Minimal Scene Descriptions from Structure from Motion Models

## Song Cao and Noah Snavely

Department of Computer Science, Cornell University

## Introduction

## - How much data do we need to describe a location?

- Context: 3D scene reconstructions by Structure from Motion
- Goal: Compute compact representations of SfM reconstructions for location recognition - Benefits: Reduce the memory and computational cost of a location recognition system - Take-home message: We can summarize an SfM model with $<2 \%$ of points, while keeping reasonable recognition performance, aided by selecting distinctive points.

input from Structure from Motion
- An image set $\mathcal{I}$ of size $m$ and 3D point set $\mathcal{P}$ of size $n(n \gg m)$
- Visibility matrix $M$ of size $m \times n: M_{i j}=\left\{\begin{array}{l}1, \text { point } P_{j} \text { is visible in image } I_{i} \\ 0, \text { otherwise }\end{array}\right.$
- A descriptor mean for each 3D point


## Obiectives

- Goal: Compute a small subset $\mathcal{P}^{\prime}$ of $\mathcal{P}$ that captures as much data as possible
- Previous Approach [1]: K-cover algorithm - greedy algorithm that maximizes coverage - Our Approach: an point selection algorithm that considers
- 1. coverage: any new image has a high probability of seeing a large number of points in $\mathcal{P}$

2. distinctiveness: the descriptors in $\mathcal{P}^{\prime}$ are sufficiently distinct from one another

## Why Distinctiveness?


Large portion of
descriptors are
confusing!

- Select points that both
ensure coverage and
distinct reduces errors
in matching process


## Maximizing Expected Coverage

- Treat visibility as probabilistic event: $P_{j}$ is visible in each database image $l_{i}$ with probability $p_{i j}$
- Goal: to find a subset $\mathcal{P}^{\prime}$ that maximizes the probabilities of each image seeing $\geq K$ points in $\mathcal{P}^{\prime}$

$$
S\left(\mathcal{P}^{\prime}\right)=\sum_{i \in \mathcal{I}} \operatorname{Pr}\left(v_{i, \mathcal{P}^{\prime}} \geq K\right)
$$

- Gain of adding point $P_{j}: G\left(j, \mathcal{P}^{\prime}\right)=S\left(\mathcal{P}^{\prime} \cup\left\{P_{j}\right\}\right)-S\left(\mathcal{P}^{\prime}\right)$
- Bootstrapping problem: If image $I_{i}$ sees fewer than $K-1$ points in $\mathcal{P}^{\prime}$, then the gain for adding any new point to $\mathcal{P}^{\prime}$ w.r.t. $\boldsymbol{l}_{i}$ is zero
- Initial point set: We first need to cover each image with K points to yield a non-zero gain

Selecting an Initial Set of Distinctive Points

- 1. Gain of adding point $P_{j}$ by K-cover (KC) algorithm [1]

$$
G_{K c}\left(j, \mathcal{P}^{\prime}\right)=\sum_{l_{i} \in \mathcal{I} \backslash C} M_{i j}
$$

2. Weight factor for encouraging distinctiveness $\left(d_{\min }(j)\right.$ is the nearest distance from $P_{j}$ to current selected $\mathcal{P}^{\prime}$ )

$$
w_{d}\left(d_{\min }(j)\right)=\left\{\begin{array}{cc}
d_{\min }(j) / d, & d_{\min }(j)<d \\
1, & d_{\min }(j) \geq d
\end{array}\right.
$$

- 3. Greedily select the point with highest weighted gain

$$
G_{K C D}\left(j, \mathcal{P}^{\prime}\right)=w_{d}\left(d_{\min }(j)\right) G_{K C}\left(j, \mathcal{P}^{\prime}\right)
$$

4. Repeat Step 3 until all images are covered by at least $K$ points


## Probabilistic K-cover Algorithm

- 1. Assuming constant $p$ for each $p_{i j}$, the number - . Assuming constant $p$ for each $p_{i j}$, the number
of points in the chosen subset $\mathcal{P}^{\prime}$ image $I_{i}$ sees of points in the chosen subs
follows binomial distribution

$$
\operatorname{Pr}\left(v_{i, \mathcal{P}^{\prime}}=K^{\prime}\right)=\binom{C_{i}}{K^{\prime}} p^{K^{\prime}}(1-p)^{C_{i}-K^{\prime}}
$$

- 2. Gain of adding point $P_{j}$ (e.g. dotted red v.s. red on the right)

$$
G_{K C P}\left(j, \mathcal{P}^{\prime}\right)=\sum_{i \in \mathcal{I} \backslash C} p_{i j} \operatorname{Pr}\left(v_{i, \mathcal{P}^{\prime}}=K-1\right)
$$

- 3. Greedily choose the point $P_{j^{*}}$ that maximizes $G_{K C P}\left(j, \mathcal{P}^{\prime}\right)$ and update $\operatorname{Pr}\left(v_{i, \mathcal{P}^{\prime}}=K^{\prime}\right)$

4. Repeat from Step 3 until a specified percentage of images are covered.

Datasets

| Dataset | \# DB Imgs | \# 3D Points | \# Queries |
| :---: | :---: | :---: | :---: |
| Dubrovnik [1] | 6,044 | $1,886,884$ | 800 |
| Aachen [2] | 4,479 | $1,980,036$ | 369 |
| Landmarks [3] | 205,813 | $38,190,865$ | 10,000 |

Registration Performance

- Methods: the $K$-cover algorithm (KC)[1], our initial point set selection algorithm only (KCD), and our full approach including the probabilistic $K$-cover algorithm (KCP)
- Compare the performances of scene descriptions with the same number of points

| Dubrovnik Dataset [1] |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| \# query images: 800, registered by full set: $99.50 \%$ |  |  |  |  |
| K | $12(9)$ | $20(12)$ | $30(20)$ | $50(35)$ |
| \# points | 5,788 | 10,349 | 17,241 | 31,752 |
| \% points | $0.31 \%$ | $0.55 \%$ | $0.91 \%$ | $1.68 \%$ |
| KC | $58.00 \%$ | $77.06 \%$ | $86.00 \%$ | $91.81 \%$ |
| KCD | $62.88 \%$ | $78.88 \%$ | $87.38 \%$ | $92.50 \%$ |
| KCP | $64.25 \%$ | $79.13 \%$ | $87.25 \%$ | $93.38 \%$ |
| Aachen Dataset [2] |  |  |  |  |
| \# query images: 369, registered by full set: $88.08 \%$ |  |  |  |  |
| K | $30(20)$ | $50(32)$ | $80(52)$ | $100(65)$ |
| \# points | 13,299 | 23,675 | 40,377 | 52,161 |
| \% points | $0.67 \%$ | $1.20 \%$ | $2.04 \%$ | $2.63 \%$ |
| KC | $50.95 \%$ | $62.06 \%$ | $66.40 \%$ | $71.27 \%$ |
| KCD | $54.20 \%$ | $63.14 \%$ | $69.38 \%$ | $72.36 \%$ |
| KCP | $56.37 \%$ | $64.23 \%$ | $70.19 \%$ | $73.98 \%$ |

Landmarks Dataset [3]
\# query images: 10,000 , registered by full set: $94.33 \%$ \# query images: 10,000 , registered by full set: $94.33 \%$ $\begin{array}{ccccc}\text { K } & 6(4) & 9(6) & 12(9) & 20(12) \\ \text { \# points } & 140,306 & 222,161 & 311,035 & 571,864\end{array}$ $\begin{array}{ccccc}\text { \# points } & 140,306 & 222,161 & 311,035 & 571,864 \\ \% \text { points } & 0.37 \% & 0.58 \% & 0.81 \% & 1.50 \%\end{array}$

| \% points | $0.37 \%$ | $0.58 \%$ | $0.81 \%$ | $1.50 \%$ |
| :---: | :---: | :---: | :---: | :---: |
| KC | $44.84 \%$ | $59.86 \%$ | $69.56 \%$ | $81.06 \%$ |
| KCD | $45.45 \%$ | $61.26 \%$ | $70.59 \%$ | $81.04 \%$ |


| KCD | $45.45 \%$ | $61.26 \%$ | $70.59 \%$ | $81.04 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| KCP | $\mathbf{4 5 . 9 0 \%}$ | $\mathbf{6 1 . 5 0 \%}$ | $\mathbf{7 1 . 8 7 \%}$ | $81.45 \%$ |

## Reference

1] Y. Li, N. Snavely, and D. Huttenlocher. Location recognition using prioritized feature matching. In ECCV, 2010.
${ }^{2}$ T. Sattler, T. Weyand, B. Leibe, and L. Kobbelt. Image retrieval for image-based localization revisited. In BMVC, 2012.
[3] Y. Li, N. Snavely, D. Huttenlocher, and P. Fua. Worldwide pose estimation using 3d point clouds. In ECCV, 2012

