MINDSET IN PROFESSIONAL DEVELOPMENT: EXPLORING EVIDENCE OF DIFFERENT MINDSETS

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This exploratory case study investigated the role of mindset (i.e., fixed mindset vs. growth mindset) as elementary teachers participated in a professional development focusing on mathematics. Data were collected on two participants with opposing mindsets. Attention was given to their interactions in collaborative group settings as well as their views regarding working with successful and struggling mathematics students. Results indicated that although the two participants engaged in the same professional development activities, their engagement led to different interactions within those activities.

Keywords: Teacher Beliefs; Teacher Education-Inservice

Introduction

To support all students in developing deep mathematical understanding, "students [must] have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential" (National Council of Teachers of Mathematics [NCTM], 2014, p. 59). Although the literature abounds with descriptions of effective teaching and learning (e.g., Franke, Kazemi, & Battey, 2007;NCTM, 2000, 2014; National Research Council, 2001), teacher educators recognize that for most mathematics teachers the classroom practices described in the literature represent a reconceptualization of mathematics teaching (Sowder, 2007). Professional development (PD) is a key mechanism for supporting teachers' development of effective teaching practices (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010;Sowder, 2007) and equity in mathematics.

PD experiences that are sustained, focused on worthwhile tasks, and provide immersion experiences in productive teaching practices can challenge teachers to transform their practice (Elmore, 2002; Hawley & Valli, 1999; Loucks-Horsley et al., 2010). Despite these known characteristics of effective PD, supporting change in mathematics classrooms continues to be daunting (Franke et al., 2007). Cooney (1999) posited that teachers' belief in teaching as an act of giving information to students represents an obstacle to change in the mathematics classroom. Sowder (2007) stated, "Many of teachers' core beliefs need to be challenged before change can occur" (p. 160). Recognizing that a teacher's beliefs influence his or her perceptions of effective instruction (NCTM, 2014;Pajares, 1992), much research on PD has sought to address changes in teachers' beliefs and changes in instructional practice, with varying results (see Philipp (2007)).

Recently, *mindset* has emerged as a term used to describe the belief that either mathematics ability can be cultivated in all students or mathematics ability cannot be changed (Dweck, 2006). NCTM (2014) reported research showing that "believing in, and acting on, growth mindsets versus fixed mindsets can make an enormous difference in what students accomplish" (p. 64). Responding to these different mindsets may be an important notion for progressing the field forward. Just as teachers' differing beliefs influence their PD experiences, we wondered how teachers' differing mindsets

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influence their engagement in PD experiences. The purpose of this study was to examine how teachers with opposing mindsets interacted with the content of a PD. Specifically, the research question that guided our work was: How do elementary teachers of different mindsets interact in a mathematics-focused PD?

Theoretical Framework

Dweck and Leggett(1988)described a social-cognitive model of motivation and personality that framed a theory of implicit conceptions of the nature of ability based on work in goal orientation and behavioral patterns. This implicit theory and its research base continues to support work in fields including educational psychology and mathematics education (e.g., Aronson, Fried, & Good, 2002; Blackwell, Trzesniewski, & Dweck, 2007; Dupeyrat & Mariné, 2005). The evidence supporting its generalization to other domains (Dweck, Chiu, & Hong, 1995; Dweck & Leggett, 1988) and the instrument used to measure its constructs (Dweck et al., 1995) provided the theoretical framework for this study.

Dweck and Leggett (1988) posited that an individual's implicit assumptions about the nature of an ability lead directly to the type of goals he pursues regarding that ability and the behaviors he exhibits when faced with challenges (Dweck et al., 1995; Dweck & Leggett, 1988). They described the *entity* and *incremental theories*. Individuals assuming an entity theory tended to view attributes as fixed, uncontrollable entities and adopted performance-oriented goals to gain or avoid judgment regarding the ability. In contrast, individuals espousing an incremental theory tended to view attributes as malleable and subscribed to learning goals focused on improvement of the ability (Dweck, 1986; Dweck & Leggett, 1988). These mindsets and their associated goals created "a framework for interpreting and responding to events" (Dweck & Leggett, 1988, p. 260) that promoted observable behavioral patterns when the ability under consideration is challenged. Maladaptive, *helpless* responses characterized by lowered performance and the avoidance of challenges were associated with entity theories. Their adaptive counterparts, *mastery-oriented* responses, were associated with incremental theories and characterized by the pursuit of challenges and persistence when faced with failure (Diener & Dweck, 1980; Dweck, 1975; Dweck & Leggett, 1988).

Although the tenets of implicit theories were initially established through the characterization of an individual's own intelligence, Dweck and Leggett(1988) proposed a framework through which its generalization to other attributes and domains occurred, culminating in the validation of an instrument used to assess individuals' implicit theories for multiple attributes (Dweck et al., 1995). The authors predicted that for any attribute of personal significance, "viewing it as a fixed trait will lead to a desire to document the adequacy of that trait, whereas viewing it as a malleable quality will foster a desire to develop that quality" (Dweck & Leggett, 1988, p. 266). Additional evidence supported that the model holds for generalization to traits beyond the self, such as the character and attributes of other people and the world (Dweck & Leggett, 1988; Erdley & Dweck, 1993). This application of the model suggested further observable characteristics of an individual's interactions based on their implicit theories. Those with fixed mindsets should be seen to reject change in themselves and others and draw simplified conclusions from brief experiences. In contrast, those with growth mindsets should be seen to encourage growth in other individuals and organizations and experience a sense of control relative to their environment (Dweck & Leggett, 1988).

Methodology

To examine the role of mindset in PD, we used case study methodology, specifically a holistic, multiple-case design (Yin, 2014). We viewed this as an exploratory case study, with the purpose "to identify research questions or procedures to be used in a subsequent research study, which might or might not be a case study" (Yin, 2014, p. 238).

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This study occurred within a PD project serving 82 kindergarten through sixth grade mathematics teachers. The project was in its second year of external funding and represented a partnership between a university and four rural school districts in a southeastern state. Although the project included academic year meetings and demonstration lessons, the project component of focus in this report was the ten-day summer institute occurring during the second year of the project in which teachers met in grade-level groups (i.e., K–2, 3–4, 5–6) and engaged in immersion and practice-based experiences (Loucks-Horsley et al., 2010), with a mathematical focus on fractions and their operations.

The twelve-item Likert style mindset survey included Dweck et al.'s (1995) nine items related to mindset in relation to intelligence, morality, and world. (For evidence of reliability and validity see Dweck et al.) In addition, we created three survey items pertaining to a teacher's point of view of their students' mathematical abilities. The addition of these items is supported by the work of Dweck and Leggett (1988).

We developed four semi-structured interview protocols along with an observation protocol for data collection. Interview questions related to participants' strong academic students, struggling students, and their role as mathematics teachers working with these students. The observation protocol, employed during observation of participants in PD activities, consisted of six categories: evaluation of situation, dealing with setbacks, challenges, effort, criticism, and success of others. In each category, observable behaviors for each mindset were recorded. Participants were video-recorded during institute activities.

On the first day of the summer institute all teachers completed the mindset survey, which was scored following Dweck and colleagues' (1995) protocol. Four participants representing each mindset were interviewed and then two teachers, Ms. Fitzgerald (fixed mindset) and Ms. Gorman (growth mindset) were selected. Both Ms. Fitzgerald and Ms. Gorman (pseudonyms) participated in the grades 5–6 PD group, which had as its primary focus the modeling of multiplication and division with fractions. At the time of the study, Ms. Fitzgerald, a Caucasian female, had completed eight years as a classroom teacher and Ms. Gorman, an African American female, had completed nine years of classroom teaching.

To analyze the data, we drew on the organization and analysis procedures described by Yin (2014). We developed a case study database for each participant, organizing and compiling all data chronologically. We began by employing an inductive strategy for analyzing participant interviews. This strategy led to the development and refinement of a set of codes representing relevant concepts. In addition, we relied on our theoretical framework to guide the analysis of interviews and observation data, identifying evidence of both fixed and growth mindsets. The validity and reliability of our findings are supported by the use of multiple sources of evidence (construct validity), replication logic (external validity), and a case study protocol (reliability).

Results

In this section, we present the results from our analysis. Each case will be presented separately, organized according to the three themes that emerged during analysis: intelligence, goals, and challenges.

Ms. Fitzgerald (Fixed Mindset)

Intelligence. When asked if intelligence was something that could change, Ms. Fitzgerald stated, "I don't think so. I think it's something that you are born with and what you learn through school is what you learn." During her interviews, Ms. Fitzgerald distinguished between students who were naturally talented at mathematics and those who were talented at literacy:

> I think that you are either literature based or you're mathematically based. . . . The literature-based [students] are slower learners. . . . The mathematically based [students], they can do it through

> computations. They can do it with manipulatives. They can usually draw diagrams.

She also spoke of instructional practices related to students of different ability levels.

> I would give [students in the high ability group] a higher level task and give my inclusion group a lower level task. But all that was aiming towards the same goal. . . . For my inclusion group, I might pull a fourth grade task on perimeter versus my high group, I might pull a sixth grade task. And then my middle group, keep them on target with a fifth grade task. But in the end, we are all working towards the same goal.

Ms. Fitzgerald indicated that students have a tendency to disengage when faced with challenging material, thus leading to behavioral issues. In addition, as she reflected on her own frustration when faced with a challenge, she said: "Now I see how my kids get frustrated. . . . I think that I would back off a little bit if a task seemed to be too difficult for them. I don't want them to get this frustrated."

Goals. Ms. Fitzgerald indicated that she was guided by performance-oriented goals for herself and her students, which was evidenced in three ways. First, Ms. Fitzgerald focused on solutions to problems. She said:

> I would let [the struggling students] do their own models first but we might go to an algorithm a little bit quicker with them and try to work backwards if they can't get the models on their own. Solve the problem and then try to create a model that matches and then hopefully later on they could go to the model first.

Ms. Fitzgerald indicated that if a student cannot draw the model right away, then she has the student find the solution using an algorithm, thus emphasizing the model as another procedure to be learned. Then, the student can use the solution as a means for drawing the correct model.

Ms. Fitzgerald also placed a great emphasis on testing and algorithmic proficiency. She stated:

> I go straight to the algorithm and I would like for my kids to have different ways to do it because when it comes time for [the state's constructed response assessments], they can't just do the algorithm. They have to be able to draw some type of model or picture.

She saw her instructional role in working with the students as guiding them to the answer.

> I have to step away and let them do their thing, not tell them the right answer, step back, let them struggle a little bit, and guide themselves to the correct answer. And if they get it wrong, it's ok. . . . I might give them an easier task the next day, something that's still within . . . the same unit, but something that may be a little easier, not as confusing for them. After that, lead into some instruction because they always have to have some instruction.

Challenges. During the PD, Ms. Fitzgerald faced the challenge of developing models (i.e., either pictorial or concrete representations) for fraction problems involving multiplication and division. Ms. Fitzgerald consistently demonstrated helpless responses. She made statements such as, "I couldn't tell you because I don't know," and "Yeah, but that's probably not right," which indicated a negative self-cognition. In addition, Ms. Fitzgerald offered statements that seemed to be intended to divert away from the discussion such as, "I'm more of an algorithm person," and, "I need to go to the K-2 class and then maybe I can do something in there." When selected to respond during the institute, she would offer statements such as "We're nowhere yet" and "I'm thinking I don't know."

Ms. Fitzgerald expressed continued frustration when faced with the challenge of drawing models. "I can't read the models. I can't draw the models. So I was very frustrated. That made it hard to participate in the tasks and the problem sets." Similarly, Ms. Fitzgerald stated, "Because I've tried – like our problem sets, you have to draw the model, no algorithm and I just look at it and go – oh, I'm defeated again today, I can't do my homework." When asked how she might overcome the challenge, Ms.

Fitzgerald said, “Um, just forge ahead and really try to wrap my brain around it.” She also indicated that more time should be spent “breaking [the model] down and simplifying it to where everybody can understand it.”

Ms. Gorman

Intelligence. When asked if intelligence was something that could change, Ms. Gorman stated, “I look at this as saying people can make mistakes, and you can learn from your mistakes, and you can change.” She explained that although some students understand procedures more quickly, *all* students can gain a stronger understanding of mathematics through struggling with difficult material and exploring and explaining their thinking.

> I think that in the long run they would have a better understanding of math, they would have the critical thinking skills to think through it, whereas some of the others . . . they could do the procedures but couldn't explain why. . . . I had some [high ability] kids who, at the beginning of the year, 'The answer is 24', 'How do you know that?', 'I just know.' They knew the procedures of it, they could tell you the answer, but couldn't explain. It didn't mean anything. . . . I was almost excited when they would get something incorrect because that gave me the chance to help them think through it.

Ms. Gorman’s goal of supporting all students in understanding mathematics led her to describe her instructional practices in working with low achieving students.

> Definitely take some steps back. . . . Let them struggle. . . . to a degree. . . . you don't want them to struggle so much that they're like "I'm done." But to let them struggle to see if they can figure out – a way of persevering through it. And then offer my help and then walk away.

Ms. Gorman described her own struggles in working with high-ability students:

> This past year I had five [high ability] kids, which is a lot to have in one room. They were very procedural and what I found was hard for them was, they were like 'I just know,' but they couldn't visually show me or tell me why. . . . So, with them, having more advanced type tasks, questions, or even be able to have questions to be able to ask them, I think that's one of my struggles. Even though I know I'm supposed to ask advancing and assessing questions, I think that even when I'm looking at it I have a hard time being able to pinpoint what question to ask at that moment quickly and walk away.

Goals. Ms. Gorman’s statements suggested that she was guided by learning-oriented goals, as indicated by her tendency to focus on developing mathematical understanding. She described the role of student exploration and explanation in developing this understanding and linked these to changes in her instructional practice.

> Being able to connect with how they are thinking, I would say it's a lot more engaging, just because I am pulling it out and trying to find manipulatives, and getting the kids to explain their answers, and I've been trying to share with the kids multiple ways that they can get to it, and I usually don't share until they open the door first. So, I kind of build off of their ideas.

She indicated her role in working with students was to ask good questions and serve as a guide.

> So as far as my teaching strategies go, it's just taking some steps back, making sure I'm asking the right types of questions, to get the kids to go in a certain path, so even if they're steering off the wrong way my questioning needs to be correct in that I'm pointing them back in the right direction. If they have no clue at all, you know, just helping them to find that starting point without giving too much. And without them always using my strategy.

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Finally, she acknowledged the role of drawing models in supporting the development of mathematical understanding. The models served as a tool for making sense of the mathematics and making connections.

> A lot of times I think . . . the lower kids tend to not have number sense or not really know what the numbers represent. And so with pictures, and things like that--with the labeling--I think that would help them to progress to a better understanding of what it is they're doing.

Challenges. When discussing the challenges associated with understanding the models for multiplication and division of fractions, Ms. Gorman's responses most often could be categorized as mastery-oriented responses. Ms. Gorman attributed her struggles with understanding the models to the fact that she had not been taught these models as an elementary student. She said, "I feel like I'm trying to figure some of these [visual models] out because I was not taught that way. . . . I thought I knew math. I do not know math the way I thought I knew math."

She felt it was important, however, to understand the models so that she could utilize them in her classroom.

> I don't [understand] the pattern blocks. . . . I would not use those in my classroom because I would feel like if I could not explain it to the kids with a clear understanding – because that to me would become one of the tools that they could use to figure out the problem. But they can't use something if I can't explain it efficiently.

During summer institute sessions, she frequently continued to work with one particular model until she felt sure, even as others around her moved on to a different representation.

In reference to the variety of models displayed by her peers, Ms. Gorman described herself as being overwhelmed as her lack of understanding had been exposed and she aimed to understand all of the different models.

> I'm very overwhelmed. Just in the sense of – obviously I know how to divide and multiply fractions – but having to show pictorially how to multiply and divide fractions is totally different. Because I never have really had to do that. . . . And then another thing that was overwhelming is that you are passing these posters around and you are looking and somebody like me whose mind is just constantly going and I see a different way to work out something then I'm like – ooh! You know, I want to work it out this way. And then there's another way. And then another.

She believed that to overcome the challenge she needed more practice with the models. She said, "Before I took it to the classroom I would need some more practice . . . just so I could feel comfortable in what I'm saying, so that they could try it and understand what they were doing."

Discussion and Conclusion

Supporting all students in learning meaningful mathematics is a daunting task, and one that requires a shift in how teachers view effective mathematics teaching (Sowder, 2007). PD represents the primary means towards supporting teachers in developing this view (Loucks-Horsley et al., 2010) and much is known about key components of effective PD (Elmore, 2002; Hawley & Valli, 1999; Loucks-Horsley et al., 2010). The impact of PD is often hindered, though, as a result of teachers' beliefs about instruction (Sowder, 2007). Although much research has examined the role of beliefs in teachers' PD (Philipp, 2007), the role of mindset has only recently emerged and has yet to be examined in this way.

As a result, the purpose of our study was to examine how teachers' mindsets influenced their participation in a PD setting. Throughout the PD, the teachers worked with representing the multiplication and division of fractions with models, a task to which many teachers had not been previously exposed. In this sense, Ms. Fitzgerald and Ms. Gorman were no exception. They each

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described the uncomfortable feelings associated with not knowing; yet how they responded to this challenge was quite different. Ms. Fitzgerald spoke of frustration, stating continuously that she did not understand the models and that she preferred to utilize algorithms. Ms. Gorman spoke of being overwhelmed, providing statements that demonstrated her persistence in trying to understand the variety of models available.

Additional differences between the two participants emerged when considering the purpose of understanding the models. Ms. Fitzgerald seemed to see the development of a model as an additional process for students to learn; a process that was needed in order to be successful on state assessments. Ms. Gorman indicated that the development of a model supported the development of students' understanding. Interestingly, Ms. Fitzgerald believed that students who understood mathematics made connections among the models and the algorithms. Unlike Ms. Gorman, however, she did not see this as an indication that students struggling with mathematics might benefit from tasks or activities that supported them in making those same connections. Rather, she felt these students did not hold the potential for understanding mathematics and instead needed more practice with the procedures.

These contrasting views influenced the instructional practices described by the two participants. While both mentioned the role of struggle in learning, Ms. Fitzgerald aimed to support her students in avoiding struggle, for example through the use of easier tasks. This practice was likely influenced by her belief in the inability to influence students' mathematical abilities. Rather, each student's achievement level should be identified and then appropriate tasks assigned. Ms. Gorman described the importance of productive struggle in supporting the learning process. Thus, students' mathematical abilities could be shaped by their learning experiences.

Across the three areas in our analysis (i.e., intelligence, goals, and challenges), the differences between views expressed by the two participants can be attributed to their opposing mindsets. Ms. Fitzgerald's propensity to see intelligence as unchangeable is clearly linked to her fixed mindset, along with her performance-oriented goals and helpless responses. Similarly, Ms. Gorman's view that intelligence is changeable combined with her learning-oriented goals and mastery oriented responses is clearly aligned with her growth mindset. As a result, the opposing mindsets resulted in different PD experiences.

The findings for this exploratory case study lead us to ask more questions. First, we believe that Ms. Gorman and Ms. Fitzgerald left the PD with greatly different experiences. How will the differences we observed in Ms. Gorman's and Ms. Fitzgerald's interactions during PD carry over into their classroom practice? In *Principles to Actions*(NCTM, 2014) explicit connections are drawn between growth mindsets and mathematical success for all students. Further, Blackwell, Trzesniewski, and Dweck (2007) reported being able to manipulate mindset. What interventions can we include in PD so that we orient teachers toward a growth mindset? How might findings from these questions impact future PD and even work with pre-service teachers? Finally, how can we help teachers build ways to work productively with students of different mindsets? Knowing that "believing in, and acting on, growth mindsets versus fixed mindsets can make an enormous difference in what students accomplish" (NCTM, 2014, p. 64), we must also continue to follow this investigation into the differences that can be made in what teachers accomplish when attention is given to mindsets.

References

Aronson, J., Fried, C. B., & Good, C. (2002). Reducing the effects of stereotype threat on African American college students by shaping theories of intelligence. *Journal of Experimental Social Psychology*, *38*, 113-125.

Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development*, *78*, 246-263.

Cooney, T. J. (1999). Conceptualizing teachers' ways of knowing. *Educational Studies in Mathematics, 38,* 163-187.

Diener, C. I., & Dweck, C. S. (1980). An analysis of learned helplessness: II. The processing of success. *Journal of Personality and Social Psychology, 39,* 940-952.

Bartell, T. G., Bieda, K. N., Putnam, R. T., Bradfield, K., & Dominguez, H. (Eds.). (2015). *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. East Lansing, MI: Michigan State University.

Dupeyrat, C., & Mariné, C. (2005). Implicit theories of intelligence, goal orientation, cognitive engagement, and achievement: A test of Dweck's model with returning to school adults. *Contemporary Educational Psychology*, *30*, 43-59.

Dweck, C. S. (1975).The role of expectations and attributions in the alleviation of learned helplessness. *Journal of Personality and Social Psychology, 31,* 674-685.

Dweck, C. S. (1986).Motivational processes affecting learning. *American Psychologist*, *41*, 1040-1048.

Dweck, C. S. (2006). *Mindset: The new psychology of success*. New York, NY: Random House.

Dweck, C. S., Chiu, C. Y., & Hong, Y. Y. (1995). Implicit theories and their role in judgments and reactions: A word from two perspectives. *Psychological Inquiry*, *6*, 267-285.

Dweck, C. S., & Leggett, E. L. (1988). A social-cognitive approach to motivation and personality. *Psychological Review*, *95*, 256-273.

Elmore, R. F. (2002). *Bridging the gap between standards and achievement: the imperative for professional development in education*. Washington, DC: Albert Shanker Institute.

Erdley, C. A., & Dweck, C. S. (1993). Children's implicit personality theories as predictors of their social judgments. *Child Development*, *64*, 863-878.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Reston, VA: National Council of Teachers of Mathematics.

Hawley, W. D., & Valli, L. (1999). The essentials of effective professional development: A new consensus. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 127–150). San Francisco: Jossey-Bass.

Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N., & Hewson, P. W. (2010). *Designing professional development for teachers of science and mathematics* (3rd ed.). Thousand Oaks, CA: Corwin.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62,* 307–332.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Reston, VA: National Council of Teachers of Mathematics.

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157–223). Reston, VA: National Council of Teachers of Mathematics.

Yin, R. K. (2014). *Case study research: Design and methods* (5th ed.). Thousand Oaks, CA: Sage Publications.

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