



## Article

# Microphysical Characteristics of Raindrop Size Distribution and Implications for Dual-Polarization Radar Quantitative Precipitation Estimations in the Tianshan Mountains, China

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**Abstract:** In order to improve the understanding of the microphysical characteristics of raindrop size distribution (DSD) under different rainfall rates ( $R$ ) classes, and broaden the knowledge of the impact of radar wavelengths and  $R$  classes on the QPE of dual-polarization radars in the Tianshan Mountains, a typical arid area in China, we investigated the microphysical characteristics of DSD across  $R$  classes and dual-polarimetric radar QPE relationships across radar wavelengths and  $R$  classes, based on the DSD data from a PARSIVEL<sup>2</sup> disdrometer at Zhaosu in the Tianshan Mountains during the summers of 2020 and 2021. As the  $R$  class increased, the DSD became wider and flatter. The mean value of the mass-weighted mean diameters ( $D_m$ ) increased, while the mean value of logarithm normalized intercept parameters ( $\log_{10} N_w$ ) decreased after increasing from C1 to C3, as the  $R$  class increased. The largest contributions to  $R$  and the radar reflectivity factor from large raindrops (diameter > 3 mm) accounted for approximately 50% and 97%, respectively, while 84% of the total raindrops were small raindrops (diameter < 1 mm). Dual-polarization radars—horizontal polarization reflectivity ( $Z_h$ ), differential reflectivity ( $Z_{dr}$ ), and specific differential phase ( $K_{dp}$ )—were retrieved based on the DSD data using the T-matrix scattering method. The DSD-based polarimetric radar QPE relations of a single-parameter ( $R(Z_h)$ ,  $R(K_{dp})$ ), and double-parameters ( $R(Z_h, Z_{dr})$ ,  $R(K_{dp}, Z_{dr})$ ) on the S-, C-, and X-bands were derived and evaluated. Overall, the performance of the  $R(K_{dp})$  ( $R(K_{dp}, Z_{dr})$ ) scheme was better than that of  $R(Z_h)$  ( $R(Z_h, Z_{dr})$ ) for the QPE in the three bands. Furthermore, we have for the first time confirmed and quantified the performance differences in the QPE relationship of dual-polarization radars under different schemes, radar wavelengths, and  $R$  classes in typical arid areas of China. Therefore, selecting an appropriate dual-polarization radar band and QPE scheme for different  $R$  classes is necessary to improve the QPE ability compared with an independent scheme under all  $R$  classes.

**Keywords:** raindrop size distribution; dual-polarization radar; quantitative precipitation estimation; rain rate class; Tianshan Mountains

## 1. Introduction

The microphysical characteristics of raindrop size distribution (DSD) are important for understanding the dynamic processes of precipitation [1–3]. Furthermore, DSD has an important application value in improving the parameterization scheme of microphysical

processes in numerical weather prediction models [4–6] and in enhancing the ability to estimate rainfall kinetic energy [7–9]. More importantly, DSD information is very helpful for improving the ability of quantitative precipitation estimation (QPE), whether using ground-based or spaceborne radars [10–12].

The microphysical characteristics of DSD vary with the climate region, terrain, rainfall type, season, and weather system [13–20]. China is a vast country with significant differences in climatic characteristics among its different regions. Researchers have conducted detailed surveys of DSD in the southern [21,22], eastern [23–26], northern [27–29], and Tibetan Plateau [30,31] regions of China directly affected by the monsoon system and obtained the characteristics of DSD in these regions. However, the research on Xinjiang, which accounts for one-sixth of China's total land area and is characterized by an arid climate, is insufficient. Xinjiang is not directly affected by the monsoon system and has an uneven distribution of precipitation [32]. Affected by the terrain, the Tianshan Mountains in central Xinjiang, China, and their adjacent areas are rich in precipitation, while the famous Taklimakan Desert and Gurbantunggut Desert form farther away from the Tianshan Mountains [33,34]. In recent years, several studies have partially revealed the characteristics of DSD on the Tianshan Mountains [35,36]. Several recent studies have also shown significant differences in DSD in different seasons (spring, summer, and fall), locations (western and central regions), and altitudes (foot and top stations) in the Tianshan Mountains [37–39]. However, further in-depth research is needed on the microphysical characteristics of DSD across the rainfall rate ( $R$ ) classes in the Tianshan Mountains.

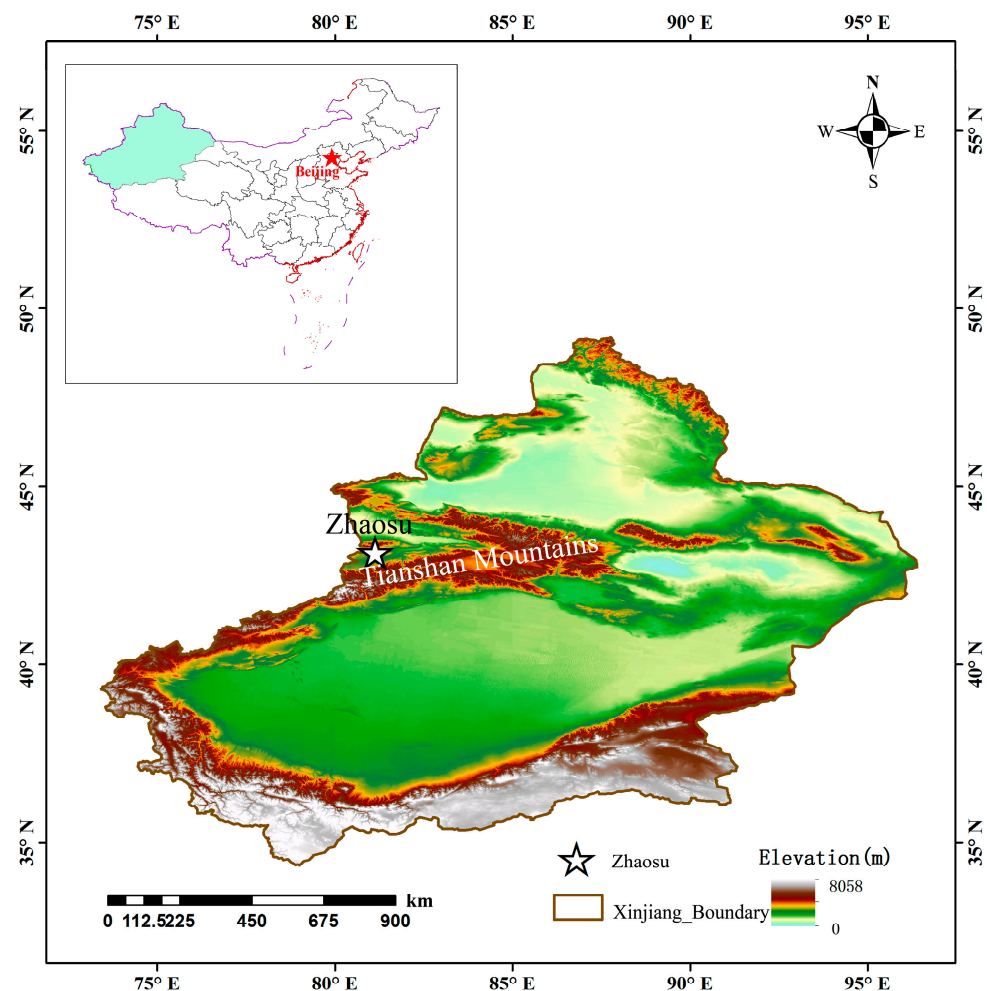
DSD information is of great significance for improving the accuracy of local ground-based radar QPEs by providing accurate microphysical characteristics of raindrops [14,20,40]. The QPE of single-polarization radars has been revealed in many studies by establishing a relationship between radar reflectivity factors and rain rates based on DSD data [41–48]. Single-polarization radar QPE relationships in the southern [22], eastern [25], northern [28], and Tibetan Plateau [31] of China have been established based on local DSD information. Similarly, based on the DSD data observed in the Tianshan Mountains, researchers have established single-polarization radar QPE relationships for different seasons, rainfall types, and altitudes [35–39]. However, the accuracy of the QPE for single-polarization radars is lower than that for dual-polarization radars [49–52]. Therefore, in recent years, the QPE relationships for dual-polarization radars have been established in different regions of China and have shown significantly better QPE capabilities than single-polarization radars [23,24,26–29,53,54]. More importantly, both the microphysical characteristics of the DSD and the QPE of the dual-polarization radar are closely related to the rain rate classes [17,18,28,55], and the QPE of the dual-polarization radar is very sensitive to the radar wavelength [24,53]. However, the previous studies mentioned above have mostly focused on the impact of one or two of the three factors, namely the  $R$  class, radar wavelength, and QPE scheme, on the QPE of dual-polarization radars. There has been relatively little comprehensive analysis of the impact of these three factors on the QPE of dual-polarization radars, and similarly, there is still a gap in these knowledge in the Tianshan Mountains, a typical arid area in China.

To reveal the microphysical characteristics of DSD under different rain rate classes and to explore the effects of radar wavelengths and rain rate classes on the QPE of dual-polarization radars in the Tianshan Mountains, we conducted this study using DSD data from the Tianshan Mountains and dual-polarization radar variables based on the T-matrix scattering method. The remainder of this paper is organized as follows: The data and methodology are presented in Section 2. Section 3 presents the microphysical characteristics of the DSD under different rain rate classes and the QPE of the dual-polarization radar with different wavelengths in the Tianshan Mountains. Section 4 provides the summary and conclusions of the study.

## 2. Data and Methodology

### 2.1. Study Area and Research Data

Tianshan Mountain is located in northwest China, as well as in Central Asia, and is not directly affected by the monsoon system. Summer is the most important rainfall period. In this study, the DSD measurements were collected at Zhaosu (1850.8 m ASL, 43.14°N, 81.13°E) over the Tianshan Mountains, China (Figure 1), during the 2020 and 2021 summer season, based on the second-generation OTT Particle Size Velocity (PARSIVEL<sup>2</sup>) disdrometer [54]. The PARSIVEL<sup>2</sup> disdrometer obtains DSD information by simultaneously recording particle sizes (32 unequal intervals from 0.062 to 24.5 mm) and fall speeds (32 unequal intervals from 0.05 to 20.8 m s<sup>-1</sup>) within a resolution of 1 min [56,57].



**Figure 1.** Location of Zhaosu (the black dot), with shading representing the topography (m) of the Tianshan Mountains.

### 2.2. Data Quality Control

The DSD data collected by the PARSIVEL<sup>2</sup> disdrometer were affected by many factors, such as the measurement accuracy and environmental conditions. Therefore, it is necessary to perform quality control before using DSD data for further analysis. In this study, the first two particle size bins were not considered because of their low signal-to-noise ratios [17,45]. Detected raindrops with very large diameters are likely to be generated by overlapping raindrops rather than by actual independent raindrops [58]; therefore, raindrops with a diameter of more than 8 mm were deleted. Furthermore, to reduce marginal effects [57], strong wind, and splashing effects [59] when measuring raindrops using a PARSIVEL<sup>2</sup> disdrometer, the theoretical raindrop fall speed–diameter relation, proposed by Atlas et al. [60] was used to constrain (within  $\pm 60\%$ ) the correlation between

the raindrop fall speed and diameter during measurement, thereby eliminating unrealistic raindrops [59,61]. In this process, the theoretical raindrop fall speed–diameter relation [60] was adjusted by considering a correction factor (1.07) for air density adjustments related to the terrain height of Zhaosu [54]. In addition, samples with raindrop numbers of <10 or with rain rates of <0.1 mm h<sup>-1</sup> were removed [14,62]. After data quality control, 14,609 DSD samples were used.

### 2.3. DSD and DSD-Based Polarimetric Radar QPE Relations

The DSD ( $N(D_i)$ , m<sup>-3</sup> mm<sup>-1</sup>) for raindrop per unit volume per unit diameter interval can be calculated according to Equation (1):

$$N(D_i) = \sum_{j=1}^{32} \frac{n_{ij}}{A_{eff}(D_i) \cdot \Delta t \cdot V(D_i) \cdot \Delta D_i} \quad (1)$$

where  $D_i$  (mm) represents the equivalent spherical raindrop diameter of the  $i$ th size class;  $n_{ij}$  is the number of drops within the  $i$ th size class and the  $i$ th velocity bin;  $\Delta t$  (s) represents the sampling time resolution (60 s in this study);  $\Delta D_i$  (mm) is the diameter interval of the  $i$ th size class; and  $A_{eff}(D_i)$  (m<sup>2</sup>) is the effective sampling area calculated according to Equation (2):

$$A_{eff}(D_i) = 180 \times 10^{-6} \cdot (30 - 0.5 \cdot D_i) \quad (2)$$

$V(D_i)$  (m s<sup>-1</sup>) is the raindrop velocity at the  $i$ th size class [18,45,54,60], which can be expressed as

$$V(D_i) = (9.65 - 10.3 \cdot \exp(-0.6 \cdot D_i)) \cdot \delta(h) \quad (3)$$

where  $\delta(h)$  represents the correction factor for air density adjustments (1.07), and  $h$  (m) is the terrain height of Zhaosu.

The rain rate  $R$  (mm h<sup>-1</sup>), liquid water content  $LWC$  (g m<sup>-3</sup>), total number concentration of raindrops  $N_t$  (m<sup>-3</sup>), median volume diameter  $D_0$  (mm), radar reflectivity factor  $Z$  (mm<sup>6</sup> m<sup>-3</sup>), the normalized intercept parameter  $N_w$  (mm<sup>-1</sup> m<sup>-3</sup>), and the mass-weighted mean diameter  $D_m$  (mm) are expressed by Equations (4)–(10), respectively:

$$R = \frac{6\pi}{10^4} \cdot \sum_{i=1}^{32} N(D_i) \cdot D_i^3 \cdot V(D_i) \cdot \Delta D_i \quad (4)$$

$$LWC = \frac{\pi}{6000} \cdot \sum_{i=1}^{32} N(D_i) \cdot D_i^3 \cdot \Delta D_i \quad (5)$$

$$N_t = \sum_{i=1}^{32} N(D_i) \cdot \Delta D_i \quad (6)$$

$$\sum_{i=1}^{D_0} N(D_0) \cdot D_i^3 \cdot \Delta D_i = \sum_{i=D_0}^{32} N(D_0) \cdot D_i^3 \cdot \Delta D_i \quad (7)$$

$$Z = \sum_{i=1}^{32} N(D_i) \cdot D_i^6 \cdot \Delta D_i \quad (8)$$

$$N_w = \frac{4^4}{\pi \cdot \rho_w} \cdot \frac{10^3 \cdot W}{D_m^4} \quad (9)$$

$$D_m = \frac{\sum_{i=1}^{32} N(D_i) \cdot D_i^4 \cdot \Delta D_i}{\sum_{i=1}^{32} N(D_i) \cdot D_i^3 \cdot \Delta D_i} \quad (10)$$

The gamma model describing DSD [13] is given by Equation (11).

$$N(D) = N_0 \cdot D^\mu \cdot \exp(-\Lambda \cdot D) \quad (11)$$

where  $N_0$  ( $\text{mm}^{-1-\mu} \text{m}^{-3}$ ),  $\mu$  ( $-$ ), and  $\Lambda$  ( $\text{mm}^{-1}$ ) represent the intercept parameter, the shape factor, and the slope parameter of the gamma model, respectively [19]. These three parameters were calculated using the truncated moment method [63,64] with the third–fourth–sixth moments [13–15,18,45,54], where the  $n$ th order moment  $M_n$  ( $\text{mm}^n \text{m}^{-3}$ ),  $N_0$ ,  $\mu$ , and  $\Lambda$  can be calculated according to the following Equation:

$$M_n = \int_0^{\infty} D^n \cdot N(D) \cdot dD \quad (12)$$

$$G = \frac{M_4^3}{M_3^2 M_6} \quad (13)$$

$$N_0 = \frac{M_3 \cdot \Lambda^{\mu+4}}{\Gamma(\mu+4)} \quad (14)$$

$$\mu = \frac{11 \cdot G - 8 + \sqrt{G \cdot (G + 8)}}{2(1 - G)} \quad (15)$$

$$\Lambda = (\mu + 4) \cdot \frac{M_3}{M_4} \quad (16)$$

DSD-based QPE relations of dual-polarization radars have been proven to be very helpful in improving the accuracy of QPE [23,24,26–29,53,54], and these DSD-based QPE relations are established by the dual-polarization radar variables: radar reflectivity at horizontal or vertical polarization  $Z_{h,v}$  ( $\text{mm}^6 \text{m}^{-3}$ ), differential reflectivity  $Z_{dr}$  (dB), and the specific differential phase  $K_{dp}$  ( $^{\circ} \text{km}^{-1}$ ), which can be calculated using the observed DSD based on the method of T-matrix scattering [52,65–67] as follows:

$$Z_{h,v} = \left( \frac{4 \cdot \lambda^4}{\pi^4 \cdot |K_w|^2} \right) \cdot \int_{D_{min}}^{D_{max}} |f_{hh,vv}(D)|^2 \cdot N(D) \cdot dD \quad (17)$$

$$Z_{dr} = 10 \cdot \log_{10} \left( \frac{Z_h}{Z_v} \right) \quad (18)$$

$$K_{dp} = 10^{-3} \cdot \frac{180}{\pi} \cdot \lambda \cdot \text{Re} \left\{ \int_{D_{min}}^{D_{max}} [f_h(D) - f_v(D)] \cdot N(D) \cdot dD \right\} \quad (19)$$

where  $\lambda$  (mm) and  $K_w$  ( $-$ ) represent the radar wavelength (for the S-, C-, and X-band, the values are 111.0 mm, 53.5 mm, and 33.3 mm, respectively), and the dielectric constant factor of water (here is 0.9639), respectively;  $f_{hh,vv}(D)$  and  $f_{h,v}(D)$  represent the backscattering amplitude and the forward scattering amplitude of a raindrop with horizontal and vertical polarizations, respectively. In addition, the raindrops followed the axis-ratio relationship proposed by Brandes [52].

The DSD-based QPE relationships, including  $R(Z_h)$ ,  $R(K_{dp})$ ,  $R(Z_h, Z_{dr})$ , and  $R(K_{dp}, Z_{dr})$  of the dual-polarization radar, were derived for the S-, C-, and X-bands. The dual-polarization radar QPE estimators are as follows:

$$R(Z_h) = \alpha \cdot Z_h^{\beta} \quad (20)$$

$$R(K_{dp}) = \alpha \cdot K_{dp}^{\beta} \quad (21)$$

$$R(Z_h, Z_{dr}) = \alpha \cdot Z_h^{\beta} \cdot 10^{\gamma \cdot Z_{dr}} \quad (22)$$

$$R(K_{dp}, Z_{dr}) = \alpha \cdot K_{dp}^{\beta} \cdot 10^{\gamma \cdot Z_{dr}} \quad (23)$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are coefficients in the corresponding QPE estimator.

#### 2.4. Assessing the Accuracy of QPE Estimators

The R value calculated using Equation (4), containing DSD information, was used to evaluate the performance of the QPE algorithms (Equations (20)–(23)) in the S-, C-, and X-band dual-polarization radars at Zhaosu. The correlation coefficient (CC), root mean square error (RMSE), and normalized mean absolute error (NMAE) were adopted for the evaluation of the QPE algorithms in this study, and are defined as

$$CC = \frac{\sum_{i=1}^n (R_i - R) \cdot (R_{e,i} - R_e)}{\sqrt{\sum_{i=1}^n (R_i - R)^2} \cdot \sqrt{\sum_{i=1}^n (R_{e,i} - R_e)^2}} \quad (24)$$

$$RMSE = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^n (R_{e,i} - R_i)^2} \quad (25)$$

$$NMAE = \frac{\frac{1}{n} \cdot \sum_{i=1}^n |R_{e,i} - R_i|}{R} \quad (26)$$

where  $n$  represents the number of samples;  $R_i$  and  $R$  are the individual and mean  $R$  calculated from the DSD data, respectively; and  $R_{e,i}$  and  $R_e$  represent the individual and mean  $R$  calculated from the QPE algorithms, respectively.

### 3. Results

#### 3.1. DSD Characteristics under Different Rain Rate Classes

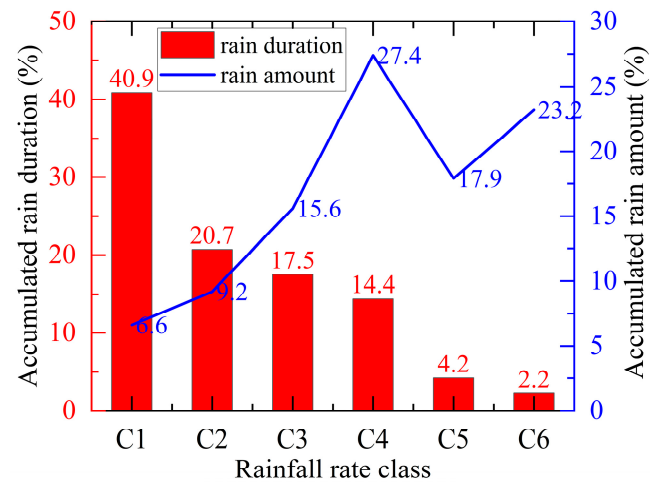
Many previous studies have revealed that the characteristics of DSD vary with rain rates ( $R$ ) [17,18,24,30,53]. To reveal the microphysical characteristics of DSD under different rain rate classes, referring to previous classification standards [36,38], all the samples were classified into six classes on the basis of  $R$ : C1: 0.1–0.5 mm h<sup>-1</sup>, C2: 0.5–1 mm h<sup>-1</sup>, C3: 1–2 mm h<sup>-1</sup>, C4: 2–5 mm h<sup>-1</sup>, C5: 5–10 mm h<sup>-1</sup>, and C6: ≥10 mm h<sup>-1</sup>. The number of samples for each class is listed in Table 1. The accumulated rain duration (red histogram) and amount (blue line) for the six  $R$  classes in Zhaosu are shown in Figure 2. As the  $R$  class increased, its contribution to the total rainfall duration decreased. Specifically, the first two classes (C1 and C2) contributed the most to the total rain duration, accounting for 40.9% and 20.7%, respectively, whereas the last two classes (C5 and C6) contributed the least to the total rain duration, contributing less than 7%. The largest contributor to the total amount of rain was the fourth class (C4), followed by the last class (C6), accounting for 27.4% and 23.2%, respectively.

**Table 1.** Several important DSD parameters for the six  $R$  classes.

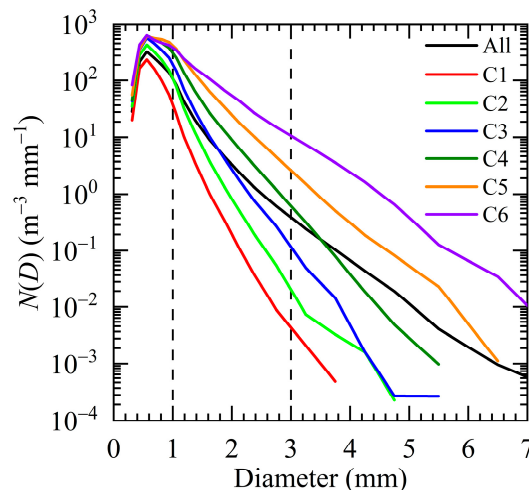
Parameters	No. of Samples	$R$ (mm h <sup>-1</sup> )	$D_0$ (mm)	$LWC$ (g m <sup>-3</sup> )	$Z$ (dBZ)	$N_t$ (m <sup>-3</sup> )
C1	5970	0.26	0.90	0.02	14.67	188
C2	3030	0.72	1.00	0.05	20.66	222
C3	2563	1.43	1.13	0.09	25.04	237
C4	2109	3.07	1.34	0.17	30.25	249
C5	612	6.91	1.72	0.32	36.21	254
C6	325	16.85	2.30	0.66	43.03	260

The DSD variations for the different  $R$  classes in Zhaosu are shown in Figure 3. As the  $R$  class increased, the raindrop spectrum widened, along with the increased concentrations of large raindrops (diameter > 3 mm), medium-size raindrops (1 ≤ diameter ≤ 3 mm), and small raindrops (diameter < 1 mm) [18,68], and reached the peak concentration at small raindrops around 0.6 mm in diameter for all classes (color lines) and all samples (black line). For small raindrops, the raindrop spectra of all the samples were between those of C1 and C2, whereas the raindrop spectrum of medium-size raindrops for all samples was similar to that of C3. For large raindrops, the raindrop spectra of all the samples

were between C4 and C5. Box-and-whisker plots of the variations in the mass-weighted mean diameter ( $D_m$ ) and normalized intercept parameter ( $\log_{10}N_w$ ) for the six  $R$  classes are shown in Figure 4. The mean  $D_m$  value increased from 0.92 mm at C1 to 2.40 mm at C6 with an increasing  $R$  class, while the mean  $\log_{10}N_w$  value first increased and then decreased with an increasing  $R$  class, reaching a maximum at C3 ( $3.63 \text{ mm}^{-1} \text{ m}^{-3}$ ) and a minimum at C6 ( $3.27 \text{ mm}^{-1} \text{ m}^{-3}$ ). Furthermore, several other important DSD parameters, such as  $R$ , median volume diameter ( $D_0$ ), liquid water content ( $LWC$ ), radar reflectivity factor ( $Z$ ), and total number concentration of raindrops ( $N_t$ ), for different  $R$  classes in Zhaosu are shown in Table 1. Their mean values increased with an increasing  $R$  class.



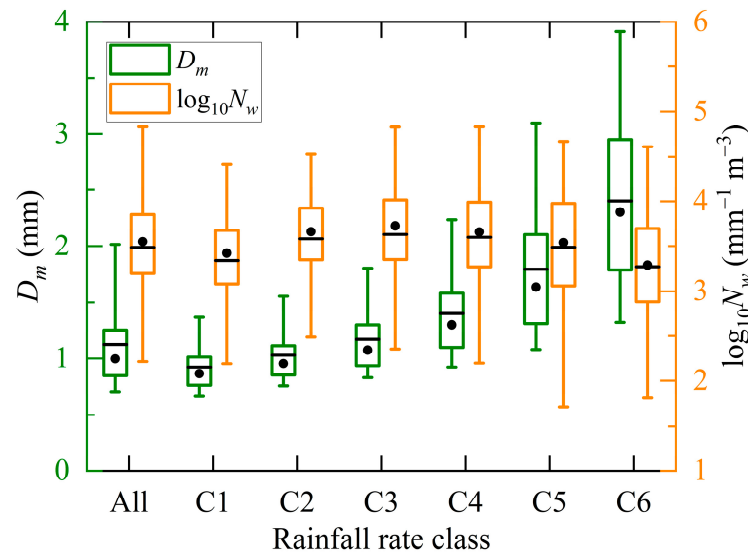
**Figure 2.** Accumulated rain duration (red histogram) and rain amount (blue line) for the six  $R$  classes in Zhaosu.



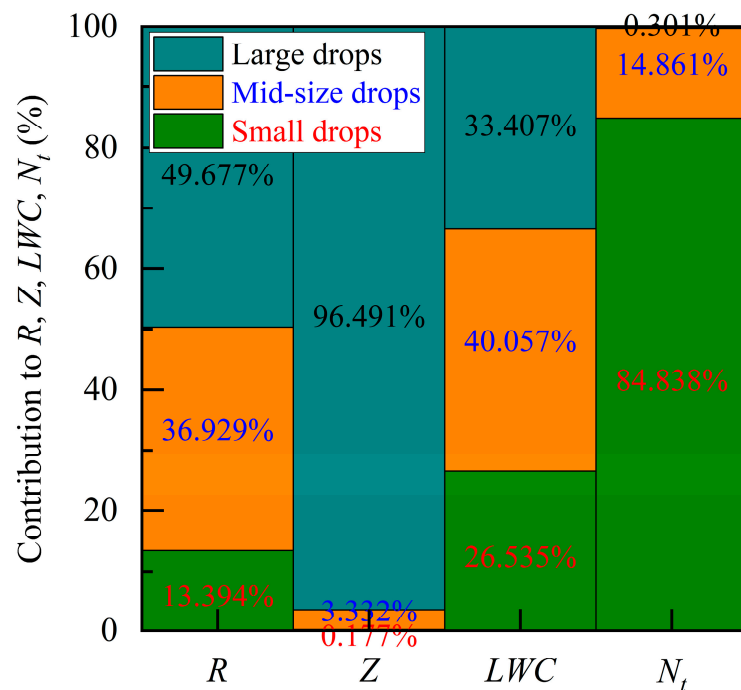
**Figure 3.** Mean DSD variations for different  $R$  classes (color lines) and all samples (black line) in Zhaosu. The two vertical dashed lines on the left and right distinguish the raindrop spectrum of small and medium-size raindrops, and the raindrop spectrum of medium-size and large raindrops, respectively.

The DSD was composed of raindrops of different sizes (large, medium, and small) and their corresponding concentrations. Therefore, discussing the contribution of raindrops of various sizes to the parameters is conducive to further understanding the DSD [37]. Figure 5 illustrates the contributions of small, medium-size, and large drops to  $R$ ,  $Z$ ,  $LWC$ , and  $N_t$  in Zhaosu. Large raindrops contributed nearly half of  $R$ ; small raindrops contributed less than 14% to  $R$ ; and medium-size raindrops contributed nearly 37% to  $R$ . The vast majority of the contributions to  $Z$  were from large raindrops (over 96%), whereas the contribution of small raindrops to  $Z$  was minimal (less than 0.2%). Most contributions to  $LWC$  came from medium-size raindrops (>40%), followed by large raindrops (>33%). The majority of the

contribution to  $N_t$  was from small raindrops (over 96%), whereas the contribution of large raindrops to  $N_t$  was minimal (approximately 0.3%).



**Figure 4.** Variations of the  $D_m$  and the  $\log_{10}N_w$  in Zhaosu for the six  $R$  classes. The line and dot of the box indicate the mean (black line) and median (black dot), respectively. The bottom (top) lines of the box indicate the 25th (75th) percentiles. The bottom (top) lines of the vertical lines out of the box indicate the 5th (95th) percentiles.



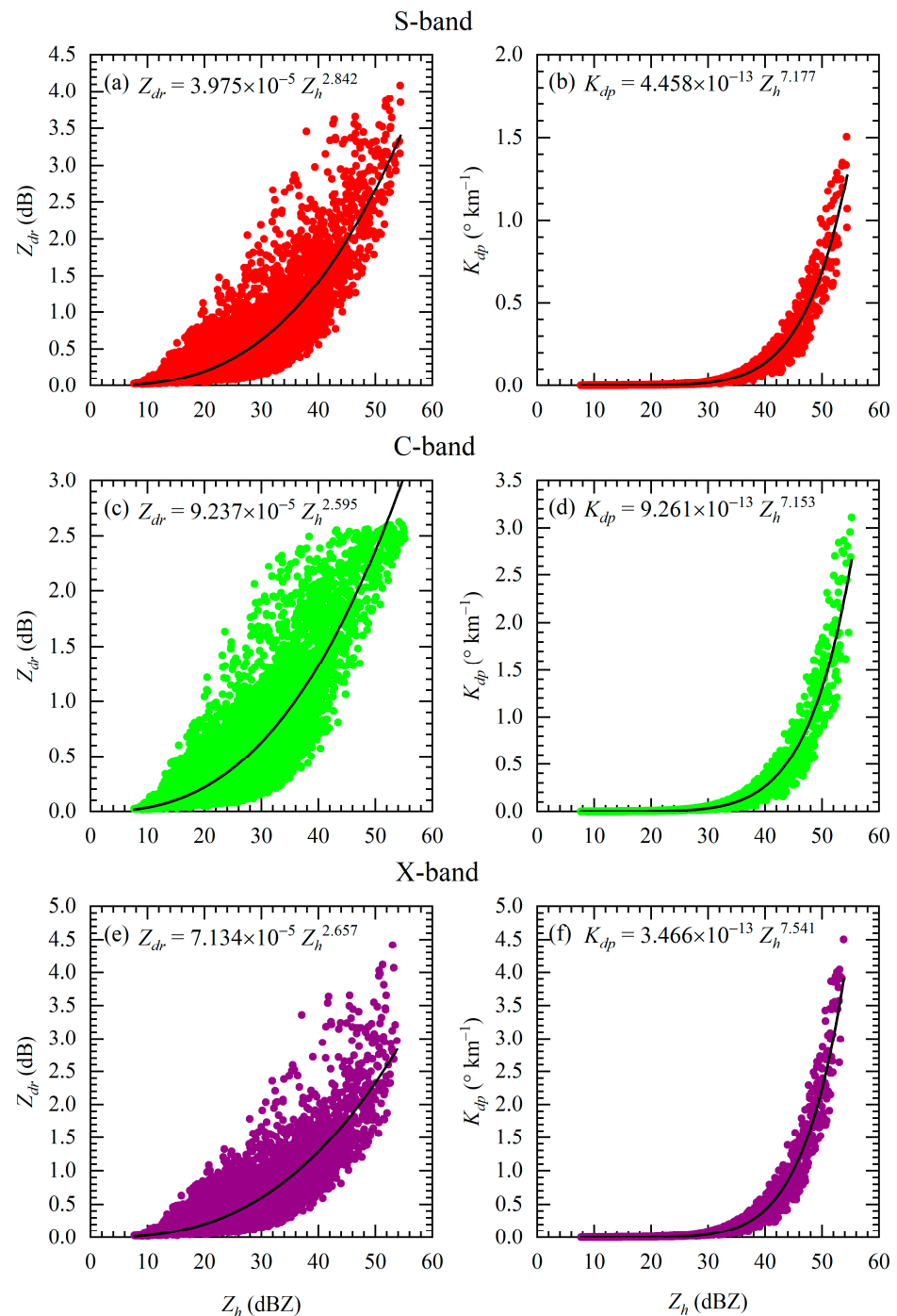
**Figure 5.** The contribution of small, medium, and large drops to  $R, Z, LWC$ , and  $N_t$  in Zhaosu.

### 3.2. Dual-Polarization Radar QPE Relations

Previous studies have revealed the advantages of using DSD information to retrieve dual-polarization radar variables for QPE based on the T-matrix scattering method [23,24,26–29,54]. The dual-polarization radar variables  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  were calculated using Equations (17)–(19). Figure 6 shows the scatterplots of  $Z_{dr}$  versus  $Z_h$  and  $K_{dp}$  versus  $Z_h$  and the power-law fitting algorithms derived for  $Z_{dr}$ – $Z_h$  and  $K_{dp}$ – $Z_h$  on the S-, C-, and X-bands. For these three-band  $Z_{dr}$ – $Z_h$  relations, the coefficient values



ranged from  $3.975 \times 10^{-5}$  to  $9.237 \times 10^{-5}$ , and the exponent values varied between 2.595 and 2.842. Specifically, the  $Z_{dr}$ - $Z_h$  relationship in the S-band (C-band) had the smallest coefficient (index) value and the largest index (coefficient) value. For the  $K_{dp}$ - $Z_h$  relations on the S-, C-, and X-bands, the coefficient values ranged from  $3.466 \times 10^{-13}$  (on the X-band) to  $9.261 \times 10^{-13}$  (on the C-band), and the exponent values varied between 7.153 (on the C-band) and 7.541 (on the X-band). From the above results, it can be seen that the  $Z_{dr}$ - $Z_h$  relation corresponding to different radar bands had obvious differences, as did the  $K_{dp}$ - $Z_h$  relation, which further illustrates the necessity of studying dual-polarization radar variables and their relationships (including the QPE) at different radar bands.



**Figure 6.** Scatterplots of  $Z_{dr}$  versus  $Z_h$ , and the  $Z_{dr}$ - $Z_h$  relations represented by black line and equation on the (a) S-band, (c) C-band, and (e) X-band. Scatterplots of  $K_{dp}$  versus  $Z_h$ , and the  $K_{dp}$ - $Z_h$  relations represented by black line and equation on the (b) S-band, (d) C-band, and (f) X-band.

The DSD-based dual-polarization radar QPE relationships ( $R(Z_h)$ ,  $R(K_{dp})$ ,  $R(Z_h, Z_{dr})$ , and  $R(K_{dp}, Z_{dr})$ ) for the S-, C-, and X-bands at Zhaosu were derived in this study and are listed in Table 2. Differences were observed in the QPE estimators for the different bands. Specifically, the two coefficients ( $\alpha$  and  $\beta$ , as shown in Equation (20)) of the  $R(Z_h)$  relation on the S- and C-bands were very similar, with a small difference compared to them on the X-band. For the  $R(K_{dp})$  relations, the difference in the  $\alpha$  coefficient was relatively large (13.053 to 27.831), while the difference in the  $\beta$  coefficient (as shown in Equation (21)) was relatively small (0.639 to 0.668) on these three bands. For the  $R(Z_h, Z_{dr})$  relations, the differences in the  $\alpha$  and  $\gamma$  coefficients were relatively large, while the difference in the  $\beta$  coefficient (as shown in Equation (22)) was relatively small. The  $\alpha$  coefficient (as shown in Equation (23)) was 23.265 on the X-band, while the  $\alpha$  coefficient on the C-band was about twice that, and the  $\alpha$  coefficient on the S-band was about three times that at the  $R(K_{dp}, Z_{dr})$  relation.

**Table 2.** The DSD-based dual-polarization radar QPE relations for S-, C-, and X-band at Zhaosu.

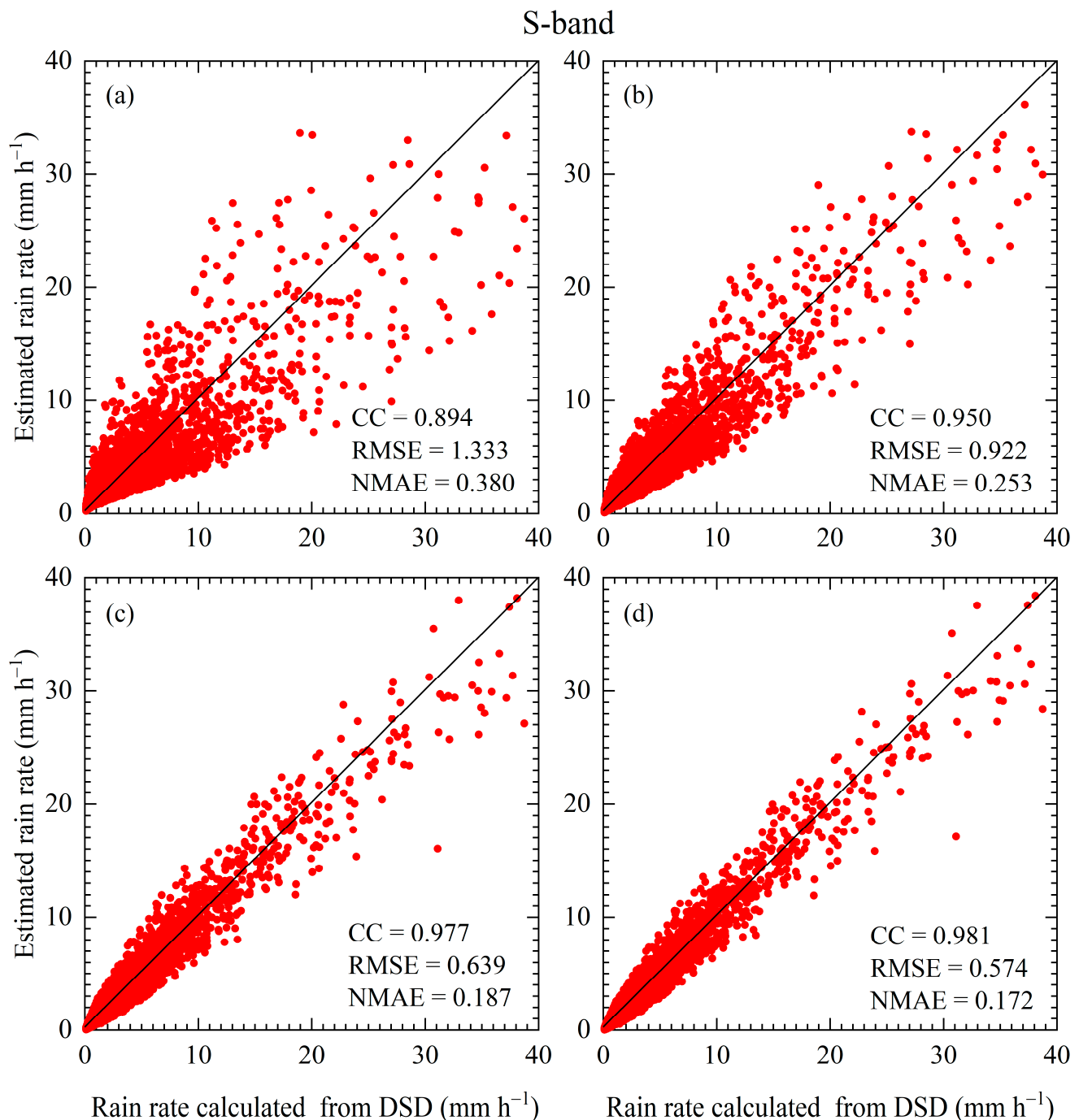
Band	$R(Z_h)$	$R(K_{dp})$	$R(Z_h, Z_{dr})$	$R(K_{dp}, Z_{dr})$
S	$R(Z_h) = 0.096 Z_h^{0.468}$	$R(K_{dp}) = 27.831 K_{dp}^{0.639}$	$R(Z_h, Z_{dr}) = 0.013 Z_h^{0.824} 10^{-0.352 Z_{dr}}$	$R(K_{dp}, Z_{dr}) = 75.719 K_{dp}^{0.845} 10^{-0.172 Z_{dr}}$
C	$R(Z_h) = 0.098 Z_h^{0.457}$	$R(K_{dp}) = 16.914 K_{dp}^{0.641}$	$R(Z_h, Z_{dr}) = 0.010 Z_h^{0.900} 10^{-0.556 Z_{dr}}$	$R(K_{dp}, Z_{dr}) = 51.816 K_{dp}^{0.890} 10^{-0.251 Z_{dr}}$
X	$R(Z_h) = 0.070 Z_h^{0.497}$	$R(K_{dp}) = 13.053 K_{dp}^{0.668}$	$R(Z_h, Z_{dr}) = 0.018 Z_h^{0.744} 10^{-0.294 Z_{dr}}$	$R(K_{dp}, Z_{dr}) = 23.265 K_{dp}^{0.816} 10^{-0.147 Z_{dr}}$

It is important to evaluate the performance of various DSD-based dual-polarization radar QPE relations in QPE applications.  $R$  calculated from DSD (Equation (4)) was used to evaluate the QPE relations [27,28,53,54,69]. In this study, three evaluation indicators—the correlation coefficient (CC), root mean square error (RMSE), and normalized mean absolute error (NMAE)—were used to evaluate the different QPE relations for different bands [27,53]. Figures 7–9 show the scatterplots of  $R$  computed from the QPE relations and the DSD information on the S-, C-, and X-bands. The performances of the double-parameter schemes ( $R(Z_h, Z_{dr})$  and  $R(K_{dp}, Z_{dr})$ ) were superior to those of the single-parameter schemes ( $R(Z_h)$  and  $R(K_{dp})$ ) for all bands, characterized by a larger CC and smaller RMSE and NMAE. The performance of the  $R(K_{dp})$  scheme is better than that of the  $R(Z_h)$  scheme for single-parameter schemes. Similarly, the  $R(K_{dp}, Z_{dr})$  scheme showed a relatively better performance than the  $R(Z_h, Z_{dr})$  scheme in the double-parameter schemes. Moreover, both single-parameter schemes performed the best for the X-band, whereas both double-parameter schemes performed the best for the C-band.

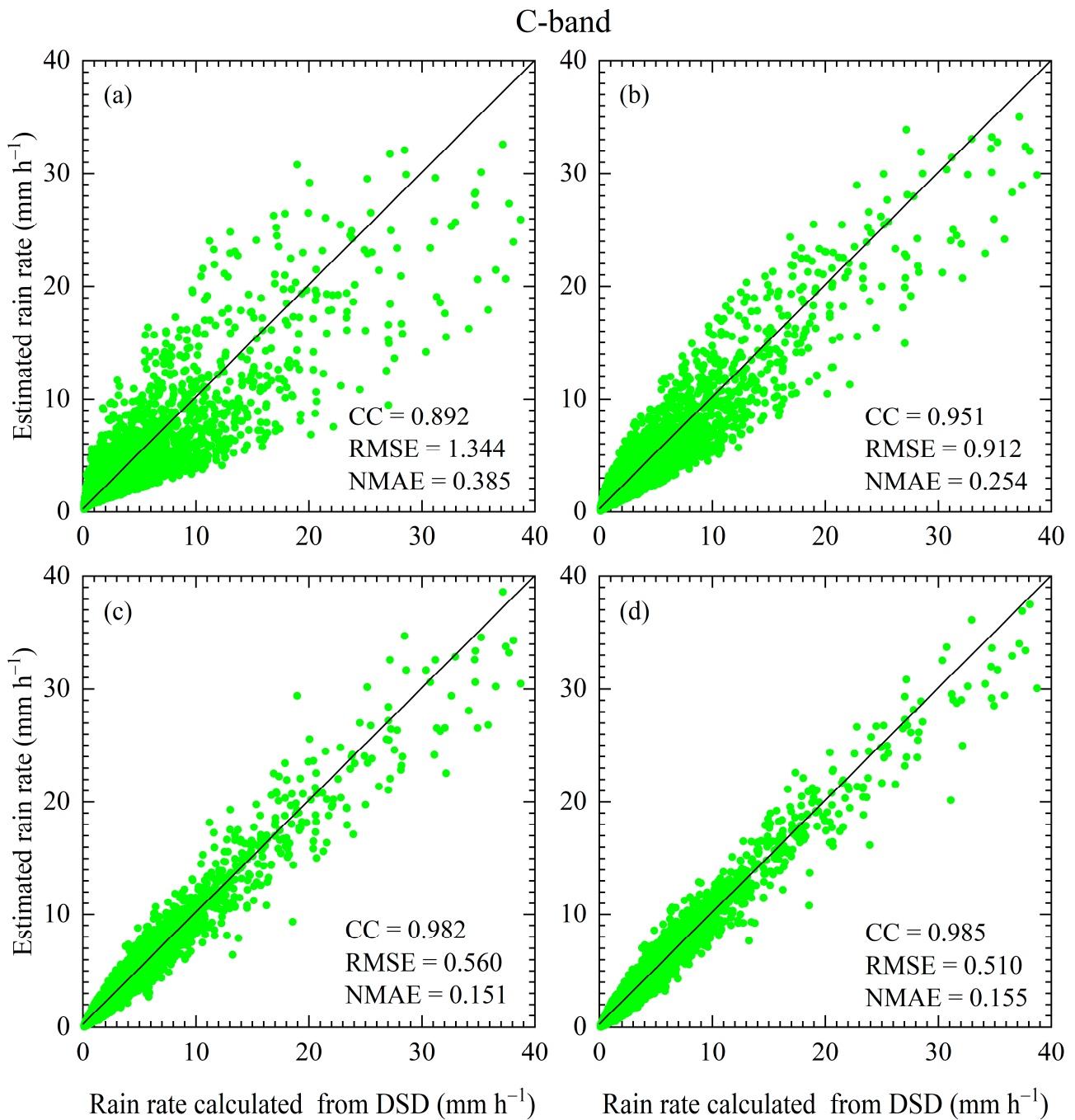
### 3.3. QPE Relations under Different Rain Rate Classes

$R$  classes have an important impact on the performance of dual-polarization radar QPE estimators [53,55,69] as well as radar bands [53,55,69]. To quantify the performance of QPE estimators under different  $R$  classes and different radar bands, we used CC, RMSE, and NMAE to evaluate the performance of these QPE estimators in detail. Before evaluating the performance of the QPE estimators, we first provided the distribution and average values of the dual-polarization radar variables required to establish these QPE estimators in Table 2 for different  $R$  classes and different radar bands, as shown in Figure 10 and Table 3.  $Z_h$  increased with an increasing  $R$  class for all bands, and the mean  $Z_h$  value in the X-band was the largest for all  $R$  classes except C6, compared to that in the S- and X-bands. The distribution of  $Z_h$  was narrowest in C2 and widest in C6 for all the bands. Similar to  $Z_h$ ,  $Z_{dr}$  also increased with an increasing  $R$  class for all bands; however, in the first two  $R$  classes (C1 and C2),  $Z_{dr}$  was largest in the X-band, whereas in the middle two  $R$  classes (C3 and C4),  $Z_{dr}$  was largest in the C-band, and in the last two  $R$  classes (C5 and C6),  $Z_{dr}$  was largest in the S-band. The distribution of  $Z_{dr}$  was narrower in the first three  $R$  classes and widened

in the last three  $R$  classes, particularly in the last  $R$  class (C6), where  $Z_{dr}$  had the widest distribution. Interestingly, during the process of increasing  $K_{dp}$  as the  $R$  class increased, the mean  $K_{dp}$  value of the next  $R$  class was about three times that of the previous  $R$  class for all bands (for example,  $11.3 \times 10^{-3} \text{ } ^\circ \text{ km}^{-1}$  in C3 and  $32.8 \times 10^{-3} \text{ } ^\circ \text{ km}^{-1}$  in C4 for S-band). The mean  $K_{dp}$  value in the C-band was about twice that in the S-band, and the mean  $K_{dp}$  value in the X-band was about three times that in the S-band for each  $R$  class (for example,  $11.3 \times 10^{-3} \text{ } ^\circ \text{ km}^{-1}$  in the S-band,  $24.2 \times 10^{-3} \text{ } ^\circ \text{ km}^{-1}$  in the C-band, and  $39.8 \times 10^{-3} \text{ } ^\circ \text{ km}^{-1}$  in the X-band for C3).



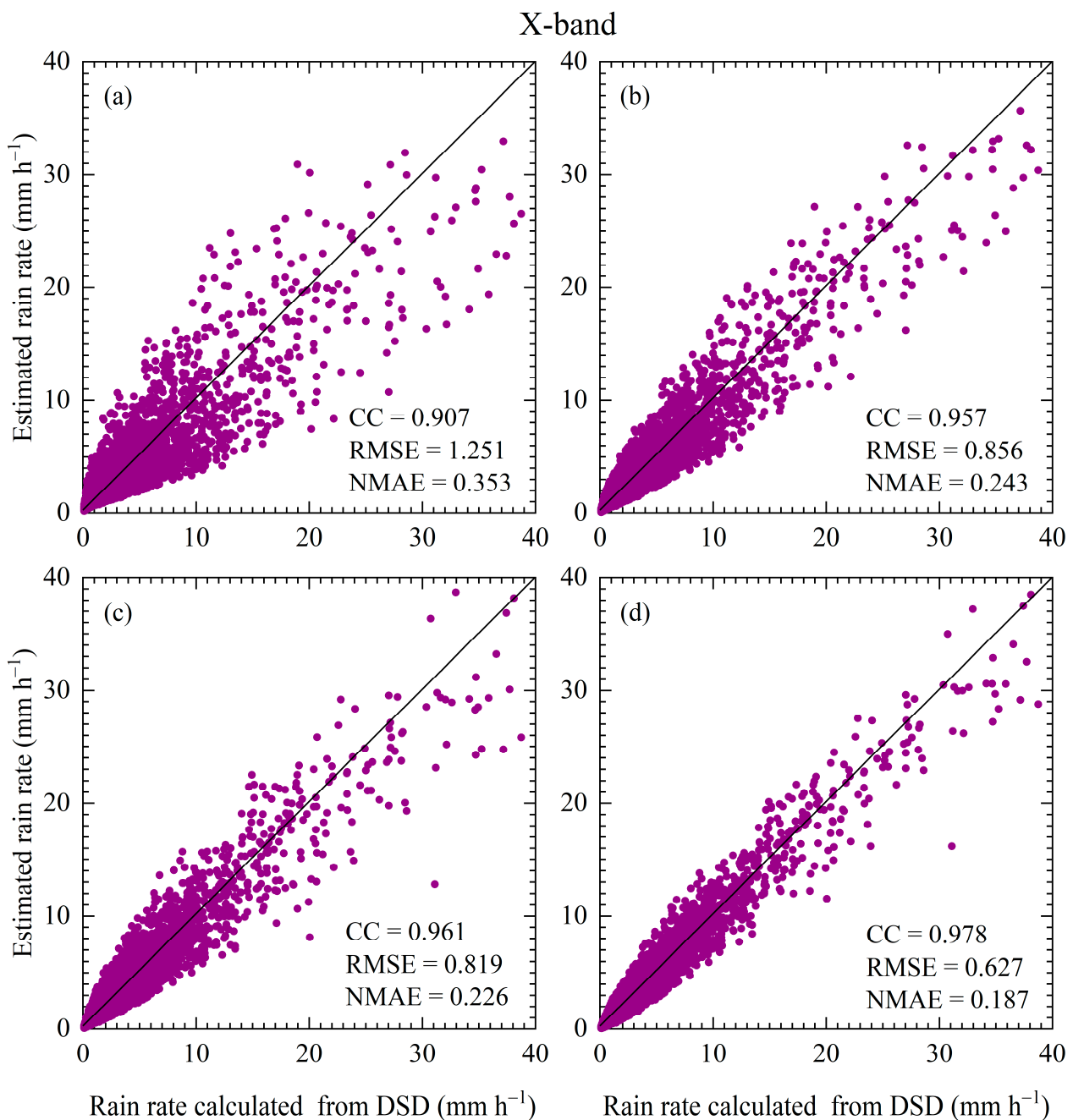
**Figure 7.** Scatterplot of  $R$  calculated from (a)  $R(Z_h)$ , (b)  $R(K_{dp})$ , (c)  $R(Z_h, Z_{dr})$ , and (d)  $R(K_{dp}, Z_{dr})$  relations versus the  $R$  computed from DSD for S-band in Zhaosu.



**Figure 8.** Scatterplot of  $R$  calculated from (a)  $R(Z_h)$ , (b)  $R(K_{dp})$ , (c)  $R(Z_h, Z_{dr})$ , and (d)  $R(K_{dp}, Z_{dr})$  relations versus the  $R$  computed from DSD for C-band in Zhaosu.

**Table 3.** The mean of  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  on the S-, C-, and X-bands for the six  $R$  classes. Red font indicates the maximum value at the same  $R$  class.

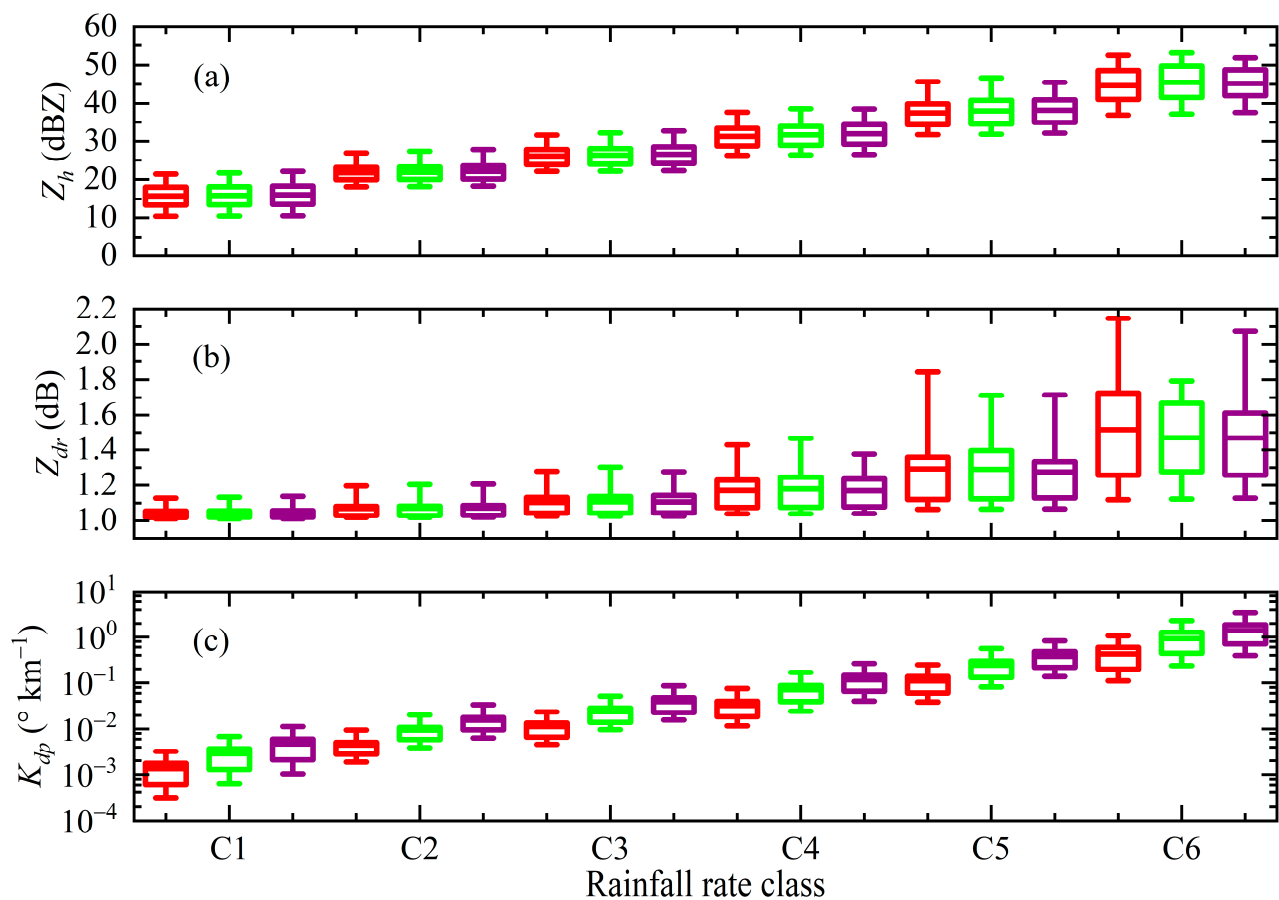
Band	$Z_h$ (dBZ)						$Z_{dr}$ ( $10^{-1}$ dB)						$K_{dp}$ ( $10^{-3}$ km $^{-1}$ )					
	C1	C2	C3	C4	C5	C6	C1	C2	C3	C4	C5	C6	C1	C2	C3	C4	C5	C6
S	15.82	21.73	26.08	31.27	37.47	44.69	10.47	10.71	11.05	11.71	12.93	15.17	1.4	4.5	11.3	32.8	110.1	426.7
C	15.93	21.89	26.31	31.64	38.03	45.51	10.48	10.72	11.09	11.79	12.90	14.71	2.9	9.5	24.2	71.5	241.7	933.6
X	16.09	22.10	26.58	31.94	38.18	45.22	10.49	10.73	11.07	11.70	12.75	14.69	4.7	15.5	39.8	116.2	379.9	1394.0



**Figure 9.** Scatterplot of  $R$  calculated from (a)  $R(Z_h)$ , (b)  $R(K_{dp})$ , (c)  $R(Z_h, Z_{dr})$ , and (d)  $R(K_{dp}, Z_{dr})$  relations versus the  $R$  computed from DSD for X-band in Zhaosu.

Figure 11 shows the CC, RMSE, and NMAE of  $R$  estimated from the dual-polarization radar QPE estimators against  $R$  calculated from the DSD under different  $R$  classes (C1–C6) and radar bands (the S-, C-, and X-bands). The performances of the four schemes for these three bands differed under different  $R$  classes. For the S-band radar, the  $R(Z_h)$  estimator had the worst performance, characterized by a relatively lower CC and higher RMSE and NMAE for all  $R$  classes, followed by the  $R(K_{dp})$  estimator. The  $R(K_{dp}, Z_{dr})$  estimator performed the best (highest CC and lowest RMSE and NMAE) for all  $R$  classes. The RMSE of all the estimators increased with an increasing  $R$  class, whereas the CC and NMAE did not monotonically increase or decrease during this process (Figure 11a–c). For the C-band radar, similar to the S-band radar, the performance of the  $R(Z_h)$  estimator remained the worst, followed by that of the  $R(K_{dp})$  estimator for all  $R$  classes. However, unlike in the

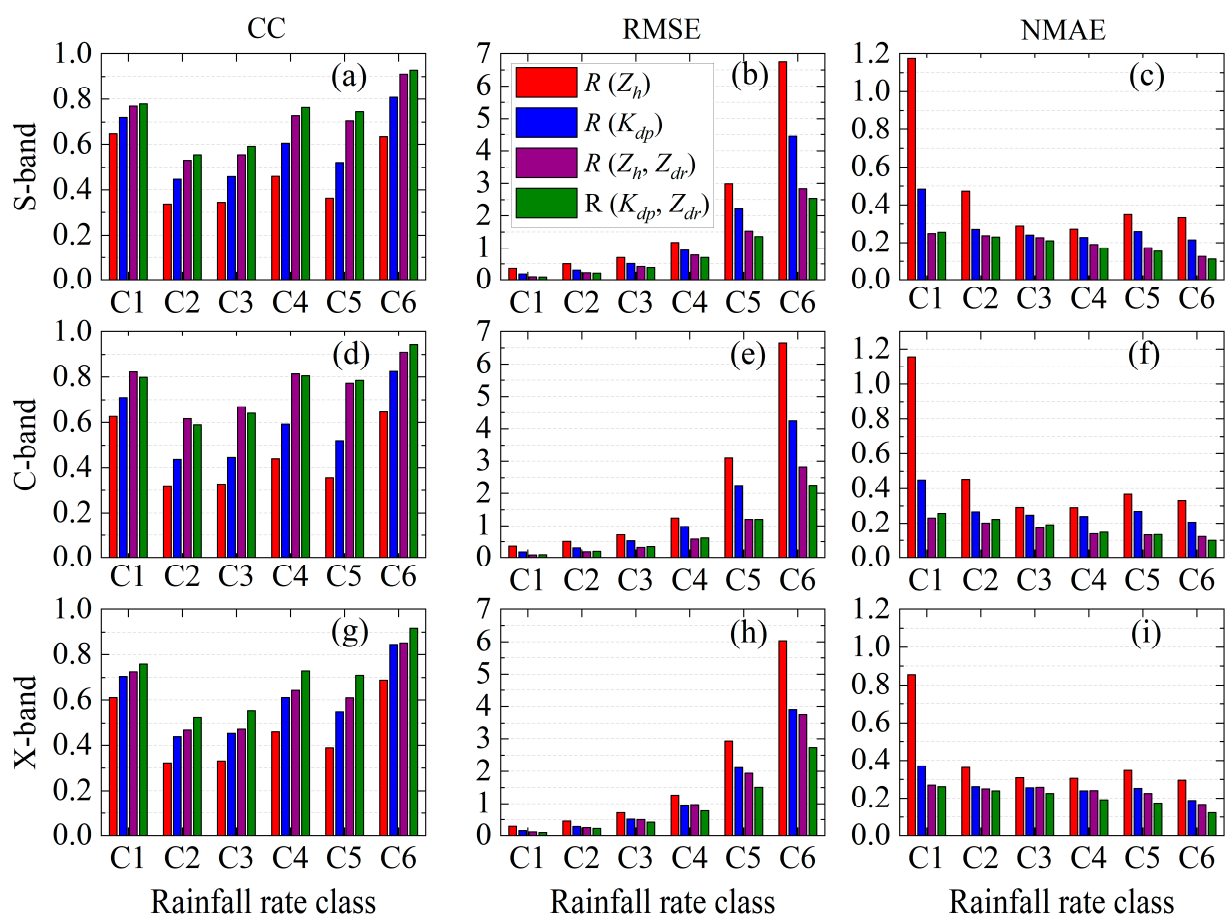
S-band radar, the performance of the  $R(K_{dp}, Z_{dr})$  estimator was not always the best for all the  $R$  classes in the C-band radar. Specifically, when the  $R$  class was between C1 and C4, the  $R(Z_h, Z_{dr})$  estimator was slightly superior to the  $R(K_{dp}, Z_{dr})$  estimator, with a higher CC and lower RMSE and NMAE; however, the opposite was true when the  $R$  class was C5 and C6 (Figure 11d–f). For the X-band radar, similar to the S-band radar, the  $R(Z_h)$  estimator exhibited the worst performance, whereas the  $R(K_{dp}, Z_{dr})$  estimator exhibited the best performance for all the  $R$  classes. However, the gap in performance between the  $R(K_{dp})$  and  $R(Z_h, Z_{dr})$  estimators for X-band radars narrowed compared to the S- and C-band radars, as reflected in the narrowing of the gap between the three evaluation parameters (CC, RMSE, and NMAE) (Figure 11d–f).



**Figure 10.** Variations of (a)  $Z_h$ , (b)  $Z_{dr}$ , and (c)  $K_{dp}$  on the S- (red), C- (green), and X-bands (purple) for the six  $R$  classes. The line of the box indicates the mean. The bottom (top) lines of the box indicate the 25th (75th) percentiles. The bottom (top) lines of the vertical lines out of the box indicate the 5th (95th) percentiles.

For the same type of QPE estimator and the same  $R$  class, the performances of the estimators in the different bands were different. Taking the  $R(Z_h, Z_{dr})$  estimator at C3 as an example, for the  $R(Z_h, Z_{dr})$  estimator at C3, the CC, RMSE, and NMAE were 0.669, 0.318 mm h<sup>-1</sup>, and 0.73 mm h<sup>-1</sup> in the C-band, respectively, while the CC, RMSE, and NMAE were 0.554 (0.471), 0.417 (0.501) mm h<sup>-1</sup>, and 0.227 (0.258) mm h<sup>-1</sup> in the S-band (X-band), respectively. Therefore, among the three bands, the C-band estimator performed the best, whereas the X-band estimator performed the worst for the  $R(Z_h, Z_{dr})$  estimator at C3. Similarly, taking the  $R(Z_h)$  estimator at C6 as an example again, for the  $R(Z_h)$  estimator at C6, the CC, RMSE, and NMAE were 0.689, 6.032 mm h<sup>-1</sup>, and 0.295 mm h<sup>-1</sup> in the X-band, respectively, while the CC, RMSE, and NMAE were 0.634 (0.646), 6.749 (6.651) mm h<sup>-1</sup>, and 0.332 (0.329) mm h<sup>-1</sup> in the S-band (X-band), respectively. Therefore, among the three bands, the X-band estimator performed best, whereas the S-

band estimator performed worst for the  $R(Z_h)$  estimator at C6. Overall, the performances of the double-parameter schemes were significantly better than those of the single-parameter schemes for all bands and  $R$  classes. Furthermore, the performance of the dual-parameter estimators in the C-band was better than that in the S- and X-bands for all  $R$  classes, and the performance of the  $R(Z_h, Z_{dr})$  estimator was better compared to the  $R(K_{dp}, Z_{dr})$  estimator at lower  $R$  classes (C1 to C4,  $R$  less than  $5 \text{ mm h}^{-1}$ ), while the performance of the  $R(K_{dp}, Z_{dr})$  estimator was better compared to the  $R(Z_h, Z_{dr})$  estimator at higher rainfall rates (C5 to C6,  $R$  greater than  $5 \text{ mm h}^{-1}$ ) for the C-band. It is worth noting that previous studies have shown the importance of selecting suitable estimators for actual dual-polarization radar QPEs, and suitable estimators need to be provided for different regions and different band radars [55,69–71]. For a dual-polarization radar QPE estimator in the Tianshan Mountains, we plan to conduct the relevant research based on dual-polarization radar observational data in the future.



**Figure 11.** The (a,d,g) CC, (b,e,h) RMSE, and (c,f,i) NMAE of  $R$  estimated from the dual-polarization radar QPE estimators against  $R$  calculated from the DSD under different  $R$  classes and different radar bands, (a–c) for S-band, (d–f) for C-band, and (g–i) for X-band, respectively.

#### 4. Summary and Conclusions

To reveal the microphysical characteristics of the raindrop size distribution (DSD) across rainfall rate ( $R$ ) classes, and more importantly, to quantify the effects of radar wavelengths, QPE estimators, and  $R$  classes on the QPE of dual polarization radars in typical arid areas of China, DSD data from a PARSIVEL<sup>2</sup> disdrometer at Zhaosu in the Tianshan Mountains during summer 2020 and 2021 were used to investigate the microphysical characteristics of DSD for six rain rate ( $R$ ) classes (C1:  $0.1\text{--}0.5 \text{ mm h}^{-1}$ , C2:  $0.5\text{--}1 \text{ mm h}^{-1}$ , C3:  $1\text{--}2 \text{ mm h}^{-1}$ , C4:  $2\text{--}5 \text{ mm h}^{-1}$ , C5:  $5\text{--}10 \text{ mm h}^{-1}$ , and C6:  $\geq 10 \text{ mm h}^{-1}$ ) and DSD-based polarimetric radar quantitative precipitation estimation (QPE) relations on the

S-, C-, and X-bands for different  $R$  classes. The analysis revealed that the first two  $R$  classes (C1 and C2) contributed the most to the total rain duration, which accounted for 40.9% and 20.7%, respectively, and the largest contributors to the total rain amount were the fourth class (C4) and the last class (C6), which accounted for 27.4% and 23.2%, respectively. The raindrop spectrum widened, characterized by an increase in the concentrations of large raindrops (diameter  $> 3$  mm), medium-size raindrops ( $1 \leq$  diameter  $\leq 3$  mm), and small raindrops (diameter  $< 1$  mm) with an increasing  $R$  class. The mean  $D_m$  value increased from 0.92 mm in C1 to 2.40 mm in C6, while the mean logarithm of  $\log_{10}N_w$  value decreased after increasing from C1 to C3 with an increasing  $R$  class. In addition, the mean values of  $R$ ,  $D_0$ ,  $LWC$ ,  $Z$ , and  $N_t$  increased with the  $R$  class. For the entire dataset, large raindrops had the largest contribution to  $R$  and  $Z$ , accounting for 50% and 97% of the total contribution, respectively, compared with small and medium-size raindrops, whereas small raindrops had the largest contribution to  $N_t$ , accounting for more than 84% of the total contribution.

Dual-polarization radar parameters including  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  were retrieved based on the DSD data using the T-matrix scattering method. The  $Z_{dr}$ - $Z_h$  and  $K_{dp}$ - $Z_h$  relations were established in a power-law fitting form on the S-, C-, and X-bands. The  $Z_{dr}$ - $Z_h$  relation corresponding to different radar bands had obvious differences as well as the  $K_{dp}$ - $Z_h$  relation. The DSD-based dual-polarization radar QPE estimators ( $R(Z_h)$ ,  $R(K_{dp})$ ,  $R(Z_h, Z_{dr})$ , and  $R(K_{dp}, Z_{dr})$ ) for the S-, C-, and X-bands were derived. For the  $R(Z_h)$  relations, the two coefficients ( $\alpha$  and  $\beta$ ) on the S- and C-bands were very similar, with a small difference compared to them on the X-band. For the  $R(K_{dp})$  relations, the difference in the  $\alpha$  coefficient was relatively large (13.053 to 27.831), while the difference in the  $\beta$  coefficient was relatively small (0.639 to 0.668) on these three bands. For the  $R(Z_h, Z_{dr})$  relations, the differences in the  $\alpha$  and  $\gamma$  coefficients were relatively large, while the difference in the  $\beta$  coefficient was relatively small. For the  $R(K_{dp}, Z_{dr})$  relations, the  $\alpha$  coefficient was 23.265 on the X-band, while the  $\alpha$  coefficient on the C-band was about twice that, and the  $\alpha$  coefficient on the S-band was about three times that. The CC, RMSE, and NMAE of  $R$  estimated from the dual-polarization radar QPE estimators against  $R$  calculated from the DSD were used to evaluate the performance of these dual-polarization radar QPE estimators. The result revealed that the performance of double-parameter estimators ( $R(Z_h, Z_{dr})$  and  $R(K_{dp}, Z_{dr})$ ) was superior to that of single-parameter estimators ( $R(Z_h)$  and  $R(K_{dp})$ ), and the performance of the  $R(K_{dp}, Z_{dr})$  estimator was superior to that of the  $R(Z_h, Z_{dr})$  estimator for all the bands. Overall, the single-parameter estimator performed the best for the X-band, whereas the double-parameter estimator performed the best for the C-band.

Furthermore, the distribution and mean values of the dual-polarization radar variables establishing these QPE estimators across  $R$  classes and radar wavelengths were determined, and the performance of these four types of estimators ( $R(Z_h)$ ,  $R(K_{dp})$ ,  $R(Z_h, Z_{dr})$ , and  $R(K_{dp}, Z_{dr})$ ) for the three bands (S-, C-, and X-bands) showed differences across the  $R$  classes. Generally, for all the  $R$  classes, the dual-parameter estimators had better performances in the C-band than the other two bands, and the performance of the  $R(Z_h, Z_{dr})$  ( $R(K_{dp}, Z_{dr})$ ) estimator was better compared to the  $R(K_{dp}, Z_{dr})$  ( $R(Z_h, Z_{dr})$ ) estimator at lower (higher)  $R$  classes for the C-band. Our conclusion emphasizes that when conducting dual-polarization radar QPE applications, it is necessary to consider both the appropriate radar wavelength and the type of estimator, as well as the impact of  $R$  classes on the accuracy of QPE. It should be noted that although this study reported promising findings, they need to be further confirmed using dual-polarization radar observations in the future.

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