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Min-sum soft symbol estimation for iterative MMSE detection

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ABSTRACT Soft symbol estimations (SSE) is the first process in soft interference cancellation minimum mean squared error (SIC-MMSE) detection for multiple-input multiple-output (MIMO) systems. SSE requires the sum of exhaustive multiplications of probability that occupies a non-negligible amount of the entire SIC-MMSE complexity. This paper proposes two approaches to reduce the complexity of SSE. The first is to find the approximation of SSE by investigating the estimation in the log-domain, which leads to a simple min-sum operation. The second is to approximate the hyperbolic tangent with low-order piecewise polynomial functions. These two approaches are then integrated. The simulation results show that the proposed methods do not degrade the performance in terms of bit error rate (BER), and that they can greatly reduce the number of multiplications, from $O(K2^K)$ to $O(K)$.

INDEX TERMS soft MIMO detection, SIC-MMSE, iterative detection, coded MIMO, soft symbol estimation

I. INTRODUCTION

TECHNIQUES to improve the error performance via iterative estimations have gained strong interest since the discovery of turbo codes [1]. The basic principle of improving the performance, almost up to the Shannon limit, lies in the iterative exchange of soft information between the two constituent decoders. As multiple antenna technologies have been rapidly developed, the principle of iterative estimation was extended to coded multiple-input multiple-output (MIMO) systems [2]. The symbol level detector can cooperatively work with the decoder in addition to the soft information exchange inside the decoder, and it has been termed joint iterative detection and decoding (JIDD).

Many studies focused on JIDD schemes utilizing minimum mean squared error (MMSE) detection because they have comparatively good performance and complexity trade-off [3]- [7]. The requirement of the MMSE method in the JIDD is the capability to handle soft information, which is attained by estimating the soft symbol values and processing them during the MMSE filtering process [5]. Therefore, this method has been termed the soft interference cancellation MMSE (SIC-MMSE) method. In the earlier stage of the studies on JIDD with SIC-MMSE schemes, studies were often focused on compensating for the performance degradation compared to JIDD with maximum likelihood (ML) detection, either by utilizing additional *a posteriori* information or additional loops [6] [7]. On the other hand, attempts were also made to reduce the computational complexity [4].

The introduction of a large-scale or massive MIMO system attracted attention to reducing the complexity of SIC-MMSE-based JIDD schemes [8]- [11]. Even though the MMSE-based scheme reduces the computational complexity compared to the ML detection, the complexity remains too high for practical implementation in massive MIMO systems. The previous studies mostly attempted to reduce the complexity of matrix operations by considering the greatly increased size of the filtering matrix. The matrix inversion process during the MMSE filtering process was approximated by either iteration-based or series expansion methods. These approximation methods, however, are only valid when the channel hardening is applicable, such as in the massive MIMO channel.

In addition, proposals were presented to reduce the complexity of the soft symbol estimation (SSE) process during SIC-MMSE detection. The soft symbol values used in the SIC-MMSE process are estimated in the form of statistical average values, and the estimation requires exhaustive probability multiplications and an averaging process. The computational burden of SSE occupies a non-negligible amount of total computational complexity [3] [12]. Therefore, attempts were made to reduce the complexity of SSE [3] [4] [7]. In addition, a compact closed form of SSE for 2^K quadrature amplitude modulation (QAM) and 2^K phase shift keying (PSK) were proposed [12].

On the other hand, it was found that log-domain probability estimation can reduce the computational complexity of soft iterative decoding of low density parity check (LDPC) code [13]–[15]. In the conventional belief propagation (BP) decoding algorithm, the check-node update computation requires the multiplication of multiple hyperbolic tangent terms [16]. Log-domain approximation of this computation, which has been termed min-sum algorithm, requires only simple binary operations [13]. In addition, the hyperbolic tangent function was approximated to a piecewise linear function, and a look-up table was used for sum-product decoding algorithm of LDPC code [16].

In this paper, we reduce the computational complexity of SSE by using two different approaches. The first is to investigate the statistical average values in the log-domain to eliminate multiplication operations, and we show that the multiplication can be realized with a simple $\min(\cdot)$ operation. Eventually, the SSE can be made by min-sum operations with reduced candidate symbols by combining the approach in [12]. The second approach is to approximate the hyperbolic tangent function by using simple low-order polynomial functions, and we propose three candidates. Finally, we combine these two approaches, and present a simple formula for SSE which can greatly reduce the computational complexity.

The rest of the paper is organized as follows. In section II, we briefly present the basic principle of JIDD with the SIC-MMSE method, and then describe the complexity-reduced SSE for SIC-MMSE presented in [12]. Section III presents the proposed methods. After presenting our two complexity-reduced approaches respectively, we combine them. We present the bit error rate (BER) simulation results of various SIC-MMSE schemes in Section IV, and demonstrate that the proposed schemes produce almost the same performance as the conventional schemes. We also compare the computational complexities of estimating SSE, and prove the greatly reduced complexity in the proposed schemes. Finally, this paper is concluded in Section V.

Notation: Bold lowercase letters represent vectors, while bold uppercase letters denote matrices. $(\cdot)^H$ of a matrix denotes Hermitian transpose. $|\cdot|$ denotes the absolute value of a real number, while $||\cdot||$ denotes the norm of a complex number. $\bar{\cdot}$ and $\mathrm{Var}[\cdot]$ denote a statistical average (or expected value) and variance operators, respectively.

II. RELATED WORKS

A. JIDD WITH SIC-MMSE

JIDD can be applied to a coded MIMO system equipped with the soft MIMO detector and channel decoder which interactively exchange soft information with each other [5] [7]. Assuming N_t transmit and N_r receive antennas, respectively, the information symbol vector, $\mathbf{s}$ consisting of $N_t \times 1$ 2^K-ary modulation symbols, is transmitted. Afterwards, the $N_r \times 1$ signal vector $\mathbf{y}$ is received as follows:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ denotes an $N_r \times N_t$ channel matrix and $\mathbf{n}$ denotes an $N_r \times 1$ complex additive white Gaussian noise vector in which each element has 0 mean and variance of σ^2. We assume that the transmit antennas equally share the energy and that the receiver knows the channel coefficients.

At the receiver, the SIC-MMSE detector estimates the transmitted symbol by using a soft MMSE filtering process. The estimated symbol at the ith layer can be represented as follows:

$$\hat{s}_i = \mathbf{w}_i \hat{\mathbf{y}}_i, \tag{2}$$

where $\mathbf{w}_i$ is a row vector of the soft MMSE filtering matrix $\mathbf{W}$ and $\hat{\mathbf{y}}_i$ denotes the ith received symbol vector after interference cancellation.

Soft information is embedded in the form of the variance of candidate symbols, inside $\mathbf{W}$. Therefore, SSE is performed to form $\mathbf{W}$. The SIC-MMSE detector first estimates the expected value, i.e., the statistical average of the ith transmitted symbol as follows:

$$\overline{s_i} = \sum_{a \in O} \frac{a}{2^K} \prod_{k=1}^{K} \left(1 + \tilde{x}_{i,k} \tanh\left(\frac{L^s_{i,k}}{2}\right)\right), \tag{3}$$

where a is a constellation symbol of O, $\tilde{x}_{i,k}$ is considered to be -1 and 1 when the kth bit of a is 0 and 1, respectively, and

$$L^s_{i,k} = L(x_{i,k}) + L^d_a(x_{i,k}), \tag{4}$$

where $L(x_{i,k})$ denotes the soft information estimated by the detector, and $L^d_a(x_{i,k})$ is the *a priori* soft information from the decoder. $L(x_{i,k})$ exists only when the soft MMSE detector has self-iteration and $L^d_a(x_{i,k})$ is set to zero at the initial stage.

The variance of the candidate symbols $\mathrm{Var}[s_i]$ can be estimated by using the first and second moments of s_i as follows:

$$\mathrm{Var}[s_i] = \overline{||s_i||^2} - ||\overline{s_i}||^2, \tag{5}$$

where

$$\overline{||s_i||^2} = \sum_{a \in O} \frac{||a||^2}{2^K} \prod_{k=1}^{K} \left(1 + \tilde{x}_{i,k} \tanh\left(\frac{L^s_{i,k}}{2}\right)\right). \tag{6}$$

The soft MMSE filtering matrix $\mathbf{W}$ can be formed via two approaches. The first is to utilize a layer dependent matrix value. The MMSE filter utilized at the ith layer of the soft detection, $\mathbf{w}_i$ is estimated as follows [6]:

$$\mathbf{w}_i = \left((\mathbf{H}\mathbf{\Sigma}_i\mathbf{H}^H + \sigma^2\mathbf{I}_{N_r})^{-1}\mathbf{h}_i\right)^H, \tag{7}$$

where $\mathbf{\Sigma}_i$ is a diagonal matrix containing the *a priori* information at the ith layer, and can be found using

$$\mathbf{\Sigma}_i = \mathrm{diag}\{\mathrm{Var}[s_1], \cdots, \mathrm{Var}[s_{i-1}], 1, \mathrm{Var}[s_{i+1}], \cdots, \mathrm{Var}[s_{N_t}]\}. \tag{8}$$

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The second approach is to use a layer-independent soft MMSE filtering matrix in order to reduce the complexity by utilizing a universal filtering matrix across the layer. The filtering matrix for this approach was defined as follows [5]:

$$\mathbf{W} = \left(\mathbf{H}^H \mathbf{H} \boldsymbol{\Sigma} + \sigma^2 \mathbf{I}_{N_t}\right)^{-1} \mathbf{H}^H, \tag{9}$$

and

$$\boldsymbol{\Sigma} = \text{diag}\{\text{Var}[s_1], \text{Var}[s_2], ..., \text{Var}[s_{N_t}]\}. \tag{10}$$

It has been reported that the first approach can be replaced with the second approach without loss in terms of error-rate performance [5]. Later, it was shown that the layer-independent method in (9) requires additional power of 0.15 dB compared to the method in (7) at BER of 10^{-4}. Despite the additional power requirement of 0.15 dB, the layer-independent soft MMSE filtering matrix required 4 times lower complexity for matrix inversion [17].

B. COMPLEXITY-REDUCED SSE

In soft MMSE detection, the first and second moments of s_i need to be estimated, which usually requires exhaustive probability multiplication and addition operations as in (3) and (6). Therefore, our previous research proposed a compact closed form of equations to reduce the computational complexity of SSE by using symmetric constellations of Gray-coded QAM and PSK [12]. Exhaustive estimation of SSE requires complexity of $O(K2^K)$, and the compact proposal reduces the complexity to $O(K^2)$ and $O((K-2)2^{K-2})$ for QAM and PSK, respectively. Here, we summarize the result for QAM in order to apply it to our proposed scheme in Section III.

Without loss of generality, we can assume that the information bits modulated into real and imaginary parts of a symbol are independent for QAM. Therefore, the real and imaginary parts of (3) and (6) can be estimated separately with different sets of bits. In other words, $\overline{s_i} = \Re(\overline{s_i}) + j\Im(\overline{s_i})$. where $j = \sqrt{-1}$, $\Re(\overline{s_i})$ and $\Im(\overline{s_i})$ are the real and imaginary parts of $\overline{s_i}$. With this decomposition into real and imaginary parts, the number of candidate symbols are reduced from 2^K to $2^{K/2}$ for the real and imaginary parts, respectively. Figure 1 shows this concept. It shows that $\Re(\overline{s_i})$ and $\Im(\overline{s_i})$ are the same as the average values estimated for $2^{K/2}$ candidates symbols which are projection of the all 2^K candidates symbols on to real and imaginary axes, respectively. Therefore,

$$\begin{aligned}\Re(\overline{s_i}) &= \sum_{a\in O} \frac{\Re(a)}{2^K} \prod_{k=1}^{K} \left(1 + \tilde{x}_{i,k} \tanh\left(\frac{L^s_{i,k}}{2}\right)\right) \\ &= \sum_{\Re(a)\in O^R} \frac{\Re(a)}{2^{K/2}} \prod_{k\in\kappa^R} \left(1 + \tilde{x}_{i,k} \tanh\left(\frac{L^s_{i,k}}{2}\right)\right),\end{aligned} \tag{11}$$

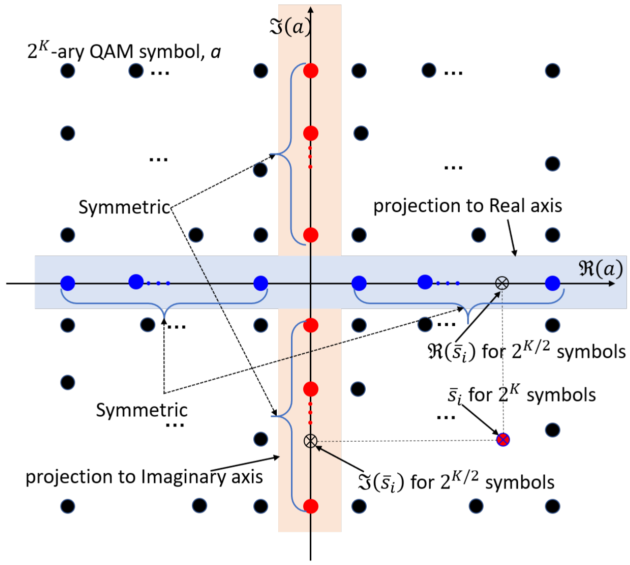

FIGURE 1. Concept of decomposing $\overline{s_i}$ into $\Re(\overline{s_i})$ and $\Im(\overline{s_i})$ for 2^K-QAM.

where $O^R = \{\Re(a)|a \in O\}$, and κ^R represents a set of bit indices determining the amplitudes of the real axis. Likewise,

$$\Im(\overline{s_i}) = \sum_{\Im(a)\in O^I} \frac{\Im(a)}{2^{K/2}} \prod_{k\in\kappa^I} \left(1 + \tilde{x}_{i,k} \tanh\left(\frac{L^s_{i,k}}{2}\right)\right), \tag{12}$$

where $O^I = \{\Im(a)|a \in O\}$, and κ^I represent a set of bit indices determining the amplitudes of the imaginary axis.

Additionally, a Gray coded QAM always has a symmetric symbol constellation, as represented in Figure 1. Bit mapping is symmetric with respect to axes, and we can usually find recursive symmetric characteristics. Utilization of the recursive symmetric bit mapping for a 2^K-QAM symbol constellation, such as in [18] led to the following simplification [12]:

$$\begin{aligned}\Re(\overline{s_i}) &= A \sum_{\alpha=1}^{\frac{K}{2}} (-1)^\alpha 2^{\frac{K}{2}-\alpha} \rho_\alpha^{R,i}, \\ \Im(\overline{s_i}) &= A \sum_{\alpha=1}^{\frac{K}{2}} (-1)^\alpha 2^{\frac{K}{2}-\alpha} \rho_\alpha^{I,i},\end{aligned} \tag{13}$$

where A is one half of the minimum distance, and

$$\begin{aligned}\rho_\alpha^{R,i} &= \prod_{\beta=1}^{\alpha} \tanh\left(\frac{L^s_{i,2\beta-1}}{2}\right), \\ \rho_\alpha^{I,i} &= \prod_{\beta=1}^{\alpha} \tanh\left(\frac{L^s_{i,2\beta}}{2}\right).\end{aligned} \tag{14}$$

Similar to the decomposition of $\overline{s_i}$, we can also decompose the estimation of the second moment in to real and imaginary

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parts, i.e., $\overline{||s_i||^2} = \overline{\Re(s_i)^2} + \overline{\Im(s_i)^2}$, and this lead to

$$\overline{\Re(s_i)^2} = \sum_{\Re(a)\in O^R} \frac{\Re(a)^2}{2^{K/2}} \prod_{k\in\kappa^R} \left(1 + \tilde{x}_{i,k} \tanh\left(\frac{L^s_{i,k}}{2}\right)\right),$$
$$\overline{\Im(s_i)^2} = \sum_{\Im(a)\in O^I} \frac{\Im(a)^2}{2^{K/2}} \prod_{k\in\kappa^I} \left(1 + \tilde{x}_{i,k} \tanh\left(\frac{L^s_{i,k}}{2}\right)\right), \tag{15}$$

If we expand (15) using the symmetric 2^K-QAM symbol constellation, then we have the following simplified estimation [12].

$$\overline{\Re(s_i)^2} = A^2 \left(\mu_0 + \sum_{\alpha=1}^{\frac{K}{2}-1} \sum_{\beta=1}^{\frac{K}{2}-\alpha} \mu_{\alpha,\beta}\varphi^{R,i}_{\alpha,\beta}\right),$$
$$\overline{\Im(s_i)^2} = A^2 \left(\mu_0 + \sum_{\alpha=1}^{\frac{K}{2}-1} \sum_{\beta=1}^{\frac{K}{2}-\alpha} \mu_{\alpha,\beta}\varphi^{I,i}_{\alpha,\beta}\right), \tag{16}$$

where

$$\mu_0 = \frac{(2^K-1)}{3},\ \ \mu_{\alpha,\beta} = (-1)^\beta 2^{K-\beta-2\alpha+1},$$

$$\varphi^{R,i}_{\alpha,\beta} = \prod_{\gamma=1}^{\beta} \tanh\left(\frac{L^s_{i,2(\alpha+\gamma)-1}}{2}\right),$$
$$\varphi^{I,i}_{\alpha,\beta} = \prod_{\gamma=1}^{\beta} \tanh\left(\frac{L^s_{i,2(\alpha+\gamma)}}{2}\right). \tag{17}$$

III. PROPOSED COMPLEXITY REDUCTION

Even though the complexity-reduced SSE described in Section II-B reduces the number of candidates, in $\prod$ operations, it still requires multiplications and hyperbolic tangent operations. In this section, we present new methods which tackle the complexity reduction problem with two approaches that differ from the conventional methods. First, we eliminate $\prod$ operations by investigating the statistical average values in the log-domain. Section III-A presents this, and shows that the product of the hyperbolic tangent terms can be replaced with a simple minimum operation, and eventually the SSE can be realized by a min-sum algorithm. Second, we replace the hyperbolic tangent function with a few piecewise low-order polynomial functions in Section III-B. Finally, Section III-C presents the integrated results of these two approaches.

A. APPROACH 1: LOG-DOMAIN INVESTIGATION OF STATISTICAL AVERAGE VALUES

The critical complexity burden to estimate $\overline{s_i}$ and $\overline{||s_i||^2}$ is in the product operations of hyperbolic tangent terms. In the first approach, the complexity is reduced by eliminating the product operations, $\prod$, by investigating the product of two hyperbolic tangents in the log-domain.

We start our derivation by considering the multiplication of two hyperbolic tangent terms, i.e., $\tanh(p)\tanh(q)$, where p and q are real numbers. The hyperbolic tangent function is a monotonically increasing function passing through the origin, and thus the signs of $\tanh(p)$ and $\tanh(q)$ are the same as those of p and q, respectively, i.e., $\text{sgn}(\tanh(p)) = \text{sgn}(p)$ and $|\tanh(p)| = \tanh(|p|)$. With these properties, the multiplication of two hyperbolic tangent terms can be expanded as follows:

$$\begin{aligned}\tanh(p)\tanh(q) &= \text{sgn}(p)\text{sgn}(q)|\tanh(p)||\tanh(q)| \\ &= \text{sgn}(p)\text{sgn}(q)\tanh(|p|)\tanh(|q|).\end{aligned} \tag{18}$$

We re-express $|\tanh(p)||\tanh(q)|$ to derive its approximation, as follows:

$$\begin{aligned}&|\tanh(p)||\tanh(q)| \\ &= \tanh\left[\tanh^{-1}\left(\tanh(|p|)\tanh(|q|)\right)\right].\end{aligned} \tag{19}$$

Because $\tanh^{-1}(x) = 1/2\log((1+x)/(1-x))$, (19) can be expanded as follows:

$$\begin{aligned}&|\tanh(p)||\tanh(q)| \\ &= \tanh\left[\frac{1}{2}\log\left(\frac{1+\tanh(|p|)\tanh(|q|)}{1-\tanh(|p|)\tanh(|q|)}\right)\right].\end{aligned} \tag{20}$$

By inserting $\tanh(x) = (e^{2x}-1)/(e^{2x}+1)$, into the above, we have the following derivation

$$\begin{aligned}&|\tanh(p)||\tanh(q)| = \tanh(|p|)\tanh(|q|) \\ &= \tanh\left[\frac{1}{2}\log\left(\frac{1+\frac{e^{2|p|}-1}{e^{2|p|}+1}\times\frac{e^{2|q|}-1}{e^{2|q|}+1}}{1-\frac{e^{2|p|}-1}{e^{2|p|}+1}\times\frac{e^{2|q|}-1}{e^{2|q|}+1}}\right)\right] \\ &= \tanh\left[\frac{1}{2}\log\left(\frac{1+\frac{e^{2(|p|+|q|)}-e^{2|p|}-e^{2|q|}+1}{e^{2(|p|+|q|)}+e^{2|p|}+e^{2|q|}+1}}{1-\frac{e^{2(|p|+|q|)}-e^{2|p|}-e^{2|q|}+1}{e^{2(|p|+|q|)}+e^{2|p|}+e^{2|q|}+1}}\right)\right] \\ &= \tanh\left[\frac{1}{2}\log\left(\frac{e^{2(|p|+|q|)}+1}{e^{2|p|}+e^{2|q|}}\right)\right] \\ &= \tanh\left[\frac{1}{2}\log\left(e^{2(|p|+|q|)}+e^0\right)-\frac{1}{2}\log\left(e^{2|p|}+e^{2|q|}\right)\right].\end{aligned} \tag{21}$$

Next, we apply a popular approximation, $\log(\sum_i e^{x_i}) \approx \max_i(x_i)$, then

$$\begin{aligned}&|\tanh(p)||\tanh(q)| \\ &\approx \tanh\left[\frac{1}{2}\max(2(|p|+|q|),0)-\frac{1}{2}\max(2|p|,2|q|)\right] \\ &= \tanh[\max(|p|+|q|,0)-\max(|p|,|q|)] \\ &= \tanh[|p|+|q|-\max(|p|,|q|)] \\ &= \tanh[\min(|p|,|q|)].\end{aligned} \tag{22}$$

Finally, by reversing back the property of $\tanh(\cdot)$ used in (18), we have

$$\begin{aligned}|\tanh(p)||\tanh(q)| &\approx \min\left(\tanh(|p|),\tanh(|q|)\right) \\ &= \min\left(|\tanh(p)|,|\tanh(q)|\right).\end{aligned} \tag{23}$$

If we expand the above results to a product of multiple hyperbolic tangent terms, i.e., $\prod_{i=1}^{N} \tanh(x_i)$, then we have the following approximation result.

$$\prod_{i=1}^{N} \tanh(x_i) = \left(\prod_{i=1}^{N} \mathrm{sgn}(x_i)\right)\left(\prod_{i=1}^{N} |\tanh(x_i)|\right) \approx \left(\prod_{i=1}^{N} \mathrm{sgn}(x_i)\right)\left(\min_{i=1}^{N} |\tanh(x_i)|\right). \quad (24)$$

Recalling that the product of sign can be implemented with a simple binary exclusive-or operation, then we find that the complexity of estimating $\prod_{i=1}^{N} \tanh(x_i)$ is now reduced to a simple minimum operation. Therefore, inserting (24) into (13)-(17) will lead to min-sum operations for SSE.

B. APPROACH 2: APPROXIMATION OF HYPERBOLIC TANGENT FUNCTION

Exact estimation of $\tanh(\cdot)$ value requires computational complexity, at least one exponential value estimation and multiple additions and a division. In this section, we approximate its value by using three simple piecewise functions. Even though there have been attempts to apply piecewise functions to implementations of exponential values, our proposal here is to find the optimum fit to the $\tanh(\cdot)$ especially for soft MMSE detection. Therefore, the results provided in this section can play an important role when implementing soft MMSE detection for coded MIMO system. We apply first-, second-, and third-order piecewise polynomial functions, $f^{(1)}(x)$, $f^{(2)}(x)$, and $f^{(3)}(x)$, respectively.

For the linear function, $f^{(1)}(x)$, we divide the positive and negative parts into three sections, respectively, On the other hand, we divide the positive and negative parts into two sections, for $f^{(2)}(x)$ and $f^{(3)}(x)$.

Our purpose is to approximate $\tanh(L^s_{i,k}/2)$ used for SSE. For this, we define $f^{(1)}(x)$ to $f^{(3)}(x)$ as follows:

$$\tanh\left(\frac{x}{2}\right) \approx f^{(1)}(x) \triangleq \begin{cases} \mathrm{sgn}(x), & |x| \geq T_2^{(1)}, \\ c_2^{(1)}x + \mathrm{sgn}(x)c_0^{(1)}, & T_1^{(1)} < |x| < T_2^{(1)}, \\ c_1^{(1)}x, & \text{else}, \end{cases} \quad (25)$$

where

$$T_1^{(1)} = \frac{c_0^{(1)}}{c_1^{(1)} - c_2^{(1)}}, \; T_2^{(1)} = \frac{1 - c_0^{(1)}}{c_2^{(1)}}. \quad (26)$$

In addition,

$$\tanh\left(\frac{x}{2}\right) \approx f^{(2)}(x) \triangleq \begin{cases} \mathrm{sgn}(x), & |x| \geq T^{(2)}, \\ \mathrm{sgn}(x)\left(c_2^{(2)}x^2 + c_1^{(2)}|x| + c_0^{(2)}\right), & \text{else}, \end{cases} \quad (27)$$

where $T^{(2)}$ is the threshold value to divide the piecewise regions and it is set for the second order polynomial value to be 1, that is,

$$T^{(2)} = \min(|x|) \text{ s.t. } c_2^{(2)}x^2 + c_1^{(2)}|x| + c_0^{(2)} = 1. \quad (28)$$

$$\tanh\left(\frac{x}{2}\right) \approx f^{(3)}(x) \triangleq \begin{cases} \mathrm{sgn}(x), & |x| \geq T^{(3)}, \\ \mathrm{sgn}(x)\left(c_3^{(3)}|x|^3 + c_2^{(3)}x^2 + c_1^{(3)}|x| + c_0^{(3)}\right), & \text{else}, \end{cases} \quad (29)$$

where $T^{(3)}$ is set to be as follows:

$$T^{(3)} = \min(|x|) \text{ s.t. } c_3^{(3)}|x|^3 + c_2^{(3)}x^2 + c_1^{(3)}|x| + c_0^{(3)} = 1. \quad (30)$$

The optimum values of coefficients, $c_j^{(l)}$ for each $f^{(l)}(x)$ can be found by minimizing the errors when estimating operations in (14) and (17), which are used for soft MMSE detection. Even though all piecewise functions, $f^{(l)}(x)$s are eventually functions of x, the arguments of the following minimizing problem will be $c_j^{(l)}$. For each $f^{(l)}(x)$, we find a set of the optimum coefficients, $\mathbf{C}^{(l)}$ satisfying the following.

$$\mathbf{C}^{(l)} = \arg\min_{c_j^{(l)} \in \mathbf{C}^{(l)}} \left(\frac{1}{n}\sum_{p=1}^{n}\left[\left(f^{(l)}(x_p)\right)^{\frac{K}{2}} - \left(\tanh\left(\frac{x_p}{2}\right)\right)^{\frac{K}{2}}\right]^2\right), \quad (31)$$

where n is the number of sampled values of x, x_ps used in (31). To find the optimum solution of the above least mean squared (LMS) problem, we set the range of x in (31) to $|x| < x_{max}$. Because $\tanh(\pm\infty) = \pm 1$, we set x_{max} for efficiency of running the optimization program in (31) as well as for minimizing the mean squared error (MSE). After we found that $\tanh(x/2) = 1 - \varepsilon$, where $\varepsilon \leq 5 \times 10^{-3}$ for $x \geq 6$, x_{max} was set to 6. It was confirmed that utilization of x_{max} greater than 6 did not alter the solution of (31), i.e., the coefficients found in Table 1. In addition, the increment of x, in (31), i.e., $x_p - x_{p-1}$ was set to 0.1. In our simulation, an increment value less than 0.1 does not alter the solution. Because the above optimization is a function of K and therefore the optimum values of $c_j^{(l)}$s are dependent on K.

Table 1 lists the optimum values found for $f^{(l)}(x)$ with 3-digit precision. In addition, $\epsilon^{(l)}$ denotes the MSE found by using the optimum coefficient values. Our results of the optimization process in (31) revealed that $c_j^{(l)}$s for $f^{(1)}(x)$ and $f^{(2)}(x)$ are different across K values from 6 to 12, and further that the threshold values to fix $f^{(l)}(x) = \mathrm{sgn}(x)$ are increased with increment of K for $f^{(1)}(x)$ and $f^{(2)}(x)$ because the error produced near the threshold value regions becomes more important as K increases. Unlike $f^{(1)}(x)$ and $f^{(2)}(x)$, coefficients with 3-digit precision for $f^{(3)}(x)$ remain constant. This is confirmed by the fact that $\epsilon^{(3)}$ is much smaller than $\epsilon^{(1)}$ and $\epsilon^{(2)}$. Figure 2 compares the estimation results of $\tanh(x/2), x \geq 0$, using various methods. In the legend, "precise estimation" indicates the estimation result using a computer built-in function, tanh.

TABLE 1. Optimum values of coefficients for $f^{(l)}(x)$ and mean squared error.

K	6	8	10	12
$c_0^{(1)}$	0.680	0.720	0.720	0.740
$c_1^{(1)}$	0.400	0.38	0.370	0.360
$c_2^{(1)}$	0.070	0.060	0.060	0.055
$T_1^{(1)}$	2.061	2.250	2.323	2.426
$T_2^{(1)}$	4.571	4.667	4.667	4.727
$\epsilon^{(1)}\ (\times 10^3)$	0.603	0.744	0.851	0.934
$c_0^{(2)}$	0.036	0.037	0.033	0.042
$c_1^{(2)}$	0.448	0.439	0.431	0.420
$c_2^{(2)}$	-0.052	-0.050	-0.048	-0.046
$T^{(2)}$	4.441	4.500	4.592	4.688
$\epsilon^{(2)}\ (\times 10^3)$	1.173	1.644	2.124	2.504
$c_0^{(3)}$	-0.017			
$c_1^{(3)}$	0.590			
$c_2^{(3)}$	-0.118			
$c_3^{(3)}$	0.008			
$T^{(3)}$	5.917			
$\epsilon^{(3)}\ (\times 10^3)$	0.025	0.041	0.058	0.074

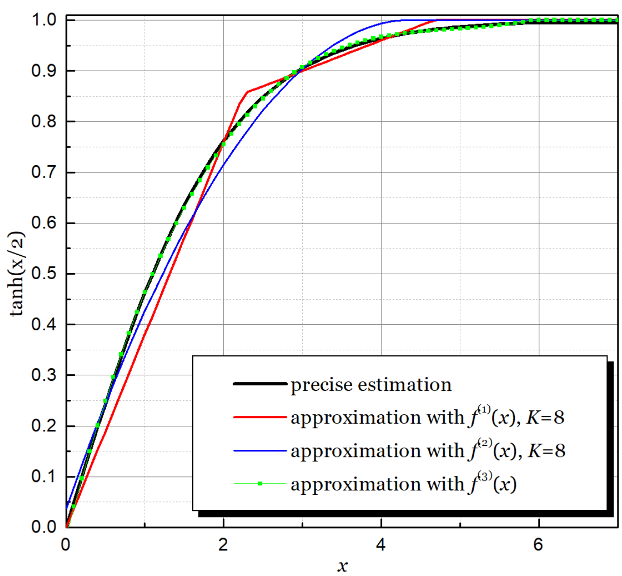

FIGURE 2. Approximation of $\tanh(x/2)$ with piecewise functions, $x \geq 0$.

By using the approximated functions to the $\tanh(\cdot)$, (14) can be approximated as follows:

$$\rho_\alpha^{R,i} \approx \prod_{\beta=1}^{\alpha} f^{(l)}\left(L_{i,2\beta-1}^s\right),$$
$$\rho_\alpha^{I,i} \approx \prod_{\beta=1}^{\alpha} f^{(l)}\left(L_{i,2\beta}^s\right). \quad (32)$$

Likewise, (17) can be also approximated as follows:

$$\varphi_{\alpha,\beta}^{R,i} \approx \prod_{\gamma=1}^{\beta} f^{(l)}\left(L_{i,2(\alpha+\gamma)-1}^s\right),$$
$$\varphi_{\alpha,\beta}^{I,i} \approx \prod_{\gamma=1}^{\beta} f^{(l)}\left(L_{i,2(\alpha+\gamma)}^s\right). \quad (33)$$

C. INTEGRATION OF APPROACHES 1 AND 2

In this section, we combine approach 1 for log-domain estimation and approach 2 with $f^{(l)}(x)$ for approximation of the hyperbolic tangent function for SSE. Hence, (14) can be approximated as follows:

$$\rho_\alpha^{R,i} \approx \left(\prod_{\beta=1}^{\alpha} \operatorname{sgn}\left(L_{i,2\beta-1}^s\right)\right)\left(\min_{\beta=1}^{\alpha}\left|f^{(l)}\left(L_{i,2\beta-1}^s\right)\right|\right),$$
$$\rho_\alpha^{I,i} \approx \left(\prod_{\beta=1}^{\alpha} \operatorname{sgn}\left(L_{i,2\beta}^s\right)\right)\left(\min_{\beta=1}^{\alpha}\left|f^{(l)}\left(L_{i,2\beta}^s\right)\right|\right). \quad (34)$$

Correspondingly, (17) can also be approximated as follows:

$$\varphi_{\alpha,\beta}^{R,i} \approx \left(\prod_{\gamma=1}^{\beta} \operatorname{sgn}\left(L_{i,2(\alpha+\gamma)-1}^s\right)\right)\left(\min_{\gamma=1}^{\beta}\left|f^{(l)}\left(L_{i,2(\alpha+\gamma)-1}^s\right)\right|\right),$$
$$\varphi_{\alpha,\beta}^{I,i} \approx \left(\prod_{\gamma=1}^{\beta} \operatorname{sgn}\left(L_{i,2(\alpha+\gamma)}^s\right)\right)\left(\min_{\gamma=1}^{\beta}\left|f^{(l)}\left(L_{i,2(\alpha+\gamma)}^s\right)\right|\right). \quad (35)$$

IV. COMPLEXITY AND PERFORMANCE COMPARISON

The purpose of the proposed schemes is to reduce the complexity of SSE, which is a function of the modulation order, K. Therefore, we first compare the BER performances of the proposed methods according to K by using a MIMO scheme with comparatively small number of antennas. The BER performance of the proposed methods for JIDD are compared with those of the conventional schemes by using 4×4 MIMO systems with 64, 256 and 1024-QAM over a frequency-flat Rayleigh fading channel. Figure 3 shows the BER performances of the various SIC-MMSE-based JIDD schemes. For the SIC-MMSE, we utilized a layer-independent soft MMSE filtering matrix in (9). A low-density parity check (LDPC) code with a length of 16200 bits and a code rate of 1/2 was used as a forward error correction scheme. In the decoder, we implement the offset min-sum decoding algorithm [15]. During the JIDD, the maximum numbers of iterations inside the decoder, between the decoder and detector, and inside the detector were set to 10, 5, and 2, respectively.

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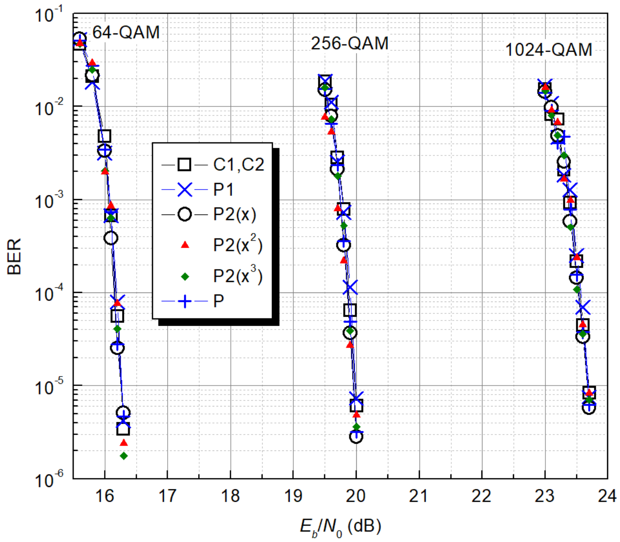

FIGURE 3. BER simulation results for a 4×4 MIMO system.

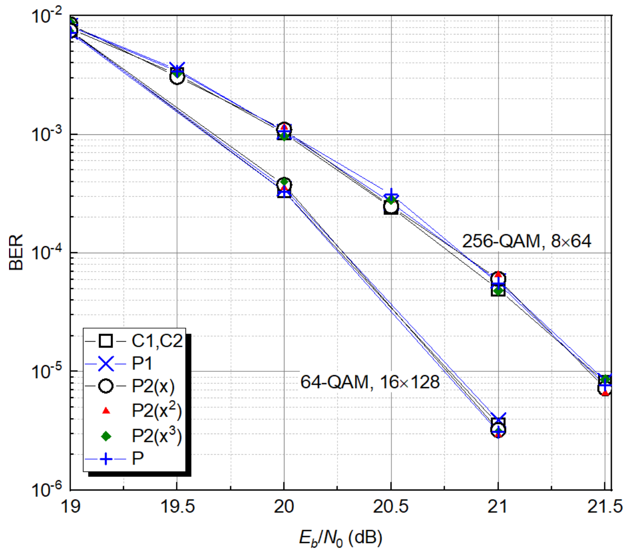

FIGURE 4. BER simulation results for 64 and 256-QAM with various number of antennas.

In Fig. 3, C1 and C2 are used to denote conventional methods, where C1 denotes the conventional exhaustive method for SSE by using (3) and (6), and C2 denotes the conventional reduced complexity method for SSE by using (13) and (16) [12]. On the other hand, P1, P2(x), P2(x^2), P2(x^3) and P are used to denote the proposed methods, where P1 represents the proposed methods for SSE with approach 1, P2(x), P2(x^2) and P2(x^3) represent SSE with approach 2 using $f^{(1)}(x)$, $f^{(2)}(x)$ and $f^{(3)}(x)$, respectively, and P represents the integration of P1 and P2(x) descried in III-C.

The BER simulation results shown in Fig. 3, exhibited the following facts. First, the proposed schemes produce almost the same BER performance as the conventional schemes, regardless of the modulation order. Second, the proposed schemes with $f^{(l)}(x)$s exhibited almost no performance differences. Third, we could apply a common set of coefficients independent of K, even though the coefficients of the polynomials found in Table 1 are dependent on K. In other words, coefficients found for the highest order modulation scheme can be universally applied to all the other lower order modulation schemes, without performance degradation. By noting the above observation, we only consider P2(x) for approach 2 when comparing the computational complexity of the proposed schemes.

In order to further confirm that the proposed schemes work efficiently for MIMO systems with lager number of antennas, we present BER performance for MIMO systems with various number of antennas. Figure 4 presents BER performance of 16×128 and 8×64 MIMO systems using 64 and 256-QAM, respectively. The proposed methods produce almost the same BER performance as the conventional methods as in the 4×4 system.

We compare the computational complexity of various SSE schemes utilized in the SIC-MMSE, in terms of the number of multiplications n_m per symbol, and thus they are represented as a function of K. Table 2 presents n_m required to estimate $\overline{s_i}$ and $\overline{||s_i||^2}$. In addition, we also represent a few additional operations other than the multiplications. The estimation of n_m is mainly counted for the multiplication of $\tanh(\cdot)$ in (14) and (17). We do not include the multiplication with coefficients of $(-1)^{\alpha}2^{\frac{K}{2}-\alpha}$ in (13) and $\mu_{\alpha,\beta}$ in (16) because they have an integer power of 2, and thus can be simply implemented by shifting operations.

TABLE 2. Comparison of computational complexities of SSE

Methods	n_m for $\overline{s_i}$	n_m for $\overline{\|\|s_i\|\|^2}$	Others
C1	$(K+2)2^K$	$(K+1)2^K$	$\tanh(\cdot)$
C2	$\frac{K^2-2K+8}{4}$	$\frac{K^3-6K^2+8K+24}{24}$	$\tanh(\cdot)$
P1	2	1	$\tanh(\cdot)$, $\|\cdot\|$, $\min(\cdot)$
P2(x)	$\frac{K^2+2K+8}{4}$	$\frac{K^3-6K^2+8K+24}{24}$	-
P	$K+2$	1	$\|\cdot\|$, $\min(\cdot)$

The results in Table 2 demonstrate that the complexities of P1 are reduced to $O(1)$ from the exponentially increasing complexities of the conventional methods, except $\tanh(\cdot)$. Even though P2(x) still requires a complexity of $O(K^3)$, it does not require any $\tanh(\cdot)$ operations. By integrating P1 with P2(x), the proposed method P achieves the complexity of $O(K)$ without any $\tanh(\cdot)$ operations. However, the proposed methods need additional binary operations to replace the product operation, and this will additionally require exclusive-or operations of the sign bits and operations for finding the minimum.

Figure 5 compares the computational complexity of the various schemes, in terms of n_m, according to the number of bits in a symbol, K. In the figure, $\tanh(\cdot)$ was assumed

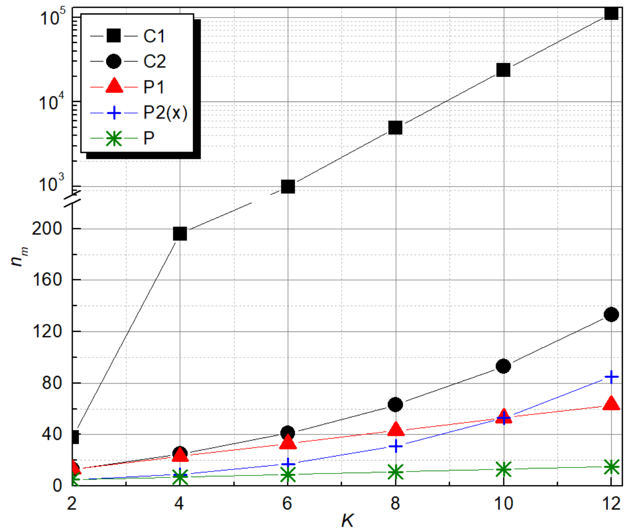

FIGURE 5. Complexity comparison of SSE.

to be estimated by using a third-order polynomial equation for the conventional scheme. It was used to estimate $\overline{s_i}$ and $\overline{||s_i||^2}$, and thus the complexity in terms of n_m for $\tanh(\cdot)$ was estimated as $5K$. The complexity reduction effect of P2(x) is greater than that of P1 when K is less than 10. The complexity reduction effect was maximized with P. For example, when $K = 8$, P1 needs only 0.88% and 68.25% of the complexity required by C1 and C2, respectively, and P2(x) needs only 0.63% and 49.21%, respectively. On the other hand, P with integration of both needs only 0.22% and 17.46% of n_m required by C1 and C2, respectively. The complexity reduction effect of the proposed methods becomes more evident as K increases.

V. CONCLUSION

The paper has presented efficient methods to reduce the complexity of SSE. The complexity reduction problem was tackled with two different approaches: the first reduces the complexity to $O(K)$ from $O(K2^K)$, and the second replaces the hyperbolic tangent estimation with a simple piecewise linear equation. We found no noticeable performance degradation of the proposed schemes compared to the conventional schemes. The proposed schemes are applicable to all SIC-MMSE-based MIMO detection schemes and can significantly reduce the complexity.

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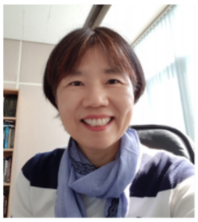

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