Minimal Electromagnetic Currents and Commutation Relations*

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It is pointed out that within the quark model the equal-time commutation relations of currents provide us with a test of minimal electromagnetic interaction of the hadrons. When compared with experiment, the modifications of the Cabibbo-Radicati and Drell-Hearn sum rules resulting from nonminimal interactions may possibly fix the magnitudes of the isoscalar and the isovector Pauli interactions.

IT has been recognized for some time that a difficulty in field theory arises in attempting to formulate the idea of minimal electromagnetic interactions in a mathematically unambiguous way.[1] Consider, for instance, a spin-$\frac{1}{2}$ particle of charge e with the Lagrangian density

$$L=-\bar{\psi}(\gamma\partial+m)\psi. \tag{1}$$

The minimal gauge-invariant Lagrangian is then obtained by the replacement

$$\partial_\alpha\to\partial_\alpha-ieA\alpha, \tag{2}$$

resulting in the usual electromagnetic interaction without a Pauli moment term. However, if we add to the Lagrangian (1) a term like

$$(2\mu/e)\partial_\alpha(\bar{\psi}\sigma_{\alpha\beta}\partial_\beta\psi), \tag{3}$$

we obtain the Pauli term by the same replacement (2), although the free equation of motion for the spin-$\frac{1}{2}$ particle remains unaltered. It appears that a difficulty of this kind cannot be settled until one finds a prescription for fixing the Lagrangian in a "minimal" way.[2] It is, therefore, desirable to define the minimal current without recourse to a Lagrangian, since the current operator itself might remain as a physically sensible operator even when the Lagrangian formalism turns out to be inadequate for the description of hadronic systems.

In this paper, we wish to point out that the current commutation relations[3]

$$[V_0^i(\mathbf{x},0),V_0^j(\mathbf{y},0)]=if_{ijk}V_0^k(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}), \tag{4}$$

or their Fourier transforms

$$[V_0^i(\mathbf{q}),V_0^j(\mathbf{q}')]=if_{ijk}V_0^k(\mathbf{q}+\mathbf{q}') \tag{5}$$

might serve as a set of conditions for defining the minimal currents.[4]

It is, of course, obvious that the minimal currents obey the commutation rules (5), since the commutation rules (5) were in fact derived from the quark model with the assumption of minimal coupling. However, one can further show that if *the currents are not of minimal type*, the commutation rules (4) are in general no longer valid. Within the quark model, this can be demonstrated as follows: We define the quark current as

$$V_\alpha^i(x)=\mathfrak{F}_\alpha^i(x)+\kappa^i\partial_\beta T_{\beta\alpha}^i(x), \tag{6}$$

where $\mathfrak{F}_\alpha^i(x)$ is the usual F-spin current density and $T_{\beta\alpha}^i(x)$ is an antisymmetric tensor current. κ^i is a constant with a dimension of length. Obviously the second term of (6) is of nonminimal type. Note also that the current be given by (6) is a specific way of introducing nontrivial gradient terms into the commutators between current densities. Given the current (6), it is now straightforward to calculate the commutation relations:

$$\begin{aligned}[V_0^i(\mathbf{q}),V_0^j(\mathbf{q}')]=if_{ijk}\{[1-\kappa^i\kappa^j(\mathbf{q}\cdot\mathbf{q}')]\mathfrak{F}_0^k(\mathbf{q}+\mathbf{q}')\\ -i(\kappa^i\mathbf{q}+\kappa^j\mathbf{q}')_l T_{0l}^k(\mathbf{q}+\mathbf{q}')\}\\ +id_{ijk}\kappa^i\kappa^j(\mathbf{q}\times\mathbf{q}')_l A_l^k(\mathbf{q}+\mathbf{q}'),\end{aligned} \tag{7}$$

where $T_{0i}^k(\mathbf{q})$ and $A_i^k(\mathbf{q})$ are the Fourier transforms of the tensor current $T_{0i}^k(\mathbf{x},0)$ and the spatial component of the axial-vector current $A_i^k(\mathbf{x},0)$, respectively.

Except for the term independent of κ, the right-hand side of (7) consists of the contributions from the gradient terms in the local commutation relations, and thus represents the modifications of the commutation relarelations (5). In the limit $\mathbf{q}=\mathbf{q}'=0$, since $V_0^i(\mathbf{0})=\mathfrak{F}_0^i(\mathbf{0})$, one recovers the commutation rules (5).

The modified commutation relations (7) have a direct bearing on the sum rules which are expressed in terms of experimentally observable quantities. We show this by taking the matrix element of (7) between one-proton states and deriving the sum rules. From the antisymmetric part of (7), the following modified Cabibbo-

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[1] G. Wentzel, as reported by M. Goldberger in *Proceedings of the Tenth Annual Conference on High Energy Physics at Rochester, 1960*, edited by E. C. G. Sudarshan, J. H. Tinclot, and A. C. Melissions (Interscience Publishers, Inc., New York, 1961); S. L. Glashow and M. Gell-Mann, Ann. Phys. (N.Y.) **15**, 437 (1961).

[2] Besides the difficulty mentioned here, there is another well-known difficulty of a Lagrangian formalism of higher spin ($J\geq\frac{3}{2}$) charged particle with the electromagnetic interaction. See, for instance, K. Johnson and E. C. G. Sudarshan, Ann. Phys. (N. Y.) **13**, 126 (1961).

[3] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[4] In the quark model with nonderivative coupling, the commutation relation (4) or (5) is equivalent to the assumption of minimal coupling without any ambiguities.

Radicati sum rule[5] is obtained:

$$-\tfrac{1}{3}\langle r^2\rangle_{F_1}{}^V+\left(\frac{\mu_A{}^V}{2m}\right)^2+\frac{1}{2\pi^2\alpha}\times\int_0^\infty\frac{[2\sigma_{1/2}{}^V(\nu)-\sigma_{3/2}{}^V(\nu)]}{\nu}d\nu=(\kappa^3)^2, \quad (8)$$

where κ^3 is a constant appearing in (6).

A numerical estimate of the left-hand side of (8) shows[6] that, although the contribution from the (3-3) resonance is of the wrong sign, inclusion of both low-energy nonresonant and higher resonance contributions improves the situation, and there is in fact a good chance that the left-hand side of (7) is zero. If this were the case, we would have a piece of evidence for the minimal current in the quark model.[7] A small value of κ^3, however, cannot be excluded at present, and whether such a term really exists or not is still an open question.

In a previous paper,[8] we showed that the sum rule for the magnetic moment suggested by Drell and Hearn[9] follows from (5). The modified version of this sum rule from (7) is given by

$$\left(\frac{\mu_A{}^p}{2m}\right)^2-\left(\frac{\mu_A{}^n}{2m}\right)^2+\frac{1}{8\pi^2\alpha}\int_0^\infty \times\frac{[\sigma_\perp(\nu)-\sigma_{\shortparallel}(\nu)]^p-[\sigma_\perp(\nu)-\sigma_{\shortparallel}(\nu)]^n}{\nu}d\nu=\tfrac{2}{3}\kappa^3\kappa^8G_A, \quad (9)$$

where $\mu_A{}^p$ ($\mu_A{}^n$) is the anomalous magnetic moment of the proton (neutron), $\sigma_{\shortparallel}{}^p(\nu)$ and $\sigma_\perp{}^p(\nu)$ ($\sigma_{\shortparallel}{}^n(\nu)$ and $\sigma_\perp(\nu)$) are the total cross sections for the absorption of circularly polarized photons by the proton (neutron) polarized with its spin parallel and antiparallel to the photon spin, respectively. G_A is the renormalization factor for the axial-vector current of the nucleon. In deriving the sum rule (9), a subtraction has been made with respect to the proton and neutron expectation values so that the unknown constants, such as the renormalization factors of the zeroth and the eighth components of the axial-vector currents, are eliminated.

Recently, it has been shown[10] that the sum rules (8) and (9) with $\kappa^3=\kappa^8=0$ can be derived from low-energy theorems and the assumption of unsubtracted dispersion relations for the forward scattering amplitudes of the isovector photons with the nucleon. In the course of the derivation of the theorems, however, it is necessary to make use of the commutation relations (5). It is then clear that if we assume (7) instead of (5), the low-energy theorems should also be modified so that the final sum rules are in agreement with (8) and (9).

Next, we extend our argument to the commutation relations between vector and axial-vector charge densities which are modified as follows:

$$[V_0{}^i(\mathbf{q}),A_0{}^j(\mathbf{q}')]=if_{ijk}A_0{}^k(\mathbf{q}+\mathbf{q}')+\kappa^i d_{ijk}\mathbf{q}_l\tilde{T}_{l0}{}^k(\mathbf{q}+\mathbf{q}'), \quad (10)$$

where $\tilde{T}_{\alpha\beta}{}^i(\mathbf{q})$ is the Fourier transform of a dual tensor current defined by $-i\epsilon_{\alpha\beta\gamma\delta}T_{\gamma\delta}{}^i(\mathbf{x},0)$.

Following the procedure similar to that by which the sum rules (8) and (9) are derived, one then obtains, from (10), sum rules which correspond to modifications of the sum rule derived by Fubini, Furlan, and Rossetti.[11] However, these sum rules contain, in addition to the constant κ, unknown parameters involving the renormalization factors for the dual tensor currents. Since no experimental information is available for these parameters at present, it would be rather difficult to test such sum rules.

To summarize, we have shown how the sum rules, derived under the usual assumption of minimal currents, are modified by the possible presence of additonal nonminimal terms. Through an experimental check of these sum rules, it would then seem possible to see whether or not such nonminimal currents exist.[12]

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[11] S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **40**, 1171 (1965).

[12] It is of course to be understood that even if these sum rules with nonvanishing κ are verified, it does not necessarily imply that the current is uniquely given by (6). However, this could be evidence for the existence of such nonminimal currents.