Minimization of the Difference between the Theoretical Mean of the Rayleigh Probability Density Function and the Mean Obtained from its Plot

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Abstract   In this study, the difference between the mean of the Rayleigh Probability Density Function and the mean obtained from the graph of Rayleigh Probability Density Function is minimized by changing the coefficient in the equation yielding the mean. By using various numbers of data and K values, Rayleigh Probability Density Function is plotted with the means mentioned above.

Keywords   Rayleigh Probability Density Function, Line of Sight, No Line of Sight

1. Introduction

Rayleigh Probability Density Function is used for the cases in which there is NLOS between transmitters and receivers for the communication networks and channel modeling. When there occur phase differences between multipath signals arriving the receiver, fading takes place. Rayleigh distribution function is used for multipath fading in modeling of change of voltage or power that will be received[1-8]. In Rayleigh modeling, in contrast to the Ricean, there is no any specified direction, that is, signals coming from any direction are assumed to have equal probability.

Moreover Rayleigh Modeling is used for noise analysis. If a signal coming to a receiver via reflection becomes so greater than the direct signal that it suppresses that, in this case this type of channel modeling is done by means of Rayleigh Probability Density Function.

The aim of our study is to find an expression which results in numerical values that are very close to the actual values. For this purpose after much iteration we have obtained a new coefficient that gives the desired results.

2. Theory

Rayleigh Probability Density Function changes with respect to a single parameter, either standard deviation or K[9-15]where K is the power of direct signal divided by the power of coming signal via reflection and expressed as

$$K = P^{'}los/P^{'}multipath \qquad (1)$$

$$K = 1/2\ \sigma^2 \qquad (2)$$

$$P^{'}_{los} = 0\ Watt = \ 1\ dBW \qquad (3)$$

K characterizes the environment. K values are normalized when using in the equation(4).

Rayleigh Probability Density Function is expressed as

$$f_z(z) = 2\ z\ K\ exp\big((-z^2 K)\big) \qquad (4)$$

When K represents the Power

$$L = K/10 \qquad (5)$$

and if it represents the voltage

$$L = K/20 \qquad (6)$$

is used. For both cases,

$$S = 10^L. \qquad (7)$$

Normalized Rayleigh Probability Density Function can be written as

$$f_z(z) = \ 2\ z\ S\ exp\big((-z^2 S)\big) \qquad (8)$$

S is obtained through the equations (5), (6), (7) and used in equation (8) to obtain $f_z(z)$.

Mean of this function is

$$f_z(z)(mean) = M[z] = \int_0^\infty z\, f_z\,(z)\ dz, \qquad (9)$$

And theoretical mean can be written as,

$$TM = M[z] = \left(\tfrac{1}{2}\right)\sqrt{\pi/K} = 1.2533\sigma \qquad (10)$$

And the variance is written as

$$\sigma^2 = M[z^2] - M^2[z] = (0.2146/K) \qquad (11)$$

And the Standard deviation is

$$\sigma = 0.4632/\sqrt{K} \qquad (12)$$

Table 1 shows the actual and theoretical means and percent error of theoretical mean.

Table 1. Actual and Theoretical Mean Values and Percent Errors.

for 171 data			
K voltage (dB)	AM Actual mean	TM theoretical mean	Error(%)
10	41	49.83	5.16
12	36	44.44	4.93
14	33	39.58	3.84
16	29	35.28	3.67
18	26	31.44	3.18
20	23	28.02	2.93

3. Modification of Theoretical Mean Expression

In equation (10) the coefficient ½ is not suitable. Because there is a big difference between the actual and theoretical curves when theoretical mean is used for the plot of Rayleigh Probability Density Function. Instead we suggest a change in equation (10) to achieve more reasonable results. After many iterations the coefficient ½ is replaced by 1/2.3589 to minimize the difference between the theoretical mean(TM) and the actual mean (AM) which is obtained from the graph of Rayleigh Probability Density Function.

Since TM being a function of K is the theoretical mean, if and only if K1 is used instead of K in Rayleigh Probability Density Function its actual mean becomes TM. This is illustrated in Table 4.

In Table 3 peak values corresponding to AM and TM voltage differences (errors) are given.

Equation (14) can be used for finding K1 by replacing it in place of K.

$$f_1(z) = 2 z K1 \exp\big((-z^2 K1)\big) \qquad (13)$$

$$MM = M1[z] = \left(\frac{1}{2.3589}\right)\sqrt{\pi/K} \qquad (14)$$

Where MM is the modified mean. As can be seen from Table1, and Table 2 for K=10 dB AM=41, TM=49.83 MM=42 is obtained. This shows that the equation (14) yields a value which is very close to the actual mean whereas the theoretical one (TM) is far from it.

Table 2. Actual and Modified Mean Values and Percent Errors

for 171 data			
K voltage (dB)	AM Actual mean	MM Modified mean	Error (%)
10	41	42	0.58
12	36	37	0.58
14	33	33.56	0.32
16	29	30	0.58
18	26	26.6	0.35
20	23	23.76	0.44

Since the variance and standard deviation values are derived from the mean, an erroneous mean causes wrong results in variance and standard deviation[16-23]. Therefore to obtain minimum difference between the actual and theoretical mean, we changed the coefficient value several times. As a result we reduced the percent error from 5.16 percent to 0.32 percent. The average of x number of data can be expressed as

$$MM_x = \frac{Data\ x}{171} MM_{171} \qquad (15)$$

Where MM_{171} is the average value of 171 data.

Table 3. Actual and Theoretical Peak Values and the difference between them

K (dB)	K1 (dB)	Actualpeak value off_z(z) (%)	theoretical peak value off_z(z) (%)	Voltage difference
10	7.13	1.5252	1.4503	0.0749
12	9.29	1.7112	1.6398	0.0714
14	11.13	1.9199	1.8240	0.0959
16	13.13	2.1545	2.0688	0.0857
18	15.13	2.4174	2.3317	0.08423
20	17.13	2.7117	2.6049	0.1068

Table 4 shows the variations of $f_z(z)$ and $f_{1z}(z)$.

Table 4. $f_z(z)$ and $f_{1z}(z)$ values depending on K and K1 respectively

K(dB)	$f_z(z)=2 z Sexp(-z^2S)$	K1(dB)	$f_{1z}(z) =2 z S1exp(-z^2S1)$
10	6.32 z exp(-z²3.16)	7.13	4.54 zexp(-z²2.27)
12	7.96 z exp(-z²3.98)	9.29	5.82 z exp(-z²2.91)
14	10.02 z exp(-z²5.01)	11.13	7.20 z exp(-z²3.60)
16	12.61 z exp(-z²6.30)	13.13	9.06zexp(-z²4.53)
18	15.88 zexp(-z²7.94)	15.13	11.41 zexp(-z²5.7)
20	20 z exp(-z²10)	17.13	14.37 zexp(-z²7.18)

Actual, theoretical, and modified Rayleigh Probability Density Functions are plotted according to different K and data values in Fig. 1 through Fig. 6.
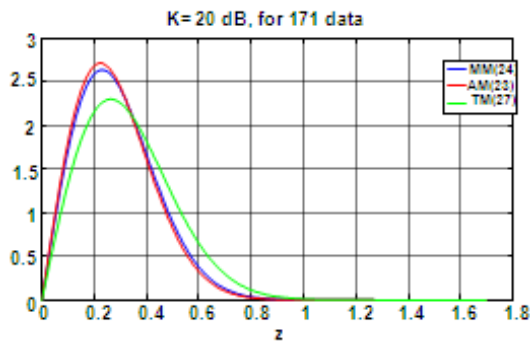

Figure 1. Plot of the Actual, Modified, and Theoretical Rayleigh Probability Density Functions
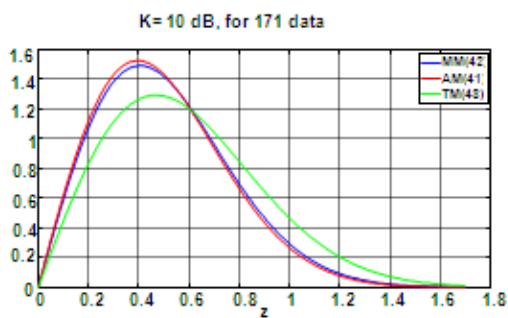

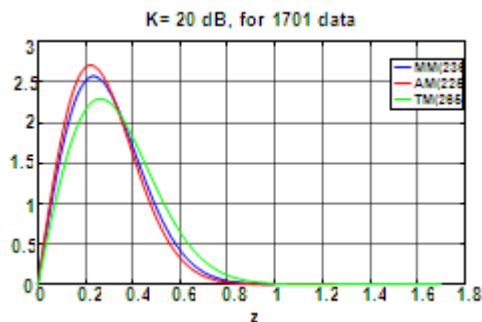**Figure 2.** Plot of the Actual, Modified, and Theoretical Rayleigh Probability Density Functions

Figure 3. Plot of the Actual, Modified, and Theoretical Rayleigh Probability Density Functions

Figure 4. Plot of the Actual, Modified, and Theoretical Rayleigh Probability Density Functions

Figure 5. Plot of the Actual, Modified, and Theoretical Rayleigh Probability Density Functions
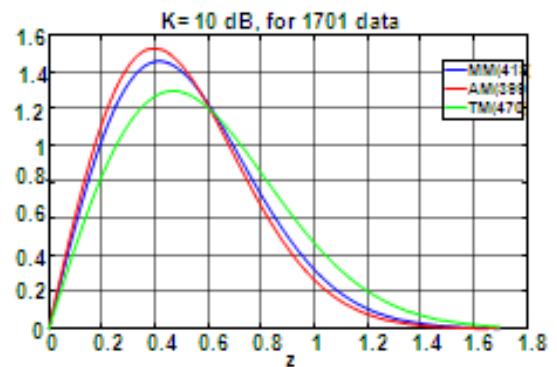

Figure 6. Plot of the Actual, Modified, and Theoretical Rayleigh Probability Density Functions
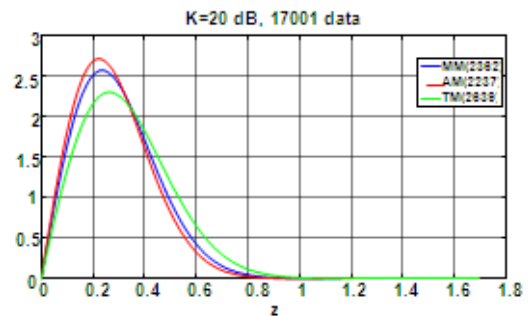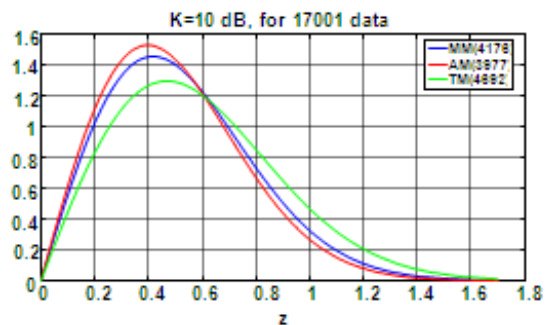
Table 5.   The mean values of $f_z(z), f_{1z}(z), f_{2z}(z)$ and their values around the means

K=20 dB, for 171 data AM				
Data number	22	23(mean)	24	25
Probability value	2.70224936	2.71177808	2.71029179	2.69828373
K=20 dB, for 171 data MM				
Data number	23	24(mean)	25	26
Probability value	2.63034350	2.63552781	2.63075854	2.61649395
K=20 dB, for 171 data TM				
Data number	26	27(mean)	28	29
Probability value	2.29304064	2.29894371	2.29814705	2.29090434

Table 6.   The mean values of $f_z(z), f_{1z}(z), f_{2z}(z)$ and their values around the means

K=10 dB, for 171 data AM				
Data number	40	41(mean)	42	43
Probability value	1.52477789	1.52529002	1.52388460	1.52061275
K=10 dB, for 171 data MM				
Data number	41	42(mean)	43	44
Probability value	1.49019207	1.49053635	1.48909413	1.48591223
K=10 dB, for 171 data TM				
Data number	47	48(mean)	49	50
Probability value	1.29257103	1.29305214	1.29236036	1.29052165

Table 7.   The mean values of $f_z(z), f_{1z}(z), f_{2z}(z)$ and their values around the means

K=20 dB, for 1701 data AM				
Data number	224	225(mean)	226	227
Probability value	2.71246757	2.71247918	2.71238249	2.71217797
K=20 dB, for 1701 data MM				
Data number	235	236(mean)	237	238
Probability value	2.57543251	2.57552531	2.57552503	2.57543207
K=20 dB, for 1701 data TM				
Data number	264	265(mean)	266	267
Probability value	2.29939718	2.29941544	2.29936765	2.29925407

Table 8.   The mean values of $f_z(z)$,$f_{1z}(z)$, $f_{2z}(z)$ and their values around the means

K=10 dB, for 1701 data AM			
Data number	398	399(mean)	400	401
Probability value	1.52533995	1.52534257	1.52532591	1.52529002
K=10 dB, for 1701 data MM			
Data number	417	418(mean)	419	420
Probability value	1.45500983	1.45501578	1.45500499	1.45497749
K=10 dB, for 1701 data TM			
Data number	469	470(mean)	471	472
Probability value	1.29305056	1.29305723	1.29305214	1.29303533

Table 9.   The mean values of $f_z(z)$,$f_{1z}(z)$, $f_{2z}(z)$ and their values around the means

K=20 dB, for 17001 data AM			
Data number	2236	2237(mean)	2238	2239
Probability value	2.71248695	2.71248756	2.71248709	2.71248554
K=20 dB, for 17001 data MM			
Data number	2361	2362(mean)	2363	2364
Probability value	2.56916919	2.56916947	2.56916883	2.56916727
K=20 dB, for 17001 data TM			
Data number	2638	2639(mean)	2640	2641
Probability value	2.29941691	2.29941708	2.29941659	2.2994154

Table 10.   The mean values of $f_z(z)$,$f_{1z}(z)$, $f_{2z}(z)$ and their values around the means

K=10 dB, for 17001 data AM			
Data number	3976	3977(mean)	3978	3979
Probability value	1.52534367	1.52534384	1.52534381	1.52534359
K=10 dB, for 17001 data MM			
Data number	4175	4176(mean)	4177	4178
Probability value	1.45292350	1.45292351	1.45292335	1.45292302
K=10 dB, for 17001 data TM			
Data number	4691	4692(mean)	4693	4694

    Table 5-Table 10 show the actual, modified and theoretical results for some data group. It can be seen from Table 1, Table 2, and Table 5 through Table 8, both TM and MM values are greater than AM values.

Table 11. Percent errors according to various K and data numbers

K voltage (dB)	for 171 data Error(%)	for 1701 data Error (%)	for 17001 data Error (%)
10	0.58	1.11	1.17
12	0.58	0.76	0.78
14	0.32	0.99	1.035
16	0.58	0.88	0.98
18	0.35	0.76	0.79
20	0.44	0.64	0.73

For a definite K value if the number of data increases percent error slightly increases as well.

4. Conclusion

In this study, to minimize the difference between the theoretical mean of Rayleigh Probability Density Function and the mean of the plot of Rayleigh function, we modified the equation (10) and obtained the equation (14) as the ultimate expression for the mean.

In actual plots and plots corresponding to theoretical means are shown together.

It is observed from the Table 11 that, for the smaller values of K, percent error increases by the number of data much more than for the larger values of K. For this particular study although the number of data increased 100 times, percent error was just only doubled.

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