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Minimum Aberration Blocking Schemes for Two-Level and Three-Level Fractional Factorial Designs

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Abstract: The concept of minimum aberration has been extended to choose blocked fractional factorial designs (FFDs). The minimum aberration criterion ranks blocked FFDs according to their treatment and block wordlength patterns, which are often obtained by counting words in the treatment defining contrast subgroups and alias sets. When the number of factors is large, there are a huge number of words to be counted, causing some difficulties in computation. Based on coding theory, the concept of minimum moment aberration, proposed by Xu (2003) for unblocked FFDs, is extended to blocked FFDs. A method is then proposed for constructing minimum aberration blocked FFDs without using defining contrast subgroups and alias sets. Minimum aberration blocked FFDs for all 32 runs, 64 runs up to 32 factors, and all 81 runs are given with respect to three combined wordlength patterns.

MSC: primary 62K15; secondary 62K10; 62K05

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1 Introduction

Fractional factorial designs (FFDs) are widely used in designing experiments. Blocking is an effective method for reducing systematic variations and therefore increasing precision of effect estimation. Experimenters would often face the problem of choosing optimally blocked FFDs.

FFDs are typically chosen according to the *maximum resolution* criterion (Box and Hunter, 1961) and its refinement, the *minimum aberration* (MA) criterion (Fries and Hunter, 1980). The study of blocking in FFDs is complicated by the presence of two defining contrast subgroups, one for defining the fraction and another for defining the blocking scheme, and therefore, resulting in two types of wordlength patterns, one for treatment and another for block. Bisgaard (1994) proposed resolution for choosing blocked FFDs. However, resolution alone is not significant enough to rank order blocked FFDs. The MA criterion can be applied to the treatment and block wordlength patterns separately. However, MA designs with respect to one wordlength pattern may not have MA with respect to the other wordlength pattern. One approach, as done by Sun, Wu and Chen (1997) and Mukerjee and Wu (1999), is to consider the concept of admissible blocking schemes, but it is often to have too many admissible designs. Another approach is to combine the treatment and block wordlength patterns into one single wordlength pattern so that the criterion of MA can be applied to it in the usual way; see Sitter, Chen and Feder (1997), Chen and Cheng (1999), Zhang and Park (2000), and Cheng and Wu (2002).

Sun, Wu and Chen (1997) provided collections of admissible blocked FFDs with 8, 16, 32, 64, and 128 runs up to 9 factors. Sitter, Chen and Feder (1997) provided collections of MA blocked FFDs with all 8 and 16 runs, 32 runs up to 15 factors, 64 runs up to 9 factors, and 128 runs up to 9 factors. Chen and Cheng (1999) developed a theory to characterize MA blocked FFDs in terms of their blocked residual designs and gave collections of MA blocked FFDs with all 8 and 16 runs, and 32 runs up to 20 factors. Cheng and Wu (2002) compared MA blocked FFDs with respect to different combined wordlength patterns for 8, 16, 32, 64, and 128 runs up to 9 factors; they also provided collections of MA and admissible blocked FFDs with all 27 runs, and 81 runs up to 10 factors.

The MA criterion ranks blocked FFDs according to the treatment and block wordlength patterns, which are often obtained by counting words in the treatment defining contrast subgroups and alias sets. When the number of factors is large, there are a huge number of words to be counted, causing some difficulties in computation. For example, when an FFD with 32 runs and 20 factors is arranged in 8 blocks, there are $2^{15} - 1 = 32,767$ words in the treatment defining contrast subgroup and 7 block effects, each block effect being confounded with $2^{15} = 32,768$ treatment effects. When an FFD with 81 runs and 20 factors is arranged in 27 blocks, there are $(3^{16}-1)/(3-1) = 21,523,360$ words in the treatment defining contrast subgroup and 13 block effects, each block effect being confounded with $3^{16} = 43,046,721$ treatment effects. It is very time consuming to count all these words or effects. This explains, partially at least, why MA blocked FFDs are available only up to 9 or 10 factors in most cases in the literature.

The purpose of this paper is to construct more MA blocked FFDs with a large number of factors. This is challenging due to aforementioned computational difficulties. Based on coding theory, we propose new methods to compare and rank blocked FFDs without using defining contrast subgroups and alias sets. The idea was originally due to Xu (2003), who proposed the concept of *minimum moment aberration* and established its equivalence to MA for unblocked FFDs. We extend the concept of minimum moment aberration to blocked FFDs. Then we propose a construction method to obtain MA blocked FFDs for all 32 runs, 64 runs up to 32 factors, and all 81 runs.

In Section 2, we review basic concepts and definitions for unblocked and blocked FFDs, and optimality criteria for choosing blocked FFDs. In Section 3, we review the minimum moment aberration criterion for unblocked FFDs and extend it to blocked FFDs. Section 4 describes the construction method. Tables of MA blocked FFDs with 32, 64 and 81 runs are given in Section 5 with comments. Concluding remarks are given in Section 6.

2 Basic concepts and definitions

2.1 Unblocked and blocked FFDs

Consider an experiment to study n treatment variables, each having s levels. A full factorial design requires s^n runs to be performed and is rarely used in practice for $n > 7$ treatment variables. For economic reasons, fractional factorial designs (FFDs), which consist of a fraction of the full factorial design, are commonly used.

A *regular* FFD, denoted by s^{n-k}, is an s^{-k}th fraction of the s^n full factorial design in which the fraction is determined by k treatment defining words. The k treatment defining words form the *treatment defining contrast subgroup*, which consists of s^k elements. Each element other than the identity I is called a word, and the number of letters in a word is called its length. Words $w, w^2, \ldots, w^{s-1}$ represent the same treatment contrast; therefore, they are viewed as the same.

There are in total $(s^k - 1)/(s - 1)$ distinct words in the treatment defining contrast subgroup. The *resolution* is defined to be the length of the shortest word in the treatment defining contrast subgroup. The larger the resolution, the better the design. This is called the *maximum resolution* criterion, proposed by Box and Hunter (1961). For $i = 1, \ldots, n$, let $A_{i,0}$ denote the number of words of length i in its treatment defining contrast subgroup. Then $\sum_{i=1}^{n} A_{i,0} = (s^k - 1)/(s - 1)$. As often done in the literature, we only consider designs with resolution III or higher so that $A_{1,0} = A_{2,0} = 0$. The vector $W_t = (A_{3,0}, \ldots, A_{n,0})$ is called the *treatment wordlength pattern*.

To reduce systematic variations and to increase the accuracy of the effect estimation, blocking is commonly used in the design of experiments. For blocking to be effective, the units should be arranged so that the within-block variation is much smaller than the between-block variation. When studying blocked FFDs, we assume that there are no interactions between block variables and treatment variables. This assumption states that the treatment effects do not vary from block to block.

To arrange an s^{n-k} FFD in s^p blocks of size s^{n-k-p}, we can choose p independent columns $b_1, \ldots, b_p$ as block defining words. The p block defining words form the *block defining contrast subgroup*, which consists of $n_p = (s^p - 1)/(s - 1)$ distinct words, each representing a block effect. A blocking scheme is *infeasible* if at least one of the treatment main effects is confounding with some block effects; otherwise, it is *feasible*. Let $b_1, \ldots, b_p, b_{p+1}, \ldots, b_{n_p}$ be the n_p block effects, where $b_{p+1}, \ldots, b_{n_p}$ are "generalized interactions" of $b_1, \ldots, b_p$. Because the treatment defining contrast subgroup has s^k elements, each block effect is confounded with s^k treatment words (or effects). For $i = 1, \ldots, n$, let $A_{i,1}$ denote the number of treatment words of length i that are confounded with some block effects. Then $\sum_{i=1}^{n} A_{i,1} = s^k n_p$. A nonzero $A_{1,1}$ indicates that some treatment main effects are confounded with some block effects. Therefore, a blocking scheme is feasible if and only if $A_{1,1} = 0$. The vector $W_b = (A_{2,1}, \ldots, A_{n,1})$ is called the *block wordlength pattern*. Let $A_{0,1} = 0$ for convenience.

Example 1. Consider a 2^{6-2} design in 2^2 blocks with treatment defining words $E = ABC$ and $F = ABD$, and block defining words $b_1 = ACD$ and $b_2 = BCD$. The treatment defining contrast subgroup is

$$I = ABCE = ABDF = CDEF, \tag{1}$$

and the treatment wordlength pattern is $W_t = (0, 3, 0, 0)$. The block defining contrast subgroup is

$$I, b_1 = ACD, b_2 = BCD, b_3 = b_1 b_2 = AB. \tag{2}$$

Combining (1) and (2) yields the following alias sets

$$\begin{aligned} b_1 &= ACD = BDE = BCF = AEF, \\ b_2 &= BCD = ADE = ACF = BEF, \\ b_3 &= AB = CE = DF = ABCDEF. \end{aligned}$$

Therefore, the block wordlength pattern is $W_b = (3, 8, 0, 0, 1)$.

Example 2. Consider a 3^{5-2} design in 3^2 blocks with treatment defining words $D = ABC$ and $E = AB^2$, and block defining words $b_1 = AB$ and $b_2 = AC^2$. The treatment defining contrast subgroup is

$$I = ABCD^2 = A^2B^2C^2D = AB^2E^2 = A^2BE = AC^2DE = A^2CD^2E^2 = BC^2DE^2 = B^2CD^2E, \tag{3}$$

For a three-level design, words w and w^2 (e.g., $ABCD^2$ and $A^2B^2C^2D$) represent the same treatment contrast; therefore, they are viewed as the same word. Hence the treatment wordlength pattern is $W_t = (1, 3, 0)$. The distinct words in the block defining contrast subgroup are

$$b_1 = AB, b_2 = AC^2, b_3 = b_1b_2 = A^2BC^2, b_4 = b_1b_2^2 = BC. \tag{4}$$

Combining (3) and (4) yields the following alias sets

$$\begin{aligned} b_1 &= AB = A^2B^2CD^2 = C^2D = A^2E^2 = B^2E = A^2BC^2DE = BCD^2E^2 = AB^2C^2DE^2 = ACD^2E, \\ b_2 &= AC^2 = A^2BD^2 = B^2CD = A^2B^2C^2E^2 = BC^2E = A^2CDE = D^2E^2 = ABCDE^2 = AB^2D^2E, \\ b_3 &= A^2BC^2 = B^2D^2 = ACD = C^2E^2 = AB^2C^2E = BCDE = ABD^2E^2 = A^2B^2CDE^2 = A^2D^2E, \\ b_4 &= BC = AB^2C^2D^2 = A^2D = ACE^2 = A^2B^2CE = ABDE = A^2BC^2D^2E^2 = B^2DE^2 = C^2D^2E. \end{aligned}$$

Therefore, the block wordlength pattern is $W_b = (10, 9, 12, 5)$.

2.2 Review of optimality criteria

Because resolution alone cannot determine the best design, Fries and Hunter (1980) further proposed the MA criterion as its refinement. For two unblocked s^{n-k} designs D_1 and D_2, let r be the smallest integer such that $A_{r,0}(D_1) \neq A_{r,0}(D_2)$. Then D_1 is said to have less aberration than D_2 if $A_{r,0}(D_1) < A_{r,0}(D_2)$. If there is no design with less aberration than D_1, then D_1 has MA. In short, the MA criterion sequentially minimizes $A_{1,0}, A_{2,0}, \ldots, A_{n,0}$.

The extension of MA to blocked FFDs is not unique due to the presence of two wordlength patterns. One popular approach is to combine the treatment and block wordlength patterns into one sequence and then apply the MA criterion to the combined wordlength pattern in the usual way. By arguing the relative importance of confounded effects, the following three combined wordlength patterns have been proposed in the literature

$$W_{scf} = (A_{3,0}, A_{2,1}, A_{4,0}, A_{3,1}, A_{5,0}, A_{4,1}, \ldots), \quad (5)$$

$$W_1 = (A_{3,0}, A_{4,0}, A_{2,1}, A_{5,0}, A_{6,0}, A_{3,1}, \ldots), \quad (6)$$

$$W_2 = (A_{3,0}, A_{2,1}, A_{4,0}, A_{5,0}, A_{3,1}, A_{6,0}, \ldots), \quad (7)$$

where $A_{i,1}$ is ranked after $A_{i+1,0}$ in W_{scf}, after $A_{2i,0}$ in W_1, and after $A_{2i-1,0}$ in W_2 for $i = 2, 3, \ldots$. The first sequence was proposed by Sitter, Chen, and Feder (1997), the second by Cheng and Wu (2002), and the third by Chen and Cheng (1999), Zhang and Park (2000), and Cheng and Wu (2002). For the pros and cons of the sequences, see Cheng and Wu (2002). Three MA criteria result from sequentially minimizing the corresponding combined wordlength patterns. MA blocked FFDs under the W sequence are called MA W designs. In this paper, we construct MA blocked FFDs with respect to all three W criteria.

Recall that when an s^{n-k} design is arranged in s^p blocks, there are $(s^k - 1)/(s - 1)$ words in the treatment defining contrast subgroup and $(s^p - 1)/(s - 1)$ block effects, and $s^k(s^p - 1)/(s - 1)$ treatment effects are confounded with some block effects. When k is large, it is very time consuming to count all these words, causing some difficulties in computation. In the next section we propose a new method for comparing blocked FFDs without using the treatment defining contrast subgroups and alias sets.

3 Minimum moment aberration

In Section 3.1, we review the concept of minimum moment aberration for unblocked FFDs due to Xu (2003). Then we extend the approach to blocked FFDs in Section 3.2.

3.1 Minimum moment aberration for unblocked FFDs

For a design of N runs and n factors, let $X = (x_{ik})$ be the $N \times n$ treatment matrix, where each row corresponds to a run and each column to a factor. For positive integers t, define power moments

$$K_{t,0} = N^{-2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\delta_{ij})^t, \quad (8)$$

where δ_{ij} is the number of coincidences between the ith and jth rows, i.e., the number of k's such that $x_{ik} = x_{jk}$.

Since the power moments $K_{t,0}$ measure the similarity among runs (i.e., rows), it is natural that a good design should have small power moments. The smaller the $K_{t,0}$, the better the design. Xu (2003) proposed the *minimum moment aberration* criterion which sequentially minimizes $K_{1,0}, K_{2,0}, \ldots, K_{n,0}$.

For a prime power s, let $GF(s)$ be the finite field of s elements. It is well known that an s^{n-k} FFD is a linear code of length n and dimension $n-k$ over $GF(s)$ in coding theory. If x_i and x_j are two row vectors of a linear code, then their difference $x_i - x_j$ (in $GF(s)$) is also a row vector. Therefore, for an s^{n-k} FFD, (8) can be simplified as

$$K_{t,0} \quad = \quad N^{-1}\sum_{i=1}^{N}[\delta_0(x_i)]^t, \tag{9}$$

where x_i is the ith row of X, $\delta_0(x_i)$ is the number of zeros in x_i and $N = s^{n-k}$. For an introduction to coding theory, see MacWilliams and Sloane (1977), van Lint (1999), and Hedayat, Sloane and Stufken (1999, Chap. 4).

Remark 1. The power moments $K_{t,0}$ were first introduced by Xu (2003) for nonregular designs. A regular design is determined by a defining contrast subgroup whereas a nonregular design is not. The definition of $K_{t,0}$ in (8) is equivalent to the definition in Xu (2003) up to some constants, and is consistent with the definition in Xu (2004).

For an integer $k > 0$, let $\binom{x}{k} = x(x-1)\cdots(x-k+1)/k!$, with $\binom{x}{0} = 1$ and $\binom{x}{k} = 0$ if $k < 0$. For integers $k, j \geq 0$, let $S(k,j)$ be a Stirling number of the second kind, i.e., the number of ways of partitioning a set of k elements into j nonempty sets. It is well known that $S(k,j) = (1/j!)\sum_{i=0}^{j}(-1)^{j-i}\binom{j}{i}i^k$ for $k \geq j \geq 0$. For integers $t, i \geq 0$, let $Q_t(i;n,s) = (-1)^i\sum_{j=0}^{t} j!S(t,j)s^{-j}(s-1)^{j-i}\binom{n-i}{j-i}$ and

$$c_t(i;n,s) = (s-1)\sum_{j=0}^{t}(-1)^j\binom{t}{j}n^{t-j}Q_j(i;n,s). \tag{10}$$

It is easy to show that $S(t,t) = 1$, $Q_t(t;n,s) = (-1)^t s^{-t} t!$ and $Q_t(i;n,s) = 0$ when $i > n$. Therefore,

$$c_t(t;n,s) = s^{-t}(s-1)t!. \tag{11}$$

By applying MacWilliams identities and Pless power moment identities, two fundamental results in coding theory (see, e.g., MacWilliams and Sloane (1977, Chap. 5)), Xu (2003) established the connection between the power moments $K_{t,0}$ and the wordlength patterns $A_{i,0}$ and the equivalence of minimum aberration and minimum moment aberration. The following two theorems are from Xu (2003, 2004).

Theorem 1. *For an s^{n-k} FFD and positive integers t,*

$$K_{t,0} = \sum_{i=0}^{t} c_t(i; n, s) A_{i,0}, \tag{12}$$

where $c_t(i; n, s)$ are constants defined in (10)*, $A_{0,0} = 1/(s-1)$ and $A_{i,0} = 0$ when $i > n$.*

Theorem 2. *Sequentially minimizing $K_{1,0}, K_{2,0}, \ldots, K_{n,0}$ is equivalent to sequentially minimizing $A_{1,0}, A_{2,0}, \ldots, A_{n,0}$. Therefore, minimum moment aberration is equivalent to minimum aberration.*

The first four identities of (12) are

$$K_{1,0} = s^{-1}(s-1)[A_{1,0} + nA_{0,0}], \tag{13}$$

$$K_{2,0} = s^{-2}(s-1)[2A_{2,0} + (2n+s-2)A_{1,0} + n(n+s-1)A_{0,0}], \tag{14}$$

$$\begin{aligned} K_{3,0} = {} & s^{-3}(s-1)[6A_{3,0} + 6(n+s-2)A_{2,0} + (3n^2 + 6ns + s^2 - 9n - 6s + 6)A_{1,0} \\ & + n(n^2 + 3ns + s^2 - 3n - 3s + 2)A_{0,0}], \end{aligned} \tag{15}$$

$$\begin{aligned} K_{4,0} = {} & s^{-4}(s-1)[24A_{4,0} + 12(2n+3s-6)A_{3,0} + 2(6n^2 + 18ns + 7s^2 - 30n - 36s + 36)A_{2,0} \\ & + (4n^3 + 18n^2 s + 14ns^2 + s^3 - 24n^2 - 54ns - 14s^2 + 44n + 36s - 24)A_{1,0} \\ & + n(n^3 + 6n^2 s + 7ns^2 + s^3 - 6n^2 - 18ns - 7s^2 + 11n + 12s - 6)A_{0,0}]. \end{aligned} \tag{16}$$

In general, by (11) and (12), we have

$$K_{t,0} = s^{-t}(s-1)t! A_{t,0} + c_t(t-1; n, s) A_{t-1,0} + \cdots + c_t(0; n, s) A_{0,0}. \tag{17}$$

Then it is clear that sequentially minimizing $K_{t,0}$ for $t = 1, 2, \ldots$ is equivalent to sequentially minimizing $A_{t,0}$ for $t = 1, 2, \ldots$. This explains why Theorem 2 is true.

3.2 Minimum moment aberration for blocked FFDs

Now consider an s^{n-k} FFD in s^p blocks. Still let X be the $N \times n$ treatment matrix with $N = s^{n-k}$. Let Y be the $N \times p$ matrix representing p independent block defining words. For positive integers t, define power moments

$$K_{t,1} = N^{-1} \sum_{i=1}^{N} [\delta_0(x_i)]^t \delta_b(y_i), \tag{18}$$

where x_i and y_i are the ith row of X and Y, respectively, $\delta_0(x_i)$ is the number of zeros in x_i, and

$$\delta_b(y_i) = \begin{cases} n_p = (s^p - 1)/(s-1) & \text{if } y_i \text{ is a vector of 0's} \\ n_p - s^{p-1} & \text{otherwise} \end{cases}$$

Remark 2. Let Z be the $N \times n_p$ matrix generated by p independent block defining words, where each column represents a block effect. Then $\delta_b(y_i)$ is indeed the number of zeros in the ith row of Z.

Similar to the unblocked case, the following theorem shows that power moments $K_{t,1}$ are related to wordlength patterns $A_{i,1}$.

Theorem 3. *For an s^{n-k} FFD in s^p blocks and positive integers t,*

$$s\, K_{t,1} - n_p\, K_{t,0} = \sum_{i=0}^{t} c_t(i; n, s) A_{i,1}, \tag{19}$$

where $n_p = (s^p - 1)/(s-1)$, $c_t(i; n, s)$ are constants defined in (10)*, and $A_{i,1} = 0$ when $i > n$.*

The proof of Theorem 3 involves generalized MacWilliams identities and Pless power moment identities and will appear elsewhere.

The first three identities of (19) are

$$sK_{1,1} - n_pK_{1,0} = s^{-1}(s-1)[A_{1,1} + nA_{0,1}], \tag{20}$$

$$sK_{2,1} - n_pK_{2,0} = s^{-2}(s-1)[2A_{2,1} + (2n+s-2)A_{1,1} + n(n+s-1)A_{0,1}], \tag{21}$$

$$\begin{aligned} sK_{3,1} - n_pK_{3,0} = {} & s^{-3}(s-1)[6A_{3,1} + 6(n+s-2)A_{2,1} + (3n^2 + 6ns + s^2 - 9n - 6s + 6)A_{1,1} \\ & + n(n^2 + 3ns + s^2 - 3n - 3s + 2)A_{0,1}]. \end{aligned} \tag{22}$$

In general, we have

$$sK_{t,1} - n_pK_{t,0} = s^{-t}(s-1)t!A_{t,1} + c_t(t-1; n, s)A_{t-1,1} + \cdots + c_t(0; n, s)A_{0,1}. \tag{23}$$

Analogue to (5), (6) and (7), we can define the following three sequences of combined power moments:

$$\hat{W}_{scf} = (K_{3,0}, K_{2,1}, K_{4,0}, K_{3,1}, K_{5,0}, K_{4,1}, \ldots), \tag{24}$$

$$\hat{W}_1 = (K_{3,0}, K_{4,0}, K_{2,1}, K_{5,0}, K_{6,0}, K_{3,1}, \ldots), \tag{25}$$

$$\hat{W}_2 = (K_{3,0}, K_{2,1}, K_{4,0}, K_{5,0}, K_{3,1}, K_{6,0}, \ldots), \tag{26}$$

where $K_{i,1}$ is ranked after $K_{i+1,0}$ in $\hat{W}_{scf}$, after $K_{2i,0}$ in $\hat{W}_1$, and after $K_{2i-1,0}$ in $\hat{W}_2$ for $i = 2, 3, \ldots$. We have three minimum moment aberration criteria for blocked FFDs by sequentially minimizing the corresponding sequences.

The next theorem establishes the equivalence of minimum moment aberration and minimum aberration for blocked FFDs.

Theorem 4. *For feasible blocking schemes,*

(i) *sequentially minimizing* $K_{3,0}, K_{2,1}, K_{4,0}, K_{3,1}, K_{5,0}, K_{4,1}, \ldots$ *is equivalent to sequentially minimizing* $A_{3,0}, A_{2,1}, A_{4,0}, A_{3,1}, A_{5,0}, A_{4,1}, \ldots$.

(ii) *sequentially minimizing* $K_{3,0}, K_{4,0}, K_{2,1}, K_{5,0}, K_{6,0}, K_{3,1}, \ldots$ *is equivalent to sequentially minimizing* $A_{3,0}, A_{4,0}, A_{2,1}, A_{5,0}, A_{6,0}, A_{3,1}, \ldots$.

(iii) *sequentially minimizing* $K_{3,0}, K_{2,1}, K_{4,0}, K_{5,0}, K_{3,1}, K_{6,0}, \ldots$ *is equivalent to sequentially minimizing* $A_{3,0}, A_{2,1}, A_{4,0}, A_{5,0}, A_{3,1}, A_{6,0}, \ldots$.

Proof. (i) Recall that for feasible blocking schemes, $A_{1,1} = 0$, in addition to the usual conditions $A_{1,0} = A_{2,0} = A_{0,1} = 0$. From (13), (14) and (20), we obtain $K_{1,0} = s^{-1}n$, $K_{2,0} = s^{-2}n(n + s - 1)$ and $K_{1,1} = s^{-2}nn_p$. Because $A_{1,0} = A_{2,0} = 0$, (15) indicates that minimizing $K_{3,0}$ is equivalent to minimizing $A_{3,0}$. Because $K_{2,0} = s^{-2}n(n + s - 1)$ and $A_{0,1} = A_{1,1} = 0$, (21) indicates that minimizing $K_{2,1}$ is equivalent to minimizing $A_{2,1}$. Given $A_{3,0}$, (16) indicates that minimizing $K_{4,0}$ is equivalent to minimizing $A_{4,0}$. Given $A_{3,0}$ and $A_{2,1}$, $K_{3,0}$ is determined by (15), then (22) indicates that minimizing $K_{3,1}$ is equivalent to minimizing $A_{3,1}$. In general, given $A_{3,0}, \ldots, A_{t-1,0}$, it follows from (17) that minimizing $K_{t,0}$ is equivalent to minimizing $A_{t,0}$; given $A_{3,0}, \ldots, A_{t,0}$ and $A_{2,1}, \ldots, A_{t-1,1}$, $K_{t,0}$ is determined by (17), then it follows from (23) that minimizing $K_{t,1}$ is equivalent to minimizing $A_{t,1}$.

(ii) and (iii) The proofs are similar to (i) and therefore omitted. □

The main advantage of minimum moment aberration over minimum aberration is that it is much more efficient to compute power moments $K_{t,0}$ and $K_{t,1}$ than wordlength patterns $A_{t,0}$ and $A_{t,1}$ when n is large. Therefore, we use minimum moment aberration to rank blocked FFDs in the computation.

Furthermore, we can compute wordlength patterns $A_{t,0}$ and $A_{t,1}$ from power moments $K_{t,0}$ and $K_{t,1}$. From (17) and (23), we have, for $t = 1, 2, \ldots, n$,

$$A_{t,0} = s^t[(s-1)t!]^{-1}[K_{t,0} - \sum_{i=0}^{t-1} c_t(i; n, s)A_{i,0}], \quad (27)$$

$$A_{t,1} = s^t[(s-1)t!]^{-1}[s\,K_{t,1} - n_p\,K_{t,0} - \sum_{i=0}^{t-1} c_t(i; n, s)A_{i,1}]. \quad (28)$$

Using (27), we can compute $A_{1,0}$, $A_{2,0}, \ldots, A_{n,0}$ recursively from $K_{1,0}, K_{2,0}, \ldots, K_{n,0}$. Using (28), we can compute $A_{1,1}$, $A_{2,1}, \ldots, A_{n,1}$ recursively from $K_{1,1}, K_{1,0}$, $K_{2,1}, K_{2,0}, \ldots, K_{n,1}, K_{n,0}$.

Example 3. Consider the 2^{6-2} design in 2^2 blocks from Example 1 with treatment defining words $E = ABC$ and $F = ABD$, and block defining words $b_1 = ACD$ and $b_2 = BCD$. Here the parameters are $n = 6$, $s = 2$, $N = 2^{6-2} = 16$, $p = 2$, $n_p = (2^2 - 1)/(2 - 1) = 3$, and $n_p - s^{p-1} = 1$. Table 1 gives the treatment matrix X, block matrix Y, and computed $\delta_0(x_i)$ and $\delta_b(y_i)$ values. It is straightforward to compute $K_{t,0}$ and $K_{t,1}$ according to (9) and (18) as follows:

$$K_{1,0} = 3, K_{2,0} = 10.5, K_{3,0} = 40.5, K_{4,0} = 172.5, K_{5,0} = 805.5, K_{6,0} = 4060.5,$$

$$K_{1,1} = 4.5, K_{2,1} = 16.5, K_{3,1} = 70.5, K_{4,1} = 340.5, K_{5,1} = 1789.5, K_{6,1} = 9916.5.$$

From (27) and (28), we obtain

$$A_{1,0} = 0, A_{2,0} = 0, A_{3,0} = 0, A_{4,0} = 3, A_{5,0} = 0, A_{6,0} = 0,$$

$$A_{1,1} = 0, A_{2,1} = 3, A_{3,1} = 8, A_{4,1} = 0, A_{5,1} = 0, A_{6,1} = 1.$$

The wordlength patterns obtained via (27) and (28) agree with that obtained via counting words in Example 1.

In the construction of blocked FFDs, we can always select p columns other than any column of the treatment matrix X. It is possible that the resulting blocking scheme is infeasible. The next theorem provides an efficient way to screen out infeasible blocking schemes.

Theorem 5. *When an s^{n-k} FFD is arranged in s^p blocks,*

$$K_{1,1} \geq s^{-2} n\, n_p,$$

with equality if and only if the blocking scheme is feasible.

Proof. Combining (13) and (20) yields

$$K_{1,1} = s^{-2}(s-1)[A_{1,1} + nA_{0,1} + n_p A_{1,0} + nn_p A_{0,0}].$$

Because $A_{0,1} = A_{1,0} = 0$ and $A_{0,0} = (s-1)^{-1}$,

$$K_{1,1} = s^{-2}(s-1)[A_{1,1} + nn_p(s-1)^{-1}] \geq s^{-2} nn_p.$$

The equation holds if and only if $A_{1,1} = 0$, i.e., the blocking scheme is feasible. □

An example will be given in the next section to illustrate the use of Theorem 5.

4 Construction method

Let $N = s^{n-k}$ and $m = (N-1)/(s-1)$. An s^{n-k} FFD can be viewed as n columns of an $N \times m$ matrix H, where H is a saturated FFD with N runs, m factors and s levels. Let G consist of all nonzero $(n-k)$-tuples $(u_1, \ldots, u_{n-k})^T$ from $GF(s)$ in which the first nonzero u_i is 1. G is called the generator matrix and H is formed by taking all linear combinations of the rows of G. For example, for $s = 2$ and $n - k = 4$, the generator matrix G and design matrix H are given in Tables 2 and 3, respectively.

The construction of optimal blocking schemes relies on the complete catalog of non-equivalent s^{n-k} FFDs. Chen, Sun and Wu (1993) developed an algorithm and constructed all non-equivalent FFDs with 8, 16, 32 and 27 runs, and 64 runs of resolution IV or higher. Xu (2004) extended their algorithm and constructed all non-equivalent FFDs with 27 and 81 runs, 243 runs of resolution IV or higher, and 729 runs of resolution V or higher.

Given an s^{n-k} FFD, which corresponds to n columns in H, there are $m-n$ remaining columns to be used as block columns. There are $\binom{m-n}{p}$ ways to choose p columns as possible block generators. We use Theorem 5 to screen out infeasible blocking schemes. For each feasible blocking scheme, we compute the power moments $K_{t,0}$ and $K_{t,1}$. Then we use three minimum moment aberration criteria to find three optimal blocking schemes.

Given n, s, and k, we first use Xu's (2004) algorithm to generate all non-equivalent s^{n-k} FFDs and use the minimum moment aberration criterion to rank them. Then for each p, $1 \le p \le n-k-1$, we obtain optimal blocking schemes by searching over all possible blocking schemes for all non-equivalent s^{n-k} FFDs. Because all three MA criteria minimize $A_{3,0}$ first, there is no need to search over designs with larger $A_{3,0}$ whenever feasible blocking schemes for a design with smaller $A_{3,0}$ are available. In particular, when feasible blocking schemes for resolution IV designs exist, it is not necessary to search over resolution III designs. This enables us to find MA blocked FFDs with 64 runs up to 32 factors, because all 64-run FFDs with resolution IV or higher are known.

For an optimal blocking scheme, we report the treatment and block wordlength patterns and the number of clear effects. A main effect or two-factor interaction is *clear* if it is not aliased with any other main effect or two-factor interaction and it is not confounded with any block effect (Sun, Wu and Chen, 1997). We should avoid using defining contrast subgroups and alias sets, because there are many words to be counted when k is large. We compute the treatment and block wordlength patterns from the power moments according to (27) and (28). For unblocked FFDs, Xu (2004) introduced a method for finding clear effects using power moments. His method can be extended easily to blocked FFDs and therefore is used to find the number of clear effects. The details are omitted.

Example 4. According to Chen, Sun and Wu (1993), the MA unblocked 2^{6-2} design consists of treatment columns 1, 2, 4, 8, 7, and 11. To arrange it into 2^3 blocks, we can choose 3 columns from the remaining 9 columns: 3, 5, 6, 9, 10, 12, 13, 14, and 15. There are $\binom{9}{3} = 84$ combinations: $(3,5,6),(3,5,9),\ldots,(13,14,15)$. For illustration, consider two blocking schemes corresponding to the first two combinations $(3,5,6)$ and $(3,5,9)$. Using Table 3, it is straightforward to compute that $K_{1,1} = 12$ for the first scheme whereas $K_{1,1} = 10.5$ for the second scheme. The lower bound in Theorem 5 is 10.5. Therefore, the first scheme is infeasible whereas the second is feasible. Indeed, the second blocking scheme is optimal under all three MA criteria.

5 Tables of optimal blocking schemes

Using the construction method described in the last section, we obtain MA blocked FFDs with all 32 runs, 64 runs up to 32 factors, and all 81 runs; see Tables 5, 6 and 8. Table 4 and Table 7 shows generator matrices, where the columns are in Yates order and independent columns are in boldface. Previously, Sitter, Chen and Feder (1997) gave MA W_{scf} designs with 32 runs up to 15 factors, and 64 runs up to 9 factors. Cheng and Wu (2002) gave MA W_1 and W_2 designs with 81 runs up to 10 factors.

The tables of designs show the designs, treatment columns, treatment wordlength patterns (W_t), block columns, block wordlength patterns (W_b), the number of clear main effects ($C1$), and the number of clear two-factor interactions ($C2$). The designs are labeled as $n-k.i/\mathrm{B}p(W)$, where i denotes the rank order of the unblocked s^{n-k} FFD under the MA criterion, p denotes the number of block variables, and W denotes the MA W-criterion, where W_2W_{scf} means both W_2 and W_{scf} criteria, and "all" means all three criteria. To save space, independent columns are omitted in the treatment columns, and the wordlength patterns are truncated as $W_t = (A_{3,0}, A_{4,0}, A_{5,0}, A_{6,0})$ and $W_b = (A_{2,1}, A_{3,1}, A_{4,1}, A_{5,1})$.

Example 5. Consider choosing 2^{7-2} designs in 2^2 blocks. Table 5 lists two designs 7-2.1/B2(W_1) and 7-2.3/B2(W_2W_{scf}). The first design is optimal under W_1 criterion whereas the second is optimal under both W_2 and W_{scf} criteria. For the first design, the treatment columns given in Table 5 are 15 and 19; therefore, it consists of columns 1, 2, 4, 8, 16, 15 and 19. Label the 7 treatment columns as A–G, where A, B, C, D and E correspond to 5 independent columns 1, 2, 4, 8 and 16. According to Table 4, this design has treatment defining words $F = ABCD$ and $G = ABE$. The block columns are 7 and 25, representing block defining words $b_1 = ABC$ and $b_2 = ADE$. Table 5 shows that the treatment wordlength pattern is $W_t = (0, 1, 2, 0, \ldots)$ and the block wordlength pattern is $W_b = (1, 6, 4, 0, \ldots)$. All 7 main effects are clear (i.e., $C1$=7) and there are 14 clear two-factor interactions (i.e., $C2$=14).

Example 6. Consider choosing 3^{8-4} designs in 3^3 blocks. Table 8 lists one design 8-4.1/B3(all), which is optimal under all three criteria. The treatment columns given in Table 8 are 22, 9, 24 and 31; therefore, it consists of columns 1, 2, 5, 14, 22, 9, 24 and 31. Label the 8 treatment columns as A–H, where A, B, C, and D correspond to 4 independent columns 1, 2, 5 and 14. According to Table 7, this design has treatment defining words $E = ABCD$, $F = AB^2C$, $G = AC^2D$ and $H = AB^2D$. The block columns are 4, 8 and 15, representing block defining words $b_1 = AB^2$, $b_2 = ABC$ and $b_3 = AD$. Table 8 shows that the treatment wordlength pattern is $W_t = (0, 10, 16, 4, \ldots)$ and the block wordlength pattern is $W_b = (28, 56, 200, 264, \ldots)$. All 8 main effects are clear (i.e., $C1$=8) and there are 8 clear two-factor interactions (i.e., $C2$=8).

It is interesting to know when MA blocked FFDs under different criteria are different. We observed that in most cases, MA blocked FFDs under three criteria are the same. (This occurs for all 8 and 27 runs, and all 16 runs except for two cases, noted by Cheng and Wu (2002).) When MA blocked FFDs under three criteria are not all the same, one of the following four situations occurs.

1. MA blocked FFDs under W_1 and W_2 are the same, but are different from those under W_{scf}.
2. MA blocked FFDs under W_2 and W_{scf} are the same, but are different from those under W_1.
3. MA blocked FFDs under W_1 and W_{scf} are the same, but are different from those under W_2.
4. MA blocked FFDs under three criteria are all different.

Situation 1 occurs once for both 32-run and 64-run designs; see 6-1.1/B1(W_1W_2) and 6-1.2/B1(W_{scf}) for 32 runs, and 7-1.1/B2(W_1W_2) and 7-1.3/B2(W_{scf}) for 64 runs. Situation 1 does not occur for 81-run designs. Situation 2 occurs 12 times for 32-run designs, 35 times for 64-run designs, and twice for 81-run designs. Situation 3 does not occur. Situation 4 occurs only once for 64-run designs; see 12-6.1/B2(W_1), 12-6.11/B2(W_2), and 12-6.13/B2(W_{scf}).

When MA blocked FFDs are different, MA W_1 designs tend to have larger $C1$ or $C2$ values in most cases. MA W_1 designs have larger $C1$ values in the following two cases: 81 runs with

$(n, p) = (11, 1), (11, 2)$. MA W_1 designs have larger $C2$ values in the following 14 cases: 32 runs with $(n, p) = (6, 2), (7, 2), (8, 3), (9, 2), (9, 3)$ (also reported by Cheng and Wu (2002)), and 64 runs with $(n, p) = (8, 3), (10, 3), (11, 2), (11, 3), (12, 2), (12, 3), (13, 2), (13, 3), (13, 4)$.

However, MA W_1 designs have smaller $C2$ values than MA W_2 and W_{scf} designs in the following six cases: 64 runs with $(n, p) = (14, 2), (14, 3), (15, 3), (16, 2), (16, 3), (17, 2)$. This is surprising because Cheng and Wu (2002, page 272) wrote "it is expected that the number of clear main effects ($C1$) and the number of clear 2fi's ($C2$) for an MA W_1 design should be larger than or equal to the corresponding numbers for an MA W_2 designs." Note that we do not use $C1$ and $C2$ to select designs. It is possible to find other MA blocked FFDs with different $C1$ or $C2$ values. Cheng and Wu (2002) reported two MA blocked 3^{6-2} designs in 3^2 blocks, one with $C2 = 16$ and another with $C2 = 18$.

It is also interesting to know when MA blocked FFDs originate from MA unblocked FFDs. When MA blocked FFDs are the same for all three criteria, MA blocked FFDs originate from MA unblocked FFDs in all cases except for the following 22 cases: $n-(n-5).i$/B4(all) with $n = 7$–10 and 21-16.2/B3(all) for 32 runs; $n-(n-6).i$/B5(all) with $n = 7$–20 and 7-1.2/B4(all) for 64 runs, and 10-6.2/B3(all) and 11-7.2/B3(all) for 81 runs.

When MA blocked FFDs are different under different criteria, MA W_1 designs originate from MA unblocked FFDs except for one case, i.e., 15-9.2/B3(W_1), MA W_2 designs originate from MA unblocked FFDs when they are the same as MA W_1 designs, and MA W_{scf} designs do not originate from MA unblocked FFDs.

6 Concluding remarks

We extend the concept of minimum moment aberration to blocked FFDs and establish the equivalence between minimum moment aberration and MA for blocked FFDs. We propose a method for constructing MA blocked FFDs and obtain optimal blocking schemes with respect to three MA criteria for all 32 runs, 64 runs up to 32 factors, and all 81 runs.

As argued by Chen and Cheng (1999), Zhang and Park (2000), and Cheng and Wu (2002), MA W_{scf} designs are not recommended when they are different from MA W_1 and W_2 designs. The choice of MA W_1 designs and MA W_2 designs depends on the situation. Cheng and Wu (2002) suggested that if a follow-up experiment is planned, MA W_2 designs are recommended; otherwise, MA W_1 designs are recommended. See Cheng and Wu (2002) for further discussions.

Although MA W_{scf} designs are not recommended for blocking, they are useful. An s^{n-k} FFD in s^p blocks can be viewed as a mixed $(s^p)s^n$ design, i.e., one factor at s^p levels and n factors at s levels. Then the MA W_{scf} criterion is also known as the MA criterion (of type 0) proposed by Wu and Zhang (1993), Zhang and Shao (2001) and Mukerjee and Wu (2001) for mixed FFDs. Consequently, we obtain many mixed MA designs from MA W_{scf} designs for free. In particular, Tables 5, 6, and 8 give all MA $4^1 2^n$, $8^1 2^n$ and $16^1 2^n$ designs with 32 runs, MA $4^1 2^n$, $8^1 2^n$, $16^1 2^n$ and $32^1 2^n$ designs with 64 runs and $n \leq 32$, and all MA $9^1 3^n$ and $27^1 3^n$ designs with 81 runs. Previously, Wu and Zhang (1993) gave all MA $4^1 2^n$ designs with 16 runs, and MA $4^1 2^n$ designs with 32 runs and $n \leq 9$; Wu and Hamada (2000) further gave MA $4^1 2^n$ designs with 64 runs and $n \leq 9$; Zhang and Shao (2001) gave MA $9^1 3^n$ designs with 27 runs and $n \leq 8$.

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References

Bisgaard, S., 1994. A note on the definition of resolution for blocked 2^{k-p} designs. Technometrics 36, 308–311.

Box, G.E.P., Hunter, J.S., 1961. The 2^{k-p} fractional factorial designs. Technometrics 3, 311–351; 449–458.

Chen, H., Cheng, C.S., 1999. Theory of optimal blocking of 2^{n-m} designs. Ann. Statist. 27, 1948–1973.

Chen, J., Sun, D.X., Wu, C.F.J., 1993. A catalogue of two-level and three-level fractional factorial designs with small runs. Internat. Statist. Rev. 61, 131–145.

Cheng, S.W., Wu, C.F.J., 2002. Choice of optimal blocking schemes in two-level and three-level designs. Technometrics 44, 269–277.

Fries, A., Hunter, W.G., 1980. Minimum aberration 2^{k-p} designs. Technometrics 22, 601–608.

Hedayat, A.S., Sloane, N.J.A., Stufken, J., 1999. Orthogonal Arrays: Theory and Applications. Springer, New York.

MacWilliams, F.J., Sloane, N.J.A., 1977. The Theory of Error-Correcting Codes. North-Holland, Amsterdam.

Mukerjee, R., Wu, C.F.J., 1999. Blocking in regular fractional factorials: a projective geometric approach. Ann. Statist. 27, 1256–1271.

Mukerjee, R., Wu, C.F.J., 2001. Minimum aberration designs for mixed factorials in terms of complementary sets. Statist. Sinica 11, 225–239.

Sitter, R.R., Chen, J., Feder, M., 1997. Fractional resolution and minimum aberration in blocked 2^{n-p} designs. Technometrics 39, 382–390.

Sun, D.X., Wu, C.F.J., Chen, Y., 1997. Optimal blocking schemes for 2^p and 2^{n-p} designs. Technometrics 39, 298–307.

van Lint, J.H., 1999. Introduction to Coding Theory. 3rd ed. Springer, New York.

Wu, C.F.J., Hamada, M., 2000. Experiments: Planning, Analysis and Parameter Design Optimization. Wiley, New York.

Wu, C.F.J., Zhang, R., 1993. Minimum aberration designs with two-level and four-level factors. Biometrika 80, 203–209.

Xu, H., 2003. Minimum moment aberration for nonregular designs and supersaturated designs. Statist. Sinica 13, 691–708.

Xu, H., 2004. A catalogue of three-level fractional factorial designs. UCLA Statistics Electronic Publications, preprint 373 (http://preprints.stat.ucla.edu/).

Zhang, R., Park, D., 2000. Optimal blocking of two-level fractional factorial designs. J. Statist. Plann. Inference 91, 107–121.

Zhang, R., Shao, Q., 2001. Minimum aberration $(s^2)s^{n-k}$ designs. Statist. Sinica 11, 213–223.

Table 1: A 2^{6-2} design in 2^2 blocks

	X						Y			
Run	A	B	C	D	E	F	b_1	b_2	$\delta_0(x_i)$	$\delta_b(y_i)$
1	0	0	0	0	0	0	0	0	6	3
2	0	0	0	1	0	1	1	1	4	1
3	0	0	1	0	1	0	1	1	4	1
4	0	0	1	1	1	1	0	0	2	3
5	0	1	0	0	1	1	0	1	3	1
6	0	1	0	1	1	0	1	0	3	1
7	0	1	1	0	0	1	1	0	3	1
8	0	1	1	1	0	0	0	1	3	1
9	1	0	0	0	1	1	1	0	3	1
10	1	0	0	1	1	0	0	1	3	1
11	1	0	1	0	0	1	0	1	3	1
12	1	0	1	1	0	0	1	0	3	1
13	1	1	0	0	0	0	1	1	4	1
14	1	1	0	1	0	1	0	0	2	3
15	1	1	1	0	1	0	0	0	2	3
16	1	1	1	1	1	1	1	1	0	1

Table 2: The generator matrix for 16-run designs

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
B	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
C	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
D	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

Table 3: The design matrix for 16-run designs

Run	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
3	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
4	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
5	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
6	0	1	1	0	0	1	1	1	1	0	0	1	1	0	0
7	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
8	0	1	1	1	1	0	0	1	1	0	0	0	0	1	1
9	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
10	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0
11	1	0	1	1	0	1	0	0	1	0	1	1	0	1	0
12	1	0	1	1	0	1	0	1	0	1	0	0	1	0	1
13	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
14	1	1	0	0	1	1	0	1	0	0	1	1	0	0	1
15	1	1	0	1	0	0	1	0	1	1	0	1	0	0	1
16	1	1	0	1	0	0	1	1	0	0	1	0	1	1	0

Table 4: Generator matrices for 32- and 64-run designs

	1	**2**	3	**4**	5	6	7	**8**	9	10	11	12	13	14	15	**16**	17	18	19	20	21
A	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
B	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
C	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1
D	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	22	23	24	25	26	27	28	29	30	31	**32**	33	34	35	36	37	38	39	40	41	42
A	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
B	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1
C	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0
D	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1
E	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
F	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1

	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
A	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
B	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
C	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
D	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
E	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
F	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

For 32 runs use the first 5 rows and 31 columns; for 64 runs use the entire matrix. The independent columns are in boldface and numbered 1, 2, 4, 8, 16 and 32.

Table 5: MA blocking schemes for 32-run designs

Design	Treatment	W_t	Block	W_b	$C1$	$C2$
5-0.1/B1(all)		0 0 0	31	0 0 0 1	5	10
5-0.1/B2(all)		0 0 0	7 25	0 2 1 0	5	10
5-0.1/B3(all)		0 0 0	3 12 21	2 4 1 0	5	8
5-0.1/B4(all)		0 0 0	3 5 9 17	10 0 5 0	5	0
6-1.1/B1(W_1W_2)	31	0 0 0 1	7	0 2 0 0	6	15
6-1.2/B1(W_{scf})	15	0 0 1 0	19	0 1 1 0	6	15
6-1.1/B2(W_1)	31	0 0 0 1	3 13	1 4 1 0	6	14
6-1.3/B2(W_2W_{scf})	7	0 1 0 0	11 21	0 4 2 0	6	9
6-1.1/B3(all)	31	0 0 0 1	3 12 21	3 8 3 0	6	12
6-1.1/B4(all)	31	0 0 0 1	3 5 9 17	15 0 15 0	6	0
7-2.1/B1(all)	15 19	0 1 2 0	21	0 2 2 0	7	15
7-2.1/B2(W_1)	15 19	0 1 2 0	7 25	1 6 4 0	7	14
7-2.3/B2(W_2W_{scf})	7 11	0 3 0 0	13 19	0 7 4 0	7	6
7-2.1/B3(all)	15 19	0 1 2 0	7 11 17	5 12 6 2	7	12
7-2.2/B4(all)	31 7	0 2 0 1	3 5 9 17	21 0 33 0	7	0
8-3.1/B1(all)	15 19 21	0 3 4 0	25	0 3 4 0	8	13
8-3.1/B2(all)	15 19 21	0 3 4 0	7 25	1 10 8 0	8	12
8-3.1/B3(W_1)	15 19 21	0 3 4 0	7 9 17	8 16 11 12	8	8
8-3.2/B3(W_2W_{scf})	31 7 11	0 5 0 2	3 13 20	7 18 10 12	8	4
8-3.2/B4(all)	31 7 11	0 5 0 2	3 5 9 17	28 0 65 0	8	0
9-4.1/B1(all)	15 19 21 25	0 6 8 0	30	0 4 8 0	9	8
9-4.1/B2(W_1)	15 19 21 25	0 6 8 0	3 29	4 8 16 8	9	8
9-4.3/B2(W_2W_{scf})	31 7 11 21	0 9 0 6	6 26	2 14 9 12	9	0
9-4.1/B3(W_1)	15 19 21 25	0 6 8 0	3 5 24	12 16 32 24	9	8
9-4.3/B3(W_2W_{scf})	31 7 11 21	0 9 0 6	3 13 20	9 27 18 27	9	0
9-4.3/B4(all)	31 7 11 21	0 9 0 6	3 5 9 17	36 0 117 0	9	0
10-5.1/B1(W_1)	15 19 21 25 30	0 10 16 0	3	2 4 6 8	10	0
10-5.2/B1(W_2W_{scf})	31 7 11 21 25	0 15 0 15	13	0 10 0 12	10	0
10-5.1/B2(W_1)	15 19 21 25 30	0 10 16 0	3 5	6 12 18 24	10	0
10-5.2/B2(W_2W_{scf})	31 7 11 21 25	0 15 0 15	3 13	3 20 13 24	10	0
10-5.1/B3(W_1)	15 19 21 25 30	0 10 16 0	3 5 17	17 24 36 64	10	0
10-5.3/B3(W_2W_{scf})	31 7 11 21 13	0 16 0 12	5 10 19	12 36 30 60	10	0
10-5.2/B4(all)	31 7 11 21 25	0 15 0 15	3 5 9 17	45 0 195 0	10	0
11-6.1/B1(all)	31 7 11 21 25 13	0 25 0 27	14	0 13 0 25	11	0
11-6.1/B2(all)	31 7 11 21 25 13	0 25 0 27	5 19	4 26 19 50	11	0
11-6.1/B3(W_1)	31 7 11 21 25 13	0 25 0 27	3 5 17	25 0 145 0	11	0
11-6.2/B3(W_2W_{scf})	31 7 11 21 13 14	0 26 0 24	5 10 19	15 48 48 112	11	0
11-6.1/B4(all)	31 7 11 21 25 13	0 25 0 27	3 5 9 17	55 0 305 0	11	0
12-7.1/B1(all)	31 7 11 21 25 13 14	0 38 0 52	19	0 17 0 44	12	0
12-7.1/B2(all)	31 7 11 21 25 13 14	0 38 0 52	5 19	5 34 28 88	12	0
12-7.1/B3(W_1)	31 7 11 21 25 13 14	0 38 0 52	3 5 17	30 0 217 0	12	0
12-7.2/B3(W_2W_{scf})	31 7 11 21 13 14 26	0 39 0 48	5 10 19	18 64 72 192	12	0
12-7.1/B4(all)	31 7 11 21 25 13 14	0 38 0 52	3 5 9 17	66 0 457 0	12	0
13-8.1/B1(all)	31 7 11 21 25 13 14 19	0 55 0 96	22	0 22 0 72	13	0
13-8.1/B2(all)	31 7 11 21 25 13 14 19	0 55 0 96	6 26	6 44 40 144	13	0
13-8.1/B3(all)	31 7 11 21 25 13 14 19	0 55 0 96	3 5 17	36 0 310 0	13	0
13-8.1/B4(all)	31 7 11 21 25 13 14 19	0 55 0 96	3 5 9 17	78 0 660 0	13	0
14-9.1/B1(all)	31 7 11 21 25 13 14 19 22	0 77 0 168	26	0 28 0 112	14	0
14-9.1/B2(all)	31 7 11 21 25 13 14 19 22	0 77 0 168	6 26	7 56 56 224	14	0
14-9.1/B3(all)	31 7 11 21 25 13 14 19 22	0 77 0 168	3 9 17	42 0 434 0	14	0
14-9.1/B4(all)	31 7 11 21 25 13 14 19 22	0 77 0 168	3 5 9 17	91 0 924 0	14	0
15-10.1/B1(all)	31 7 11 21 25 13 14 19 22 26	0 105 0 280	28	0 35 0 168	15	0
15-10.1/B2(all)	31 7 11 21 25 13 14 19 22 26	0 105 0 280	3 5	21 0 252 0	15	0
15-10.1/B3(all)	31 7 11 21 25 13 14 19 22 26	0 105 0 280	3 5 9	49 0 588 0	15	0
15-10.1/B4(all)	31 7 11 21 25 13 14 19 22 26	0 105 0 280	3 5 9 17	105 0 1260 0	15	0
16-11.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28	0 140 0 448	3	8 0 112 0	16	0
16-11.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28	0 140 0 448	3 5	24 0 336 0	16	0
16-11.1/B3(all)	31 7 11 21 25 13 14 19 22 26 28	0 140 0 448	3 5 9	56 0 784 0	16	0
16-11.1/B4(all)	31 7 11 21 25 13 14 19 22 26 28	0 140 0 448	3 5 9 17	120 0 1680 0	16	0

Table 5: Continued

Design	Treatment	W_t	Block	W_b	$C1$	$C2$
17-12.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3	8 140 112 448	5	8 8 112 112	0	0
17-12.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3	8 140 112 448	5 9	24 24 336 336	0	0
17-12.1/B3(all)	31 7 11 21 25 13 14 19 22 26 28 3	8 140 112 448	5 9 17	56 56 784 784	0	0
18-13.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5	16 148 224 560	9	8 16 120 224	0	0
18-13.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3 5	16 148 224 560	9 17	24 48 360 672	0	0
18-13.1/B3(all)	31 7 11 21 25 13 14 19 22 26 28 3 5	16 148 224 560	6 9 17	57 112 832 1568	0	0
19-14.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9	24 164 344 784	17	8 24 136 344	0	0
19-14.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9	24 164 344 784	15 17	24 73 408 1024	0	0
19-14.1/B3(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9	24 164 344 784	6 10 17	59 168 928 2408	0	0
20-15.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17	32 188 480 1128	30	8 32 161 480	0	0
20-15.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17	32 188 480 1128	6 27	25 98 472 1424	0	0
20-15.1/B3(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17	32 188 480 1128	6 10 18	62 224 1073 3360	0	0
21-16.1/B1(W_1)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 30	40 220 641 1608	6	9 41 184 632	0	0
21-16.2/B1(W_2W_{scf})	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15	40 221 640 1600	23	8 42 192 624	0	0
21-16.1/B2(W_1)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 30	40 220 641 1608	6 10	27 123 552 1896	0	0
21-16.2/B2(W_2W_{scf})	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15	40 221 640 1600	6 27	26 124 560 1888	0	0
21-16.2/B3(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15	40 221 640 1600	6 10 18	66 280 1268 4480	0	0
22-17.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23	48 263 832 2224	27	8 52 232 800	0	0
22-17.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23	48 263 832 2224	6 27	27 152 672 2432	0	0
22-17.1/B3(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23	48 263 832 2224	6 10 18	71 336 1516 5824	0	0
23-18.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27	56 315 1064 3024	29	8 63 280 1008	0	0
23-18.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27	56 315 1064 3024	6 10	33 168 780 3192	0	0
23-18.1/B3(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27	56 315 1064 3024	6 10 18	77 392 1820 7448	0	0
24-19.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29	64 378 1344 4032	6	12 64 312 1344	0	0
24-19.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29	64 378 1344 4032	6 10	36 192 936 4032	0	0
24-19.1/B3(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29	64 378 1344 4032	6 10 18	84 448 2184 9408	0	0
25-20.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29 6	76 442 1656 5376	10	12 76 376 1656	0	0
25-20.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29 6	76 442 1656 5376	10 18	36 228 1128 4968	0	0
26-21.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29 6 10	88 518 2032 7032	18	12 88 452 2032	0	0
26-21.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29 6 10	88 518 2032 7032	12 18	37 264 1344 6096	0	0
27-22.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29 6 10 18	100 606 2484 9064	30	12 101 540 2472	0	0
27-22.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29 6 10 18	100 606 2484 9064	12 20	39 300 1584 7452	0	0
28-23.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29 6 10 18 30	112 707 3024 11536	12	14 112 616 3024	0	0
28-23.1/B2(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29 6 10 18 30	112 707 3024 11536	12 20	42 336 1848 9072	0	0
29-24.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29 6 10 18 30 12	126 819 3640 14560	20	14 126 728 3640	0	0
30-25.1/B1(all)	31 7 11 21 25 13 14 19 22 26 28 3 5 9 17 15 23 27 29 6 10 18 30 12 20	140 945 4368 18200	24	15 140 840 4368	0	0

Table 6: MA blocking schemes for 64-run designs

Design	Treatment	W_t	Block	W_b	$C1$	$C2$
6-0.1/B1(all)		0 0 0 0	63	0 0 0 0	6	15
6-0.1/B2(all)		0 0 0 0	15 51	0 0 3 0	6	15
6-0.1/B3(all)		0 0 0 0	7 25 42	0 4 3 0	6	15
6-0.1/B4(all)		0 0 0 0	3 12 21 37	3 8 3 0	6	12
6-0.1/B5(all)		0 0 0 0	3 5 9 17 33	15 0 15 0	6	0
7-1.1/B1(W_1)	63	0 0 0 0	7	0 1 1 0	7	21
7-1.2/B1(W_2W_{scf})	31	0 0 0 1	39	0 0 2 0	7	21
7-1.1/B2(W_1W_2)	63	0 0 0 0	7 25	0 3 3 0	7	21
7-1.3/B2(W_{scf})	15	0 0 1 0	19 37	0 2 3 1	7	21
7-1.1/B3(all)	63	0 0 0 0	7 25 42	0 7 7 0	7	21
7-1.2/B4(all)	31	0 0 0 1	3 12 21 33	5 12 7 4	7	16
7-1.2/B5(all)	31	0 0 0 1	3 5 9 17 33	21 0 35 0	7	0
8-2.1/B1(all)	31 39	0 0 2 1	41	0 1 2 1	8	28
8-2.1/B2(all)	31 39	0 0 2 1	11 49	0 4 5 2	8	28
8-2.1/B3(W_1)	31 39	0 0 2 1	7 9 50	2 8 10 6	8	26
8-2.2/B3(W_2W_{scf})	31 35	0 1 0 2	7 11 49	1 10 10 4	8	21
8-2.1/B4(all)	31 39	0 0 2 1	3 12 21 33	7 18 15 10	8	21
8-2.2/B5(all)	31 35	0 1 0 2	3 5 9 17 33	28 0 69 0	8	0
9-3.1/B1(all)	31 39 41	0 1 4 2	51	0 1 4 2	9	30
9-3.1/B2(all)	31 39 41	0 1 4 2	19 46	0 6 8 5	9	30
9-3.1/B3(all)	31 39 41	0 1 4 2	6 11 49	2 14 17 8	9	28
9-3.1/B4(all)	31 39 41	0 1 4 2	3 12 21 33	9 27 26 23	9	23
9-3.4/B5(all)	31 35 13	0 3 0 4	3 5 9 17 33	36 0 123 0	9	0
10-4.1/B1(all)	31 39 41 51	0 2 8 4	42	0 2 6 4	10	33
10-4.1/B2(all)	31 39 41 51	0 2 8 4	11 53	0 8 16 8	10	33
10-4.1/B3(W_1)	31 39 41 51	0 2 8 4	5 11 48	4 16 28 24	10	29
10-4.2/B3(W_2W_{scf})	31 39 41 19	0 3 6 4	5 11 48	3 19 26 21	10	24
10-4.1/B4(all)	31 39 41 51	0 2 8 4	3 12 21 33	12 36 44 52	10	25
10-4.7/B5(all)	31 35 13 52	0 5 0 10	3 5 9 17 33	45 0 205 0	10	0
11-5.1/B1(all)	31 39 41 51 42	0 4 14 8	60	0 2 10 8	11	34
11-5.1/B2(W_1)	31 39 41 51 42	0 4 14 8	11 21	1 12 18 14	11	33
11-5.5/B2(W_2W_{scf})	31 39 41 51 11	0 6 12 4	22 42	0 13 20 12	11	25
11-5.1/B3(W_1)	31 39 41 51 42	0 4 14 8	5 11 18	5 24 38 42	11	29
11-5.2/B3(W_2W_{scf})	31 39 41 19 61	0 5 10 10	5 10 49	4 25 41 42	11	21
11-5.1/B4(all)	31 39 41 51 42	0 4 14 8	5 11 18 35	15 48 70 98	11	24
11-5.14/B5(all)	31 35 13 52 14	0 9 0 19	3 5 9 17 33	55 0 321 0	11	0
12-6.1/B1(all)	31 39 41 51 42 60	0 6 24 16	11	0 8 9 0	12	36
12-6.1/B2(W_1)	31 39 41 51 42 60	0 6 24 16	5 11	2 16 25 24	12	34
12-6.11/B2(W_2)	31 39 41 19 11 22	0 12 13 12	21 46	0 17 26 31	12	17
12-6.13/B2(W_{scf})	31 39 41 19 11 13	0 12 14 12	21 46	0 16 27 34	12	17
12-6.1/B3(W_1)	31 39 41 51 42 60	0 6 24 16	5 11 18	6 32 57 72	12	30
12-6.2/B3(W_2W_{scf})	31 39 41 51 42 21	0 8 20 14	6 11 48	5 34 58 68	12	22
12-6.1/B4(all)	31 39 41 51 42 60	0 6 24 16	5 11 18 35	18 64 105 168	12	24
12-6.17/B5(all)	31 35 13 52 14 55	0 14 0 36	3 5 9 17 33	66 0 481 0	12	0
13-7.1/B1(all)	31 39 41 51 42 21 22	0 14 28 24	52	0 8 12 17	13	20
13-7.1/B2(W_1)	31 39 41 51 42 21 22	0 14 28 24	12 52	2 18 36 57	13	18
13-7.19/B2(W_2W_{scf})	31 39 41 19 11 22 13	0 20 18 22	21 46	0 22 35 54	13	16
13-7.1/B3(W_1)	31 39 41 51 42 21 22	0 14 28 24	5 11 48	7 42 78 124	13	17
13-7.3/B3(W_2W_{scf})	31 39 41 19 21 49 62	0 15 24 32	6 11 48	6 44 80 120	13	12
13-7.1/B4(W_1)	31 39 41 51 42 21 22	0 14 28 24	3 5 9 49	23 75 156 297	13	16
13-7.3/B4(W_2W_{scf})	31 39 41 19 21 49 62	0 15 24 32	3 13 20 33	22 80 148 296	13	8
13-7.20/B5(all)	31 35 13 52 14 55 21	0 22 0 60	3 5 9 17 33	78 0 693 0	13	0
14-8.1/B1(all)	31 39 41 51 13 21 11 52	0 22 40 36	58	0 8 20 24	14	8
14-8.1/B2(W_1)	31 39 41 51 13 21 11 52	0 22 40 36	19 46	2 25 47 85	14	8
14-8.19/B2(W_2W_{scf})	31 39 41 19 11 22 13 25	0 31 24 40	21 46	0 28 46 88	14	15
14-8.1/B3(W_1)	31 39 41 51 13 21 11 52	0 22 40 36	5 25 35	9 52 105 200	14	8
14-8.4/B3(W_2W_{scf})	31 39 41 19 21 49 62 11	0 23 32 56	6 26 42	7 56 110 192	14	12
14-8.1/B4(all)	31 39 41 51 13 21 11 52	0 22 40 36	5 10 19 33	26 100 209 460	14	8
14-8.18/B5(all)	31 35 13 52 14 55 37 61	0 31 0 104	3 5 9 17 33	91 0 970 0	14	0
15-9.1/B1(all)	31 39 41 51 13 21 11 52 58	0 30 60 60	22	0 13 21 36	15	0
15-9.1/B2(W_1)	31 39 41 51 13 21 11 52 58	0 30 60 60	5 25	3 32 60 123	15	0
15-9.19/B2(W_2W_{scf})	31 35 13 21 37 62 11 19 47	0 45 0 160	14 55	0 35 60 168	15	0
15-9.2/B3(W_1)	31 39 41 51 13 21 11 52 46	0 30 61 60	5 10 19	11 66 137 297	15	0
15-9.3/B3(W_2W_{scf})	31 39 41 19 21 49 62 11 13	0 33 44 96	6 26 35	9 68 144 304	15	12
15-9.1/B4(all)	31 39 41 51 13 21 11 52 58	0 30 60 60	5 10 19 33	30 125 285 690	15	0
15-9.14/B5(all)	31 35 13 52 14 55 37 61 11	0 44 0 165	3 5 9 17 33	105 0 1321 0	15	0

Table 6: Continued

Design	Treatment	W_t	Block	W_b	$C1$	$C2$
16-10.1/B1(all)	31 39 41 51 13 21 11 52 58 22	0 43 81 96	25	0 16 27 54	16	0
16-10.1/B2(W_1)	31 39 41 51 13 21 11 52 58 22	0 43 81 96	5 25	3 41 78 177	16	0
16-10.6/B2(W_2W_{scf})	31 39 41 51 13 21 11 22 25 28	0 51 64 102	7 58	1 42 82 188	16	3
16-10.1/B3(W_1)	31 39 41 51 13 21 11 52 58 22	0 43 81 96	5 25 33	12 81 184 441	16	0
16-10.3/B3(W_2W_{scf})	31 39 41 51 13 21 11 52 22 25	0 47 72 98	7 27 33	11 82 186 444	16	4
16-10.1/B4(W_1)	31 39 41 51 13 21 11 52 58 22	0 43 81 96	5 9 17 33	45 108 426 1071	16	0
16-10.18/B4(W_2W_{scf})	31 35 13 52 14 55 21 37 11 19	0 61 0 252	3 5 17 41	35 150 364 1100	16	0
16-10.10/B5(all)	31 35 13 52 14 55 37 61 11 19	0 59 0 262	3 5 9 17 33	120 0 1761 0	16	0
17-11.1/B1(all)	31 39 41 51 13 21 11 52 58 22 25	0 59 108 150	28	0 19 36 78	17	0
17-11.1/B2(W_1)	31 39 41 51 13 21 11 52 58 22 25	0 59 108 150	19 46	3 50 102 256	17	0
17-11.3/B2(W_2W_{scf})	31 39 41 51 13 21 11 52 22 25 28	0 64 96 156	7 58	2 50 104 268	17	2
17-11.1/B3(all)	31 39 41 51 13 21 11 52 58 22 25	0 59 108 150	15 19 33	13 99 242 627	17	0
17-11.1/B4(W_1)	31 39 41 51 13 21 11 52 58 22 25	0 59 108 150	7 9 19 33	48 144 537 1524	17	0
17-11.18/B4(W_2W_{scf})	31 35 13 52 14 55 21 37 11 19 25	0 81 0 384	3 5 17 41	40 180 479 1560	17	0
17-11.9/B5(all)	31 35 13 52 14 55 37 61 11 19 21	0 79 0 394	3 5 9 17 33	136 0 2301 0	17	0
18-12.1/B1(all)	31 39 41 51 13 21 11 52 58 22 25 28	0 78 144 228	46	0 22 48 108	18	0
18-12.1/B2(all)	31 39 41 51 13 21 11 52 58 22 25 28	0 78 144 228	19 46	3 60 132 360	18	0
18-12.1/B3(W_1)	31 39 41 51 13 21 11 52 58 22 25 28	0 78 144 228	3 5 40	21 85 363 885	18	0
18-12.4/B3(W_2W_{scf})	31 35 13 52 14 55 37 61 11 19 21 44	0 102 0 588	5 25 34	15 117 285 1029	18	0
18-12.1/B4(W_1)	31 39 41 51 13 21 11 52 58 22 25 28	0 78 144 228	7 9 19 33	57 160 710 2096	18	0
18-12.12/B4(W_2W_{scf})	31 35 13 52 14 55 21 37 11 19 25 38	0 105 0 570	3 5 17 41	45 216 615 2160	18	0
18-12.4/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44	0 102 0 588	3 5 9 17 33	153 0 2958 0	18	0
19-13.1/B1(all)	31 39 41 51 13 21 11 52 58 22 25 28 46	0 100 192 336	61	0 25 64 144	19	0
19-13.1/B2(W_1)	31 39 41 51 13 21 11 52 58 22 25 28 46	0 100 192 336	3 61	8 49 200 488	19	0
19-13.2/B2(W_2W_{scf})	31 35 13 52 14 55 37 61 11 19 21 44 7	0 131 0 847	22 47	4 70 131 694	19	0
19-13.1/B3(W_1)	31 39 41 51 13 21 11 52 58 22 25 28 46	0 100 192 336	3 5 56	24 97 472 1176	19	0
19-13.2/B3(W_2W_{scf})	31 35 13 52 14 55 37 61 11 19 21 44 7	0 131 0 847	5 25 42	17 138 359 1402	19	0
19-13.1/B4(W_1)	31 39 41 51 13 21 11 52 58 22 25 28 46	0 100 192 336	3 9 20 33	67 176 928 2800	19	0
19-13.15/B4(W_2W_{scf})	31 35 13 52 14 55 21 37 11 19 25 38 7	0 135 0 823	3 5 17 41	51 252 785 2940	19	0
19-13.2/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7	0 131 0 847	3 5 9 17 33	171 0 3745 0	19	0
20-14.1/B1(W_1)	31 39 41 51 13 21 11 52 58 22 25 28 46 61	0 125 256 480	3	4 16 80 240	20	0
20-14.2/B1(W_2W_{scf})	31 35 13 52 14 55 37 61 11 19 21 44 7 62	0 164 0 1208	25	0 40 0 472	20	0
20-14.1/B2(W_1)	31 39 41 51 13 21 11 52 58 22 25 28 46 61	0 125 256 480	3 5	12 48 240 720	20	0
20-14.2/B2(W_2W_{scf})	31 35 13 52 14 55 37 61 11 19 21 44 7 62	0 164 0 1208	22 47	5 82 160 928	20	0
20-14.1/B3(W_1)	31 39 41 51 13 21 11 52 58 22 25 28 46 61	0 125 256 480	3 5 9	34 96 560 1696	20	0
20-14.2/B3(W_2W_{scf})	31 35 13 52 14 55 37 61 11 19 21 44 7 62	0 164 0 1208	5 25 42	19 162 448 1872	20	0
20-14.1/B4(W_1)	31 39 41 51 13 21 11 52 58 22 25 28 46 61	0 125 256 480	3 5 9 17	78 192 1200 3648	20	0
20-14.18/B4(W_2W_{scf})	31 35 13 52 14 55 21 37 11 19 25 38 7 26	0 170 0 1170	3 5 17 41	57 294 986 3920	20	0
20-14.2/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62	0 164 0 1208	3 5 9 17 33	190 0 4681 0	20	0
21-15.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25	0 204 0 1680	49	0 46 0 624	21	0
21-15.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25	0 204 0 1680	26 33	6 94 192 1233	21	0
21-15.1/B3(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25	0 204 0 1680	6 26 41	21 189 552 2457	21	0
21-15.1/B4(W_1)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25	0 204 0 1680	3 5 17 40	100 0 2811 0	21	0
21-15.11/B4(W_2W_{scf})	31 35 13 52 14 55 21 37 11 19 25 38 7 26 49	0 210 0 1638	3 5 17 41	63 343 1218 5145	21	0
21-15.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25	0 204 0 1680	3 5 9 17 33	210 0 5781 0	21	0
22-16.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49	0 250 0 2304	22	0 54 0 801	22	0
22-16.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49	0 250 0 2304	5 42	7 108 232 1602	22	0
22-16.1/B3(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49	0 250 0 2304	3 17 41	24 216 669 3204	22	0
22-16.1/B4(W_1)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49	0 250 0 2304	3 5 24 33	110 0 3435 0	22	0
22-16.12/B4(W_2W_{scf})	31 35 13 52 14 55 21 37 11 19 25 38 7 26 49 22	0 259 0 2240	3 5 17 41	70 392 1498 6664	22	0
22-16.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49	0 250 0 2304	3 5 9 17 33	231 0 7065 0	22	0
23-17.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22	0 304 0 3105	41	0 61 0 1033	23	0
23-17.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22	0 304 0 3105	5 42	8 123 278 2057	23	0
23-17.1/B3(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22	0 304 0 3105	3 17 41	27 246 805 4114	23	0
23-17.1/B4(W_1)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22	0 304 0 3105	3 5 9 33	121 0 4151 0	23	0
23-17.9/B4(W_2W_{scf})	31 35 13 52 14 55 21 37 11 19 25 38 7 26 49 22 28	0 315 0 3024	3 5 17 41	77 448 1820 8512	23	0
23-17.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22	0 304 0 3105	3 5 9 17 33	253 0 8551 0	23	0
24-18.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41	0 365 0 4138	38	0 70 0 1302	24	0
24-18.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41	0 365 0 4138	3 56	9 141 330 2594	24	0
24-18.1/B3(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41	0 365 0 4138	9 20 38	30 280 961 5208	24	0
24-18.1/B4(W_1)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41	0 365 0 4138	3 5 17 40	132 0 4981 0	24	0
24-18.8/B4(W_2W_{scf})	31 35 13 52 14 55 21 37 11 19 25 38 7 26 49 22 28 50	0 378 0 4032	3 5 17 41	84 512 2184 10752	24	0
24-18.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41	0 365 0 4138	3 5 9 17 33	276 0 10261 0	24	0

Table 6: Continued

Design	Treatment	W_t	Block	W_b	$C1$	$C2$
25-19.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38	0 435 0 5440	26	0 80 0 1622	25	0
25-19.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38	0 435 0 5440	9 50	10 160 390 3245	25	0
25-19.1/B3(W_1)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38	0 435 0 5440	3 5 57	66 0 2777 0	25	0
25-19.2/B3(W_2W_{scf})	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 26	0 436 0 5430	9 20 38	33 316 1143 6528	25	0
25-19.1/B4(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38	0 435 0 5440	3 5 9 48	144 0 5923 0	25	0
25-19.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38	0 435 0 5440	3 5 9 17 33	300 0 12215 0	25	0
26-20.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26	0 515 0 7062	28	0 90 0 2013	26	0
26-20.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26	0 515 0 7062	3 56	11 180 460 4026	26	0
26-20.1/B3(W_1)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26	0 515 0 7062	3 5 57	72 0 3275 0	26	0
26-20.3/B3(W_2W_{scf})	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 26 28	0 516 0 7052	9 20 38	36 356 1350 8092	26	0
26-20.1/B4(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26	0 515 0 7062	3 5 9 48	156 0 6999 0	26	0
26-20.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26	0 515 0 7062	3 5 9 17 33	325 0 14435 0	26	0
27-21.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28	0 605 0 9075	42	0 101 0 2473	27	0
27-21.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28	0 605 0 9075	3 56	12 202 539 4946	27	0
27-21.1/B3(W_1)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28	0 605 0 9075	3 12 33	78 0 3841 0	27	0
27-21.2/B3(W_2W_{scf})	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 26 28 42	0 606 0 9064	9 20 38	39 400 1584 9936	27	0
27-21.1/B4(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28	0 605 0 9075	3 5 9 33	169 0 8209 0	27	0
27-21.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28	0 605 0 9075	3 5 9 17 33	351 0 16945 0	27	0
28-22.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42	0 706 0 11548	47	0 113 0 3012	28	0
28-22.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42	0 706 0 11548	3 56	13 226 628 6024	28	0
28-22.1/B3(W_1)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42	0 706 0 11548	5 27 33	84 0 4481 0	28	0
28-22.2/B3(W_2W_{scf})	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 26 28 42 56	0 707 0 11536	9 20 38	42 448 1848 12096	28	0
28-22.1/B4(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42	0 706 0 11548	3 5 24 33	182 0 9577 0	28	0
28-22.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42	0 706 0 11548	3 5 9 17 33	378 0 19769 0	28	0
29-23.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47	0 819 0 14560	50	0 126 0 3640	29	0
29-23.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47	0 819 0 14560	3 56	14 252 728 7280	29	0
29-23.1/B3(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47	0 819 0 14560	5 17 33	91 0 5187 0	29	0
29-23.1/B4(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47	0 819 0 14560	3 5 17 33	196 0 11102 0	29	0
29-23.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47	0 819 0 14560	3 5 9 17 33	406 0 22932 0	29	0
30-24.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50	0 945 0 18200	56	0 140 0 4368	30	0
30-24.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50	0 945 0 18200	3 56	15 280 840 8736	30	0
30-24.1/B3(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50	0 945 0 18200	5 9 17	98 0 5978 0	30	0
30-24.1/B4(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50	0 945 0 18200	5 9 17 33	210 0 12810 0	30	0
30-24.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50	0 945 0 18200	3 5 9 17 33	435 0 26460 0	30	0
31-25.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50 56	0 1085 0 22568	59	0 155 0 5208	31	0
31-25.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50 56	0 1085 0 22568	3 5	45 0 2940 0	31	0
31-25.1/B3(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50 56	0 1085 0 22568	3 5 9	105 0 6860 0	31	0
31-25.1/B4(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50 56	0 1085 0 22568	3 5 9 17	225 0 14700 0	31	0
31-25.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50 56	0 1085 0 22568	3 5 9 17 33	465 0 30380 0	31	0
32-26.1/B1(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50 56 59	0 1240 0 27776	3	16 0 1120 0	32	0
32-26.1/B2(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50 56 59	0 1240 0 27776	3 5	48 0 3360 0	32	0
32-26.1/B3(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50 56 59	0 1240 0 27776	3 5 9	112 0 7840 0	32	0
32-26.1/B4(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50 56 59	0 1240 0 27776	3 5 9 17	240 0 16800 0	32	0
32-26.1/B5(all)	31 35 13 52 14 55 37 61 11 19 21 44 7 62 25 49 22 41 38 26 28 42 47 50 56 59	0 1240 0 27776	3 5 9 17 33	496 0 34720 0	32	0

Table 7: Generator matrix for 81-run designs

	1	**2**	3	4	**5**	6	7	8	9	10	11	12	13	**14**	15	16	17	18	19	20
A	1	0	1	1	0	1	0	1	1	1	0	1	1	0	1	0	1	1	0	1
B	0	1	1	2	0	0	1	1	2	0	1	1	2	0	0	1	1	2	0	0
C	0	0	0	0	1	1	1	1	1	2	2	2	2	0	0	0	0	0	1	1
D	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	0	1	1	1	0	1	1	1	0	1	1	0	1	0	1	1	1	0	1	1
B	1	1	2	0	1	1	2	0	1	1	2	0	0	1	1	2	0	1	1	2
C	1	1	1	2	2	2	2	0	0	0	0	1	1	1	1	1	2	2	2	2
D	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2

The independent columns are in boldface and numbered 1, 2, 5 and 14.

Table 8: MA blocking schemes for 81-run designs

Design	Treatment	W_t	Block	W_b	$C1$	$C2$
4-0.1/B1(all)		0 0	22	0 0 1	4	12
4-0.1/B2(all)		0 0	8 18	0 4 0	4	12
4-0.1/B3(all)		0 0	3 6 15	6 4 3	4	6
5-1.1/B1(all)	22	0 0 1	9	0 2 1 0	5	20
5-1.1/B2(all)	22	0 0 1	3 24	1 7 3 1	5	19
5-1.1/B3(all)	22	0 0 1	3 6 15	10 10 15 4	5	10
6-2.1/B1(all)	22 9	0 2 2 0	24	0 3 4 1	6	18
6-2.1/B2(all)	22 9	0 2 2 0	6 18	2 12 10 8	6	18
6-2.1/B3(all)	22 9	0 2 2 0	3 6 15	15 20 43 28	6	11
7-3.1/B1(all)	22 9 24	0 5 6 1	31	0 5 10 3	7	15
7-3.1/B2(all)	22 9 24	0 5 6 1	6 18	3 20 25 27	7	14
7-3.1/B3(all)	22 9 24	0 5 6 1	3 6 15	21 35 100 99	7	11
8-4.1/B1(all)	22 9 24 31	0 10 16 4	34	0 8 20 8	8	8
8-4.1/B2(all)	22 9 24 31	0 10 16 4	6 18	4 32 50 72	8	8
8-4.1/B3(all)	22 9 24 31	0 10 16 4	4 8 15	28 56 200 264	8	8
9-5.1/B1(all)	22 9 24 31 34	0 18 36 12	39	0 12 36 18	9	0
9-5.1/B2(all)	22 9 24 31 34	0 18 36 12	4 20	9 30 117 162	9	0
9-5.1/B3(all)	22 9 24 31 34	0 18 36 12	4 8 17	36 84 360 594	9	0
10-6.1/B1(all)	22 9 24 31 34 39	0 30 72 30	3	3 12 39 102	10	0
10-6.1/B2(all)	22 9 24 31 34 39	0 30 72 30	3 6	12 48 156 408	10	0
10-6.2/B3(all)	22 9 24 31 34 3	2 28 57 65	4 8 17	45 118 602 1203	5	0
11-7.1/B1(W_1)	22 9 24 31 34 39 3	3 42 111 132	6	3 18 63 180	4	0
11-7.2/B1(W_2W_{scf})	22 9 24 31 3 25 13	3 48 84 177	37	1 24 60 177	2	0
11-7.1/B2(W_1)	22 9 24 31 34 39 3	3 42 111 132	6 15	13 69 252 729	4	0
11-7.3/B2(W_2W_{scf})	22 9 24 25 7 12 18	3 54 63 195	3 20	10 81 234 735	2	0
11-7.2/B3(all)	22 9 24 31 3 25 13	3 48 84 177	4 8 15	55 162 942 2226	2	1
12-8.1/B1(all)	22 9 24 31 3 25 13 37	4 72 144 354	6	3 30 75 336	0	0
12-8.1/B2(all)	22 9 24 31 3 25 13 37	4 72 144 354	6 18	14 100 360 1272	0	0
12-8.1/B3(all)	22 9 24 31 3 25 13 37	4 72 144 354	4 8 15	66 216 1413 3816	0	0
13-9.1/B1(all)	22 9 24 31 3 25 13 37 6	7 102 219 690	18	3 38 115 546	0	0
13-9.1/B2(all)	22 9 24 31 3 25 13 37 6	7 102 219 690	7 16	16 130 526 2055	0	0
13-9.1/B3(all)	22 9 24 31 3 25 13 37 6	7 102 219 690	4 8 15	78 279 2043 6216	0	0
14-10.1/B1(all)	22 9 24 31 3 25 13 37 6 18	10 140 334 1236	7	3 52 161 819	0	0
14-10.1/B2(all)	22 9 24 31 3 25 13 37 6 18	10 140 334 1236	7 16	18 168 736 3182	0	0
14-10.1/B3(all)	22 9 24 31 3 25 13 37 6 18	10 140 334 1236	4 8 15	91 354 2863 9676	0	0

Table 8: Continued

Design	Treatment	W_t	Block	W_b	$C1$	$C2$
15-11.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7	13 192 495 2055	35	3 64 225 1233	0	0
15-11.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7	13 192 495 2055	12 15	21 208 1005 4785	0	0
15-11.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7	13 192 495 2055	4 8 15	105 442 3903 14520	0	0
16-12.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35	16 256 720 3288	12	4 80 294 1784	0	0
16-12.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35	16 256 720 3288	12 15	24 256 1340 6960	0	0
16-12.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35	16 256 720 3288	4 8 15	120 544 5204 21120	0	0
17-13.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12	20 336 1014 5072	38	4 96 390 2536	0	0
17-13.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12	20 336 1014 5072	4 15	40 210 2079 9256	0	0
17-13.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12	20 336 1014 5072	4 8 15	136 660 6804 29926	0	0
18-14.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38	24 432 1404 7608	15	9 72 648 3276	0	0
18-14.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38	24 432 1404 7608	4 15	45 252 2673 12816	0	0
18-14.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38	24 432 1404 7608	4 8 15	153 792 8748 41436	0	0
19-15.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15	33 504 2052 10884	16	9 99 756 4653	0	0
19-15.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15	33 504 2052 10884	4 19	45 351 3141 18243	0	0
19-15.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15	33 504 2052 10884	4 8 17	171 936 11124 56088	0	0
20-16.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16	42 603 2808 15537	19	9 126 909 6282	0	0
20-16.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16	42 603 2808 15537	4 19	46 450 3780 24654	0	0
20-16.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16	42 603 2808 15537	4 8 17	190 1098 13932 74712	0	0
21-17.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19	51 729 3717 21819	23	9 154 1107 8244	0	0
21-17.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19	51 729 3717 21819	4 26	48 550 4590 32418	0	0
21-17.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19	51 729 3717 21819	4 8 17	210 1279 17226 98028	0	0
22-18.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23	60 883 4824 30063	26	10 182 1332 10668	0	0
22-18.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23	60 883 4824 30063	4 26	51 652 5573 41904	0	0
22-18.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23	60 883 4824 30063	4 8 17	231 1480 21062 126846	0	0
23-19.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26	70 1065 6156 40731	34	10 213 1620 13577	0	0
23-19.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26	70 1065 6156 40731	11 30	55 756 6735 53504	0	0
23-19.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26	70 1065 6156 40731	4 8 17	253 1701 25500 162089	0	0
24-20.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34	80 1278 7776 54308	30	12 240 1926 17232	0	0
24-20.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34	80 1278 7776 54308	11 30	60 864 8082 67584	0	0
24-20.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34	80 1278 7776 54308	4 8 17	276 1944 30600 204744	0	0
25-21.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30	92 1518 9702 71540	33	12 276 2310 21462	0	0
25-21.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30	92 1518 9702 71540	4 17	92 680 11216 78736	0	0
25-21.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30	92 1518 9702 71540	4 8 17	300 2208 36432 255948	0	0
26-22.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33	104 1794 12012 93002	40	13 312 2730 26572	0	0
26-22.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33	104 1794 12012 93002	4 8	100 768 13248 97504	0	0
26-22.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33	104 1794 12012 93002	4 8 17	325 2496 43056 316888	0	0

Table 8: Continued

Design	Treatment	W_t	Block	W_b	$C1$	$C2$
27-23.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40	117 2106 14742 119574	4	27 216 3888 29916	0	0
27-23.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40	117 2106 14742 119574	4 8	108 864 15552 119664	0	0
27-23.1/B3(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40	117 2106 14742 119574	4 8 17	351 2808 50544 388908	0	0
28-24.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4	144 2322 18630 149490	8	27 270 4320 37692	0	0
28-24.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4	144 2322 18630 149490	8 17	108 1080 17280 150768	0	0
29-25.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8	171 2592 22950 187182	17	27 324 4860 46332	0	0
29-25.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8	171 2592 22950 187182	10 17	109 1296 19413 185346	0	0
30-26.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17	198 2916 27810 233514	20	27 379 5508 56025	0	0
30-26.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17	198 2916 27810 233514	10 20	111 1513 21951 224235	0	0
31-27.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20	225 3295 33318 289539	10	28 434 6237 67032	0	0
31-27.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20	225 3295 33318 289539	10 27	114 1732 24896 268272	0	0
32-28.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10	253 3729 39555 356571	27	28 492 7101 79454	0	0
32-28.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10	253 3729 39555 356571	11 21	118 1953 28254 318326	0	0
33-29.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27	281 4221 46656 436025	21	30 546 8028 93750	0	0
33-29.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27	281 4221 46656 436025	11 21	123 2178 32031 375222	0	0
34-30.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27 21	311 4767 54684 529775	29	30 609 9114 109725	0	0
34-30.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27 21	311 4767 54684 529775	11 28	129 2406 36243 439872	0	0
35-31.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27 21 29	341 5376 63798 639500	32	31 672 10290 127900	0	0
35-31.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27 21 29	341 5376 63798 639500	11 28	136 2640 40896 513112	0	0
36-32.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27 21 29 32	372 6048 74088 767400	11	36 720 11502 148968	0	0
36-32.1/B2(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27 21 29 32	372 6048 74088 767400	11 28	144 2880 46008 595872	0	0
37-33.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27 21 29 32 11	408 6768 85590 916368	28	36 792 12942 171972	0	0
38-34.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27 21 29 32 11 28	444 7560 98532 1088340	36	37 864 14490 197880	0	0
39-35.1/B1(all)	22 9 24 31 3 25 13 37 6 18 7 35 12 38 15 16 19 23 26 34 30 33 40 4 8 17 20 10 27 21 29 32 11 28 36	481 8424 113022 1286220	39	39 936 16146 226980	0	0