MINDLIN CYLINDRICAL PANELS WITH TWIST AND DOUBLE CURVATURE

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Summary An open cylindrical panel, which has a twist around the lengthways direction and double curvature in the radial and lengthways directions, is a better shell model of turbine blades. In order to analyse vibration characteristics accurately, a precise relationship between strains and displacement components of the shell model is derived on the general shell theory and the first order shear deformation theory. By using the principle of virtual work and the Rayleigh Ritz method, the governing equation for free vibration of the model is presented. The effects of parameters such as curvature and twist on vibration are studied.

INTRODUCTION

A comprehensive summary about the research of blades before 1980 was made by Rao [1-3]. It is known that three models such as beam, plate and shell were adopted for studying the dynamics of blades, where the beam was the most common model and it is also used for the research of blades now. Although the plate and the shell models are more approximate to the geometric configurations of blades, and there is a little related work reported. Since 80s more complicated and precise shell models have been presented. For example, Leissa, Lee *et al.* used twisted plate, shallow cylindrical shell and doubly curved shallow shell models for the vibrations of turbo machinery blades and studied their vibration performance by Ritz method [4-7]. Introducing a *pb-2* Ritz method, Lim *et al.* studied the problem using several models such as the trapezoidal plate, the shallow cylindrical shell and the shallow conical shell [8-11]. Based on an exact strain-deformation relationship on the general shell theory, Tsuiji *et al.* presented a pretwisted thin plate and thin cylindrical models [12,13] and Hu *et al.* presented a curved and twisted thin cylindrical model [14]. Sakiyama *et al.* further studied the vibration of the cylindrical panel with non-uniform thickness [15].
The purpose of this work is to extend the research of thin shell model and to introduce an accurate strain-deformation relationship of the model under considering the influence of shear deformation and rotary inertia. By the use of the principle of virtual work the energy equation of the panel is given out, and then the governing equations are achieved by the Rayleigh-Ritz method with orthonormal polynomials as admissible displacement components. The vibration characteristics and the effects on it are studied briefly.

THEORETICAL FORMULATION

The profile, geometric parameters and the coordinate system of the new blade model is shown in Figure 1 where the cylindrical panel rotates around the curvilinear axis x at a twist rate k, and has two curvature (*1/R* and *1/r*). Based on the accurate relation between strains and displacement components for the 3D panel [14] and the first order shear deformation theory, a precise relationship for the Mindlin cylindrical panel is achieved.

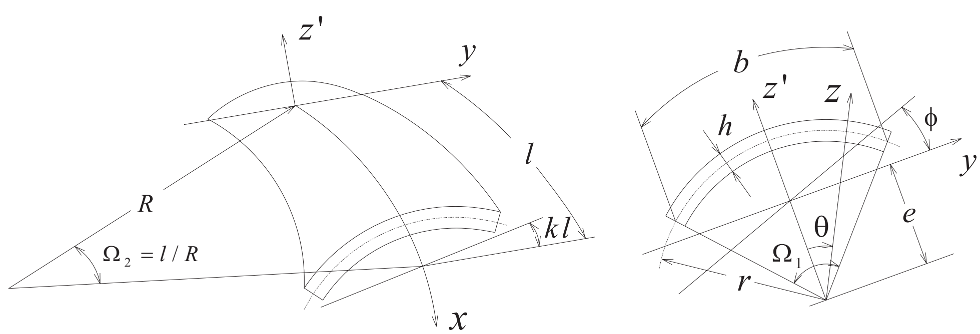

Figure 1. A schematic diagram of a Mindlin cylindrical panel

$$\left\{ \begin{matrix} \varepsilon_{\xi\xi} & \varepsilon_{\eta\eta} & \gamma_{\xi\eta} & \gamma_{\xi\zeta} & \gamma_{\eta\zeta} \end{matrix} \right\}^T = \boldsymbol{\mathcal{Z}}_1 \boldsymbol{\mathcal{G}}^{(1)} \boldsymbol{\mathcal{R}}_1 + \boldsymbol{\mathcal{Z}}_2 \boldsymbol{\mathcal{G}}^{(2)} \boldsymbol{\mathcal{R}}_2, \tag{1}$$

$$\boldsymbol{\mathcal{R}}_1^T = \left\{ \frac{\partial u}{\partial x} \quad \frac{\partial u}{\partial y} \quad u \quad \frac{\partial v}{\partial x} \quad \frac{\partial v}{\partial y} \quad v \quad \frac{\partial w}{\partial x} \quad \frac{\partial w}{\partial y} \quad w \right\}, \qquad \boldsymbol{\mathcal{R}}_2^T = \left\{ \frac{\partial \Theta_1}{\partial x} \quad \frac{\partial \Theta_1}{\partial y} \quad \Theta_1 \quad \frac{\partial \Theta_2}{\partial x} \quad \frac{\partial \Theta_2}{\partial y} \quad \Theta_2 \right\}, \tag{2}$$

where $\boldsymbol{\mathcal{G}}^{(1)}$ and $\boldsymbol{\mathcal{G}}^{(2)}$ are matrices relative to the geometric parameters, matrices $\boldsymbol{\mathcal{Z}}^{(1)}$ and $\boldsymbol{\mathcal{Z}}^{(2)}$ are related to the thickness of panel only, and *u, v, w* are linear and Θ_1, Θ_2 are angular displacement components.
The energy equation of motion is given out by the principle of virtual work,

$$\delta\Pi = \iiint_{vol} \varepsilon^T \delta\sigma \, dV_{ol} - \iiint_{vol} \rho\,\omega^2 \boldsymbol{U}\,\delta\boldsymbol{U}\, dV_{ol}, \tag{3}$$

Considering the Rayleigh-Ritz method, a set of orthonormal polynomials is selected as displacement functions as shown in the follows,

$$u = \sum_{i=1}^{N_u}\sum_{j=1}^{M_u} a_{ij}^u\,\Phi_i^u(x)\,\Psi_j^u(y), \quad v = \sum_{k=1}^{N_v}\sum_{l=1}^{M_v} a_{kl}^v\,\Phi_k^v(x)\,\Psi_l^v(y), \quad w = \sum_{m=1}^{N_w}\sum_{n=1}^{M_w} a_{mn}^w\,\Phi_m^w(x)\,\Psi_n^w(y),$$

$$\Theta_1 = \sum_{p=1}^{N_{\Theta_1}}\sum_{q=1}^{M_{\Theta_1}} a_{pq}^{\Theta_1}\,\Phi_p^{\Theta_1}(x)\,\Psi_q^{\Theta_1}(y), \quad \Theta_2 = \sum_{r=1}^{N_{\Theta_2}}\sum_{s=1}^{M_{\Theta_2}} a_{rs}^{\Theta_2}\,\Phi_r^{\Theta_2}(x)\,\Psi_s^{\Theta_2}(y), \tag{4}$$

where $\Phi_P^S(x)$ and $\Psi_Q^S(y)\,(S = u,\, v,\, w,\, \Theta_1,\, \Theta_2;\;\, P,\, Q = 1,\, 2,\, 3, \cdots)$ are polynomials generated by the Gram-Schmidt process and their first polynomials $\Phi_1(x)$ and $\Psi_1(y)$ are defined according to the geometric boundary conditions of panel. Substituting equations (4) into equation (3) and the independent property of the coefficients in equation (4) yield the following governing equation,

$$\left|\boldsymbol{K} - \lambda^2\boldsymbol{M}\right| = \boldsymbol{0}, \tag{5}$$

and then the eigenproblem of blades can be solved.

CONCLUSIONS

Using the general shell theory and the first order shear deformation theory, an accurate relationship between strains and displacement components for the Mindlin cylindrical panel with twist and two curvature is derived. The governing equation for free vibration is formulated by the principle of virtual work and the Rayleigh-Ritz method with a set of orthonormal polynomials. Vibration frequencies and modes are achieved for Mindlin cylindrical panels with different geometric parameters such as the twist, the curvature and the aspect ratio, and then the characteristics of the model is revealed.

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