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Microwave background constraints on extended inflation voids

Andrew R. Liddle and David Wands
Astronomy Centre, Division of Physics and Astronomy, University of Sussex, Brighton BN1 9QH

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SUMMARY

Extended inflation ends via the nucleation of bubbles of true vacuum, and in principle the earliest bubbles to nucleate can be expanded by the last stages of inflation to astrophysically interesting sizes, giving large voids empty of any matter. Several authors have suggested a link with the observed bubbly large-scale structure in the Universe. We examine in detail the constraints on the bubble distribution from the microwave background observations, particularly *COBE*, for different types of dark matter. In a baryon-dominated universe, we obtain upper limits on the extended inflation parameters similar to those previously given, but these become somewhat stronger when one includes dark matter. In any scenario, extended inflation cannot by itself be responsible for the bubbly large-scale structure.

1 INTRODUCTION

Recent observations of large-scale structure, which have discovered the existence of unexpectedly large coherent structure, are posing some awkward questions for standard galaxy formation scenarios. The presence of the great wall and of voids of sizes upwards of 25 h^{-1} Mpc in the CfA survey (de Lapparent, Geller & Huchra 1986; Geller & Huchra 1989), and indications of yet larger voids in other surveys (Kirshner *et al.* 1987; Kauffmann & Fairall 1991), are already difficult to reproduce in most galaxy formation models, even without the further possibility of a large-scale periodicity at a characteristic scale around 130 h^{-1} Mpc (Broadhurst *et al.* 1990). The APM (Maddox *et al.* 1990) and QDOT (Saunders *et al.* 1991) surveys also indicate more structure on large angular scales than would be expected in many models. Meanwhile, the extraordinary smoothness of the microwave background witnessed by *COBE* (Smoot *et al.* 1991), *RELIKT* (Klypin *et al.* 1987) and other experiments places strong constraints on the amplitude of perturbations on various scales. These observations have led towards the investigation of models in which the power spectrum departs from the Harrison–Zel'dovich form, and also the possibility that the statistics of the density field are non-Gaussian (Messina *et al.* 1990; Coles & Jones 1991; Salopek & Bond 1991); this latter would for instance be a natural feature in models where the seed perturbations arise from the breaking of global symmetries, such as textures (Turok 1989; Park, Spergel & Turok 1991) or global monopoles (Bennett & Rhie 1990). A more radical possibility would be an explosion scenario (Ikeuchi 1981; Ostriker & Cowie 1981) in which large-scale structure can have a partly non-gravitational origin.

A recent addition to the collection of inflationary universe models, known as extended inflation (La & Steinhardt 1989a), provides the possibility of extra perturbations over and above the Harrison–Zel'dovich-like spectrum generated by quantum fluctuations (Kolb, Salopek & Turner 1990) as in all inflationary models (Guth & Pi 1982; Bardeen, Steinhardt & Turner 1983). The qualitative difference between extended inflation and more conventional scenarios such as the chaotic inflation model is that extended inflation comes to an end via a first-order phase transition (Kolb 1991), whereby bubbles of true vacuum nucleate quantum mechanically within the sea of false vacuum driving inflation. These bubbles then expand at the speed of light, collide and thermalize. However, the earliest bubbles to form can be swept up in the subsequent inflationary expansion and be stretched to sizes large enough to be of astrophysical interest. Such large bubbles will be unable to thermalize before, say, decoupling and thus provide density perturbations in addition to those generated by quantum fluctuations.

The process of bubble nucleation transfers the energy formerly spread evenly throughout the interior of the bubble into the bubble walls. As the bubbles expand, the energy in the surrounding regions is swept up, predominantly into kinetic energy of the by now relativistic bubble walls. It is important to note that, unlike new or chaotic inflation, the interiors of these bubbles are completely devoid of matter of any type. When the bubble collides with neighbouring bubbles, the energy in the walls is released and presumably transformed back into relativistic particles, which are then able to stream back into the bubble interior at close to the speed of light. If the typical bubble size (which depends rather sensitively on the model parameters) is fairly small, then this matter should rapidly thermalize and restore homogeneity (apart from the residual quantum fluctuations which now provide density perturbations), inflation having in the meantime solved the horizon problem and the others for which inflation is currently the conventional solution (Guth 1981). However, causality dictates that if there are rare large bubbles, they certainly cannot thermalize in less time than is required for light to cross the bubble, so for instance a bubble larger than the horizon size at decoupling will not have thermalized by that time. These considerations lead to constraints on the model parameters which were indeed sufficient to rule out the original extended inflation model (Weinberg 1989), though a variety of improved models have since been devised (Accetta & Trester 1989; La, Steinhardt & Bertschinger 1989; Steinhardt & Accetta 1990; Barrow & Maeda 1990; Holman, Kolb & Wang 1990; Holman *et al.* 1991a, b).

This still leaves the possibility, suggested by several authors (La & Steinhardt 1989b; La *et al.* 1989; La 1991; Kolb 1991), that there could be enough large bubbles to be interesting for large-scale structure formation while evading any microwave background constraints. Such large bubbles would after all naturally correspond to empty voids of closely spherical shape at the present time. Indeed, their method of formation is somewhat reminiscent both of an explosion scenario and of the Voronoi model utilizing gravitational expansion of underdensities (Yoshuka & Ikeuchi 1989; van der Weigaert & Icke 1989; Coles & Barrow 1990), though of course occurring at much earlier stages. In this paper we aim to assess in more depth than previous authors the constraints on such a scenario imposed by the smoothness of the microwave background in order to ascertain whether or not these large bubbles can indeed produce interesting effects. We point out that some of our analysis of dynamics of bubbles is similar to that made by La (1991), though we would not agree with all of his conclusions.

In Section 2, we consider the void spectrum from the original extended inflation model. The calculation of the spectrum of void sizes at the end of extended inflation is fairly clear-cut, but the subsequent filling and evolution is a much more tricky question to deal with and depends to some extent on the type of matter in the Universe – particularly whether any dark matter is hot or cold. We discuss this in some detail as a safe constraint on extended inflation parameters from the microwave background experiments depends on the degree to which this evolution can smooth the matter distribution.

Section 3 examines the microwave constraints on this spectrum, and Section 4 how these might be altered when one goes to a more complicated but viable model (the simplest model being ruled out by these constraints), and discusses whether a sufficient number density of voids is allowed to be of significance for large-scale structure. Our final concluding section comments on the effect of these results on extended inflation model building.

Before continuing this discussion, it is worth recalling some standard results (Hogan, Kaiser & Rees 1982; Kaiser & Silk 1986). First of all, we shall assume that the post inflation universe is at the critical density $\Omega = 1$, and we shall throughout write the Hubble constant as 100 h km s^{-1} Mpc^{-1}. We shall generally use comoving distances; that is, we refer to a distance at a given time by the size that distance would have at the present epoch if solely subject to stretching by the expansion of the Universe. Distances usually carry a factor of h^{-1} indicating the uncertainty in the overall scale.

By a redshift of about 1300 more than 90 per cent of the baryonic matter in the Universe has recombined into neutral atoms. But there is still some residual ionization and the photons of the cosmic microwave background we observe today come from a later redshift. Their last scattering surface has a finite thickness Δz, well approximated by a Gaussian of width 80 centred upon $z = 1080$ (Jones & Wyse 1985). The comoving thickness of the surface corresponding to that width is 6.7 h^{-1} Mpc. The scales we shall typically deal with allow us to assume the last scattering surface (henceforth lss) is sharp.

A length of 1 h^{-1} Mpc on the last scattering surface subtends an angle of about 0.5 arcmin. The horizon size at this decoupling is about 190 h^{-1} Mpc, subtending about 1°.5. The horizon size when matter comes to dominate the Universe is somewhat smaller, around 20 h^{-2} Mpc (note the different power of h), while the present horizon distance $a(t_{\rm pres}) \int dt/a(t)$ is around 6000 h^{-1} Mpc.

2 THE VOID SPECTRUM AND ITS EVOLUTION

The history of first-order phase transitions in inflationary cosmology goes back to Guth's original paper in 1981. The mechanism by which inflation takes place is that the Universe becomes trapped in a false vacuum state, promoting exponential expansion. This inflationary phase is brought to an end by quantum tunnelling to a true vacuum state, leading to the nucleation of true vacuum bubbles which expand and collide. Unfortunately the model fails, because although bubble nucleation is an exponential process, the exponential expansion of the space generically dominates and the phase transition never completes (Guth & Weinberg 1983). Alternatively, if the bubble nucleation rate is sufficiently high for the transition to complete, insufficient inflation occurs. The relevant parameter is the nucleation rate per Hubble volume per Hubble time $\varepsilon = \Gamma/H^4$, Γ being the nucleation rate per unit volume per unit time which depends only on microphysical parameters. In Guth's model, both numerator and denominator are constant. A sufficiently high ε, greater than some critical value $\varepsilon_{\rm cr}$, completes the phase transition.

Extended inflation (La & Steinhardt 1989a) remedies this state of affairs by examining extensions to general relativity in which the false vacuum dominated solutions are slower than exponential but still fast enough to provide inflation, such as a rapid power law. The quantity ε above now becomes an increasing function of time; it can start low enough to allow sufficient inflation and grow to values which allow the transition to complete. The original extended inflation model was based on Jordan–Brans–Dicke theory, but was found to contradict observational data combining the microwave background and solar system experiments (Weinberg 1989; La *et al.* 1989). Several enhanced models have since appeared, and we shall use the terminology of extended inflation to refer to all such models. One can readily see that by making the bubble nucleation rate Γ an increasing function of time, as in double field inflation (Adams & Freese 1991), one has an alternative method of completing the transition. Our method of constraint can also be applied there, but we do not attempt this here as existing models appear very unnatural.

As just stated, the microwave background has already been used to rule out the original extended inflationary model; however, this was only done by using a rather *ad hoc* criterion that the fraction of the volume of the Universe in bubbles so big as to still be thermalizing at recombination be less than 10^{-F}, where F is perhaps 4. In this paper we examine the microwave limits in much more detail, followed by discussion of the implications for more general extended inflationary models. In particular we consider the possibility that these models can allow enough large voids to be relevant for large-scale structure.

The key uncertainty in evaluating the constraints on the void distribution is in the dynamics of void filling, which determines the size of a void at any given epoch. This depends on physics which is not well understood, and also on the nature of any dark matter present in the Universe. The initial phase of void filling occurs when the bubbles collide, during which the energy trapped as a coherent scalar field in the bubble walls is freed. One expects initially the propagation of coherent scalar waves at the speed of light into the bubble interior, but this energy may rapidly be converted into incoherent particles and radiation through scalar decays, restoring a conventional matter content to the Universe. During the radiation-dominated era, the adiabatic sound speed of baryonic matter is $c/\sqrt{3}$, and although conditions at the edge of the bubble wall are out of thermal equilibrium we can still expect this to be a reasonable representation of the propagation speed into the void.

There also exists the possibility of various instabilities on the surface of the material entering the void, perhaps forming black holes. Of course, these holes will also be travelling into the void relativistically, and may evaporate sufficiently rapidly via the Hawking process that there is no problem, but the entire situation is hard to assess. We shall ignore such possibilities which may in themselves be sufficient to rule out extended inflation.

2.1 Growth of bubbles during inflation

As described above, the era of extended inflation ends at time t_e with the rapid nucleation of small bubbles whose size is at most of the same order as the Hubble length at t_e and these are then rapidly thermalized. In this paper we are primarily interested in the comparatively small number of bubbles nucleated sometime earlier in the inflationary era which have time to grow, swept up in the power-law expansion, to sizes very much greater than the Hubble length $[x \gg H^{-1}(t_{\rm end})]$ at the end of inflation. If they are sufficiently large these voids could survive until the epoch of recombination and may thus distort the microwave background.

The original extended inflation model (La & Steinhardt 1989a) is implemented in a Brans–Dicke theory (Brans & Dicke 1961), with Brans–Dicke parameter ω. The enhanced working models described earlier can often be cast in a form resembling Brans–Dicke theory with a variable ω. Inflation is driven by the false-vacuum energy $\rho_{\rm v}$ of a scalar field trapped in a metastable vacuum state, giving (after an initial quasi-exponential phase) a scale factor increasing as a rapid power law (Mathiazhagan & Johri 1984)

$$a(t) \approx (\mathscr{C}t)^{\omega+1/2} \tag{1}$$

where the constant $\mathscr{C}$ is given by

$$\mathscr{C}^2 = \frac{8\pi\rho_{\rm v}}{3m_{\rm Pl}^2(3+2\omega)(5+6\omega)/12}. \tag{2}$$

During inflation, the Hubble parameter is

$$H(t) \approx \left(\omega+\frac{1}{2}\right)\frac{1}{t}. \tag{3}$$

Ultimately the inflationary era is brought to an end via bubble nucleation. If a bubble nucleates at time t_1 with zero initial radius, it then grows at the speed of light within the expanding Universe to a size x by a time t.

$$x(t>t_1) = a(t)\int_{t_1}^{t}\frac{dt}{a(t)}. \tag{4}$$

Thus at time t its size in relation to the Hubble radius is

$$H(t)\,x = \left[\left(\frac{t}{t_1}\right)^{\omega-1/2} - 1\right] + \mathscr{O}\left(\frac{1}{\omega}\right). \tag{5}$$

One can see how rapidly the bubble can be stretched to superhorizon sizes.

2.2 The bubble spectrum

At time t_1, the bubble nucleation probability per unit comoving volume per unit time is

$$\frac{dN}{dt_1} = \Gamma\, a^3(t_1). \tag{6}$$

We can use equation (5) to change variables from time t_1 to the size x at a fixed subsequent time t, thus giving the range of bubble sizes at that time

$$\frac{dN}{dx} = -\frac{\Gamma\, a^3(t)}{[1+(Hx)]^{4+4/\omega}}. \tag{7}$$

Following the notation of La (1991), we will consider the number of voids $N_{\rm B}$ we expect to find at the end of inflation in a sphere whose radius $l_{\rm H}$ corresponds to the physical size of our present horizon at that time. A convenient dimensionless measure of size is relative to the Hubble length $H_{\rm e}^{-1} \equiv H(t_{\rm e})^{-1}$ when inflation ends at time $t_{\rm e}$. Remember that for any voids of astrophysical interest today $H_{\rm e}x \gg 1$, for example

$$H_{\rm e} l_{\rm H} \approx 4\times 10^{25}\left(\frac{T_{\rm GUT}}{10^{15}\ h\ {\rm GeV}}\right).$$

In terms of this length, we have

$$\frac{dN_{\rm B}}{d(H_{\rm e}x)} = -\frac{4\pi}{3}\,\varepsilon_{\rm e}\,\frac{(H_{\rm e}l_{\rm H})^3}{(H_{\rm e}x)^{4+4/\omega}} \tag{8}$$

where $\varepsilon_{\rm e}$ is the nucleation rate per Hubble volume per Hubble time, $\Gamma/H_{\rm e}^4$, at the end of inflation. During extended inflation, ε increases as $H(t)$ falls, thus favouring increased bubble nucleation at late times. After ε passes a critical value, which is not particularly well known but for which heuristic arguments give the estimate $\varepsilon_{cr} \approx 3/4\pi$, nucleation begins to exponentially outstrip inflation and shortly thereafter inflation comes to an end. We shall assume the above value of ε_{cr} – note that if the true value is lower, the number of bubbles reduces in proportion. As bubble nucleation will actually complete when $\varepsilon \simeq \varepsilon_{cr}$, the spectrum of large bubbles is

$$\frac{dN_B}{d(H_e x)} \simeq -\frac{(H_e l_{11})^3}{(H_e x)^{4+4/\omega}}. \tag{9}$$

Note that this result differs from that quoted by La *et al.* (1989), where they introduce a normalization constant $4/\omega$ so that when this spectrum is integrated over all bubble sizes, these bubbles occupy the whole of space at the end of inflation. We feel that this is not justified as most of the Universe is filled by small bubbles, $H_e x \sim 1$, for which this spectrum is not valid, and that the normalization above is more appropriate within the range of validity of this expression. Thus we are left with a dependence on ε_{cr}. [There is another lesser objection that the time corresponding to the end of inflation is somewhat vaguely defined as some regions may be undergoing thermalization before other regions even complete the phase transition. Thus there is no time when the integrated bubble spectrum is equal exactly to one. This point becomes important only for very small bubble sizes. One may also worry about the neglect of bubble overlap, again not a problem for large bubbles (La 1991).]

One might worry that our integrated volume in bubbles may exceed 1 in some circumstances. If the integration (carried out over regions where the spectrum is valid) does exceed 1, we believe this indicates that the value of ε_{cr} has been taken to be too high, as after all its value is tied to the completion of the phase transition. As ε is not constant in extended inflation, it may be also that the value ε_{cr} calculated for exponential inflation is not quite the quantity we require here, though it should be a good approximation.

By integrating this spectrum, one obtains an expression for the number of voids *larger* than size $H_e x$ at the end of inflation:

$$N_B(>H_e x) \simeq \frac{1}{3+4/\omega}(H_e l_{11})^3 (H_e x)^{-3-4/\omega}. \tag{10}$$

2.3 Void evolution

It is far from trivial to relate this spectrum at the end of inflation to the one we would expect to see at decoupling. Void filling certainly proceeds initially at close to the speed of light, but the important question is when void filling stops. We can identify three possibilities (*cf.* La 1991); an excellent discussion of the evolution of different types of perturbation is given by Efstathiou (1990).

(1) *Void filling proceeds at the adiabatic sound speed always.* In a universe without dark matter (admittedly a scenario usually at odds with the assumption that $\Omega = 1$), the adiabatic sound speed is $c/\sqrt{3}$ during radiation domination, but drops rapidly once matter domination sets in, and finally becomes negligible after recombination. The characteristic scale for void filling under such circumstances is therefore the horizon size at the time t_{eq} of matter–radiation equality, about 20 h^{-2} Mpc (we provide a more accurate calculation in the Appendix). This ties in with the usual statement that this is the time at which matter perturbations can begin to grow.

(2) *Void filling in a universe with cold dark matter.* In a universe with cold dark matter, one of course expects the distribution of the dark matter to initially reflect the bubbly structure. When the cold dark matter is non-relativistic, which may be true immediately or at some intermediate epoch between the end of inflation and t_{eq}, it will effectively cease to flow into the voids. However, because the dark matter is not interacting directly with baryonic matter, this visible matter will continue to fill the voids until t_{eq}. In this case, one can expect greater bubbly structure in the cold dark matter than in the visible matter, which will have a distribution more like that in case (1) above. The uneven distribution of the dark matter will lead to additional Sachs–Wolfe distortions (as discussed below) to the microwave background over and above those of case (1).

(3) *Void filling in a universe with hot dark matter.* In a universe with hot dark matter, the dark matter decouples from normal matter whilst relativistic. How far it travels independently of matter depends primarily on when it becomes non-relativistic, and hence on its mass. For light neutrinos ~ 30 eV, the filling is not dissimilar to case (1), but lighter particles can give greater degrees of filling.

There is also one further aspect of void dynamics of extreme importance in interpreting the void spectrum at decoupling, namely that the voids expand faster than the Hubble flow (Bertschinger 1985, and references therein; Thompson & Vishniac 1987). Once void filling has ceased, gravitation will provide the dominant dynamical effect, and a Newtonian analysis reveals a rate of void expansion in a matter-dominated regime of $t^{4/5}$ for completely empty voids, rather than the $t^{2/3}$ of the background. (Strictly speaking, this is an adiabatic and self-similar solution, but other circumstances give similar results.) As the horizon size at decoupling is much larger than the typical voids of interest even without the faster expansion, this Newtonian result will be an accurate representation. The consequence of this faster expansion is that in order to have a radius of say 25 h^{-1} Mpc today, it need only have a radius of a factor $z^{-1/5} \sim 1/4$ that size at decoupling, thus reducing the area seen on the microwave background by a factor of 16. While unimportant for the size of microwave distortions from a given void spectrum, this factor is crucial in interpreting what our results mean for the observed Universe today.

Before we can impose constraints at decoupling, one must examine the evolution of this void spectrum. As described earlier, we will assume that while the Universe is radiation-dominated, matter and radiation stream into the void, reducing its comoving size by some 'shrinkage factor' $\Delta = \xi L_{eq}$, where $L_{eq} = 20$ h^{-2} Mpc is the comoving horizon size at matter–radiation equality and ξ depends on the matter content. Clearly only voids whose initial comoving size is greater than Δ will survive until t_{eq} but given that they do, and that further filling is negligible, these non-linear underdensities will expand during the matter-dominated era. Their physical size grows as $t^{4/5}$ corresponding to an increase in comoving size proportional to $z^{1/5}$.

In a baryon-dominated universe void filling can continue up until recombination ($z_{rec}=1300$) allowing only slight growth before last scattering ($z_{lss}=1080$) which corresponds to a negligible growth factor

$$g=\left(\frac{z_{rec}}{z_{lss}}\right)^{1/5}=1.03. \tag{11}$$

On the other hand, in the case of a CDM-dominated universe voids increase their comoving size by a factor

$$g=\left(\frac{z_{eq}}{z_{lss}}\right)^{1/5}=1.85\ h^{2/5}. \tag{12}$$

To obtain the spectrum at last scattering, one therefore converts the above result into comoving coordinates and then imposes the void-filling and expansion corrections on to the spectrum.

$$r=g\left(\frac{L_H}{l_H}x-\Delta\right), \tag{13}$$

$$\frac{dN_B}{dr}\simeq-\frac{1}{gL_H}(H_e l_H)^{-4/\omega}\left(\frac{L_H}{(r/g)+\Delta}\right)^{4+(4/\omega)}, \tag{14}$$

writing the current comoving horizon distance as L_H. We obtain the number of bubbles larger than size r as

$$N_B(>r)\simeq\frac{1}{3+(4/\omega)}\left(4.3\times10^{25}\frac{T_{GUT}}{10^{15}\ h\ \mathrm{GeV}}\right)^{-4/\omega}\times\left(\frac{L_H}{(r/g)+\Delta}\right)^{3+(4/\omega)}. \tag{15}$$

This therefore is the spectrum of bubble sizes we wish to constrain. One should be careful to note, however, that as the initial bubble nucleation process is a probabilistic process, some care must be taken in interpreting this spectrum near its tail as a given realization of the bubble spectrum will poorly sample the distribution at large radii. Clearly, then, this spectrum is not particularly useful for individual questions such as 'what is the size of the largest bubble inside our horizon?', but it is appropriate for questions concerning collections of bubbles such as 'above what size of bubble is there a 95 per cent chance of at least one intersecting the last scattering surface?', as statistics regarding collections of bubbles will be good.

With this estimate of the number of bubbles, we can also calculate what fraction of the Universe at last scattering is still filled by voids. This yields

$$f\simeq g^3\kappa(\omega)(H_e l_H)^{-4/\omega}\left(\frac{L_H}{\Delta}\right)^{4/\omega} \tag{16}$$

where

$$\kappa(\omega)=\frac{\omega}{4}\left(\frac{1}{1+(4/\omega)}\right)\left(\frac{2}{2+(4/\omega)}\right)\left(\frac{3}{3+(4/\omega)}\right). \tag{17}$$

For sample values $\omega=20$, $T_{GUT}=10^{15}\ h$ GeV, $g=1$ and $\Delta\sim40\ h^{-1}$ Mpc, we get $f\sim7\times10^{-5}$ at last scattering. Note that this can increase by a factor of up to $(1+z_{lss})^{3/5}=66$ by the present due to void expansion since last scattering. (Void mergers can reduce this number somewhat, but are likely to be rare as the voids are initially separated by at least 2Δ.)

Previous studies have sought to place constraints on the parameters of extended inflation by suggesting that the isotropy of the microwave background requires that at the end of inflation no more than 10^{-4} of the volume of the Universe should be in bubbles greater than the decoupling horizon size. Our discussion above suggests that void filling may substantially reduce the anisotropy such voids create, while smaller voids can contribute to the anisotropy as they need not have thermalized completely by last scattering. Also one must be careful about statistics of voids $>190\ h^{-1}$ Mpc, as these can be rare enough to avoid the lss by chance. Thus here we aim to provide a more concrete assessment of the microwave background limits and find safe constraints.

3 CONSTRAINING EXTENDED INFLATION

Voids on the last scattering surface of course lead to temperature anisotropies in the microwave background. We shall here concentrate on the data from the *COBE* DMR experiment (Smoot *et al.* 1991) which has produced full sky maps with a pixel size of 2°.8, corresponding to a comoving size of about 300 h^{-1} Mpc on the last scattering surface. Because we are interested in rare features, sky coverage is the most important required feature of experimental data. Davies *et al.* (1991) offer similar sensitivities, but only about 10 per cent sky coverage. Other microwave-background experiments have much smaller resolutions (e.g. Readhead *et al.* 1989), but these are generally restricted to studying one small patch of sky; for our purposes there is a trade-off between pixel size (determining the void size that could be detected) and sky coverage (which limits the number density at that size). Normally we shall demand a 95 per cent confidence that an observable void intersects the covered region. We find that the *COBE* experiment gives the tightest constraint.

It is important to have a picture of what we expect to see on the lss. If a void has size greater than the horizon size at last scattering, 190 h^{-1} Mpc, then no light has had time to cross it and so it will appear dark. This leads directly to a $\delta T/T$ of -1. (Though the slightly hotter shell will lead to an enhanced photon flux from the regions neighbouring the shell, it is easily shown that the void contribution dominates.) However, this anisotropy may be on a scale much less than the *COBE* beam width, so the observed $\delta T/T$ involves a weighting by the area of the void to that of the beam. One could examine this in more detail, but it turns out that because one can have many voids with a reasonable probability that none this large intersects the lss, the constraint one obtains is weaker than that from smaller voids.

If the void is smaller than 190 h^{-1} Mpc, then all lines of sight through the void intersect the incoming far wall, as shown in Fig. 1. Ignoring for the moment a Doppler contribution from the inrushing wall, one can see photons emitted from the far wall (those generated at the edge see few electrons and so are not constrained by the adiabatic sound speed). As the walls may not be substantially hotter than the background temperature, these actually redshift down to approximately the same temperature as the background universe, and hence the anisotropy induced in this way may

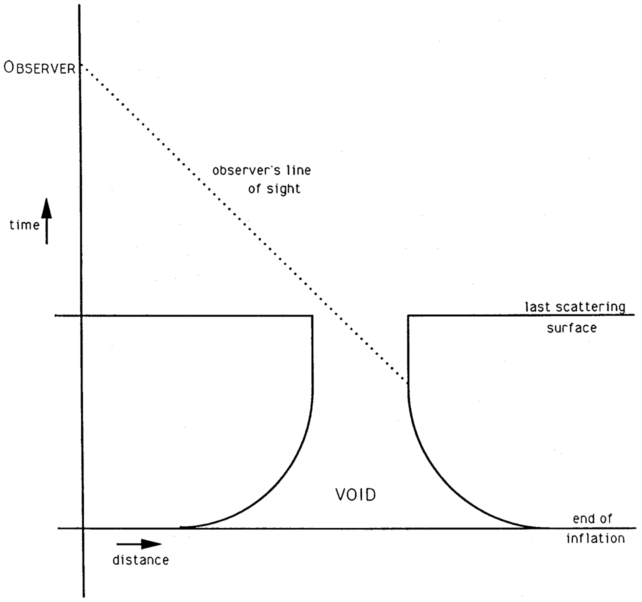

Figure 1. A schematic space–time diagram showing void evolution up to last scattering in a baryon- or HDM-dominated universe. The void shown had a comoving size at the end of inflation less than the decoupling horizon size, and although the void itself is empty an observer at the present time will see the plasma on the far side of the bubble. The higher temperature of this far wall seen at an earlier time can be compensated by the greater redshift. The line of sight need not intersect the far wall only when the void was initially larger than the horizon size at decoupling.

actually be quite small.* As the details are very hard to handle, we shall assume that these direct anisotropies may not be the dominant effect; of course, if they are then our constraints would be strengthened.

A safe estimate of the distortion caused by the voids comes from anisotropies in the gravitational potential, Φ (Sachs & Wolfe 1967). Photons coming from the regions of differing gravitational potential suffer a different red(blue)shift before reaching us and exhibit a temperature anisotropy

$$\frac{\delta T}{T} = -\frac{\Phi}{3} \simeq -\frac{1}{3}\left(\frac{\delta\rho}{\rho}\right)\left(\frac{r}{L_{\rm lss}}\right)^2, \qquad (18)$$

where $\delta\rho/\rho = -1$ for voids ($L_{\rm lss}$ being the comoving horizon size at last scattering). The full Sachs–Wolfe equation also contains terms due to Doppler shifts from motions on the last scattering surface; an unusual feature of this scenario is that the Doppler term has the same sign as the gravitational potential term and so cannot counter it. It could be large on lines of sight which intersect the far wall during rapid filling, but the bulk of lines of sight intersect once the wall has slowed down; consequently we shall also neglect this term.

The observed temperature deviation in a beam whose radius $r_{\rm b}$ is much larger than the void is reduced by a factor $(r/r_{\rm b})^2$, and the large pixel size used by *COBE* necessarily limits us to constraining only larger voids. As a result we will restrict ourselves to estimates of the anisotropy due only to the gravitational potential. Smoot *et al.* (1991) report pixel variations in $\delta T/T \lesssim 10^{-4}$. With $r_{\rm b} = 147\ h^{-1}$ Mpc this means they could detect any void with a radius greater than $r_{\rm v} = 22\ h^{-1}$ Mpc at decoupling. In practice, the constraints we obtain depend only weakly on the minimum resolvable radius.

One must also take into account the probability, given voids of this size, that one would intersect the lss. The Sachs–Wolfe term becomes small once the void centre is greater than half the void radius from the lss, as the sectioned area of the void becomes smaller. Hence the comoving volume within which we would get an observable effect if we placed a void centre is just $4\pi L_{\rm H}^2 r_{\rm v}$. This is to be compared with a horizon volume of $(4\pi/3)L_{\rm H}^3$, so the ratio of volumes is just $r_{\rm v}/2000\ h^{-1}$ Mpc. As no void is observed, we demand that the number $N_{\rm B}$ of voids inside the horizon is less than that which gives a 95 per cent confidence level that at least one is on the lss; that is

$$\left(1 - \frac{r_{\rm v}}{2000\ h^{-1}\ {\rm Mpc}}\right)^{N_{\rm B}} > 0.05 \qquad (19)$$

or

$$N_{\rm B} < \frac{\ln 0.05}{\ln(1 - r_{\rm v}/2000\ h^{-1}\ {\rm Mpc})} \sim \frac{6000\ h^{-1}\ {\rm Mpc}}{r_{\rm v}}. \qquad (20)$$

The constraint on the filling fraction for voids of this size is simply

$$f < \left(\frac{r_{\rm v}}{6000\ h^{-1}\ {\rm Mpc}}\right)^2. \qquad (21)$$

Note that the allowed filling fraction increases with void size; this simply reflects the increasing possibility that large voids may by chance avoid intersecting the lss. If we require only a confidence level of 67 per cent the filling fraction falls by a further factor 3.

For $r_{\rm v} = 22\ h^{-1}$ Mpc, we obtain $N_{\rm B} < 273$ at the 95 per cent confidence level, implying a filling fraction of such voids at recombination equal to

$$f < 1.4 \times 10^{-5}. \qquad (22)$$

Note that this implies that the chance of finding such a void (which will have grown in size up to 88 h^{-1} Mpc by the present) in a survey such as CfA or QDOT, which have radii up to 150 h^{-1} Mpc, is less than 1 per cent.

Clearly this is an entirely model-independent constraint on voids of a given size present at last scattering. Because extended inflation predicts a particular spectrum of voids we can produce an integrated anisotropy constraint on voids down to a size $r_{\rm v}$ which at the 95 per cent confidence limit yields

$$\frac{1}{3}\int_{r_{\rm v}}^{\infty} \frac{4\pi L_{\rm H}^2 r}{(4\pi/3)L_{\rm H}^3}\left(\frac{dN_{\rm B}}{dr}\right) dr < 1, \qquad (23)$$

which can be integrated to give

$$\frac{1}{3 + (4/\omega)}(H_{\rm e} l_{\rm H})^{-4/\omega}\left(\frac{r_{\rm v}}{(r_{\rm v}/g) + \Delta} + \frac{g}{2 + (4/\omega)}\right) \times \left(\frac{L_{\rm H}}{(r_{\rm v}/g) + \Delta}\right)^{2 + (4/\omega)} < 1, \qquad (24)$$

using equation (14) and $H_{\rm e} l_{\rm H} = 4.3 \times 10^{25}(T_{\rm GUT}/10^{15}\ h\ {\rm GeV})$.

* We thank Bernard Jones for convincing us of this.

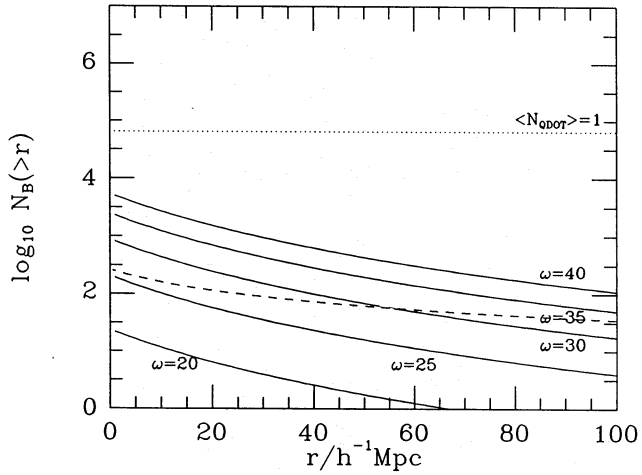

Figure 2. Plot of spectrum of voids surviving at last scattering in a baryon-dominated universe assuming void shrinkage length $\Delta=40$ h^{-1} Mpc, for various values of ω. The dashed line shows the microwave background constraint that can be applied if we can resolve voids on the last scattering surface down to a minimum radius r_v. The dotted line shows the number of voids required so that the expected number within a survey of the size of the QDOT survey is at least one.

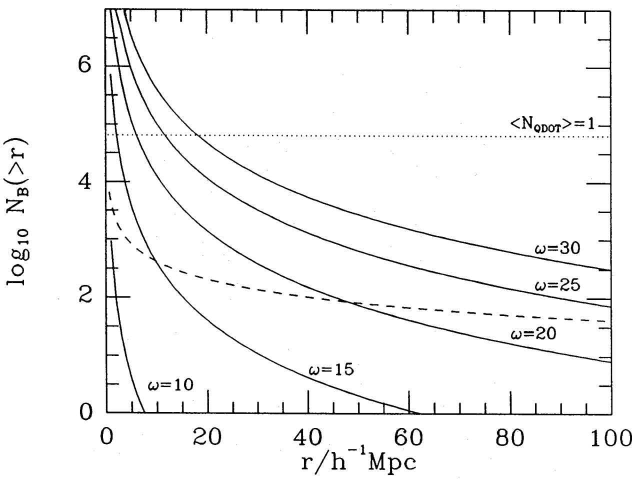

Figure 3. As Fig. 2, plotting the spectrum of voids at last scattering for various values of ω in a CDM-dominated model in the limiting case that $\Delta \to 0$. This is just the original void spectrum at the end of inflation with the void sizes given in terms of their comoving radius at last scattering.

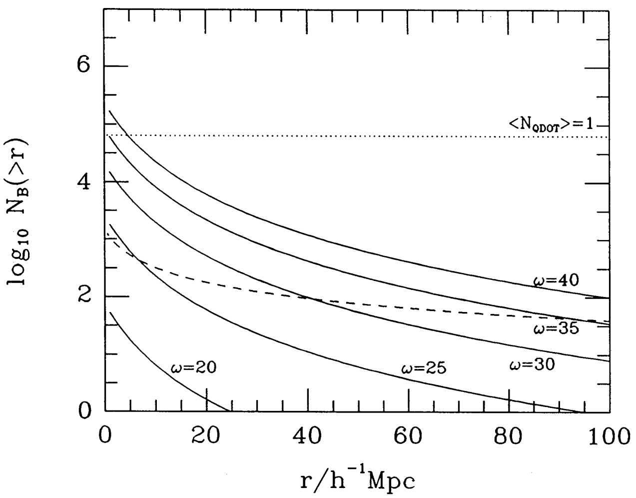

Figure 4. As Fig. 2, plotting the spectrum of voids at last scattering for various values of ω in a HDM-dominated universe with a void shrinkage length $\Delta=9$ h^{-1} Mpc.

Figs 2, 3 and 4 show this constraint plotted as a function of the smallest resolvable void radius for the baryon-, CDM- and HDM-dominated models alongside the spectra of voids predicted by extended inflation models. The key point is that when the extended inflation spectrum becomes fairly flat, the tight constraints on large voids lead to strong upper limits on the number of smaller voids, which are not in themselves observable and may grow to interesting sizes by the present.

We now discuss the constraints for the different choices of matter content. These are summarized in the Conclusions.

3.1 Extended inflation in a baryon-dominated universe

Before examining the case of a post-inflationary universe dominated by baryonic matter ($\Omega_b=1$) it is first worth remarking that this model, which is usually considered to be ruled out by constraints from nucleosynthesis, might be permissible in a universe with a large number of voids. Such an inhomogeneous distribution of matter at the time of nucleosynthesis would considerably alter the calculations of primordial nuclear abundances and this will itself provide stringent limits on the allowed void spectrum. We will not investigate such constraints here other than to point out that an $\Omega_b=1$ model is not automatically ruled out, though the bulk of this matter would not be luminous. Our real motivation for studying this case is as a reference point between the choices of dark matter.

In order to apply the above constraint to extended inflation models we must estimate some of the quantities involved. It has already been shown that before recombination in a baryon-dominated universe voids grow by a factor $g=1.03$. In the Appendix we show that the shrinkage length in such a universe is slightly larger than the horizon size at equality, with $\Delta=(47-37)$ h^{-1} Mpc over the range $h=(0.5-1.0)$. Using these values and taking $T_{GUT}=10^{15}$ h GeV, Fig. 5 shows how we can constrain ω as a function of the minimum detectable void radius r_v. Because the void spectrum is quite flat the constraint on ω varies remarkably little with r_v. With $r_v=22$ h^{-1} Mpc and assuming $T_{GUT}=10^{15}$ h GeV we require $\omega<29$.

Fig. 6 shows how this constraint can be relaxed somewhat for $T_{GUT}>10^{15}$ h GeV. The relationship between ω_{max} and $\ln(T_{GUT})$ is very nearly linear.

$$\omega \le 28+0.5 \ln(T_{GUT}/10^{15}\, h\, \mathrm{GeV}) \tag{25}$$

from equation (24) assuming $r_v=22$ h^{-1} Mpc and $\Delta=40$ h^{-1} Mpc.

In this case one can actually obtain a stronger constraint from voids within the lss, as discussed later in this section. This limit gives $\omega<24$ for our canonical value of T_{GUT}. With this value of ω and $h=1.0$ this predicts only about 170 voids of *any* size within our present horizon. The probability of seeing even one extended-inflation void within a QDOT-type survey is less than 0.3 per cent. Thus we can rule out

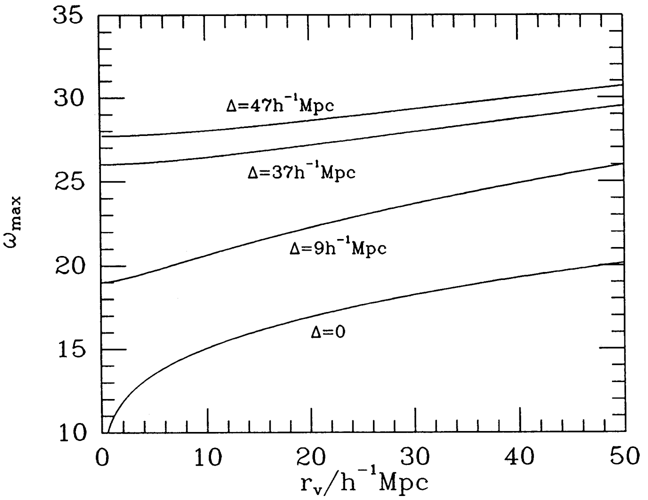

Figure 5. Plot of the maximum possible value of ω against the smallest void radius that is observable on the last scattering surface in all-sky surveys of the microwave background for various different void shrinkage lengths (assuming $T_{\rm GUT}=10^{15}$ GeV). $\Delta=37$–$47\ h^{-1}$ Mpc is the range of shrinkage lengths possible in a baryon-dominated model, $\Delta=9\ h^{-1}$ Mpc is typical of HDM, and $\Delta=0$ represents the constraint in a CDM-dominated universe. We estimate $r_{\rm v}=22\ h^{-1}$ Mpc from the *COBE* experiment.

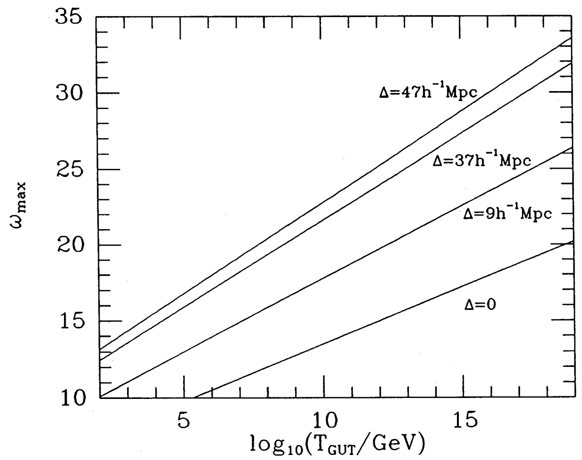

Figure 6. Plot of the maximum possible value of ω against the temperature of the grand unified phase transition that drives inflation for the void shrinkage lengths used in Fig. 5. To derive this constraint we have taken the smallest observable void radius on the last scattering surface as $r_{\rm v}=22\ h^{-1}$ Mpc.

extended inflation voids as being able to produce any observable large-scale 'bubbly' structure in a baryon-dominated model.

3.2 Extended inflation in a CDM-dominated universe

When we talk about voids in this case we are referring to non-linear underdensities in the dominant gravitating matter – invisible dark matter which decoupled from baryonic matter with non-relativistic velocities long before $t_{\rm eq}$. The void shrinkage length here is much smaller than in the baryon-dominated case but the baryons themselves can continue streaming into the voids right up until recombination. Thus the baryons will be much more evenly distributed at recombination than the underlying dark matter.

Because the void shrinkage length Δ is considerably smaller than in the baryon-dominated case, many more voids survive until $t_{\rm eq}$ and they can then grow by a factor $g\simeq 1.85$ before last scattering. Thus the constraints on ω from the microwave-background isotropy become much stronger (see Fig. 5). The constraint on ω with respect to $T_{\rm GUT}$, as in the baryon-dominated case, is almost linear.

$$\omega \lesssim 17+0.3\ln(T_{\rm GUT}/10^{15}\ h\,{\rm GeV}) \qquad (26)$$

for $r_{\rm v}=22\ h^{-1}$ Mpc. In the limit $\Delta\to 0$ we require $\omega<18$ for $r_{\rm v}=22\ h^{-1}$ Mpc (taking $T_{\rm GUT}=10^{15}\ h$ GeV) and for smaller $r_{\rm v}$ the constraint can become much tighter. On the other hand, if $\omega=18$ there may still be a large number of much smaller voids within the present horizon if Δ is very small. But even in the limiting case where $\Delta=0$, we would expect to see only ≈ 15 bubbles whose present radius is greater than about $5\ h^{-1}$ Mpc in a complete survey out to $150\ h^{-1}$ Mpc (though the number of voids greater than some size r diverges for small r, $N_{\rm B}\sim r^{-3}$). In this case the filling fraction of voids greater than $5\ h^{-1}$ Mpc radius today is still less than 3 per cent, which is substantially greater than that in the other cases but still nowhere near the observed void filling fraction.

3.3 Extended inflation in a HDM-dominated universe

The natural alternative to a universe dominated by cold dark matter is one ruled by invisible matter which is still relativistic when it decouples from visible matter. The most likely candidate for hot dark matter (HDM) is one or more species of light neutrino ($5.5\ h^2$ eV$<m_\nu<1$ MeV). These will decouple from baryonic matter when $T\simeq 1$ MeV and remain relativistic until $T_\nu \lesssim m_\nu$. Assuming all three generations of neutrinos have similar masses, a neutrino mass $m_\nu\simeq 30$ eV would provide the density needed to close the Universe ($\Omega_\nu\simeq 1$). The free-streaming length by last scattering is then $\Delta\simeq 18\ (30\ {\rm eV}/m_\nu)$ Mpc. Notice that this distance carries no power of h; if $h=0.5$ for instance the shrinkage length to compare with earlier examples is $9\ h^{-1}\ (30\ {\rm eV}/m_\nu)$ Mpc. This gives an intermediate case between CDM- and baryon-dominated shrinkage lengths. Because filling continues right up until last scattering (and beyond) there is no opportunity for the voids to grow before the microwave background photons decouple and so $g=1$.

For $m_\nu=30$ eV we can constrain $\omega<(23$–$25)$ for $h=(0.5$–$1.0)$. Even taking $\omega=25$ and $h=0.5$ we would not expect to see any voids of any size within a QDOT-type survey. If the HDM is lighter than 30 eV the void shrinkage length increases and, although this slightly weakens the constraint on ω, the number of voids within our horizon rapidly decreases so we can rule out any significant bubble structure due to extended inflation voids in a universe dominated by hot dark matter particles lighter than 30 eV. If the particles are heavier than 30 eV then the predictions tend towards those of a CDM-dominated universe and while constraints on ω become tighter, the number of voids surviving until the present becomes larger.

3.4 Voids within the last scattering surface

Up until now, we have concentrated our discussion on voids straddling the lss. There is, however, an additional effect, discussed previously by Thompson & Vishniac (1987) caused by the faster-than-average expansion of voids relative to the background. A photon from the lss traversing such a void will experience a change in gravitational potential between falling into the void and climbing back out, leading to a redshift over and above that caused by the cosmological expansion (Rees & Sciama 1968). Thompson & Vishniac constructed a phenomenological model with voids typical of the CfA scale, and showed that even if we have enough voids to match the observed Universe, this effect remains well below observational limits, including those made since their paper. However, the anisotropy grows as the cube of the size of the void, so the very largest voids from the spectrum may significantly distort photons that pass through them, and thus this effect merits some discussion here. It is also interesting to note that the assumptions of spherical symmetry and completely empty voids, which were worrying in their original phenomenological approach, are very well justified in this instance.

The anisotropy generated by a void at redshift z with comoving size r *at decoupling*, assumed to expand as $t^{4/5}$, is given by

$$\frac{\delta T}{T} \simeq \left(\frac{r}{L_{\rm lss}}\right)^3 \left(\frac{1+z}{1+z_{\rm lss}}\right)^{9/10} \left(\frac{16}{5} - \frac{16}{3}\cos^2\psi_0\right) \quad \cos\psi_0, \qquad (27)$$

where ψ_0 is an angular impact parameter (Thompson & Vishniac 1987). A feature of this type of distortion is that when one averages over all values of the impact parameter of the photon on the void, the anisotropy vanishes, and hence such a distortion can never be seen if the void is entirely contained within a single pixel of the observations, or indeed even if any angularly symmetric portion of the void (such as half or a quarter) was in a single $2\overset{\circ}{.}8$ pixel. Thus voids constrained in this way should be sufficiently large that they both contribute a $\delta T/T > 10^{-4}$ and occupy an angular scale of, say, twice the typical pixel size. The former constraint requires voids of size greater than 70 h^{-1} Mpc in order to be visible at low redshifts (which dominate the volume inside the horizon). The latter requires that they be closer than a redshift of 7. Imposing a 95 per cent confidence level of having a void both this large and this close leads to a constraint of $\omega < 24$ in the baryon-dominated case, rather stronger than that given above. In cases with dark matter this constraint is similar to that obtained before, these cases already containing a greater suppression of larger voids. For hot dark matter, we estimate $\omega < 22$ and for cold dark matter $\omega < 18$. However, we would emphasize that although giving similar results this calculation is a lot more dependent on the details of void evolution than the more robust constraints from the Sachs–Wolfe effect, and hence we have greater faith in the limits from that calculation.

4 MORE GENERAL EXTENDED INFLATION MODELS

The extended inflation model based on the pure Brans–Dicke theory is unable to reconcile the smoothness of the microwave background with measurements of the rate of variation of the gravitational 'constant'. Solar-system bounds give $\omega > 500$. Various models have been proposed in order to circumvent this problem, and these can normally be cast in a form similar to Brans–Dicke theory with enhancements to the action. Strategies for evading the ω constraint fall into two general classes.

4.1 Class 1: disguising the value of ω

In the first such class, ω is a constant, but one arranges that the consequences of a low ω are not observable today. The archetypal method of carrying this out is to introduce a potential for the Brans–Dicke field Φ, a so-called induced-gravity model (Accetta & Trester 1989; La *et al.* 1989). During extended inflation, Φ is displaced from the minimum of its potential, and behaves like a free field, giving conventional extended inflation. After inflation the field rolls into the minimum of the potential, which is located so as to reproduce the present gravitational constant. The potential ensures that at late times the gravitational 'constant' *is* constant, thus reproducing observations despite the low ω.

In such a model, the constraint on ω is exactly that we found in the previous section; the void spectrum predictions of this model are indistinguishable from the original model. (This conclusion could be altered somewhat if at some stage during inflation the potential for Φ begins to have a significant dynamical effect on Φ.) By introducing a potential, the present value of the gravitational constant is fixed microphysically by the location of the potential minimum, rather than dynamically as in a pure Brans–Dicke theory, but whether one prefers the gravitational constant to be determined microphysically or dynamically seems a matter of personal taste. A more serious criticism against this model is that in order for the potential energy of the Φ field not to interfere with the inflation (by dominating the potential energy of the inflation in its metastable vacuum), the potential must be made very flat, which involves the introduction of a small parameter, perhaps of order 10^{-16} (Adams, Freese & Guth 1991). This detracts from the hope that extended inflation may avoid the fine-tuning problems commonly present in other types of inflationary model.

Another model of this class is that of Holman *et al.* (1990). Actually, the principle is rather the converse – they make a large ω look like a small one during inflation, as opposed to the above which made a small ω look large today. They investigated a model in which the Brans–Dicke coupling to the inflation need not match that to visible matter, with a ratio $\beta = \beta_{\rm I}/\beta_{\rm V}$. The effect is to alter the constraint on ω from the microwave background to one on ω/β^2; thus a sufficiently large β salvages the microwave background even for $\omega > 500$. In this case, the bubble spectrum predictions are essentially those of Section 3 where ω is replaced by ω/β^2, and so the limits on how many voids can be produced remain the same.

Holman *et al.* (1991b) have implemented extended inflation in the context of a scale-invariant theory. One can pick out an effective value of ω, but such a term does not last until the present so one can choose ω as one wishes during extended inflation, again reproducing the type of bubble spectrum constrained by this paper. They do require recourse to physics at the Planck scale, however. This scenario can lead to alternative consequences in that the possibility exists for a period of slow-rolling inflation after bubble nucleation brings extended inflation to an end. Such a subsequent expansion will of course greatly enhance the number of large bubbles, and one can expect much stronger constraints on ω if a significant amount of slow-rolling inflation occurs. We shall not, however, investigate this further here.

4.2 Class 2: making ω variable

The second means by which the ω problem can be circumvented is by enhancing the action to the extent that ω is no longer constant. Such a model is provided by hyperextended inflation (Barrow & Maeda 1990; Steinhardt & Accetta 1990), where the Lagrangian is taken to be

$$\mathscr{L} = -f(\phi)\,\mathscr{R} + \tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + 16\pi\mathscr{L}_{\text{matter}} \tag{28}$$

where ϕ is a scalar field coupling to gravity. This can be written in Brans–Dicke form via the field redefinition $\Phi = f(\phi)$ to give

$$\mathscr{L} = -\Phi\mathscr{R} + \frac{\omega(\Phi)}{\Phi}\,\partial_\mu\Phi\,\partial^\mu\Phi + 16\pi\mathscr{L}_{\text{matter}} \tag{29}$$

where $\omega(\Phi) = f/(2f')^2$, $f' = df/d\phi$. This is just a Brans–Dicke theory where ω is now a function of Φ.

While the two forms of the action above are of course equivalent, they lead to rather different scenarios on philosophical grounds. Barrow & Maeda (1990) took equation (29) as their basic Lagrangian, and assumed $\omega(\Phi)$ to be given by a truncated Taylor series $\omega(\Phi) = \omega_m\Phi^m + \omega_0$ for some m. On the other hand, Steinhardt & Accetta (1990) took equation (28) as theirs, writing $f(\phi)$ as a truncated Taylor series. Once this Taylor series has more than two terms, one cannot even in general write $\omega(\Phi)$ in closed form, as one requires $\phi = f^{-1}(\Phi)$ and f may not be analytically invertible. Hence despite the equivalence of the theories, one's opinion as to which of $\omega(\Phi)$ and $f(\phi)$ should have a simple form leads to substantially different modelling.

Nevertheless, the strategy here is clear – let ω be small enough during inflation to satisfy the microwave background constraint and let it grow subsequently such as to evade the lower bound on ω today. In fact, in Steinhardt & Accetta's version the dynamics is rather more complicated, with ω starting small, increasing and then decreasing to below 1/2 to bring inflation to an end; finally it is forced large in the post-inflationary era. For a viable scenario, they arrange that bubbles can be produced in significant numbers only in the last of the above stages and they predict a bubble spectrum somewhat different from the extended inflation one we have discussed previously.

With a variable ω one must be more careful in deciding when to apply our constraints. The voids from which the constraint arises have initial comoving sizes of about $(20\ h^{-1} + \Delta)$ Mpc. Such bubbles nucleate considerably before the end of inflation – there are $\sim 54 + \ln(T_{\text{GUT}}/10^{15}\ h$ GeV) e-foldings of inflation subsequently. Without a detailed calculation, it seems reasonable to suppose that one need only ensure that ω obeys the constraint at this time and one has considerable leeway in letting ω grow during the rest of inflation. In the Barrow & Maeda model, where ω increases monotonically, these considerations are conceptually simple. The Steinhardt & Accetta case is substantially trickier because of its relatively complicated dynamics, and we reserve a full discussion of the constraints on these models for a future publication.

5 CONCLUSIONS AND DISCUSSION

We have used results from the *COBE* all-sky survey of the microwave background to produce a definite limit on the maximum number density of large voids present at last scattering. Our bound, that there are no voids on the last scattering surface whose comoving radii exceed 22 h^{-1} Mpc, is almost certainly a rather conservative limit but we have used this as it is determined solely by the peculiar gravitational potential of the void and should be largely independent of the detailed dynamics of the voids and their environment at decoupling. Using this constraint we have deduced somewhat tighter bounds on ω than previously quoted for a universe dominated by dark matter. In particular, if the Universe is dominated by cold dark matter which has a negligible free-streaming length before matter–radiation equality, we constrain $\omega < 18$. In addition, voids within the lss can be seen if their size is greater than about 70 h^{-1} Mpc and they are within a redshift of about 7.

Previous studies have suggested $\omega \leq 25$ by requiring that less than 10^{-4} of the volume of the Universe is filled by voids whose comoving size at the end of inflation was greater than the horizon size at decoupling ($L_{\text{lss}} = 190\ h^{-1}$ Mpc). We have shown that bubbles much smaller than this could be detected on the last scattering surface. Because the void shrinkage length can be very small, a large number of voids would survive until last scattering, but the void spectrum is dominated by small voids ($r < 1$ kpc). The filling fraction of much larger bubbles ($r \geq 1\ h^{-1}$ Mpc), even allowing for maximum growth up until the present (unhindered by void mergers), is limited to less than a few per cent. Therefore extended inflation voids alone in a CDM universe cannot produce the very large bubbly structures observed in maps like the CfA survey. If voids are important for structure formation it must be on very small scales.

In a universe dominated by hot dark matter or baryons, our constraints on ω are weaker as the comoving sizes of all bubbles are reduced by the void shrinkage length at last scattering, Δ. In an $\Omega_b = 1$ model we can only constrain $\omega < 29$ from the Sachs–Wolfe effect, but there is a stronger constraint $\omega < 24$ from voids within the lss. Saturating this constraint gives a filling fraction in voids at last scattering of $f \sim 5 \times 10^{-4}$. But this filling fraction is dominated by very rare, very large voids (present radius $\geq 100\ h^{-1}$ Mpc) and the chances of seeing a void of any size within a local (150 h^{-1} Mpc) survey is less than 0.3 per cent. We also note that the constraints made by Weinberg (1989) and La *et al.* (1989) are based on the large void tail of the spectrum, at sizes greater than 190 h^{-1} Mpc, where the number density of such voids is of order one per horizon volume, and their limits can never be made statistically good.

We also discussed the relevance of our constraints for various specific models designed to evade the ω-problem. Recalling that all our constraints derive from the bubble distribution, our conclusion that insufficient number densities of voids arise to be astrophysically interesting always holds, even if some way of avoiding the current limits on ω is devised. Ultimately, the reason why extended inflation voids are proving unsatisfactory in this respect is because they are completely empty. It is much more efficient to grow a void from a much smaller level of underdensity such as $\delta\rho/\rho \sim 10^{-n}$ for some n, which can become deeper and lead to a large empty region between last scattering and the present while causing a much lower anisotropy than a true vacuum void designed to create the same present structure.

We note that our constraints could be substantially altered if the Universe undergoes significant re-ionization due to early formation of non-linear structure, which could shift the last scattering surface to a later redshift of say 50. They would also be strongly influenced if the critical value of the nucleation rate ε_{cr} which completes the phase transition is substantially different from $3/4\pi$. We have not considered the possibility that the dark matter may reside in a cosmological constant.

It is worth briefly mentioning other means by which voids can affect the microwave background. The inhomogeneity of the matter distribution will lead to a spectrum which is not a pure blackbody in regions of the sky where voids are present. However, any voids satisfying the anisotropy constraint will not give rise to visible spectral distortions. There is a further potential source of spectral distortion in the dissipation of energy during void filling. Sunyaev & Zel'dovich (1970) showed that if energy is fed into the microwave background within a certain redshift interval before recombination, electromagnetic interactions are too slow to allow thermalization to take place before last scattering. Such considerations have been used to constrain certain types of adiabatic density perturbations (Barrow & Coles 1991; Daly 1991) using data from the *COBE* FIRAS experiment (Mather *et al.* 1990). Voids which are of such a size as to complete their filling during this redshift window are presumably the dominant contributors to this effect. We shall not consider this effect any further here, beyond commenting that if the void spectrum satisfies the bounds we have already given we would expect this effect to be small.

Final note. As we were completing this paper, we received a preprint from Goldwirth & Zaglauer (1991), which discusses the growth of bubbles during extended inflation in detail. Their treatment suggests that if the inflation scale is near the Planck mass, then bubbles which nucleate within the first few e-foldings of inflation may experience less growth than one might expect from the usual arguments. Should only just enough inflation to solve the horizon and flatness problems have occurred, then this could affect bubbles of the size we have constrained, weakening our constraints on ω without having much effect on the conclusion that the voids are astrophysically unimportant. In the more generic case where the amount of inflation is not tuned to just solve these problems by today, this effect is unimportant.

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APPENDIX: VOID FILLING AT THE ADIABATIC SOUND SPEED

We will split this integral between the radiation-dominated as the plasma shock front propagates into the true vacuum at the adiabatic sound speed

$$v_s = \frac{c}{\sqrt{3}} \left(\frac{1}{1+(3\rho_B/4\rho_{rad})} \right)^{1/2} \tag{A1}$$

from the end of inflation right up until recombination at $z_{rec} = 1300$. The void shrinkage length in present comoving coordinates is then

$$\Delta = a_0 \int_0^{t_{rec}} \frac{v_s\, dt}{a(t)}. \tag{A2}$$

We will split this integral between the radiation-dominated era

$$\frac{\rho_b}{\rho_{rad}} = \frac{a(t)}{a_{eq}} = \left(\frac{t}{t_{eq}}\right)^{1/2} \qquad t < t_{eq} \tag{A3}$$

and the matter-dominated era

$$\frac{\rho_B}{\rho_{rad}} = \frac{a(t)}{a_{eq}} = \left(\frac{t}{t_{eq}}\right)^{2/3} \qquad t > t_{eq} \tag{A4}$$

and introduce the dimensionless variable $\theta = t/t_{eq}$.

$$\Delta = L_{eq}(\xi_{rad} + \xi_m) \tag{A5}$$

where

$L_{eq} = (a_0/a_{eq})\, 2ct_{eq}$, $\theta_{rec} = t_{rec}/t_{eq} = 100\, h^3$ and

$$\xi_{rad} = \frac{1}{2\sqrt{3}} \int_0^1 \theta^{-1/2} \left(1 + \frac{3\theta^{1/2}}{4}\right)^{-1/2} d\theta \tag{A6}$$

$$= \frac{8}{3\sqrt{3}} \left(\sqrt{\frac{7}{4}} - 1\right) \sim 0.50, \tag{A7}$$

$$\xi_m = \frac{1}{2\sqrt{3}} \int_1^{\theta_{rec}} \theta^{-2/3} \left(1 + \frac{3\theta^{2/3}}{4}\right)^{-1/2} d\theta \tag{A8}$$

$$= \log \left(\frac{\theta_{rec}^{1/3} + \sqrt{(4/3) + \theta_{rec}^{2/3}}}{1 + \sqrt{(4/3) + 1}} \right) \tag{A9}$$

$$= (0.66 - 1.32) \text{ for } h = (0.5\text{–}1.0). \tag{A10}$$

Thus in a baryon-dominated universe the void shrinkage length is $\Delta = (47\text{–}37)\, h^{-1}$ Mpc over the range $h = (0.5\text{–}1.0)$.

In a post-inflationary universe ($\Omega = 1$) dominated by dark matter, so $\Omega_b \simeq 0.01$, the adiabatic sound speed of the baryonic plasma can be much greater at late times ($t \geq t_{eq}$) and the shrinkage length for visible matter is increased to $\Delta_b = 70$.