tilde{E}_{p11} \quad (19)$$

The equation for the particle cell microstresses due to an applied  $\Delta T$  are similar to the above equation with appropriate subscripts.

Particle stresses are the same as the particle cell stresses. One can also combine these microstresses to represent them in a combined stress manner (like von Mises stress).

## RESULTS AND DISCUSSION

The above equations were applied to generate composite properties of two particulate reinforced composites as discussed below.

(a) Generic particulate composite.—The first case is a generic particulate composite, where the constituent properties are assumed to be as follows:

$$E_p/E_b = 5$$

$$v_p/v_b = 0.57$$

$$\alpha_p/\alpha_b = 0.1$$

$$\rho_p/\rho_b = 2$$

$$K_p/K_b = 100$$

$$C_p/C_b = 0.68$$

The mechanical properties mentioned above are representative of constituent properties of concrete. Based on the above, the mechanical and thermal properties of the particulate composite at different particle volume ratios are computed and plotted in figures 3 and 4 respectively. These properties are normalized with respect to binder properties. The figures show the trend in the variation of composite properties with the particle volume ratio. In this particular example, normal and shear moduli increase with increasing particle volume ratio but the change in Poisson's ratio is not as pronounced. The coefficient of thermal expansion decreases with increasing fiber volume ratio, while the thermal conductivity increases as the conductivity of the particles has been assumed to be 100 times that of the binder material.

(b) Particle reinforced metal matrix composite.—Composite properties of a SiC (silicon carbide) particle reinforced aluminum matrix are also predicted. This particular composite is a potential candidate material in certain automotive applications. The properties of the particle and the aluminum matrix are given in Table I.

TABLE I.—PROPERTIES OF CONSTITUENTS OF  
Al/SiC<sub>p</sub> COMPOSITE

Property	Matrix (Al)	Particle (SiC)
Modulus, Gpa	72.1	431
Shear modulus, Gpa	26.9	181.1
Poisson's ratio	0.34	0.19
Coeff. of thermal expansion, (10 <sup>-6</sup> /K)	23.6	8.6

The normal modulus, shear modulus and coefficient of thermal expansion for this composite at various fiber volume ratios are computed and plotted in figures 5, 6 and 7. Results available from other theoretical predictions, such as Hahn-Shtrikman bounds, third-order bounds, as well as available experimental data from reference 4 are also shown on the above figures. It is seen that the comparison is extremely good. The micromechanics predictions are well within the predicted bounds. Figure 7 shows that coefficient of thermal expansion for the particulate composite reduces as the particle volume fraction increases. The micromechanics with its simplified equations can therefore be a valuable tool in analyzing particulate reinforced composite. The accuracy of the predictions of the bounds depends upon the extent to which the properties of the individual phases differ. Collectively, the results show that the simplified micromechanics equations provide an efficient computational tool to predict the effective properties of the particulate composites. The prediction of residual stresses due to a mismatch or material degradation due to temperature or cyclic loading can be accounted by following similar analyses of continuous fiber reinforced composites.

The prediction of microstresses as well as the verification of the predicted properties/stress response with a detailed three-dimensional finite element analyses, as well as boundary element analyses, has been carried out and is the subject matter of a follow-up report.

## CONCLUSION

A set of simplified micromechanics equations to predict the properties of particulate reinforced composites in terms of their constituent properties was developed. The equations for predicting microstresses in the constituents due to applied thermal or mechanical loading were also presented. These equations are currently being programmed as a separate module in a NASA developed composite mechanics code. Such a computational tool will enable the user to analyze the particulate reinforced composites by themselves or as a matrix material in a continuous fiber reinforced composite at a high computational efficiency. Preliminary comparison with other predictions and experimental data is excellent.

## APPENDIX

The spherical particle in the unit cell is converted to a cubic particle (fig. 1). The diameter of the particle is  $d$ , volume fraction of the particles is  $V_f$ , the equivalent dimension of the particle is  $d_e$ , the overall dimension of the cubic unit cell,  $s$ , are given by - (see also fig. 2)

$$\frac{\pi d^3}{6} = V_f s^3 = d_e^3 \quad (\text{A1})$$

which can be written as

$$V_f = \left(\frac{d_e}{s}\right)^3 \quad (\text{A2})$$

### Normal Modulus

Consider the unit cell is subjected to a uniaxial load in longitudinal or 1-direction. The total load on the unit cell is defined from force equilibrium to be

$$P_{pc} = \tilde{P}_p + P_b \quad (\text{A3})$$

In an average sense,

$$\sigma_{pc} A_{\text{unit cell}} = \tilde{\sigma}_p \tilde{A}_p + \sigma_b A_b \quad (\text{A4})$$

Dividing the above equation by  $A_{\text{unit cell}}$  and substituting actual areas, one gets

$$\sigma_{pc} \tilde{\sigma}_p \frac{d_e d_e s}{s^3} + \sigma_b \cdot \frac{(s^3 - d_e^2 s)}{s^3} \quad (\text{A5})$$

substituting from eqn. A2, and writing in term of modulus and strain term

$$E_{pc} \epsilon_{pc} = \tilde{E}_p \epsilon_p V_f^{0.67} + E_b \epsilon_b (1 - V_f^{0.67}) \quad (\text{A6})$$

Compatibility of longitudinal displacement requires that strain in the composite and each constituent be the same, i.e. ( $\epsilon_{pc} = \epsilon_p = \epsilon_b$ ), so the equation (A6) reduces to

$$E_{pc} = \tilde{E}_p V_f^{0.67} + E_b (1 - V_f^{0.67}) \quad (\text{A7})$$

all term in the equation (A7) are known except for the modulus of the particle cell. For the particle cell, the binder and the particle are in "series". The displacement compatibility yields

$$s \tilde{\epsilon}_p = d_e \epsilon_p + (s - d_e) \epsilon_b \quad (\text{A8})$$

rewrite and divide by  $s$

$$\frac{\tilde{\sigma}_p}{\tilde{E}_p} = \frac{d_e}{s} \frac{\sigma_p}{E_p} + \frac{(s - d_e)}{s} \frac{\sigma_b}{E_b} \quad (\text{A9})$$

The constituents in the particle cell being in "series" are subjected to the same stress, i.e.  $\sigma_p = \sigma_p = \sigma_b$ , combine this with equation (A9) and also substituting from equation (A2)

$$\frac{1}{\tilde{E}_p} = \frac{V_f^{0.33}}{E_p} + \frac{(1 - V_f^{0.33})}{E_b} \quad (\text{A10})$$

Rearranging the above equation yields

$$\tilde{E}_p = \frac{E_p E_b}{V_f^{0.33} E_b + (1 - V_f^{0.33}) E_p} \quad (\text{A11})$$

substituting equation (A11) into equation (A7) yield

$$E_{pc} = \frac{V_f^{0.67} E_b}{1 - V_f^{0.33} \left(1 - \frac{E_b}{E_p}\right)} + (1 - V_f^{0.67}) E_b \quad (\text{A12})$$

#### Coefficient of Thermal Expansion

The thermal expansion coefficient in the 1-direction can be obtained by noting that the sum of the forces in the longitudinal direction due to a temperature difference  $\Delta T$  should be zero (there is no externally applied mechanical load),

$$\alpha_{pc} \cdot \Delta T \cdot E_{pc} s^2 = \tilde{\alpha}_p \Delta T \cdot \tilde{E}_p \cdot d_c^2 + \alpha_b \cdot \Delta T \cdot E_b \cdot (s^2 - d_c^2) \quad (\text{A13})$$

rearranging terms and substituting from equation (A2) yields

$$\alpha_{pc} = \frac{1}{E_{pc}} \left[ \tilde{\alpha}_p \cdot \tilde{E}_p V_f^{0.67} + \alpha_b \cdot E_b \cdot (1 - V_f^{0.67}) \right] \quad (\text{A14})$$

The thermal expansion coefficient of the particle cell can be computed as follows. The total displacement in the particle cell due to a  $\Delta T$  should be

$$\tilde{\alpha}_p \cdot \Delta T \cdot s = \alpha_p \cdot \Delta T \cdot d_e + \alpha_b \cdot \Delta T \cdot E_b \cdot (s - d_e) \quad (\text{A15})$$

rearranging and substituting from equation (A2)

$$\tilde{\alpha}_p = \alpha_p \cdot V_f^{0.33} + \alpha_b (1 - V_f^{0.33}) \quad (\text{A16})$$

Summarizing the equation for thermal expansion coefficient

$$\alpha_{pc} = \frac{1}{E_{pc}} \left[ \tilde{\alpha}_p \tilde{E}_p \cdot V_f^{0.67} + \alpha_b \cdot E_b \cdot (1 - V_f^{0.67}) \right] \quad (A17)$$

where

$$\tilde{\alpha}_p = \alpha_b - V_f^{0.33} (\alpha_b - \alpha_p)$$

and

$$\tilde{E}_p = \frac{E_b}{1 - V_f^{0.33} \left( 1 - \frac{E_b}{E_p} \right)}$$

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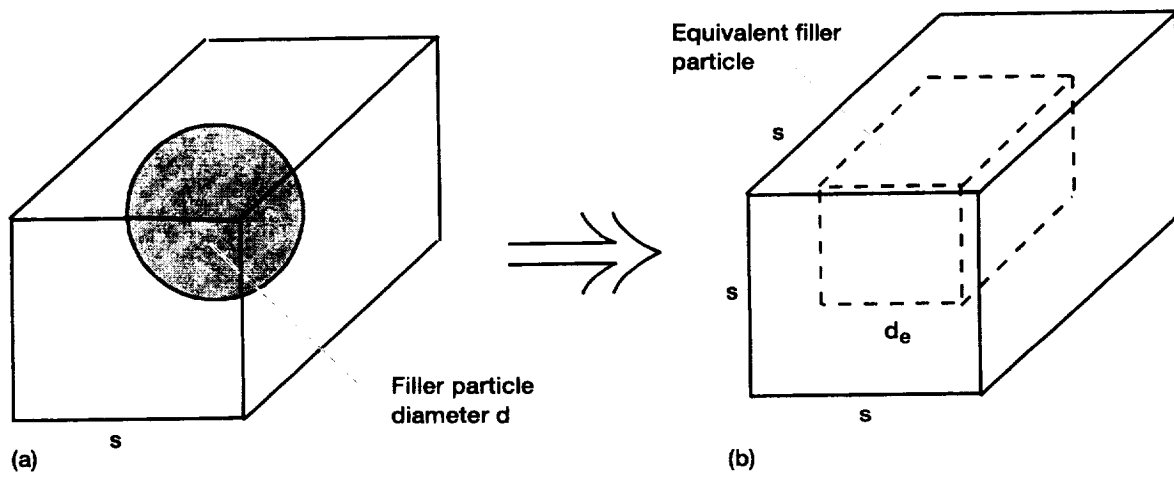


Figure 1.—Unit cell of the particulate composite.

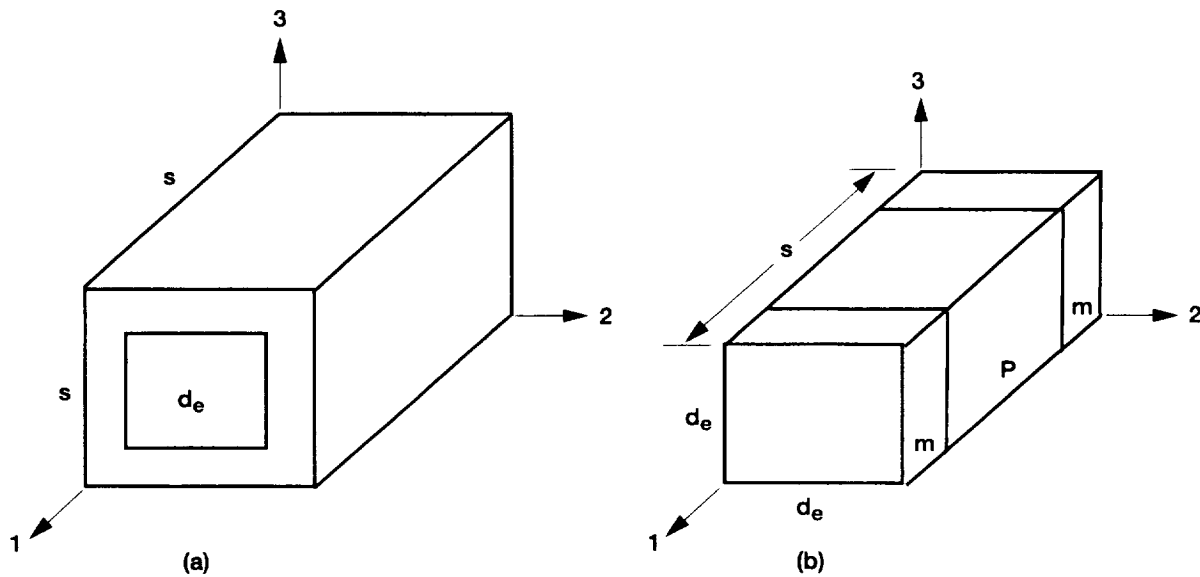


Figure 2.—Decomposition of unit cell-matrix (binder) and particle cells. (a) Matrix cell. (b) Particle cell.



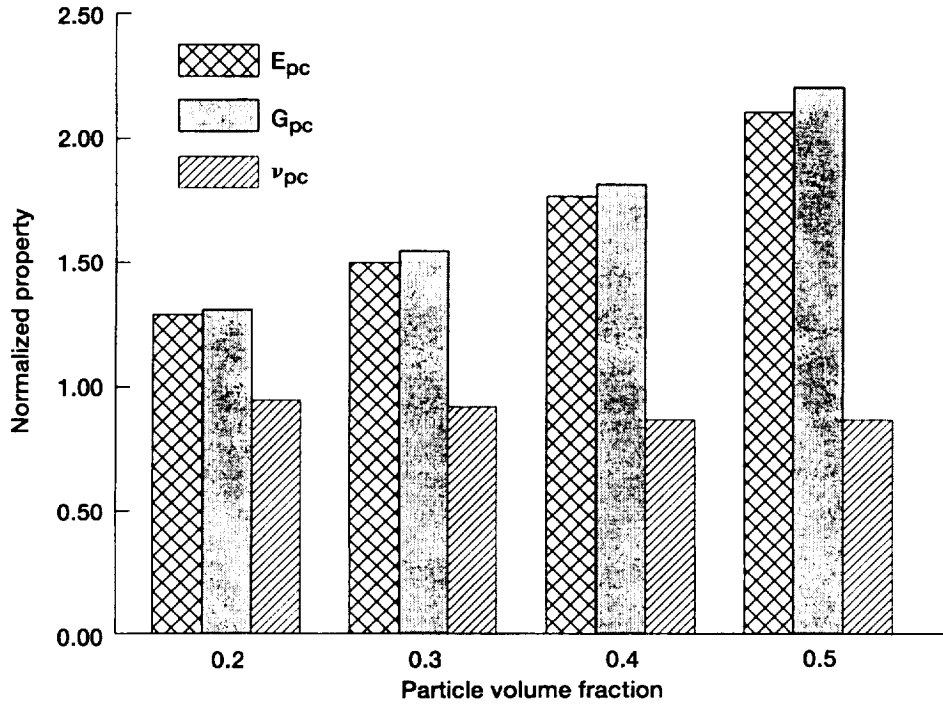


Figure 3.—Mechanical properties of a particulate composite. (Normalized w.r.t. binder properties).

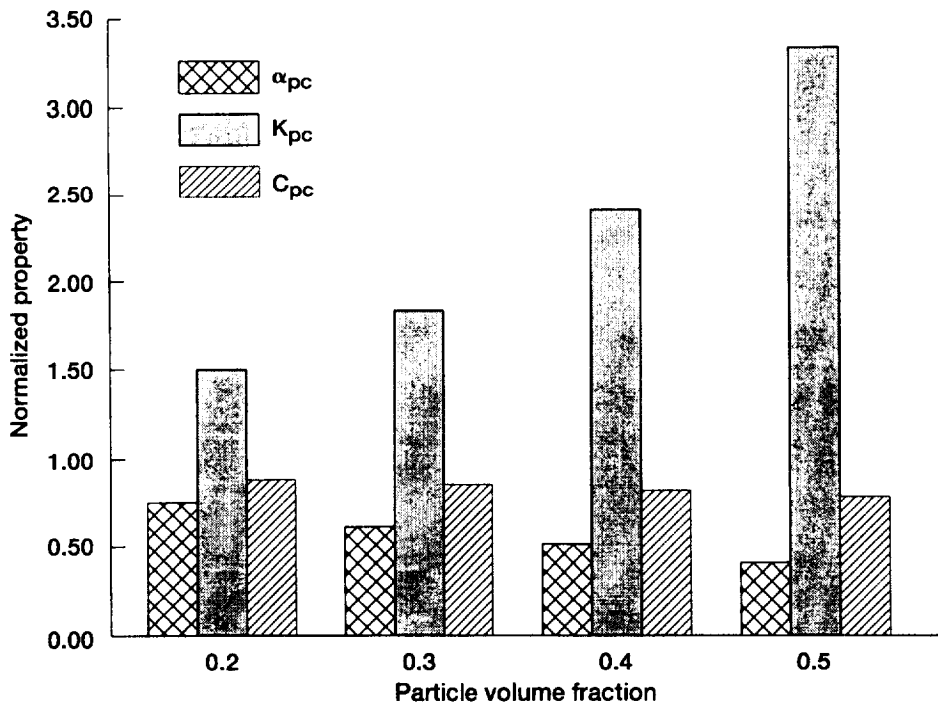


Figure 4.—Thermal properties of a particulate composite. (Normalized w.r.t. binder properties).

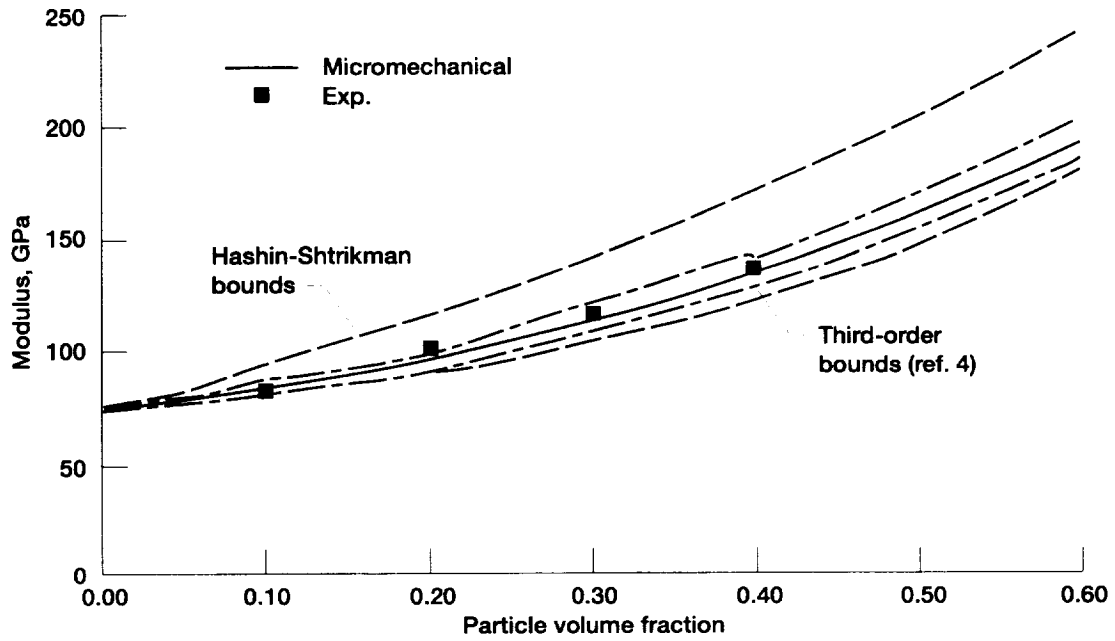


Figure 5.—Modulus of Al/SiC<sub>p</sub> composite.

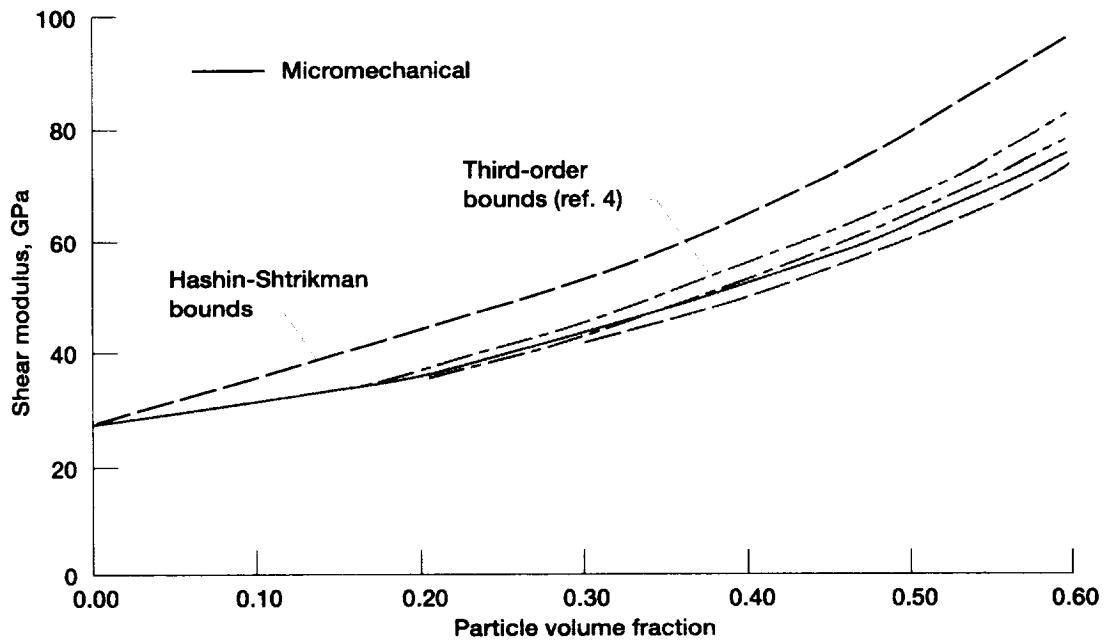


Figure 6.—Shear modulus of Al/SiC<sub>p</sub> composite.

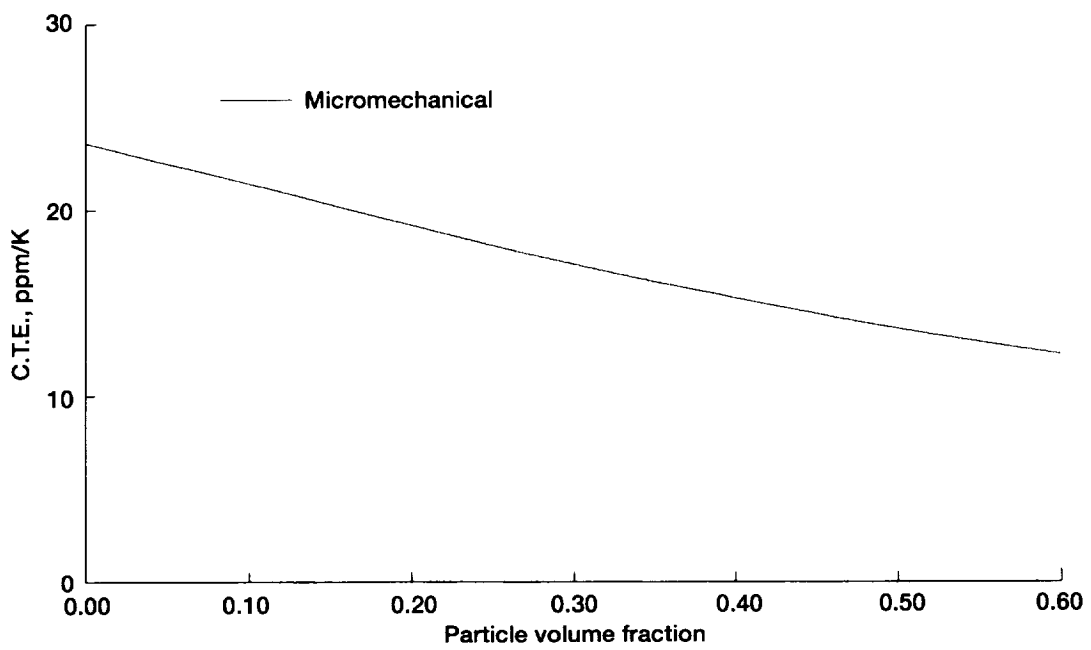


Figure 7.—Coefficient of thermal expansion of Al/SiC<sub>p</sub>.

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