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Abstract

We consider a heterogeneous network (HetNet) of base stations (BSs) connected via a backhaul network of routers and wired/wireless links with limited capacity. The optimal provision of such networks requires proper resource allocation across the radio access links in conjunction with appropriate traffic engineering within the backhaul network. In this paper we propose an efficient algorithm for joint resource allocation across the wireless links and the flow control within the backhaul network. The proposed algorithm, which maximizes the minimum rate among all the users and/or flows, is based on a decomposition approach that leverages both the Alternating Direction Method of Multipliers (ADMM) and the weighted-MMSE (WMMSE) algorithm. We show that this algorithm is easily parallelizable and converges globally to a stationary solution of the joint optimization problem. The proposed algorithm can also be extended to deal with per-flow quality of service constraint, or to networks with multi-antenna nodes.

Index Terms

Heterogeneous Networks, ADMM Algorithm, Software Defined Networking, Cross-layer Optimization, Small Cell, Limited Backhaul

1. INTRODUCTION

With the advent of cloud computing technologies and the mass deployment of low power base stations (BSs), the cellular radio access networks (RAN) has undergone a major structural change. The traditional high powered single-hop access mode between a serving BS and its users is being replaced by a mesh network consisting of a large number of wireless access

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points connected by either wireline or wireless backhaul links as well as network routers [2]. New concepts such as heterogeneous network (HetNet) or software defined air interface that capture these changes have been proposed and studied recently (see [3], [4] and references therein). Such cloud-based, software defined RAN (SD-RAN) architecture has been envisioned as a future 5G standard, and is expected to achieve 1000x performance improvement over the current 4G technology within the next ten years [4].

The success of the software defined radio access networks will depend critically on our ability to jointly provision the backhaul traffic and mitigate interference in the air interface. In recent years, interference management has been a major focus of the wireless communication research [5], [6]. For instance, various downlink interference management techniques have been developed under the assumption that the wireless user data can be routed to the transmitting BSs without any cost to the backhaul network. Unfortunately, such idealized assumption is only reasonable for traditional networks with a small number of networked BSs for which traffic engineering is straightforward. In the next generation RAN, there will be a large number of BSs, many of which may be connected to the core network without carrier-grade backhaul, e.g., WIFI access points with digital subscriber line (DSL) connections. The increased heterogeneity, network size and backhaul constraints make interference management for future cloud based RANs a challenging task.

As a multi-commodity flow problem, backhaul traffic engineering involves multi-hop routing from the source nodes (e.g., the cloud centers with backhaul connection) to the destination nodes (e.g., the users requesting content). The resulting optimal solution must guarantee the requested quality of service (QoS) for each end-to-end flow (or commodities in the terminology of traffic engineering) while satisfying the capacity constraints for all the wireless and/or wired links used by the flows. Compared to the traditional multi-commodity routing in wireline networks [7], [8], traffic engineering in the wireless setting is much more challenging due to several reasons. First, the link capacity between two nearby nodes is a nonconvex function of the transmit power budget, channel strength, as well as the underlying physical layer coding/decoding techniques used. Second, the amount of traffic that can be carried on neighboring links is interdependent due to the multiuser interference caused by nearby nodes. Third, multiple parallel channels between two nodes may be available for transmission. To respond to these new challenges, novel RAN management methods must be developed for joint wireless resource allocation in the air interface and traffic engineering within the multi-hop backhaul network. These methods

together will be a central component of the newly proposed software defined networking (SDN) concept [4], [9], which advocates centralized network provisioning for cloud based radio access networks.

The impact of the finite bandwidth of backhaul networks on wireless resource allocation has been studied recently in the context of joint processing between BSs, e.g., [10]–[13]. However, these works do not consider multi-hop routing between the source and the destination nodes. The joint optimization of the backhaul flow routing and the power allocation for wireless network has also been considered in the framework of cross-layer network utility maximization (NUM) problem, see e.g. [14]–[17] and some tutorial papers [18]–[20]. However, since the capacity of wireless links is nonconvex in the presence of multiuser interference, the authors of [14], [19] considered only the orthogonal wireless links which effectively reduced the problem to convex one. In [15], [16], [18], [20], the interference was considered in a fast fading environment but the proposed algorithms required solving difficult subproblems. In [17], the network was approximated by a deterministic channel model [21] through which an approximate optimal solution was derived. A similar joint optimization problem was also investigated in [22] for a wireless sensor network whereby a distributed algorithm capable of converging to the stationary solution is proposed. However, this approach is valid only for the setting with single antenna nodes, and requires the utility function to be strongly convex.

In this paper, we propose an efficient algorithm for joint backhaul traffic engineering and physical layer interference management for a large-scale SD-RAN. In particular, we leverage the Alternating Direction Method of Multipliers (ADMM) [23], [24] and the WMMSE interference management algorithm [25] to tackle the joint resource allocation and traffic engineering problem. The resulting algorithm is significantly more efficient than the subgradient-based methods [14], [19]. The proposed algorithm has simple closed-form updates in each step and is well suited for distributed and parallel implementation. Moreover, the proposed algorithm can be extended to deal with per-flow quality of service constraint, or to networks with multi-antenna nodes. Since not all the QoS requirements can be met simultaneously, techniques from sparse optimization [26], [27] are used to dynamically select the subset of users being served. The efficacy and the efficiency of the proposed algorithms are demonstrated via extensive simulations.

Notations: We use \mathbf{I} to denote the identity matrix, and $\mathbf{0}$ to denote a zero vector or matrix. The superscripts ‘ T ’, ‘ H ’ and ‘ $*$ ’, respectively, stand for the transpose, the conjugate transpose

TABLE I
A LIST OF NOTATIONS

\mathcal{V}	The set of nodes in the network	\mathcal{N}	The set of routers
\mathcal{B}	The set of BSs	\mathcal{U}	The set of mobile users
\mathcal{L}	The set of links	M	Number of total commodities in the system
\mathcal{L}^w	The set of wired links	\mathcal{L}^{wl}	The set of wireless links
C_l	The capacity for a wired link $l \in \mathcal{L}^w$	K	Number of tones on each wireless link
$r_m(l)$	Transmit rate for commodity m on link l	r_m	Data rate for commodity m
$D(m)$	The destination node for commodity m	$S(m)$	The source node for commodity m
p_{ds}^k	The precoder from BS s to user d on tone k	$I(l)$	The set of interferer to wireless link l

and the complex conjugate. The indicator function for a set \mathcal{A} is denoted by $1_{\mathcal{A}}(x)$, that is, $1_{\mathcal{A}}(x) = 1$ if $x \in \mathcal{A}$, and $1_{\mathcal{A}}(x) = 0$ otherwise. The projection function to the nonnegative orthant is denoted by $(x)^+$, i.e., $(x)^+ \triangleq \max\{0, x\}$. Also, the notation $0 \leq a \perp b \geq 0$ means that $a, b \geq 0$ and $ab = 0$. Some other notations are summarized in Table I.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Let \mathcal{V} denote the set of nodes in a HetNet, comprised of a set of network routers \mathcal{N} , a set of BSs \mathcal{B} , and a set of mobile users \mathcal{U} . Let \mathcal{L} denote the set of directed links that connect the nodes of \mathcal{V} . In addition, we assume that there are M source-destination pairs, denoted by $\{(S(m), D(m))\}_{m=1}^M$. For each $m = 1, \dots, M$, a data flow of rate $r(m) \geq 0$ is to be sent from the source node $S(m)$ to the destination node $D(m)$ over the network.

The set of directed links \mathcal{L} consists of both wired and wireless links. The wired links connect routers in \mathcal{N} and BSs in \mathcal{B} , and is denoted as $\mathcal{L}^w \triangleq \{(s, d) \mid (s, d) \in \mathcal{L}, \forall s, d \in \mathcal{N} \cup \mathcal{B}\}$. Here (s, d) denotes the directed link from node s to node d . Assume each wired link $l \in \mathcal{L}^w$ has a fixed capacity, C_l . Then the total flow rate on link l is constrained by

$$\sum_{m=1}^M r_l(m) \leq C_l, \forall l \in \mathcal{L}^w, \quad (1)$$

where $r_l(m) \geq 0$ denotes the nonnegative flow rate on link l for commodity m .

The wireless links provide single-hop connections between the BSs to the mobile users. We assume that each BS divides the spectrum into K orthogonal frequency subchannels, and refer to these subchannels as *wireless links*. Thus, the set of wireless links can be represented as

$$\mathcal{L}^{wl} \triangleq \{(s, d, k) \mid (s, d, k) \in \mathcal{L}, \forall s \in \mathcal{B}, \forall d \in \mathcal{U}, k = 1 \sim K\}$$

with (s, d, k) being the wireless link from node s to node d on subchannel k . For subchannel k , BS $s \in \mathcal{B}$ applies a linear scalar precoder $p_{ds}^k \in \mathbb{C}$ to the transmitted complex unit-norm symbol

of mobile user $d \in \mathcal{U}$, so each mobile user can be served by multiple BSs. Assuming that each mobile user treats the interference from other BSs as noise, the total flow rate constraint on the wireless link $l = (s, d, k) \in \mathcal{L}^{wl}$ is expressed as

$$\sum_{m=1}^M r_l(m) \leq \bar{r}_l(\mathbf{p}) \triangleq \log \left(1 + \frac{|h_{ds}^k|^2 |p_{ds}^k|^2}{\sum_{(s', d', k) \in I(l) \setminus \{l\}} |h_{ds'}^k|^2 |p_{d's'}^k|^2 + \sigma_d^2} \right), \quad \forall l \in \mathcal{L}^{wl}, \quad (2)$$

where $\mathbf{p} \triangleq \{p_{ds}^k \mid \forall (s, d, k) \in \mathcal{L}^{wl}\}$; $h_{ds}^k \in \mathbb{C}$ is the channel tap for the wireless link $l = (s, d, k)$; σ_d^2 is the variance of AWGN noise at mobile user d ; $I(l) \subseteq \mathcal{L}^{wl}$ is the set of links interfering with link l :

$$I(l) \triangleq \{(s', d', k) \in \mathcal{L}^{wl} \mid h_{ds'}^k \neq 0, (s, d, k) = l\}. \quad (3)$$

Note that in this definition, link l itself is included in $I(l)$, i.e., we have $l \in I(l)$. Each BS $s \in \mathcal{B}$ has a total power budget $\bar{p}_s \geq 0$, satisfying

$$\sum_{k=1}^K \sum_{d: (s, d, k) \in \mathcal{L}^{wl}} |p_{ds}^k|^2 \leq \bar{p}_s, \quad \forall s \in \mathcal{B}. \quad (4)$$

Each node in the network should follow the flow conservation constraint, i.e., the total incoming flow of node $v \in \mathcal{V}$ equals the total outgoing flow of that node,

$$\sum_{l \in \text{In}(v)} r_l(m) + 1_{\{S(m)\}}(v) r_m = \sum_{l \in \text{Out}(v)} r_l(m) + 1_{\{D(m)\}}(v) r_m, \quad m = 1 \sim M, \quad \forall v \in \mathcal{V} \quad (5)$$

where $\text{In}(v)$ and $\text{Out}(v)$ denote the set of links going into and coming out of a node v respectively.

In this paper, we are interested in maximizing the minimum flow rate of all commodities, while jointly performing the following tasks 1): route M commodities from node $S(m)$ to node $D(m)$, $m = 1 \sim M$; and 2) design the linear precoder at each BS. This problem can be formulated as

$$\begin{aligned} \max_{\mathbf{p}, \mathbf{r}} \quad & r & (6) \\ \text{s.t.} \quad & r \geq 0, \quad r_m \geq r, \quad r_l(m) \geq 0, \quad m = 1 \sim M, \quad \forall l \in \mathcal{L} \\ & (1), (2), (4), \text{ and } (5), \end{aligned}$$

where $\mathbf{r} \triangleq \{r, r_l(m), r_m \mid \forall l \in \mathcal{L}, m = 1 \sim M\}$. Adopting the min-rate utility results in a

fair rate allocation, and such utility has been adopted by many recent works in both the SDN and wireless communities; see [25], [28] and the references therein. At this point, it is important to note that by solving problem (6), we automatically select a subset of BSs in \mathcal{B} to serve each user. That is, for a given commodity m for user d , it is possible that there exist $r_{(s,d,k)}(m) > 0$ and $r_{(q,d,l)}(m) > 0$ with $s \neq q$, and $(s, d, k), (q, d, l) \in \mathcal{L}^{wl}$. Allowing cooperation among the BSs is in agreement with the envisioned next generation cellular networks [4], which will rely heavily on various BS cooperation schemes such as joint processing to improve the transmission rate. Here, for simplicity, we don't take joint processing between BSs into consideration.

Problem (6) is difficult to solve because of the following reasons:

- i) It is a nonconvex problem where the nonconvexity comes from the rate constraints on the wireless links
- ii) The conventional approaches such as the bisection procedure for solving the max-min rate power allocation (beamformer) design [29] cannot be applied here, due to the existence of the conservation constraints and the presence of multiple frequency tones.
- iii) The size of the problem can be huge, as a result even if we consider the simplest scenario in which there are no mobile users (or equivalently the nonconvex wireless rate constraints are not present), the resulting problem may still be difficult to solve in real time.

In the following, we propose an efficient distributed algorithm to compute a stationary solution of the problem (6).

3. JOINT TRAFFIC ENGINEERING AND INTERFERENCE MANAGEMENT

In this section, we propose a distributed algorithm that solves problem (6) to a stationary solution. We emphasize that this problem is nonconvex due to the flow rate constraints on wireless links, i.e., (2).

A. Algorithm Outline

A special case of the considered problem model is known as M -pair interference channel with the following settings i) the number of BSs is the same as that of the users, i.e., $|\mathcal{B}| = |\mathcal{U}| = M$; ii) there are only wireless links for each wireless transmitter and user pair, i.e. $|\mathcal{N}| = 0$; iii) each wireless transmitter and user pair serve, respectively, as the source and the destination node of a commodity. For this special case, it has been shown that the minimum rate maximization problem is NP-hard when both transmitter and user are equipped with no less than 2 antennas

[25]. However, when the wireless transmitters are equipped with multiple antennas (resp. single antenna) while mobile users are equipped with only one antenna (resp. multiple antennas), the nonconvex minimum rate maximization problem has been shown to be polynomial time solvable for the single tone case of $K = 1$ [29]–[31]. However, this is no longer true if there is more than one frequency tone.

In the following, we will propose an efficient algorithm that can solve problem (6) to a stationary solution. The proposed algorithm is a combination of two algorithms: 1) the max-min WMMSE algorithm developed in [25] for minimum rate maximization in M -pair interference channel; 2) the ADMM algorithm that is used to distributively solve the multi-commodity routing problem. Central to the proposed approach is the utilization of a rate-MSE relationship, stated below [25].

Lemma 1: For a given $l = (s, d, k) \in \mathcal{L}^{wl}$, $\bar{r}_l(\mathbf{p})$ can be equivalently expressed as

$$\bar{r}_l(\mathbf{p}) = \max_{u_l, w_l} E_l(u_l, w_l, \mathbf{p}) \triangleq \max_{u_l, w_l} c_{1,l} + c_{2,l} p_{ds}^k - \sum_{n=(s',d',k) \in I(l)} c_{3,ln} |p_{d's'}^k|^2 \quad (7)$$

where $(c_{1,l}, c_{2,l}, c_{3,ln})$ are given by $c_{1,l} = 1 + \log(w_l) - w_l(1 + \sigma_d^2 |u_l|^2)$, $c_{2,l} = 2w_l \text{Re}\{u_l^* h_{ds}^k\}$, and $c_{3,ln} = w_l |u_l|^2 |h_{d's'}^k|^2$.

Note that Lemma 1 reformulates $\bar{r}_l(\mathbf{p})$ by introducing two extra sets of variables $\mathbf{u} \triangleq \{u_l \mid l \in \mathcal{L}^{wl}\}$ and $\mathbf{w} \triangleq \{w_l \mid l \in \mathcal{L}^{wl}\}$, with one pair of variables $\{u_l, w_l\}$ for each wireless link l . The term inside the maximization operator is the MSE for estimating the message transmitted on link l . Given Lemma 1, we reformulate problem (6) by replacing $\bar{r}_l(\mathbf{p})$ in (6) with its MSE. We call such new constraint a *rate-MSE constraint*. Then, we consider the following problem with two extra optimization variable sets \mathbf{u} and \mathbf{w} instead:

$$\max r \quad (8)$$

$$\text{s.t. } r \geq 0, r_m \geq r, r_l(m) \geq 0, m = 1 \sim M, \forall l \in \mathcal{L},$$

$$(1), (4), \text{ and } (5),$$

$$\sum_{m=1}^M r_l(m) \leq c_{1,l} + c_{2,l} p_{ds}^k - \sum_{n=(s',d',k) \in I(l)} c_{3,ln} |p_{d's'}^k|^2, \forall l \in \mathcal{L}^{wl}. \quad (9)$$

Why do we include these extra optimization variables \mathbf{u} and \mathbf{w} ? First we observe that for any given $\{\mathbf{r}, \mathbf{p}\}$, the optimal \mathbf{u} (resp. \mathbf{w}) for (7) can be obtained while \mathbf{w} (resp. \mathbf{u}) is held fixed.

Network Max-Min WMMSE (N-MaxMin) Algorithm:

- 1: **Initialization** Generate a feasible set of variables $\{\mathbf{r}, \mathbf{p}\}$, and let $t = 1$.
- 2: **Repeat**
- 3: $\mathbf{u}^{(t)}$ is updated by (10)
- 4: $\mathbf{w}^{(t)}$ is updated by (11)
- 5: $\{\mathbf{r}^{(t)}, \mathbf{p}^{(t)}\}$ is updated by solving the problem (8) via Algorithm 1 in Table III
- 6: $t = t + 1$
- 7: **Until** Desired stopping criteria is met

TABLE II
NETWORK MAX-MIN WMMSE (N-MAXMIN) ALGORITHM

Moreover, these optimal solutions can be expressed in closed form for any $l \in \mathcal{L}^{wl}$:

$$u_l = \left(\sum_{(s',d',k) \in I(s,d,k)} |h_{ds'}^k|^2 |p_{d's'}^k|^2 + \sigma_d^2 \right)^{-1} h_{ds}^k p_{ds}^k, \quad (10)$$

$$w_l = \left(1 - (h_{ds}^k p_{ds}^k)^* u_l \right)^{-1}. \quad (11)$$

These expressions suggest that the set of variables \mathbf{u} and \mathbf{w} can be updated independently and locally at each mobile user if the interference plus noise and local channel state information are locally known to the users. Moreover, when \mathbf{u} and \mathbf{w} are fixed, the problem for updating $\{\mathbf{r}, \mathbf{p}\}$ is convex (note that (7) is a convex quadratic problem on the precoders \mathbf{p}) and can be solved in polynomial time. Hence, we propose to apply the alternating optimization technique to solve problem (8); see the N-MaxMin Algorithm in Table II for a detailed description.

The following result states that the iterates $\{\mathbf{r}^{(t)}, \mathbf{p}^{(t)}\}$ generated by the above algorithm converge to a stationary solution of the original problem (6). The proof of this result is relegated to Appendix A.

Theorem 1: The sequence $\{\mathbf{r}^{(t)}, \mathbf{p}^{(t)}\}$ generated by the N-MaxMin Algorithm converges to a stationary solution of problem (6). Moreover, every global optimal solution of problem (6) corresponds to a global optimal solution of the reformulated problem (8), and they achieve the same objective value.

Remark 1: The N-MaxMin Algorithm (Table II) and its convergence analysis (Theorem 1) extend easily to the multi-antenna case. The key is to use the matrix version of Lemma 1 in [25].

B. A Brief Review of ADMM Algorithm

The second ingredient for the proposed approach is to use the ADMM algorithm to update $\{\mathbf{r}, \mathbf{p}\}$ in the N-MaxMin Algorithm. Unlike the computation of \mathbf{u} and \mathbf{w} , the updates for $\{\mathbf{r}, \mathbf{p}\}$ do not have closed forms. We can use off-the-shelve toolboxes, but this is not very efficient. In the sequel, we first use variable splitting to decompose the problem and then solve it using ADMM. The resulting algorithm has closed form updates in each step and is well suited for parallel and distributed implementation.

We now briefly review the ADMM algorithm. Consider the following structured convex problem [24],

$$\begin{aligned} \min_{\mathbf{x} \in \mathcal{C}^n, \mathbf{z} \in \mathcal{C}^m} \quad & f(\mathbf{x}) + g(\mathbf{z}) \\ \text{s.t.} \quad & \mathbf{Ax} + \mathbf{Bz} = \mathbf{c} \\ & \mathbf{x} \in \mathcal{C}_1, \mathbf{z} \in \mathcal{C}_2 \end{aligned} \quad (12)$$

where $\mathbf{A} \in \mathbb{C}^{k \times n}$, $\mathbf{B} \in \mathbb{C}^{k \times m}$, $\mathbf{c} \in \mathbb{C}^k$; f and g are convex functions; \mathcal{C}_1 and \mathcal{C}_2 are non-empty convex sets. The partial augmented Lagrangian function for problem (12) can be expressed as

$$L_\rho(\mathbf{x}, \mathbf{z}, \mathbf{y}) = f(\mathbf{x}) + g(\mathbf{z}) + \text{Re}(\mathbf{y}^H(\mathbf{Ax} + \mathbf{Bz} - \mathbf{c})) + (\rho/2)\|\mathbf{Ax} + \mathbf{Bz} - \mathbf{c}\|_2^2 \quad (13)$$

where $\mathbf{y} \in \mathbb{C}^k$ is the Lagrangian dual variable associated with the linear equality constraint, and $\rho > 0$ is some constant. The ADMM algorithm solves problem (12) by iteratively performing three steps in each iteration t :

$$\mathbf{x}^{(t)} = \arg \min_{\mathbf{x} \in \mathcal{C}_1} L_\rho(\mathbf{x}, \mathbf{z}^{(t-1)}, \mathbf{y}^{(t-1)}) \quad (\text{primal update for the first block variable}) \quad (14a)$$

$$\mathbf{z}^{(t)} = \arg \min_{\mathbf{z} \in \mathcal{C}_2} L_\rho(\mathbf{x}^{(t)}, \mathbf{z}, \mathbf{y}^{(t-1)}) \quad (\text{primal update for the second block variable}) \quad (14b)$$

$$\mathbf{y}^{(t)} = \mathbf{y}^{(t-1)} + \rho(\mathbf{Ax}^{(t)} + \mathbf{Bz}^{(t)} - \mathbf{c}) \quad (\text{dual variable update}). \quad (14c)$$

The practical efficiency of ADMM can be attributed to the fact that in many applications, the subproblems (14a) and (14b) are solvable in closed-form. The convergence and the optimality of the algorithm is summarized in the following lemma [23].

Lemma 2: Assume that the optimal solution set of problem (12) is non-empty, and $\mathbf{A}^T \mathbf{A}$ and $\mathbf{B}^T \mathbf{B}$ are invertible. Then the sequence of $\{\mathbf{x}^{(t)}, \mathbf{z}^{(t)}, \mathbf{y}^{(t)}\}$ generated by (14a), (14b), and (14c) is bounded and every limit point of $\{\mathbf{x}^{(t)}, \mathbf{z}^{(t)}\}$ is an optimal solution of problem (12).

C. An ADMM Approach for Updating $\{\mathbf{r}, \mathbf{p}\}$

In the following, we will first reformulate the subproblem for $\{\mathbf{r}, \mathbf{p}\}$ into the form of (12), so that the ADMM can be applied. Then we will show that each step of the resulting algorithm is easily computable and amenable for distributed implementation. To this end, we will appropriately split the variables in the coupling constraints (5) and (9).

We first observe that each flow rate $r_l(m)$ on link $l = (s, d) \in \mathcal{L}^w$ (or $l = (s, d, k) \in \mathcal{L}^{wl}$) for commodity m is shared among *two* flow conservation constraints, one for node s and the other for node d . To induce separable subproblems and enable distributed computation, we introduce two local auxiliary copies of $r_l(m)$, namely $\hat{r}_l^s(m)$ and $\hat{r}_l^d(m)$, and store one at node s and the other at node d . Similarly, we introduce two local auxiliary copies for each commodity rate, denoted as $\hat{r}_m^{S(m)}$, $\hat{r}_m^{D(m)}$, $m = 1 \sim M$, and store them at the source and the destination node of each commodity, respectively. That is, we have introduced the following auxiliary variables:

$$\hat{r}_m^{S(m)} = r_m, \hat{r}_m^{D(m)} = r_m, m = 1 \sim M; \quad (15a)$$

$$\hat{r}_l^s(m) = r_l(m), \hat{r}_l^d(m) = r_l(m), \forall l = (s, d) \in \mathcal{L}^w; \quad (15b)$$

$$\hat{r}_l^s(m) = r_l(m), \hat{r}_l^d(m) = r_l(m), \forall l = (s, d, k) \in \mathcal{L}^{wl}. \quad (15c)$$

The flow rate conservation constraints on node $v \in \mathcal{V}$ can then be rewritten as

$$\sum_{l \in \text{In}(v)} \hat{r}_l^v(m) + 1_{\{S(m)\}}(v) \hat{r}_m^v = \sum_{l \in \text{Out}(v)} \hat{r}_l^v(m) + 1_{\{D(m)\}}(v) \hat{r}_m^v, m = 1 \sim M. \quad (16)$$

In addition, for the rate-MSE constraint, we introduce several copies of the transmit precoder on a given wireless link $l = (s, d, k) \in \mathcal{L}^{wl}$, i.e.

$$p_{d's', ds}^k = p_{ds}^k, \forall l \in I(s', d', k) \subset \mathcal{L}^{wl}. \quad (17)$$

Intuitively, by doing such variable splitting, each variable $p_{d's', ds}^k$ will only appear in *a single* rate-MSE constraint. For a given link $l = (s, d, k) \in \mathcal{L}^{wl}$, its rate-MSE constraint only depends on the set of precoders $\{p_{ds, d's'}^k \mid \forall (s', d', k) \in I(l)\}$, as can be seen below

$$\sum_{m=1}^M r_l(m) \leq c_{1,l} + c_{2,l} p_{ds, ds}^k - \sum_{n=(s', d', k) \in I(l)} c_{3,ln} |p_{ds, d's'}^k|^2, \forall l \in \mathcal{L}^{wl}. \quad (18)$$

Moreover, for the analysis of the convergence result, another auxiliary variable \hat{r} is introduced

such that $r = \hat{r}$.

Using these new variables, the updating step for $\{\mathbf{r}, \mathbf{p}\}$ is equivalently expressed as

$$\begin{aligned} & \max (r + \hat{r})/2 \\ & \text{s.t. } r = \hat{r}, r \geq 0, r_m \geq r, r_l(m) \geq 0, m = 1 \sim M, l \in \mathcal{L} \\ & (1), (4), (15), (16), (17) \text{ and } (18). \end{aligned} \quad (19)$$

It is important to note that the constraints of problem (19) (except the linear equality constraints $r = \hat{r}$, (15) and (17)) are now separable between two optimization variable sets *i*) the tuple $\{\mathbf{r}, \hat{\mathbf{p}}\}$ where $\hat{\mathbf{p}} \triangleq \{p_{d's', ds}^k \mid \forall l = (s, d, k), l' = (s', d', k) \in \mathcal{L}^{wl}, l \in I(l')\}$, and *ii*) the tuple $\{\hat{\mathbf{r}}, \mathbf{p}\}$ where $\hat{\mathbf{r}} \triangleq \{\hat{r}, \hat{r}_m^{S(m)}, \hat{r}_m^{D(m)}, \hat{r}_l^s(m), \hat{r}_l^d(m) \mid m = 1 \sim M, \forall l = (s, d) \text{ or } (s, d, k) \in \mathcal{L}\}$. Additionally, the objective function is linear and separable over \mathbf{r} and $\hat{\mathbf{r}}$. Therefore the ADMM algorithm can be used to solve problem (19). The resulting algorithm, described in Table III, is referred to as Algorithm 1. Note that the partial augmented Lagrange function for problem (19) is given by

$$\begin{aligned} L_{\rho_1, \rho_2}(\mathbf{r}, \hat{\mathbf{p}}, \hat{\mathbf{r}}, \mathbf{p}; \boldsymbol{\delta}, \boldsymbol{\theta}) &= (r + \hat{r})/2 + \delta(\hat{r} - r) - \frac{\rho_1}{2}(\hat{r} - r)^2 \\ &+ \underbrace{\sum_{m=1}^M \left[\delta_m^{S(m)}(\hat{r}_m^{S(m)} - r_m) + \delta_m^{D(m)}(\hat{r}_m^{D(m)} - r_m) - \frac{\rho_1}{2}(\hat{r}_m^{S(m)} - r_m)^2 - \frac{\rho_1}{2}(\hat{r}_m^{D(m)} - r_m)^2 \right]}_{\text{enforcing linear constraints (15a)}} + \sum_{m=1}^M \\ &\underbrace{\sum_{\substack{l=(s,d) \in \mathcal{L} \\ l=(s,d,k) \in \mathcal{L}^{wl}}} \left[\delta_l^s(m)(\hat{r}_l^s(m) - r_l(m)) + \delta_l^d(m)(\hat{r}_l^d(m) - r_l(m)) - \frac{\rho_1}{2}(\hat{r}_l^s(m) - r_l(m))^2 - \frac{\rho_1}{2}(\hat{r}_l^d(m) - r_l(m))^2 \right]}_{\text{enforcing linear constraints (15b) and (15c)}} \\ &+ \underbrace{\sum_{\substack{l=(s,d,k) \in \mathcal{L}^{wl} \\ n=(s',d',k) \in I(s,d,k)}} \left[\theta_{ln}(p_{d's'}^k - p_{ds,d's'}^k) - \frac{\rho_2}{2}(p_{d's'}^k - p_{ds,d's'}^k)^2 \right]}_{\text{enforcing linear constraints (17)}}, \end{aligned}$$

where we have used δ , $\{\delta_m^{S(m)}\}$, $\{\delta_m^{D(m)}\}$, $\{\delta_l^s(m)\}$, $\{\delta_l^d(m)\}$ and $\{\theta_{ln}\}$ to denote the Lagrangian multipliers for various equality constraints, and have collected these multipliers to the vectors $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$; $\rho_1 > 0$ and $\rho_2 > 0$ are some constant coefficients for, respectively, the linear equality constraints (15) and (17). For notational simplicity, let us stack all the elements of \mathbf{r} and $\hat{\mathbf{r}}$ to the following vectors

$$\begin{aligned} \mathbf{r}_{\text{stack}} &\triangleq [r, \{r_m\}_{m=1 \sim M}, \{r_l(m)\}_{m=1 \sim M, l \in \mathcal{L}}]^T \\ \hat{\mathbf{r}}_{\text{stack}} &\triangleq [\hat{r}, \{\hat{r}_m^{S(m)}\}_{m=1 \sim M}, \{\hat{r}_m^{D(m)}\}_{m=1 \sim M}, \{\hat{r}_l^s(m)\}_{m=1 \sim M, l=(s,d) \in \mathcal{L}}, \{\hat{r}_l^d(m)\}_{m=1 \sim M, l=(s,d) \in \mathcal{L}}]^T. \end{aligned}$$

Similarly, stack all the elements of \mathbf{p} and $\hat{\mathbf{p}}$ by

$$\begin{aligned}\mathbf{p}_{\text{stack}} &\triangleq \{p_{ds}^k, \forall (s, d, k) \in \mathcal{L}^{wl}\} \\ \hat{\mathbf{p}}_{\text{stack}} &\triangleq \{\{p_{d's',ds}^k, \forall (s, d, k) \in I(s', d', k')\}, \forall (s, d, k) \in \mathcal{L}^{wl}\}.\end{aligned}$$

Then the equality relationships (15a)–(15c) and (17) can be compactly expressed as

$$\mathbf{C}\mathbf{r}_{\text{stack}} = \hat{\mathbf{r}}_{\text{stack}}, \quad \mathbf{D}\mathbf{p}_{\text{stack}} = \hat{\mathbf{p}}_{\text{stack}}, \quad (20)$$

where

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{I} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{I} \end{bmatrix}^T; \quad \mathbf{D} = \text{blkdg}[\{\mathbf{1}_{ds}^k\}_{(s,d,k) \in \mathcal{L}^{wl}}] \quad (21)$$

where $\text{blkdg}\{\cdot\}$ is the block diagonalization operator; $\mathbf{1}_{ds}^k$ is an all one column vector of size equal to the total number of links with which $l = (s, d, k)$ interferes, given by

$$|\bar{I}(l)| \triangleq |\{(s', d', k) \mid (d, s, k) \in I(s', d', k)\}|.$$

Using the notation in (20), we can simplify the above expression to

$$\begin{aligned}L_{\rho_1, \rho_2}(\mathbf{r}, \hat{\mathbf{p}}, \hat{\mathbf{r}}, \mathbf{p}; \boldsymbol{\delta}, \boldsymbol{\theta}) &= r + \left[\boldsymbol{\delta}^T (\hat{\mathbf{r}}_{\text{stack}} - \mathbf{C}\mathbf{r}_{\text{stack}}) - \frac{\rho_1}{2} \|\hat{\mathbf{r}}_{\text{stack}} - \mathbf{C}\mathbf{r}_{\text{stack}}\|^2 \right] \\ &\quad + \left[\boldsymbol{\theta}^H (\mathbf{D}\mathbf{p}_{\text{stack}} - \hat{\mathbf{p}}_{\text{stack}}) - \frac{\rho_2}{2} \|\mathbf{D}\mathbf{p}_{\text{stack}} - \hat{\mathbf{p}}_{\text{stack}}\|^2 \right].\end{aligned}$$

Moreover, by appealing to the standard analysis for ADMM algorithm (Lemma 2), and using the fact that $\mathbf{C}^T\mathbf{C}$ and $\mathbf{D}^T\mathbf{D}$ are both full rank matrices, we easily see that Algorithm 1 converges to the optimal solutions of problem (19).

For the detailed step-by-step specification of Algorithm 1, we refer the readers to Appendix B. The main message from the derivation therein is that each step in Algorithm 1 can be computed distributedly in closed-form. More specifically, Step 3 of the algorithm is decomposable *among all links* in the system (wireless and wired), while Step 4 of the algorithm is decomposable *among all the nodes* in the system (also see Section 4-A for elaboration). These properties allow the entire algorithm to be easily implemented in a parallel fashion. Fig. 1 provides a flow chart showing the relationship among different algorithms

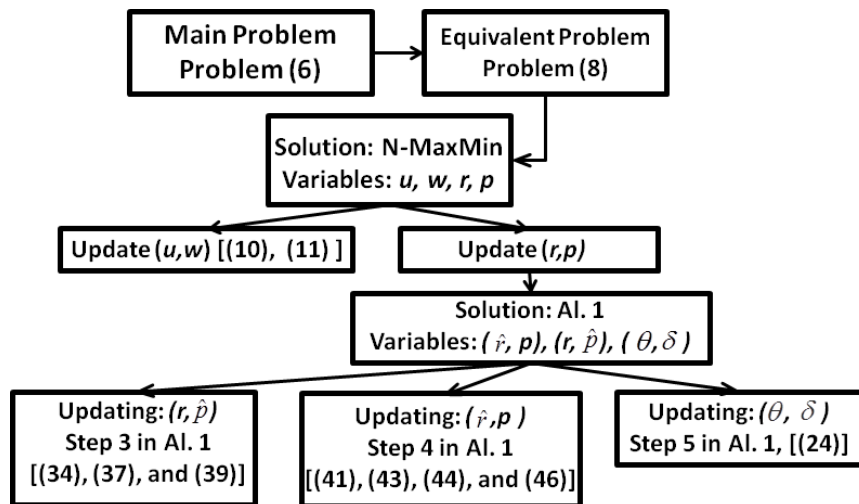


Fig. 1. Flow chart of the proposed solution approach (6).

4. DISTRIBUTED IMPLEMENTATION AND EXTENSIONS

A. Distributed Implementation and Information Exchange

In this section, we briefly elaborate how the N-MaxMin algorithm can be implemented in a distributed manner. Let us first look at the implementation for the backhaul network (i.e., the update for \mathbf{r} and $\hat{\mathbf{r}}$ when ignoring the wireless links). Suppose there is a master node in the system. Consider the update of the optimization variable \mathbf{r} in Step 3 of Algorithm 1 (cf. Step (i) in Appendix B-1). In this step, to update $\{r, r_m \mid m = 1 \sim M\}$, the source node and destination node of each commodity m , $m = 1 \sim M$, should respectively send $\left(\hat{r}_m^{S(m)} - \frac{\delta_m^{S(m)}}{\rho_1}\right)$ and $\left(\hat{r}_m^{D(m)} - \frac{\delta_m^{D(m)}}{\rho_1}\right)$ to the assumed master node. After the master node applies (34) to update $\{r, r_m \mid m = 1 \sim M\}$, it would transmit r_m back to node $S(m)$ and $D(m)$. To update $r_l(m)$, $m = 1 \sim M$, $\forall l \in \mathcal{L}$, the procedure is decoupled across *each link* (cf. step (ii) in Appendix B-1). Therefore without loss of generality, we can let the destination node of each link $l = (s, d) \in \mathcal{L}$ perform the bisection updating step (37). Thus, the source node of link l should transmit M real values, $\left(\hat{r}_l^s(m) - \frac{\delta_l^s(m)}{\rho_1}\right)$, $\forall m = 1 \sim M$, to the destination node of that link. After updating $r_l(m)$, $m = 1 \sim M$, the destination node of the link would transmit them back to the source node. After \mathbf{r} is computed, the second block variables $\hat{\mathbf{r}}$ and the Lagrange dual variable δ can be updated in each node, see (41), (43), (44), and (24).

Next we discuss the implementation for the wireless part, i.e., the update for \mathbf{p} and $\hat{\mathbf{p}}$. We assume that *i*) each mobile user has local channel state information from all interfering BSs;

Algorithm 1: ADMM for (19):

1: **Initialize** all primal variables $\mathbf{r}^{(0)}, \hat{\mathbf{r}}^{(0)}, \mathbf{p}^{(0)}, \hat{\mathbf{p}}^{(0)}$ (not necessarily a feasible solution for problem (19)); Initialize all dual variables $\boldsymbol{\delta}^{(0)}, \boldsymbol{\theta}^{(0)}$; set $t = 0$

2: **Repeat**

3: Solve the following problem and obtain $\mathbf{r}^{(t+1)}, \hat{\mathbf{p}}^{(t+1)}$:

$$\begin{aligned} \max_{\mathbf{r}, \hat{\mathbf{p}}} \quad & L_{\rho_1, \rho_2}(\mathbf{r}, \hat{\mathbf{p}}, \hat{\mathbf{r}}^{(t)}, \mathbf{p}^{(t)}; \boldsymbol{\delta}^{(t)}, \boldsymbol{\theta}^{(t)}) \\ \text{s.t.} \quad & r \geq 0, r_m \geq r, r_l(m) \geq 0, m = 1 \sim M, l \in \mathcal{L}, \\ & \text{(1) and (18)} \end{aligned} \quad (22)$$

This step can be *solved in parallel across all links*, cf. (34), (37), and (39).

4: Solve the following problem and obtain $\hat{\mathbf{r}}^{(t+1)}, \mathbf{p}^{(t+1)}$:

$$\begin{aligned} \max_{\hat{\mathbf{r}}, \mathbf{p}} \quad & L_{\rho_1, \rho_2}(\mathbf{r}^{(t+1)}, \hat{\mathbf{p}}^{(t+1)}, \hat{\mathbf{r}}, \mathbf{p}; \boldsymbol{\delta}^{(t)}, \boldsymbol{\theta}^{(t)}) \\ \text{s.t.} \quad & \text{(4) and (16)} \end{aligned} \quad (23)$$

This problem can be *solved in parallel across all nodes*, cf. (41), (43), (44), and (46).

5: Update the Lagrange dual multipliers $\boldsymbol{\delta}^{(t+1)}$ and $\boldsymbol{\theta}^{(t+1)}$ by

$$\begin{aligned} \boldsymbol{\delta}^{(t+1)} &= \boldsymbol{\delta}^{(t)} - \rho_1(\hat{\mathbf{r}}_{\text{stack}}^{(t+1)} - \mathbf{C}\mathbf{r}_{\text{stack}}^{(t+1)}), \\ \boldsymbol{\theta}^{(t+1)} &= \boldsymbol{\theta}^{(t)} - \rho_2(\mathbf{D}\mathbf{p}_{\text{stack}}^{(t+1)} - \hat{\mathbf{p}}_{\text{stack}}^{(t+1)}). \end{aligned} \quad (24)$$

6: $t = t + 1$

7: **Until** Desired stopping criterion is met

TABLE III
SUMMARY OF THE PROPOSED ALGORITHM I

and *ii*) u_l and w_l are updated according to (10) and (11) respectively at the receiver side of link $l \in \mathcal{L}^{wl}$. Let us first look at the update for $\hat{\mathbf{p}} \cup \{r_l(m) \mid m = 1 \sim M, \forall l \in \mathcal{L}^{wl}\}$ (cf. (38)). Recall that this step is decoupled over each wireless link, and all necessary information needed for the computation (such as $\mathbf{u}, \mathbf{w}, \mathbf{p}$ and the channel state information) is available at each user except $(r_l^s(m) - \frac{\delta_l^s(m)}{\rho_1})$, $m = 1 \sim M$. It follows that this update can be processed at the mobile users d , provided that for wireless link $l = (s, d, k) \in \mathcal{L}^{wl}$, the BS s sends $(r_l^s(m) - \frac{\delta_l^s(m)}{\rho_1})$, $m = 1 \sim M$ to mobile user d . After mobile user d updates $r_l(m)$, $m = 1 \sim M$, it sends them back to BS s . Next we analyze the step that update \mathbf{p} (cf. (45)). In order to solve this problem locally at each BS $s \in \mathcal{B}$, the mobile users whose transmissions interfere with the users associated with BS s , i.e.,

$$d' \in \{d' \mid (s', d', k) \in \bar{I}(s, d, k), \forall d, k = 1 \sim K, \text{ s.t. } (s, d, k) \in \mathcal{L}^{wl}\} \quad (25)$$

should send $(p_{d's',ds}^k + \frac{\theta_{(s',d',k),(s,d,k)}}{\rho_2})$, $\forall (s', d', k) \in \mathcal{L}^{wl}$ with BS s . After BS s obtains the updated p_{ds}^k by (46), it can broadcast these quantities back to those mobile users.

Given the information exchanges described above, Algorithm 1 (and therefore, the N-MaxMin Algorithm) can be implemented in a distributed and parallel manner.

B. Extension with Per-user QoS Requirements

For a subset $\mathcal{Q} \subseteq \{1, \dots, M\}$ of the end-to-end commodity pairs, we may require the flow rates to be no less than \underline{r}_q . For the rest of the commodities $\mathcal{Q}^c \triangleq \{1, \dots, M\} \setminus \mathcal{Q}$, we can maximize their minimum achievable rate. This gives rise to the following formulation:

$$\begin{aligned} & \max r \\ & \text{s. t. } r \geq 0, r_l(m) \geq 0, m = 1 \sim M, \forall l \in \mathcal{L}, \\ & r_q \geq \underline{r}_q, \forall q \in \mathcal{Q}, r_m \geq r, \forall m \in \mathcal{Q}^c, \\ & (1), (2), (4), \text{ and } (5). \end{aligned} \tag{26}$$

Different from problem (6), this QoS constrained formulation is not always feasible for any given tuple of QoS constraints $\{\bar{r}_q\}_{q \in \mathcal{Q}}$. Therefore, the N-MaxMin algorithm proposed in Table II cannot be directly applied. To circumvent this difficulty, we introduce an extra optimization variable set

$$\boldsymbol{\alpha} \triangleq \{\alpha_q \geq 0 \mid q \in \mathcal{Q}\}.$$

The variable α_q can be interpreted as the QoS violation for the q th QoS constraint. Using this set of new variables, we replace the ‘‘hard’’ QoS constraint $r_q \geq \underline{r}_q, \forall q \in \mathcal{Q}$ with the following set of ‘‘soft’’ constraints

$$r_q \geq \underline{r}_q - \alpha_q, \forall q \in \mathcal{Q}.$$

In this way problem (26) is always feasible. Hence, our goal becomes one that selects the *maximum number* of commodities in the set \mathcal{Q} to satisfy the QoS requirements, in addition to the joint optimization for power allocation and routing. In another word, besides optimizing \mathbf{p} and \mathbf{r} , we would like to find a vector $\boldsymbol{\alpha}$ that has the maximum number of zeros.

Mathematically, to induce zeros in $\boldsymbol{\alpha}$, an extra regularization term that penalizes the nonzero terms in $\boldsymbol{\alpha}$ should be added to the objective function of problem (26): $\max r - \|\boldsymbol{\alpha}\|_0$. Here the ℓ_0 norm measures the number of nonzero elements within a vector. Follow the conventional

sparse optimization strategy [26], [27], we then relax the difficult ℓ_0 norm to the convex ℓ_1 norm, and consider the following problem instead

$$\begin{aligned}
& \max r - \sum_{q \in \mathcal{Q}} \alpha_q \\
& \text{s.t. } r \geq 0, r_l(m) \geq 0, m = 1 \sim M, \forall l \in \mathcal{L}, \\
& \quad \alpha \geq 0, r_q + \alpha_q \geq \underline{r}_q, \forall q \in \mathcal{Q}, r_m \geq r, \forall m \in \mathcal{Q}^c \\
& \quad (1), (2), (4), \text{ and } (5).
\end{aligned} \tag{27}$$

This problem can be solved to a stationary solution by applying a modified N-MaxMin algorithm. In particular, the block variables are \mathbf{u} , \mathbf{w} , and $\{\mathbf{r}, \mathbf{p}, \alpha\}$. We observe that the updating procedures for \mathbf{u} and \mathbf{w} are the same as in (10) and (11). To update $\{\mathbf{r}, \mathbf{p}, \alpha\}$, we can apply the ADMM algorithm developed in Sec. 3-C for problem (19) with a few minor modifications (omitted here due to space limitations).

5. SIMULATION RESULTS

In this section, we report some numerical results on the performance of the proposed algorithms as applied to a network with 57 BSs and 11 network routers. We have tested both the efficacy and the efficiency of the proposed algorithms. The topology and the connectivity of this network are shown in Fig. 2. For the backhaul links of this network, a fixed capacity is assumed, and is same in both directions. These link capacities are given as follows:

- links between routers and those between gateway BSs and the routers: 1 (Gnats/s);
- 1-hop to the gateways: 100 (Mnats/s);
- 2-hop to the gateways: [10,50] (Mnats/s);
- 3-hop to the gateways: [2,5] (Mnats/s);
- More than 4-hop to the gateways: 0 (nats/s).

The number of subchannels is $K = 3$ and each subchannel has 1 MHz bandwidth. The power budget for each BS is chosen equally by $\bar{p} = p_s, \forall s \in \mathcal{B}$, and $\sigma_d^2 = 1, \forall d \in \mathcal{U}$. The wireless links follow the Rayleigh distribution with $CN(0, (200/\text{dist})^3)$, where dist is the distance between BS and the corresponding user. The source (destination) node of each commodity is randomly selected from network routers (mobile users), and all simulation results are averaged over 100 randomly selected end-to-end commodity pairs. Below we refer to one

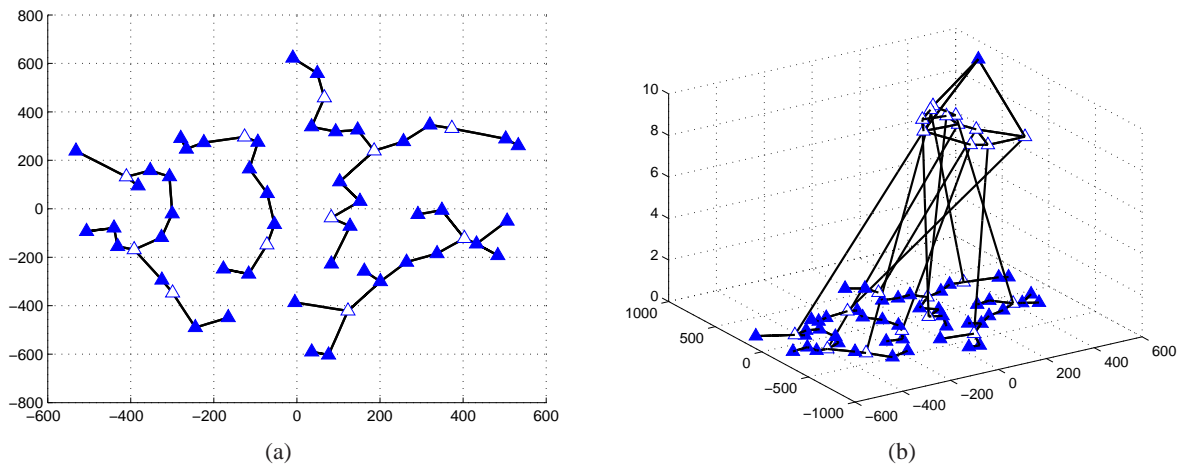


Fig. 2. The considered network consists of 57 BSs and 11 routers. Fig. 2 (a) plots the locations and the connectivity of all the BSs. Here, the solid triangles denote BSs, which only connect to other BSs, and the hollow triangles denote the gateway BSs that are connected to routers and other BSs. Fig. 2 (b) plots the connections between BSs and routers, which are displayed in the upper part of the graph.

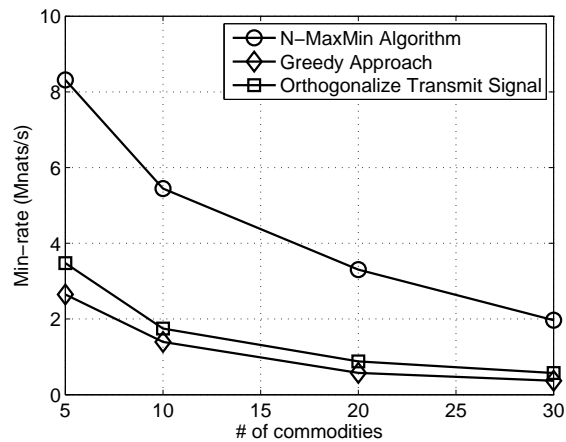


Fig. 3. The minimum rate achieved by N-MaxMin algorithm and the two heuristic algorithms for different number of commodities. We have $\bar{p} = 20\text{dB}$.

round of the N-MaxMin iteration as an *outer iteration*, and one round of Algorithm 1 for solving (\mathbf{r}, \mathbf{p}) as an *inner iteration*.

In the first experiment, we assume that all mobile users can be served by BSs within 300 meters and are interfered by all BSs. For this problem, the parameters of N-MaxMin algorithm are set to be $\rho_1 = 0.1$ and $\rho_2 = 0.001$; the termination criterion is

$$\frac{(r^{(t+1)} + \hat{r}^{(t+1)}) - (r^{(t)} + \hat{r}^{(t)})}{r^{(t)} + \hat{r}^{(t)}} < 10^{-3}$$

$$\max\{\|\mathbf{C}\mathbf{r}_{\text{stack}}^{(t)} - \hat{\mathbf{r}}_{\text{stack}}^{(t)}\|_{\infty}, \|(\mathbf{D}\mathbf{p}_{\text{stack}}^{(t)})^2 - (\hat{\mathbf{p}}_{\text{stack}}^{(t)})^2\|_{\infty}\} < 5 \times 10^{-4} \quad (28)$$

where $(\cdot)^2$ represents elementwise square operation.

For comparison purpose, the following two heuristic algorithms are considered.

- **Heuristic 1 (greedy approach):**

We assume that each mobile user is served by a single BS on a specific frequency tone. For each user, we pick the BS and channel pair that has the strongest channel as its serving BS and channel. After BS-user association is determined, each BS uniformly allocates its power budget to the available frequency tones as well as to the served users on each tone. With the obtained power allocation and BS-user association, the capacity of all wireless links are available and fixed, so the minimum rate of all commodities can be maximized by solving a multi-commodity routing problem (which is essentially problem (6) with only backhaul links and network routers).

- **Heuristic 2 (orthogonal wireless transmission):**

For the second heuristic algorithm, each BS uniformly allocates its power budget to each frequency tone. To obtain a tractable problem formulation, we further assume that each active wireless link is interference free. By doing this each wireless link rate constraints of problem (6) now becomes convex. To impose this interference free constraint, additional variables $\beta_l \in \{0, 1\}$, $\forall l \in \mathcal{L}^{wl}$ are introduced, where $\beta_l = 1$ if wireless link l is active, otherwise $\beta_l = 0$. In this way, there is no interference on wireless link l if $\sum_{n \in I(l)} \beta_n = 1$. To summarize, we solve the following optimization problem:

$$\begin{aligned}
 & \max r \\
 & \text{s.t. } r_m \geq r, r_l(m) \geq 0, m = 1 \sim M, \forall l \in \mathcal{L} \\
 & \sum_{m=1}^M r_l(m) \leq \beta_l \log \left(1 + \frac{|h_{ds}^k|^2 \bar{p}_s / K}{\sigma_d^2} \right), \forall l = (s, d, k) \in \mathcal{L}^{wl} \\
 & \sum_{n \in I(l)} \beta_n = 1, \beta_l \in \{0, 1\}, \forall l, n \in \mathcal{L}^{wl}, \tag{29}
 \end{aligned}$$

(1) and (5).

Since the integer constraints on $\{\beta_l \mid \forall l \in \mathcal{L}^{wl}\}$ are also intractable, we relax it to $\beta_l \in [0, 1]$. In this way the problem becomes a large-scale LP, whose solution represents an upper bound value of problem (29).

In Fig. 3, we show the minimum rate performance of different algorithms when $\bar{p} = 20\text{dB}$ and $M = 5 \sim 30$. We observe that the minimum rate achieved by the N-MaxMin algorithm is more than twice of those achieved by the heuristic algorithms.

In the second set of numerical experiments, we evaluate the proposed N-MaxMin algorithm using different number of commodity pairs and different power budgets at the BSs. Here we use the same settings as in the previous experiment, except that all mobile users are interfered by the BSs within a distance of 800 meters, and that we set $\rho_2 = 0.005$ (resp. $\rho_2 = 0.001$) when $\bar{p} = 10\text{ dB}$ (resp. $\bar{p} = 20\text{ dB}$). The minimum rate performance for the N-MaxMin algorithm and the required number of inner iterations are plotted in Fig. 4. Due to the fact that the obtained $\{\mathbf{r}, \mathbf{p}\}$ is far from the stationary solution in the first few outer iterations, there is no need to complete Algorithm 1 at the very beginning. Hence, we limit the number of inner iterations to be no more than 500 for the first 5 outer iterations. After the early termination of the inner Algorithm 1, we use the obtained \mathbf{p} to update \mathbf{u} and \mathbf{w} by (10) and (11), respectively.

In Fig. 4(a)–(b), we see that when $\bar{p} = 10\text{ dB}$, the minimum rate converges at about the 10th outer iteration when the number of commodities is up to 30, while less than 500 inner iterations are needed per outer iteration. Moreover, after the 10th outer iteration, the number of inner ADMM iterations reaches below 100. In Fig. 4(c)–(d), the case with $\bar{p} = 20\text{dB}$ is considered. Clearly the required number of outer iterations is slightly more than that in the case of $\bar{p} = 10\text{dB}$, since the objective value and the feasible set are both larger. However, in all cases the algorithm still converges fairly quickly.

In the last set of numerical experiments, we demonstrate how parallel implementation can speed up Algorithm 1 considerably. To illustrate the benefit of parallelization, we consider a larger network (see Fig. 5 (b)) which is derived by merging two identical networks shown in Fig. 2 (a). The new network consists of 126 nodes (12 network routers and 114 BSs).

For simplicity, we removed all the wireless links, so constraints (2) and (4) of problem (6) are absent. This reduces problem (6) to a network flow problem (a very large linear program).

We implement Algorithm 1 using the Open MPI package, and compare its efficiency with the commercial LP solver, Gurobi [32]. For the Open MPI implementation, we use 4 computation cores for each basic BS set as illustrated in Fig. 5 (a), and use 1 additional computation core for all the network routers shown in Fig. 5 (b). Since we have two identical subnetworks (connected by a common set of routers), we have in total 9 computation cores. We choose $\rho_1 = 0.01$ and let the BSs serve as the destination nodes for commodities. Table IV compares the computation

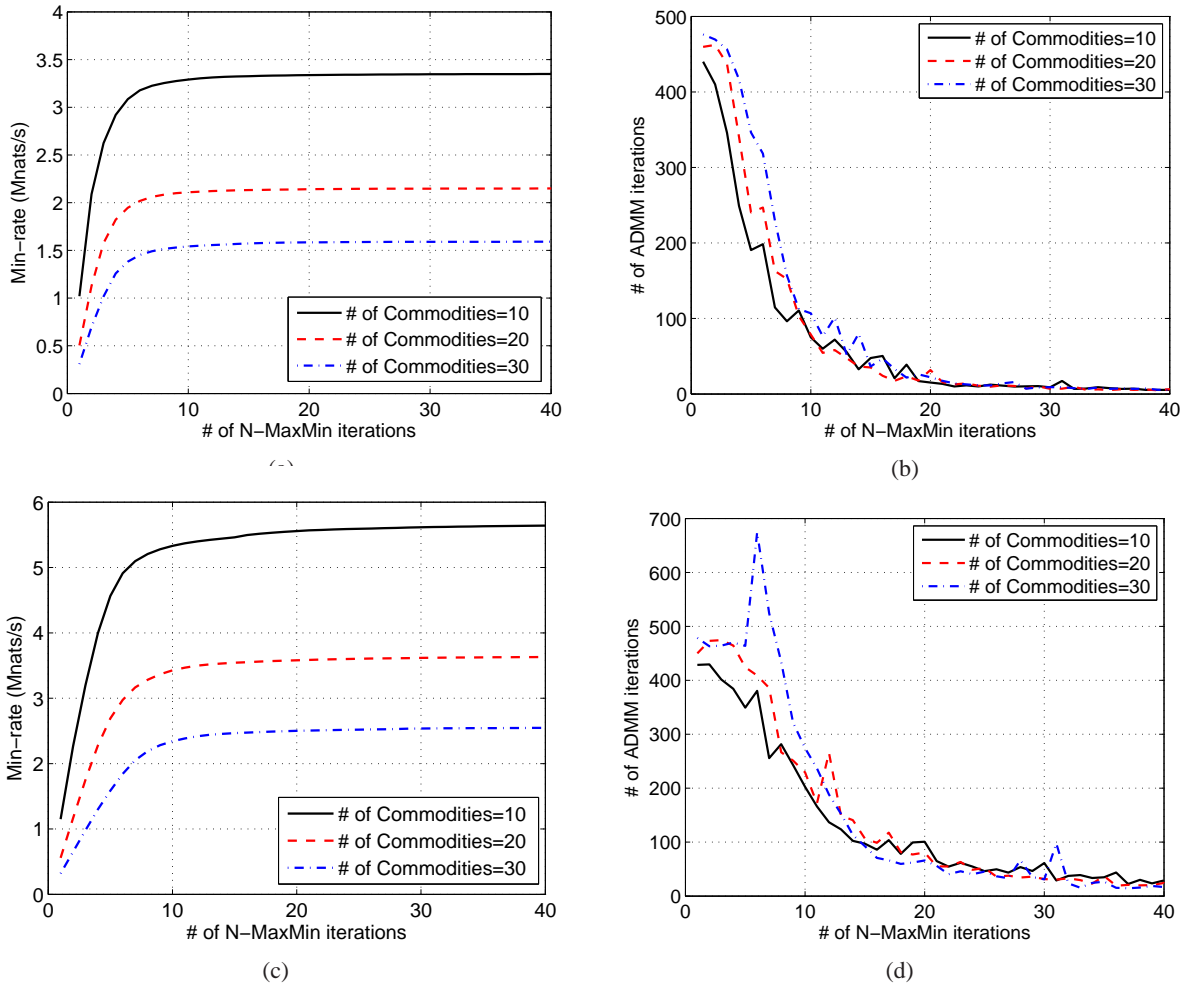


Fig. 4. The minimum rate performance and the required number of iterations for the proposed N-MaxMin algorithm. In [(a)(b)] $\bar{p} = 10\text{dB}$ and in [(c)(d)] $\bar{p} = 20\text{dB}$. In [(a)(c)], the obtained minimum rate versus the iterations of N-MaxMin is plotted. In [(b)(d)], the required number of inner ADMM iterations is plotted against the iteration for the outer N-MaxMin algorithm.

time required for different implementation of Algorithm 1 and that of Gurobi. We observe that parallel implementation of Algorithm 1 leads to more than 5 fold improvement in computation time computed on SunFire X4600 server with AMD Opteron 8356 2.3GHz CPUs. We also note that when the problem size increases, the performance of Gurobi becomes worse than that achieved by the parallel implementation of Algorithm 1. Thus, the proposed algorithm (implemented in parallel) appears to scale nicely to large problem sizes.

6. CONCLUDING REMARKS

In this paper, we have considered the joint backhaul traffic engineering and interference management problem for a SD-RAN. In the considered problem, the resources in both the

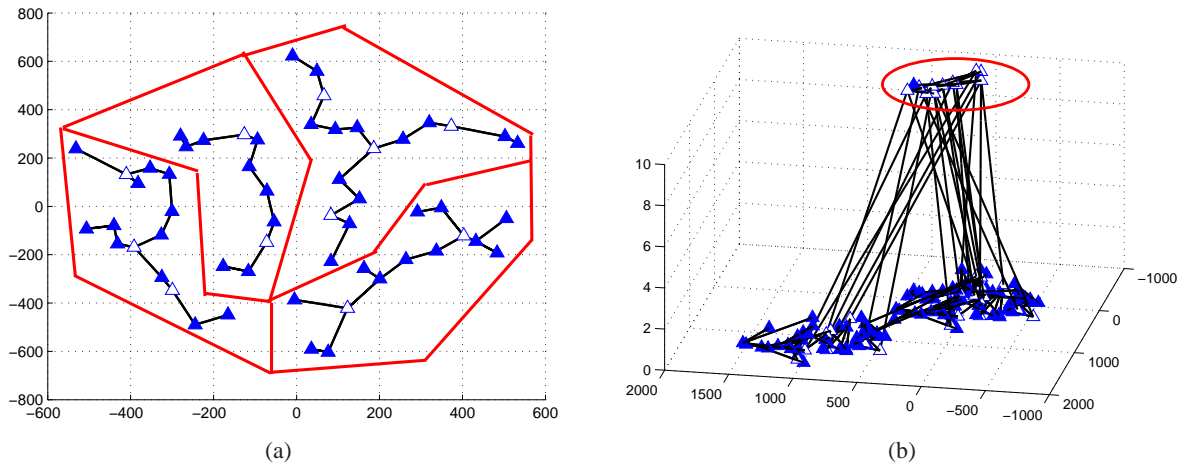


Fig. 5. The considered network consists of 114 BSs and 12 routers. Each computation core is responsible for one group of nodes shown in the figure. Fig. 5 (a) plots the locations and the connectivity of a single basic BS set (consists of 57 BSs). The solid triangles denote the BSs, which only connect to other BSs, and the hollow triangles denote the BSs serving the gateways that are connected to routers and other BSs. Fig. 5 (b) displays the connections between the BSs and routers.

# of Commodities	50	100	200	300
Time (s) (Sequential)	1.04	2.03	4.73	8.53
Time (s) (Parallel)	0.20	0.37	0.75	1.10
Time (s) (Gurobi)	0.20	0.64	1.65	2.51
# of Variables	1.4×10^4	2.9×10^4	5.8×10^4	8.7×10^4
# of Constraints	2.1×10^4	4.2×10^4	8.4×10^4	1.3×10^5

TABLE IV

COMPARISON OF COMPUTATION TIME USED BY DIFFERENT IMPLEMENTATIONS OF ALGORITHM 1 FOR THE ROUTING ONLY PROBLEM. THE SIZE OF THE PROBLEMS SOLVED ARE SPECIFIED USING A RANGE OF METRICS (TOTAL NUMBER OF COMMODITIES, VARIABLES AND CONSTRAINTS).

fixed backhaul links and the wireless radio access links are optimized. Although the problem is nonconvex, large-scale, and the optimization variables are coupled in various constraints, our proposed algorithm is capable of efficiently computing a high-quality solution in a distributed manner. Key to the efficiency of the proposed algorithm is the use of the well-known rate-MSE relationship, which helps transform the original problem into a form that is amenable to alternating optimization. In each iteration of the algorithm, two separate subproblems are solved, one admits a closed-form solution, while the other can be solved efficiently by using

the ADMM algorithm. The proposed algorithm is scalable to large networks since all its steps can be computed in closed-form independently and in parallel across all nodes of the network. Simulation results show that the proposed algorithm significantly outperforms heuristic algorithms in terms of the achieved min-rate. As a future work, we plan to investigate the use of stochastic WMMSE algorithm [33] to reduce the amount of channel state information.

APPENDIX

A. Proof of Theorem 1:

This proof follows a similar argument as in [25], so here we only provide the main steps of the proof. For the following discussion, we denote the KKT solutions of problem (6) as $\{\mathbf{r}^*, \mathbf{p}^*; \boldsymbol{\delta}^*, \boldsymbol{\theta}^*, \boldsymbol{\epsilon}^*, \boldsymbol{\kappa}^*\}$ where $\boldsymbol{\delta}^*, \boldsymbol{\theta}^*, \boldsymbol{\epsilon}^*$, and $\boldsymbol{\kappa}^*$ respectively denotes the corresponding Lagrangian dual variables for the nonnegativeness constraints $\{r \geq 0, r_m \geq r, r_l(m) \geq 0 \mid l \in \mathcal{L}, m = 1 \sim M\}$, as well as (1), (2), (4) and (5). For problem (8), the KKT solutions are similarly denoted as $\{\hat{\mathbf{r}}, \hat{\mathbf{p}}, \hat{\mathbf{u}}, \hat{\mathbf{w}}; \hat{\boldsymbol{\delta}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\epsilon}}, \hat{\boldsymbol{\kappa}}\}$, where $\hat{\boldsymbol{\delta}}$ now is the Lagrangian dual variables for constraints (1) and (9).

Step 1: If $\mathbf{x}^* \triangleq \{\mathbf{r}^*, \mathbf{p}^*; \boldsymbol{\delta}^*, \boldsymbol{\theta}^*, \boldsymbol{\epsilon}^*, \boldsymbol{\kappa}^*\}$ is an arbitrary KKT solution of problem (6), $\{\hat{\mathbf{r}}, \hat{\mathbf{p}}, \hat{\mathbf{u}}, \hat{\mathbf{w}}; \hat{\boldsymbol{\delta}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\epsilon}}, \hat{\boldsymbol{\kappa}}\}$ chosen as

$$\mathbf{y}^* \triangleq \{\mathbf{r}^*, \mathbf{p}^*, \mathbf{u}(\mathbf{p}^*), \mathbf{w}(\mathbf{p}^*); \boldsymbol{\delta}^*, \boldsymbol{\theta}^*, \boldsymbol{\epsilon}^*, \boldsymbol{\kappa}^*\}$$

is also a KKT solution of problem (8). The converse statement is also true. Here $\mathbf{u}(\mathbf{p}^*)$ and $\mathbf{w}(\mathbf{p}^*)$ are the \mathbf{u} and \mathbf{w} obtained by (10) and (11) for a given \mathbf{p}^* .

Since some of the constraints of problem (6) and problem (8) are the same, i.e., (1), (4), and (5), the corresponding feasibility and the complementary slackness conditions of these constraints are of the same form for both problems. Hence, if \mathbf{x}^* can satisfy these constraints for problem (6), \mathbf{y}^* can satisfy those of problem (8). Hence, we should only consider the

remaining KKT conditions given below. For problem (6), we have

$$-2\epsilon_s^* p_{ds}^{k^*} + \sum_{n=(s',d',k) \in \bar{I}(s,d,k)} \theta_{d's'}^{k^*} \nabla_{p_{ds}^k} \bar{r}_n(\mathbf{p}^*) = 0, \quad \forall l = (s, d, k) \in \mathcal{L}^{wl}, \quad (30a)$$

$$\delta^* + \sum_{m=1}^M \delta_m^* = 1, \quad (30b)$$

$$\delta_m^* + \kappa^{S(m)^*}(m) - \kappa^{D(m)^*}(m) = 0, \quad m = 1 \sim M, \quad (30c)$$

$$\delta_l^*(m) - \theta_l^* + \sum_{v:l \in \text{In}(v)} \kappa^{v^*}(m) - \sum_{v:l \in \text{Out}(v)} \kappa^{v^*}(m) = 0, \quad \forall v \in \mathcal{V}, \quad m = 1 \sim M, \quad (30d)$$

$$0 \leq \theta_l^* \perp \bar{r}_l(\mathbf{p}^*) - \sum_{m=1}^M r_l^*(m) \geq 0, \quad \forall l \in \mathcal{L}^{wl}. \quad (30e)$$

For problem (8), we have

$$-2\hat{\epsilon}_s \hat{p}_{ds}^k + \sum_{n=(s',d',k) \in I(s,d,k)} \hat{\theta}_{d's'}^k \nabla_{p_{ds}^k} E_n(\hat{u}_n, \hat{w}_n, \hat{\mathbf{p}}) = 0, \quad (31a)$$

$$\hat{\theta}_{ds}^k \nabla_{w_l} E_l(\hat{u}_l, \hat{w}_l, \hat{\mathbf{p}}) = 0, \quad (31b)$$

$$\hat{\theta}_{ds}^k \nabla_{w_l} E_l(\hat{u}_l, \hat{w}_l, \hat{\mathbf{p}}) = 0, \quad \forall l = (s, d, k) \in \mathcal{L}^{wl}, \quad (31c)$$

$$\hat{\delta} + \sum_{m=1}^M \hat{\delta}_m = 1, \quad (31d)$$

$$\hat{\delta}_m + \hat{\kappa}^{S(m)}(m) - \hat{\kappa}^{D(m)}(m) = 0, \quad m = 1 \sim M, \quad (31e)$$

$$\hat{\delta}_l(m) - \hat{\theta}_l + \sum_{v:l \in \text{In}(v)} \hat{\kappa}^v(m) - \sum_{v:l \in \text{Out}(v)} \hat{\kappa}^v(m) = 0, \quad \forall v \in \mathcal{V}, \quad m = 1 \sim M, \quad (31f)$$

$$0 \leq \hat{\theta}_l \perp E_l(\hat{u}_l, \hat{w}_l, \hat{\mathbf{p}}) - \sum_{m=1}^M \hat{r}_l(m) \geq 0, \quad \forall l \in \mathcal{L}^{wl}. \quad (31g)$$

Obviously, by comparing (30b)~(30d) and (31d)~(31f), we can conclude that \mathbf{y}^* can satisfy (31d)~(31f). For (31b) and (31c), by the optimality of (10) and (11), they are also true for \mathbf{y}^* . Moreover, since $\bar{r}_l(\mathbf{p}^*) = E_l(u_l(\mathbf{p}^*), w_l(\mathbf{p}^*), \mathbf{p}^*)$, it follows from Lemma 1 that (31g) can be satisfied with \mathbf{y}^* .

For the last KKT condition of problem (8), i.e., (31a), let us first split the Lagrange multiplier θ^* into two subsets

$$\mathcal{A} \triangleq \{l \mid \theta_l^* > 0, \forall l \in \mathcal{L}\}, \quad \bar{\mathcal{A}} \triangleq \{l \mid \theta_l^* = 0, \forall l \in \mathcal{L}\}.$$

Then by the same argument as Proposition 1 in [25], (31a) is also satisfied by \mathbf{y}^* . The reverse statement of step 1 can be argued similarly.

Step 2: Every global optimal solution of problem (6) corresponds to a global optimal solution of problem (8), and they achieve the same objective value.

To show this step, we recall that the network is connected and the link capacities are positive. It follows that the optimal value r^* must be strictly greater than 0. Hence, the corresponding Lagrangian dual variable δ^* is always 0 by the complementarity condition, and the KKT condition (30b) becomes $\sum_{m=1}^M \delta_m^* = 1$. The argument is the same for $\hat{\delta}$, so $\sum_{m=1}^M \hat{\delta}_m = 1$. With this fact, we can use the proof of Proposition 3 in [25] to show the desired result.

Step 3: The proposed alternating optimization method can converge to the KKT solutions of problem (6).

Given the results of previous two steps and by Theorem 2 of [25], the final convergence result is proved. \square

B. Derivation of Updating Steps of Algorithm 1

In this section, we go over Algorithm 1 step by step and explain each of its update procedure. For notational simplicity, we ignore the superscript indices.

1) Solving Step 3 for Algorithm 1: In this step, problem (22) is solved to update $\{\mathbf{r}, \hat{\mathbf{p}}\}$. This problem can be further decomposed over the variables $\{r, r_m, r_l(m) \mid m = 1 \sim M, \forall l \in \mathcal{L}^w\}$ and $\hat{\mathbf{p}} \cup \{r_l(m) \mid m = 1 \sim M, \forall l \in \mathcal{L}^{wl}\}$.

The first subblock only has to do with the wired links. A closer look at Step 3 of Algorithm 1 reveals that its optimization problem can be solved via two completely independent subproblems, one for variables $\{r, r_m \mid m = 1 \sim M\}$ and the other for $\{r_l(m) \mid m = 1 \sim M, \forall l \in \mathcal{L}^w\}$. In the following we consider the two problems separately.

(i) Subproblem for $\{r, r_m \mid m = 1 \sim M\}$: This step updates the current minimum flow rate among all commodities, and it can be mathematically expressed as

$$\begin{aligned} \max \quad & \frac{r}{2} - \frac{\rho_1}{2} \left(\hat{r} - r - \frac{\delta}{\rho_1} \right)^2 - \frac{\rho_1}{2} \sum_{m=1}^M \left[\left(\hat{r}_m^{S(m)} - r_m - \frac{\delta_m^{S(m)}}{\rho_1} \right)^2 + \left(\hat{r}_m^{D(m)} - r_m - \frac{\delta_m^{D(m)}}{\rho_1} \right)^2 \right] \\ \text{s.t.} \quad & r_m \geq r, \quad m = 1 \sim M, \quad r \geq 0. \end{aligned} \quad (32)$$

When r is fixed, the optimal $\{r_m^*\}_{m=1}^M$ of problem (32) can be obtained by the first-order

optimality condition as follows

$$r_m^* = \frac{1}{2} \left(\hat{r}_m^{S(m)} + \hat{r}_m^{D(m)} - \frac{\delta_m^{S(m)} + \delta_m^{D(m)}}{\rho_1} + \frac{\lambda_m^*}{\rho_1} \right), \quad m = 1 \sim M, \quad (33)$$

where $\{\lambda_m^* \geq 0\}$ are the optimal Lagrange dual variables for constraints $\{r_m \geq r\}$. Due to the complementarity condition, and the fact that r_m^* is an increasing function of λ_m^* , it follows that $\lambda_m^* > 0$ only if the equality holds for $r_m^* \geq r$. Thus, we can conclude

$$r_m^* = \max \left\{ r, \frac{1}{2} \left(\hat{r}_m^{S(m)} + \hat{r}_m^{D(m)} - \frac{\delta_m^{S(m)} + \delta_m^{D(m)}}{\rho_1} \right) \right\}. \quad (34)$$

After plugging the obtained r_m^* back to the objective function of problem (32), the gradient of the objective function with respect to r is given by

$$\frac{1}{2} + \rho_1 \left(\hat{r} - \frac{\delta}{\rho_1} \right) - \rho_1 \left\{ r + 2 \sum_{m: \frac{1}{2} \left(\hat{r}_m^{S(m)} + \hat{r}_m^{D(m)} - \frac{\delta_m^{S(m)} + \delta_m^{D(m)}}{\rho_1} \right) \leq r} \left[r - \frac{1}{2} \left(\hat{r}_m^{S(m)} + \hat{r}_m^{D(m)} - \frac{\delta_m^{S(m)} + \delta_m^{D(m)}}{\rho_1} \right) \right] \right\}. \quad (35)$$

Notice that the obtained derivative is a decreasing function for $r \geq 0$. Thus, the optimal $r^* = 0$ if (35) is no more than 0 with $r = 0$. Otherwise, r^* can be obtained through bisection procedure over $r \geq 0$ such that (35) equals 0.

(ii) Subproblem for $\{r_l(m) \mid m = 1 \sim M, \forall l \in \mathcal{L}^w\}$: It turns out that for this subset of variables, the corresponding updating procedure can be performed independently *over each link*. For each link $l = (s, d) \in \mathcal{L}^w$, the following optimization problem is solved

$$\begin{aligned} \min \quad & \sum_{m=1}^M \left[\left(\hat{r}_l^s(m) - r_l(m) - \frac{\delta_l^s(m)}{\rho_1} \right)^2 + \left(\hat{r}_l^d(m) - r_l(m) - \frac{\delta_l^d(m)}{\rho_1} \right)^2 \right] \\ \text{s.t.} \quad & \sum_{m=1}^M r_l(m) \leq C_l, \quad r_l(m) \geq 0, \quad m = 1 \sim M. \end{aligned} \quad (36)$$

The optimal solution $r_l^*(m)$, $m = 1 \sim M$, of problem (36) can be obtained by the first-order optimality condition

$$r_l(m)^* = \frac{1}{2} \left(\hat{r}_l^s(m) + \hat{r}_l^d(m) - \frac{\delta_l^s(m) + \delta_l^d(m)}{\rho_1} - \frac{\lambda_l^*}{2} \right)^+ \geq 0, \quad m = 1 \sim M \quad (37)$$

where λ_l^* is the optimal Lagrange dual variable of the capacity constraint on link l . Using the

complementarity condition and the fact that the left hand side of the capacity constraint is a decreasing function of λ_l^* , it follows that $\lambda_l^* = 0$ is true only if

$$\sum_{m=1}^M \frac{1}{2} \left(\hat{r}_l^s(m) + \hat{r}_l^d(m) - \frac{\delta_l^s(m) + \delta_l^d(m)}{\rho} \right)^+ \leq C_l.$$

Otherwise, λ_l^* should be chosen such that the capacity constraint is active, and this λ_l^* can be obtained by bisection procedure over $\lambda_l^* \geq 0$.

(iii) Subproblem for $\hat{\mathbf{p}} \cup \{r_l(m) \mid m = 1 \sim M, \forall l \in \mathcal{L}^{wl}\}$: The rest of variables are related only to the wireless links, and they are in fact decoupled across the wireless links. To be more specific, the problem for the wireless link $l = (s, d, k) \in \mathcal{L}^{wl}$ is shown below

$$\begin{aligned} \min \quad & \frac{\rho_1}{2} \sum_{m=1}^M \left[\left(\hat{r}_l^s(m) - r_l(m) - \frac{\delta_l^s(m)}{\rho_1} \right)^2 + \left(\hat{r}_l^d(m) - r_l(m) - \frac{\delta_l^d(m)}{\rho_1} \right)^2 \right] \\ & + \frac{\rho_2}{2} \sum_{n=(s',d',k) \in I(l)} \left(p_{d's'}^k - p_{ds,d's'}^k - \frac{\theta_{ln}}{\rho_2} \right)^2 \\ \text{s.t.} \quad & r_l(m) \geq 0, \quad m = 1 \sim M \\ & \sum_{m=1}^M r_l(m) \leq c_{1,l} + c_{2,l} p_{ds,ds}^k - \sum_{n=(s',d',k) \in I(l)} c_{3,ln} |p_{ds,d's'}^k|^2. \end{aligned} \quad (38)$$

The optimal solution of this problem, $\{r_l^*(m), p_{ds,d's'}^{k*} \mid m = 1 \sim M, (s', d', k) \in I(l)\}$, can be obtained by the first-order conditions below

$$r_l^*(m) = \frac{1}{2} \left(\hat{r}_l^s(m) + \hat{r}_l^d(m) - \frac{\delta_l^s(m) + \delta_l^d(m) + \lambda_l^*}{\rho_1} \right)^+, \quad m = 1 \sim M, \quad (39a)$$

$$p_{ds,ds}^{k*} = \frac{\rho_2(p_{ds}^k - \frac{\theta_{ll}}{\rho_2}) + \lambda_l^* c_{2,l}}{\rho_2 + 2\lambda_l^* c_{3,ll}}, \quad (39b)$$

$$p_{ds,d's'}^{k*} = \frac{\rho_2(p_{d's'}^k - \frac{\theta_{ln}}{\rho_2})}{\rho_2 + 2\lambda_l^* c_{3,ln}}, \quad \forall n = (s', d', k) \in I(l), \quad n \neq l. \quad (39c)$$

where λ_l^* is the optimal Lagrange dual variable for the rate-MSE constraint.

After plugging the obtained optimal solutions (39) into the rate-MSE constraint of problem (38), it can be observed that the left hand side of the constraint, $\sum_{m=1}^M r_l^*(m)$, is a decreasing function of λ_l^* . Furthermore, taking the gradient of the right hand side of the rate-MSE constraint

with respect to λ_l^* gives

$$\begin{aligned} & \frac{\partial(c_{1,l} + c_{2,l}p_{ds,ds}^{k^*} - \sum_{n=(s',d',k) \in I(l)} c_{3,ln} |p_{ds,d's'}^{k^*}|^2)}{\partial \lambda_l^*} \\ &= \frac{1}{\rho_2} \left[\frac{\left(c_{2,l} - 2c_{3,ln}(p_{ds}^{k^*} - \frac{\theta_{ln}}{\rho_2})\right)^2}{\left(1 + \frac{2}{\rho_2} \lambda_l^* c_{3,ln}\right)^3} + \sum_{n=(s',d',k) \in I(l) \setminus \{l\}} \frac{\left(2c_{3,ln}(p_{ds}^k - \frac{\theta_{ln}}{\rho_2})\right)^2}{\left(1 + \frac{2}{\rho_2} \lambda_l^* c_{3,ln}\right)^3} \right] \geq 0, \end{aligned} \quad (40)$$

where the nonnegativity is due to the fact that $c_{3,ln} \geq 0, \forall n$. Hence, the right hand side of the rate-MSE constraint is an increasing function of $\lambda_l^* \geq 0$. By the complementarity condition and the monotonicity of the rate-MSE constraint, the value of λ_l^* can be computed as follows: 1) $\lambda_l^* = 0$ if the rate-MSE constraint is satisfied with $\lambda_l^* = 0$; 2) otherwise, perform a bisection search to obtain the optimal λ_l^* . For the latter case, the search will terminate when the rate-MSE constraint is active, i.e., when equality holds true.

2) **Solving Step 4 for Algorithm 1:** The corresponding problem to update $\{\hat{\mathbf{r}}, \mathbf{p}\}$, i.e., step 4 of Algorithm 1, can be decomposed into two parts. The first part has to do with the flow rate conservation constraint with optimization variable $\hat{\mathbf{r}}$, and the second part has to do with \mathbf{p} .

The first part can again be separated into two independent subproblems, one for $\hat{\mathbf{r}}$ and another for the rest of the variables in $\hat{\mathbf{r}}$.

(i) **Subproblem for $\hat{\mathbf{r}}$:** The subproblem for variable $\hat{\mathbf{r}}$ is given by the following easy unconstrained quadratic optimization problem

$$\arg \max \frac{\hat{r}}{2} - \frac{\rho_1}{2} \left(\hat{r} - r - \frac{\delta}{\rho_1} \right)^2 = r + \frac{1 + 2\delta}{2\rho_1}. \quad (41)$$

(ii) **Subproblem for $\{\hat{r}_m^{S(m)}, \hat{r}_m^{D(m)}, \hat{r}_l^s(m), \hat{r}_l^d(m)\}$:** In this subproblem, the rest of the variables in $\hat{\mathbf{r}}$ are updated, subject to the conservation constraints of flow rate. As we have discussed before, the introduction of the auxiliary local optimization variables, i.e., (15), make this subproblem decoupled over each node $v \in \mathcal{V}$ and commodity m . As such, problem (23) decomposes into a series of simpler problems, one for each tuple (m, l, v)

$$\begin{aligned} \min & \sum_{l \in \text{In}(v) \cup \text{Out}(v)} \left(\hat{r}_l^v(m) - r_l(m) - \frac{\delta_l^v(m)}{\rho_1} \right)^2 + 1_{\{S(m), D(m)\}}(v) \left(\hat{r}_m^v - r_m - \frac{\delta_m^v}{\rho_1} \right)^2 \\ \text{s.t.} & \sum_{l \in \text{In}(v)} \hat{r}_l^v(m) + 1_{\{S(m)\}}(v) \hat{r}_m^v = \sum_{l \in \text{Out}(v)} \hat{r}_l^v(m) + 1_{\{D(m)\}}(v) \hat{r}_m^v. \end{aligned} \quad (42)$$

Since problem (42) has only one equality constraint, it admits a closed-form solution. In

particular, let us denote the optimal dual Lagrangian variable as $\lambda_v^*(m)$. Using the first-order optimality condition, the optimal solution for (42) is given by

$$\hat{r}_l^{v*}(m) = \begin{cases} r_l(m) + \frac{\delta_l^v(m)}{\rho_1} - \lambda_v^*(m), & l \in \text{Out}(v) \\ r_l(m) + \frac{\delta_l^v(m)}{\rho_1} + \lambda_v^*(m), & l \in \text{In}(v) \end{cases}, \quad (43)$$

and

$$\hat{r}_m^{v*} = \begin{cases} r_m + \frac{\delta_m^v}{\rho_1} - \lambda_v^*(m), & v \in D(m) \\ r_m + \frac{\delta_m^v}{\rho_1} + \lambda_v^*(m), & v \in S(m) \end{cases} \quad (44)$$

where

$$\lambda_v^*(m) = \left[\sum_{l \in \text{Out}(v)} \left(r_l(m) + \frac{\delta_l^v(m)}{\rho_1} \right) - \sum_{l \in \text{In}(v)} \left(r_l(m) + \frac{\delta_l^v(m)}{\rho_1} \right) + \left(r_m + \frac{\delta_m^v}{\rho_1} \right) \times \left(1_{\{D(m)\}}(v) - 1_{\{S(m)\}}(v) \right) \right] \left(|\text{In}(v) \cup \text{Out}(v)| + 1_{\{S(m), D(m)\}}(v) \right)^{-1}.$$

(iii) Subproblem for p: The remaining part is for optimization variable \mathbf{p} with power budget constraints, and this updating procedure can be decoupled over each BS. For BS $s \in \mathcal{B}$, the updating rule is,

$$\begin{aligned} \min \quad & \sum_{k=1}^K \sum_{\substack{d:l=(s,d,k) \in \mathcal{L}^{wl} \\ n=(s',d',k) \in \bar{I}(l)}} \left(p_{ds}^k - p_{d's',ds}^k - \frac{\theta_{nl}}{\rho_2} \right)^2 \\ \text{s.t.} \quad & \sum_{k=1}^K \sum_{d:l=(s,d,k) \in \mathcal{L}^{wl}} |p_{ds}^k|^2 \leq \bar{p}_s. \end{aligned} \quad (45)$$

By denoting the optimal Lagrange dual variable for the power constraint as $\lambda_s^* \geq 0$ and the optimal solution of problem (45) as $\{p_{ds}^{k*} \mid (s, d, k) \in \mathcal{L}^{wl}\}$, the first-order optimality condition can be expressed as

$$p_{ds}^{k*} = \frac{\sum_{(s',d',k) \in \bar{I}(s,d,k)} p_{d's',ds}^k + \frac{\theta_{(s',d',k),(s,d,k)}}{\rho_2}}{|\bar{I}(s,d,k)| + \lambda_s^*}, \quad k = 1 \sim K. \quad (46)$$

Since the following sum

$$\sum_{k=1}^K \sum_{d:l=(s,d,k) \in \mathcal{L}^{wl}} |p_{ds}^{k*}|^2$$

is a decreasing function of λ_s^* and the complementarity condition, it follows that $\lambda_s^* = 0$ if the

corresponding constraint is already satisfied

$$\sum_{k=1}^K \sum_{d:l=(s,d,k) \in \mathcal{L}^{wl}} |P_{ds}^{k*}|^2 \leq \bar{p}_s.$$

Otherwise, λ_s^* can be chosen via a bisection search to ensure the power budget constraint is active.

To summarize, all the steps in Algorithm 1 (including the updating of the Lagrange dual variables, (24)) can be efficiently computed.

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