

# Miniature electrical filters for single electron devices

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In experiments on single electron devices, the electromagnetic noise from parts of the apparatus at temperatures higher than that of the device can dramatically increase the tunnel rates out of the Coulomb-blocked state and therefore increase the device error rate. The electrical lines must therefore be filtered adequately. We derive simple expressions for calculating the required attenuation coefficient. We describe a wide-band miniature dissipative filter functioning at cryogenic temperatures. The effective thermalization of an experiment at 30 mK can be obtained by placing four of these filters in series at temperatures ranging from 4 K to 30 mK. © 1995 American Institute of Physics.

## I. INTRODUCTION

Filtering electrical lines connecting a room-temperature apparatus to a cryoelectronic device is often of prime importance. Although anchoring the lines to electrical heat sinks reduces the noise they generate themselves, it does not prevent the electromagnetic noise coming from parts at higher temperature from propagating along them. The effective thermalization of the device at a given temperature requires that this out-of-equilibrium noise be sufficiently attenuated by filters. The filtering system must satisfy two criteria:

- (i) It must be effective over a wide band spectrum;
- (ii) it must be properly thermalized.

In principle, either reflective or dissipative filters can be used. However, reflective-type filters are not adequate because their performances are very sensitive to impedance mismatches in the rest of the line which cause spurious transmission resonances. Therefore, we must choose dissipative-type filters. To our knowledge, such dissipative filters functioning at cryogenic temperatures are not commercially available.

Filters are particularly relevant to single electronics.<sup>1,2</sup> Despite their importance, and the fact that prototypes have already been built and used successfully in experiments, the literature devoted to their construction is very sparse.<sup>3,4</sup> Single-electron devices are potential candidates as current standards. The very low error probability of electron transfer which is required makes them extremely sensitive to external noise. Martinis and Nahum<sup>5</sup> have recently pointed out that this high sensitivity is due to the quantum nature of the error processes. In this article, we first calculate the filter characteristics required for Coulomb blockade devices, using their quantum spectral density formalism. We then present a practical filter which fulfills these characteristics and has the advantage over previously described filters of being particularly compact.

## II. QUANTUM CALCULATION OF THE EFFECT OF ELECTROMAGNETIC NOISE ON COULOMB BLOCKADE

The controlled transfer of electrons in single-electron devices like the pump and the turnstile is based on a linear array of tunnel junctions.<sup>2</sup> The electrodes between two con-

secutive junctions, nicknamed "islands" for short, are electrostatically coupled to "gate" voltage sources. Once the voltages are set, the array will adopt an electronic configuration, i.e., the set of numbers of electrons having gone through the junctions, corresponding to a local minimum of the electrostatic energy of the whole circuit. By varying the voltages in an appropriate way, one can displace this local minimum in configuration space, so that after one cycle of gate voltage variation, one and only one electron has gone through the array, provided that the system always follows the local minimum.

This ideal picture will be altered, however, if at some point during the cycle, the system jumps from the selected local minimum to an unwanted one. Figure 1 illustrates the parasitic processes leading to this breakdown of Coulomb blockade in the simplest example of two junctions with equal capacitances  $C_j$  in series with a voltage source. Having only one island, this circuit is too simple to allow the controlled transfer of electrons. Nevertheless, it displays the phenomenon of Coulomb blockade since for values of the voltage  $V$  below the Coulomb threshold  $e/2C_j$  ( $e$  is the elementary charge), the system has local energy minima corresponding to zero excess electrons on the island. (We suppose here that there is no gate capacitance.) Thus, the arrival of an electron on the island will cost an energy  $\Delta E = E_{\text{final}} - E_{\text{initial}} = (e^2/4C_j) - (eV/2) > 0$ . Two classes of processes by which the system will overcome this Coulomb barrier and reach a neighboring local minimum must be distinguished.

(A) Class A processes consist of a sequence of single-electron tunneling events [arrows A and A' in Fig. 1(b)]. These processes must be activated since the intermediate state is energetically unfavorable. The activation energy can come from two sources.<sup>6</sup>

(i) The first one is the thermal reservoir constituted by the conduction electrons which we suppose in equilibrium at a temperature  $T_0 = 1/(k_B\beta_0)$ . This activation will induce an escape from the Coulomb-blocked state at a rate noted  $\Gamma_0$  which decreases exponentially with  $\beta_0$ .

(ii) The second source is the electromagnetic noise coming to the junctions from the leads which we have represented by an impedance  $Z_j(\omega)$  in Fig. 1(a). This noise, in general, is not a thermal equilibrium noise at the temperature of the junction. Nevertheless, we will suppose it is Gaussian

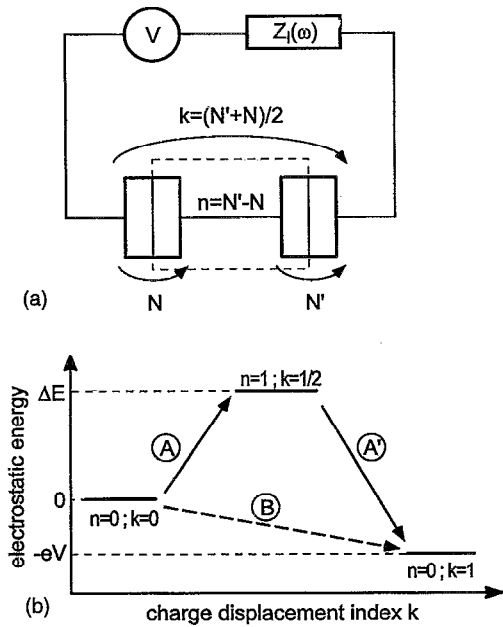


FIG. 1. (a) Example of a single-electron device with two junctions. The two degrees of freedom of this system are  $N$  and  $N'$ , the numbers of electrons transferred through each junction. Alternatively, the system can be described by the number  $n$  of excess electron on the island (box in dotted line) and the charge displacement index  $k$ .  $Z_l(\omega)$  is the impedance of the line seen from the device. (b) Electrostatic energy levels of device of (a) when the state ( $n=0, k=0$ ) is Coulomb blocked. Arrows A and B represent single electron tunneling and cotunneling processes, respectively. Process A can be induced by out-of-equilibrium photons coming from the external circuitry.

and that it can thus be completely described by an *a priori* arbitrary quantum spectral density of voltage fluctuations noted  $\mathcal{S}_V(\omega)$ .<sup>7</sup> The photons from this electromagnetic noise will be absorbed by tunneling electrons and contribute to the escape from the Coulomb-blocked state. This electromagnetic contribution  $\Gamma_{em}^1$  to the escape rate is proportional to  $\mathcal{S}_V(\omega)$ .

(B) Class B consists of higher order tunneling processes, whereby several electrons simultaneously tunnel across the junctions<sup>8</sup> [arrow B in Fig. 1(b)]. These processes, also known as “cotunneling,” will take place spontaneously, even at zero temperature and in the absence of electromagnetic noise.

It is always possible to reduce the rate of cotunneling (process B) by increasing the number of junctions involved in the process (two in our example) since the rate decreases exponentially with this number.<sup>8</sup> Similarly, the thermally activated process of type A(i) can be reduced by increasing the ratio  $\Delta E/k_B T_0$ . However, the rate of photon assisted processes of the type A(ii) depends not only on the characteristics of the device but also on the rest of the circuit. One must therefore reduce the electromagnetic noise to a level such that its contribution to the escape rate is of the same order of the rate one would observe if the entire circuit was in equilibrium at temperature  $T_0$ , i.e.,  $\Gamma_0 + \Gamma_{em0}$ . Normally, the escape rate should include all decay channels; however, with no loss of generality we can restrict ourselves to the decay channel in which an electron tunnels through the left junction, and ignore the channel in which a hole tunnels through

the right one. We thus estimate the escape rate from the Coulomb-blocked state by identifying it with the tunnel rate through the left junction.

In the following we will consider a single electron device to be effectively thermalized if the figure of merit  $\alpha = \Gamma_{em}/(\Gamma_0 + \Gamma_{em0})$  is of order 1 and we will determine the filter attenuation that satisfies this criteria. For a general device the tunnel rates have to be understood as the activated rates of the junctions involved in the first step of the dominant escape mechanism.<sup>5</sup>

We must therefore calculate the exact relationship between the ratio  $\alpha$  and the spectral density  $\mathcal{S}_V(\omega)$ .

### A. Expression of the electromagnetic contribution $\Gamma_{em}$ to the escape rate

The electromagnetic noise seen from the junction involved in the calculation of the escape rate is characterized, as we have seen, by a spectral density of voltage fluctuations  $\mathcal{S}_V(\omega)$ . For an arbitrary linear circuit at uniform temperature  $T = 1/k_B \beta$  characterized by an impedance  $Z(\omega)$  seen by the junction,  $\mathcal{S}_V(\omega)$  is given by

$$\mathcal{S}_V(\omega) = \frac{2}{\pi} \text{Re}[Z(\omega)] \frac{\hbar \omega}{\exp(\beta \hbar \omega) - 1}$$

in which  $\omega > 0$  and  $\omega < 0$  correspond to the emission and absorption of photons by the impedance to and from the junction, respectively. This spectral density is for  $\omega > 0$  the one-dimensional analogue of the black body radiation spectrum. For  $\omega < 0$ ,  $\mathcal{S}_V(\omega)$  characterizes the vacuum and/or thermal fluctuations which trigger the emission of a photon to the environment. Note that  $Z(\omega)$  is a combination of both the impedance of the line  $Z_l(\omega)$  and the capacitances of the junctions.<sup>6</sup> The spectral density enters in the quantum calculation of the tunnel rate through the autocorrelation function  $J(t) = \langle [\Phi(t) - \Phi(0)]\Phi(0) \rangle$  of the generalized flux  $\Phi(t) = \int_{-\infty}^t V(t') dt'$ , where  $V$  is the voltage across the junction. The expression of  $J(t)$  is

$$J(t) = \frac{\pi}{R_K} \int_{-\infty}^{+\infty} \mathcal{S}_V\left(\frac{E}{\hbar}\right) \frac{\exp(iE t/\hbar) - 1}{E^2} dE, \quad (1)$$

$R_K = h/e^2 \approx 25.8 \text{ k}\Omega$  being the resistance quantum.

The coupling between electromagnetic modes and a tunneling electron is described by a function  $P(E)$  giving the probability that a given tunneling transition transfers an energy comprised between  $E$  and  $E + dE$  to the electromagnetic modes. This function is related to  $J(t)$  by

$$P(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} \exp\left(J(t) + i \frac{E t}{\hbar}\right) dt. \quad (2)$$

The function  $P(E)$  is a key component in the expression of the tunnel rate:

$$\Gamma(\Delta E) = \frac{1}{R_t e^2} \int_{-\infty}^{+\infty} \frac{E}{1 - \exp(-\beta_0 E)} \times P(-\Delta E - E) dE, \quad (3)$$

where  $R_t$  is the tunnel resistance of the junction.

Inserting expression (1) in the expansion to first order in  $J(t)$  of Eq. (2), integrating Eq. (2), and inserting the result in Eq. (3), we find the following expressions for the rates:

$$\Gamma(\Delta E) = \Gamma_0(\Delta E) + \Gamma_{em}(\Delta E), \quad (4)$$

$$\Gamma_0(E) = \frac{1}{R_i} \frac{E}{e^2 \exp(\beta_0 \Delta E) - 1}, \quad (5)$$

$$\Gamma_{em}(\Delta E) = \frac{\pi}{R_K} \int_{-\infty}^{+\infty} \frac{\mathcal{S}_V(E/\hbar)}{E^2} [\Gamma_0(\Delta E - E) - \Gamma_0(\Delta E)] dE; \quad (6)$$

Expression (6) is valid<sup>4</sup> for a spectral density such that  $Z(\omega)/R_K \ll 1$ .

In the case of any physically reasonable impedance  $Z(\omega)$  in thermal equilibrium with a particular junction at the uniform temperature  $T_0$  of the electrons, the numerical integration of Eq. (6) gives a negative value for  $\Gamma_{em}/\Gamma_0$ . Since its absolute value is much lower than one we can approximate  $\alpha = (\Gamma_{em}/\Gamma_0)/(1 + \Gamma_{em}/\Gamma_0)$  by  $\Gamma_{em}/\Gamma_0$  in the following.

Given the present state-of-the-art of nanofabrication which determines typical junction capacitances,  $\Delta E$  is about 1 K. In the case of interest here, the spectral density thus remains large at frequencies higher than  $\Delta E/\hbar$ . The electronic temperature has consequently a negligible influence in Eq. (6) and can be taken equal to zero. One can thus simplify Eq. (6) and get the following expression, valid for  $\beta_0 \Delta E \gg 1$ :

$$\Gamma_{em}(\Delta E) = \frac{1}{R_i} \frac{\pi}{R_K} \int_{\Delta E}^{+\infty} \mathcal{S}_V\left(\frac{E}{\hbar}\right) \frac{E - \Delta E}{E^2} dE. \quad (7)$$

Although expressions (6) and (7) are valid for any spectral density, we restrict ourselves to physical quantum spectral densities of thermal noise which decrease as  $\exp(-\hbar\omega/k_B T)$  for  $\omega > k_B T/\hbar$  and which make integral (7) converge.

## B. Optimal attenuation

For the sake of simplicity, we approximate  $\mathcal{S}_V(\omega)$  by a step function:  $\mathcal{S}_V(\omega) = \mathcal{S}_{V0}$  for  $\omega < \omega_{\max}$  and  $\mathcal{S}_V(\omega) = 0$  for  $\omega > \omega_{\max}$ , where  $\omega_{\max} = E_{\max}/\hbar = k_B T/\hbar$ . We thereby find that  $\alpha$  is given by

$$\alpha(\mathcal{S}_{V0}, E_{\max}, \Delta E) = \frac{\pi \mathcal{S}_{V0}}{R_K} \left[ \ln\left(\frac{E_{\max}}{\Delta E}\right) - 1 + \frac{\Delta E}{E_{\max}} \right] \frac{\exp(\beta_0 \Delta E)}{\Delta E}. \quad (8)$$

For given values of  $\alpha$  and  $T_0$ , we can compute from Eq. (8) the maximum acceptable value for  $\mathcal{S}_{V0}$ . From this value we can calculate an "optimal" attenuation, i.e., an attenuation which is an upper bound of the required attenuation between a noise source and a junction and whose expression is a simple function of  $\beta_0$ ,  $\Delta E$  and of the characteristics of the noise source.

We thus consider an electric line connected to a single-electron device at very low temperature. The line includes several thermal noise sources (due, for instance, to the dissipative filters placed at higher temperature than the device). Each thermal source of noise  $i$  is characterized by its resis-

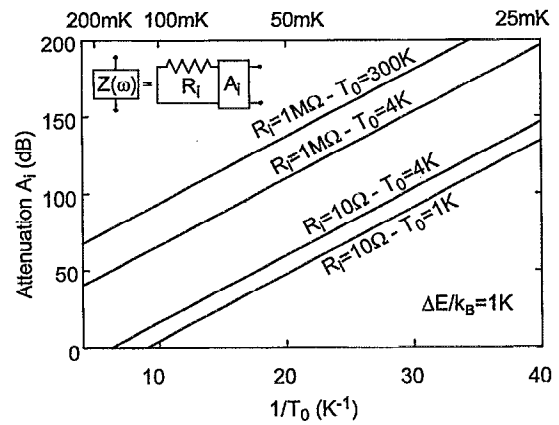


FIG. 2. Optimal attenuation  $A_i$  for the effective thermalization of a single-electron device in a Coulomb-blocked state separated from the next Coulomb-blocked state by an energy barrier corresponding to 1 K. The attenuation is given as a function of the device temperature  $T_0$  for several thermal noise sources of resistance  $R_i$ . Here, the impedance  $Z(\omega)$  seen from one junction consists of the resistance  $R_i$  combine with a frequency independent attenuating element  $A_i$ .

tance  $R_i$  at temperature  $T_i = \theta_i \Delta E/k_B$  such that  $\theta_i > 1$ . The contribution to  $\mathcal{S}_V$  from source  $i$  is  $a_i \mathcal{S}_{V_i}$ , where  $\mathcal{S}_{V_i}$  is the noise spectral density across source  $i$ , and  $a_i$  the corresponding attenuation coefficient, seen from the junction. As discussed above, we choose  $a_i = 1$  as the condition for an effective thermalization of the junction.

This condition and Eq. (8) lead to the following expression for the optimal attenuation  $A_i(\text{dB}) = -10 \log_{10}(a_i)$ :

$$A_i(\text{dB}) = \frac{10}{\ln(10)} \left[ \beta_0 \Delta E + \ln\left(2 \frac{R_i}{R_K}\right) + \ln[\theta_i \ln(\theta_i) - \theta_i + 1] \right], \quad (9)$$

a result valid for the typical case  $\beta_0 \Delta E \gg 1$ ,  $a_i R_i/R_K \ll 1$  and  $\theta_i > 2$ ; for a temperature such that  $\theta_i \approx 1$ , the numerical integration of Eq. (7) shows that the  $\ln[\theta_i \ln(\theta_i) - \theta_i + 1]$  term has to be replaced by  $-2$ .

The quantum expression (9) gives the optimal attenuation between a resistance generating noise and the junction. Figure 2 shows this optimal attenuation  $A_i(\text{dB})$  as a function of the device temperature, and for several thermal noise sources. The values of  $A_i$  are much larger than those of order  $T/T_0$  one would expect from a fully classical theory involving an Arrhenius term. Note however that a semiclassical theory involving a classical electromagnetic noise spectral density (i.e., constant for all frequencies) would have predicted a higher attenuation, since in that case expression (7) would converge only because of the high-frequency cut off in  $a_i$  due to the junction capacitances of the device. The attenuation (9) is dominated by the  $1/T_0$  term, which varies from 40 to 220 dB when  $T_0$  varies from 100 to 20 mK. The second term of Eq. (9) has a more modest influence: Its contribution is equal, for instance, to  $-39$  dB for a resistance of  $10 \Omega$  at 1 K and equal to  $+60$  dB for a resistance of  $10 \text{ M}\Omega$  at 300 K. Figure 2 shows that it can be difficult to thermalize a single electron device at a temperature lower

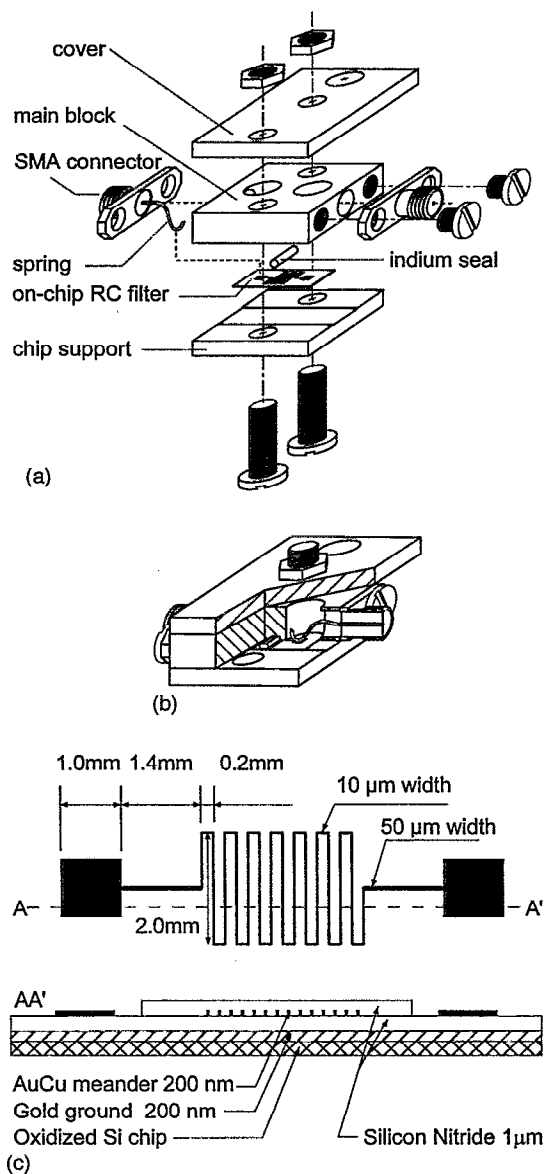


FIG. 3. Miniature filter for preventing electromagnetic noise from reaching Coulomb-blockade devices at cryogenic temperature. (a) Exploded view showing all the components of the filter. (b) Cut-away view of the filter shown on (a) after assembly. (c) Dimensions of the meander line, dielectric, and ground plane.

than 50 mK: One needs for this purpose filters with an as large as possible attenuation in a bandwidth corresponding to the temperature interval 1–300 K, i.e., 20 GHz–6 THz. These resistive filters have to be anchored at the lowest available temperatures in order to minimize the noise they themselves generate.

### III. PRACTICAL REALIZATION OF A FILTERING SYSTEM

#### A. Description of the miniature filters

The filter presented in Fig. 3 consists of a chip clamped in a brass case equipped with two “SMA” connectors. Its outer dimensions are 35 mm×25 mm×10 mm. The filtering

element is a distributed RLC meander line microfabricated on the chip. This microfabrication by optical lithography on oxidized silicon wafers involves four layers: a 200-nm-thick gold ground plane deposited by *e*-beam evaporation; a 1- $\mu$ m-thick silicon nitride dielectric layer deposited by plasma enhanced chemical vapor deposition (PECVD). Its relative dielectric constant is  $\epsilon_r=7$ ; a 200-nm-thick resistive copper-gold alloy layer, patterned in a meander line terminated by two pads [see Fig. 3(c) for dimensions]. It is deposited by thermal evaporation; and a 1- $\mu$ m-thick silicon nitride protective layer deposited by PECVD.

The connection to the pads is made through two CuBe springs soft-soldered to the “SMA” connectors. An indium wire, inserted in a groove of the main block, is pressed against the chip, when the filter is assembled. The role of this seal is to minimize the direct electromagnetic coupling between the springs. This filter was able to withstand twenty thermal shocks between room temperature and 77 K without degradation of performances and several filters have been used successfully during six runs at temperatures ranging from 4 K to 16 mK.

#### B. Performances of the filters

The power attenuation of our filters was measured as a function of frequency [see bar-shaped points on Fig. 4(a)] up to 20 GHz. The setup consists of a 50  $\Omega$  output impedance microwave generator, a 35 dB directivity reflection bridge for impedance matching, one filter and a 50  $\Omega$  input impedance spectrum analyzer. For each frequency point, we first measure the response of the filter with the analyzer. We then remove the filter and redo the measurement by connecting directly the reflection bridge to the analyzer. The attenuation is given by the difference between the results of the two measurements. The filters built with chips coming from the same deposition batch have attenuation differing at most by 6 dB. Figure 4(a) shows a typical attenuation curve of a filter with a 1.5 k $\Omega$  dc resistance of the meander. It corresponds to an alloy with a  $10^{-7}$   $\Omega$ m resistivity. Since the penetration depth is reduced to the meander thickness only at 600 GHz, the skin effect is negligible in the frequency range shown. The total line capacitance  $C$  and line inductance  $L$  are equal to 20 pF and 3.5 nH, respectively. These parameters are calculated from the geometry of the meander line, from the dielectric constant of the silicon nitride, and from the classical formula giving the impedance of a microcircuit line in the quasistatic approximation.<sup>9</sup> Compared to the resistance, the distributed inductance is negligible up to 400 GHz, and the RLC line reduces to a simple RC line below this frequency. The capacitance  $C_C$  of each pad is equal to 50 pF.

Figure 4(a) shows that the attenuation decreases above 300 MHz. As we have seen, this effect cannot be due to the distributed inductance. We believe it is due to the electromagnetic leakage between the two connecting fixtures and/or between adjacent meanders, since the attenuation at about 20 GHz of a closed case with springs but without chip [see dots on Fig. 4(a)] is lower than that of a complete filter. Furthermore, the attenuation of a complete filter in the GHz range increases by a few dB when the assembly screws are tightened. A crude model for the leakage [see Fig. 4(b)] considers

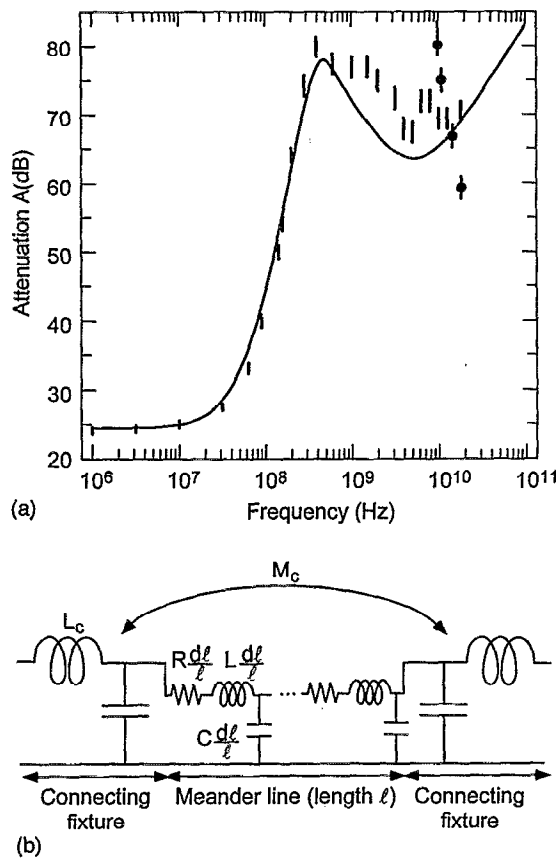


FIG. 4. (a) Power attenuation of the filter inserted in a circuit consisting of a 50  $\Omega$  generator and load. The experimental points correspond to a single complete filter (bars) and to a closed case with springs but without chip (dots). Solid line: Model of (b) with  $R=1.5$  k $\Omega$ ,  $L=3.5$  nH,  $C=20$  pF,  $C_C=50$  pF,  $L_C=1.5$  nH and  $M_C=1$  pH. (b) Simple model for the complete filter, including the distributed RC meander line and the connecting fixture (springs and pads) inducing electromagnetic leakage.

the springs as two antennas with a self-inductance  $L_C$ , coupled one to another by a mutual inductance  $M_C$ . The solid curve of Fig. 4(a) shows the behavior predicted by such a model, for fitted values of  $L_C$  and  $M_C$ . The model predicts an attenuation larger than 60 dB at frequencies higher than 20 GHz, for which measurements would be difficult to perform reliably.

### C. Practical number of filters in an electrical line

Assuming a minimum attenuation of 50 dB per filter in the frequency range of interest, i.e., up to 6 THz which is the cut-off frequency corresponding to a 300 K thermal noise, and using the results of Sec. II, one can estimate the optimum number of filters per electric line.

We consider as a first example the typical case of a voltage measurement line connecting a device at 30 mK to a preamplifier at 300 K. (We suppose that the current noise of the latter is dealt with a room temperature filtering system.) The input impedance of the amplifier is 1 M $\Omega$ , and assumed to be independent of the frequency in order to bound the generated noise from above. Equation (9) indicates that the attenuation needed for the effective thermalization of the device is equal to 195 dB. This attenuation has to be provided

by the whole circuit connected to a particular junction, which includes the rest of the device, the leads and the filters. However, we consider for proper safety margins the worst case for which only the filters participate in the attenuation of the spectral density of noise. Four of our filters are thus needed between 300 K and 30 mK. The problem is then to choose the anchoring temperatures of the filters, taking into account the available room and refrigeration power at each stage of the refrigerator, together with the noise generated by the filters themselves. The noise of a filter has to be sufficiently attenuated by those of the filters placed closer to the device, i.e., at lower temperatures. Indexing the  $n$  filters and their temperatures by  $j$  in decreasing order, and taking as an upper bound of the noise of filter  $j$  that of its dc resistance  $R$  at the same temperature  $T_j$ , a sufficient condition is that  $A_j(R=1.5$  k $\Omega, T_j) \leq (n-j) \times 50$  dB,  $A_j$  being calculated according to Eq. (9). In our example, the first of the four filters can be anchored at 4 K, because its 1.5 k $\Omega$  resistance induces at most a noise corresponding to a 139 dB attenuation. Such an attenuation can be provided by the three other filters. The second filter cannot be anchored at 1 K with confidence, because the required attenuation would be of the order of 130 dB, which is larger than the attenuation provided by the two other filters. A possibility is therefore to anchor one filter at 4 K and the three others at the temperature of the device.

The case of lines to voltage sources is very similar to the previous case. These sources at 300 K have an output impedance of the order of 100  $\Omega$ . The optimum attenuation is then about 155 dB, but the same arrangement of filters as previously described is required for proper safety margins.

## IV. CONCLUDING SUMMARY

We have shown that the effective thermalization of single-electron devices requires special cryogenic filters. We have proposed a formula giving the optimal line attenuation, as a function of device temperature for values well below 1 K. In the case of typical thermal sources, this formula predicts attenuations which can be as large as 200 dB. We have then presented dissipative compact filters functioning at cryogenic temperature. Four of these filters in series provide the required attenuation for typical Coulomb blockade devices working at 30 mK. These filters should also be useful for other kinds of very low noise experiments at low temperature, such as the bolometric detection of particles.<sup>10</sup>

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<sup>7</sup>A quantum spectral density obeys  $S_V(\omega < 0) > S_V(\omega > 0)$ , where  $\omega < 0$  and  $\omega > 0$  correspond to the emission and absorption of photons to or from the junction, respectively. For the corresponding classical density, one would expect a strict equality.

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