Sādhanā (2019) 44:149
https://doi.org/10.1007/s12046-019-1128-7

Minimum cost berth allocation problem in maritime logistics: new mixed integer programming models

BOBIN CHERIAN JOS[1], M HARIMANIKANDAN[1], CHANDRASEKHARAN RAJENDRAN[1,*] and HANS ZIEGLER[2]

[1]Department of Management Studies, Indian Institute of Technology Madras, Chennai 600 036, India
[2]School of Business, Economics and Information Systems, University of Passau, Passau, Germany
e-mail: bobincherian@gmail.com; manikandanhari@gmail.com; craj@iitm.ac.in; hans.ziegler@uni-passau.de

MS received 22 December 2018; revised 16 February 2019; accepted 18 March 2019; published online 22 May 2019

Abstract. The berth allocation problem (BAP) involves decisions on how to allocate the berth space and to sequence maritime vessels that are to be loaded and unloaded at a container terminal involved in the maritime logistics. As the berth is a critical resource in a container terminal, an effective use of it is highly essential to have efficient berthing and servicing of vessels, and to optimize the associated costs. This study focuses on the minimum cost berth allocation problem (MCBAP) at a container terminal where the maritime vessels arrive dynamically. The objective comprises the waiting time penalty, tardiness penalty, handling cost and benefit of early service completion of vessels. This paper proposes three computationally efficient mixed integer linear programming (MILP) models for the MCBAP. Through numerical experiments, the proposed MILP models are compared to an existing model in the literature to evaluate their computational performance. The computational study with problem instances of various problem characteristics demonstrates the computational efficiency of the proposed models.

Keywords. Maritime logistics; discrete berth allocation problem; minimum cost berth allocation problem; mixed integer linear programming models.

1. Introduction

Maritime transportation performs the function of moving goods or passengers between ports by sea and other waterways. International trade and global economy is quite dependent on maritime logistics. Nearly 80% of global trade by volume and over 70% of global trade by value are carried by sea, and are handled by ports worldwide [1]. UNCTAD [1] forecasts a 3.8% compound annual growth rate between 2018 and 2023. As there has been a significant growth in the sea-borne demand, priority is given to make port operations more efficient by utilizing the resources effectively.

The main function of a port terminal is to serve vessels by loading and unloading containers as well as bulk materials. The container terminal is segregated into quay area, transport area, yard area and truck area. The quay area is the one in which vessels are berthed and handled. The area in which the containers are moved within a container terminal is the transport area. Yard area facilitates the storage of containers. Truck area is meant for serving external trucks.

With the increased usage of container terminals, efficiency of a container terminal is affected if sufficient berths are not allocated to incoming vessels. Vessel and berth are the two entities considered in making these decisions. The container terminal operators make these decisions based on different priorities they have and the contractual agreement they have with vessel operators. The success of a container terminal is influenced by short berth duration of vessels and low cost handling (loading and unloading) of vessels [2]. The berthing and service schedules of vessels are strongly affected by the unexpected waiting time in a container terminal. An efficient berth planning avoids such disturbances in schedules and improves the service quality of container terminals. Competition among ports continues to increase, and therefore it is necessary to reduce costs by efficiently utilizing resources. Since berth is considered as the most important resource, allocating berth space to a set of vessels for container handling is a foremost decision to be made when vessels enter the harbour and are waiting to be berthed.

The berth allocation problems (BAPs) are classified based on the berth layout, temporal attribute, handling time attribute and performance measures [3, 4]. Readers may go through the comprehensive reviews presented by Bierwirth

*For correspondence

and Meisel [3, 4] for more insights into the BAPs. In this paper, the discrete layout of berths is considered. A key related work to this paper is the work by Hansen *et al* [5], where the discrete layout BAP with dynamic arrival times of vessels is considered with the objective of minimizing the costs associated with vessels and berths. Hansen *et al* [5] termed such a problem as an MCBAP (minimum cost berth allocation problem). The key contribution of this paper is the proposal of new and computationally efficient mixed integer linear programming models (MILPs) to solve the MCBAP.

This paper is organized as follows. Section 2 presents a literature review on various discrete BAPs. Section 3 describes the MCBAP. Section 4 contains three new MILP formulations for the BAP under study. A few modifications in the existing model and implications of the present study are presented in sections 5 and 7, respectively. Details of the computational study to evaluate the performance of MILP models are given in section 6, and the paper is concluded in section 8. The terms 'vessel' and 'ship' are used interchangeably in this paper.

2. Literature review

The main focus of this paper is the discrete BAP, and henceforth the earlier studies on this domain are now presented. In berth allocation at a container terminal with discrete layout, decisions are made on the allocation of an individual berth to a vessel and the time period during which vessels are subjected to handling operations in the allocated berth.

Dynamic berth allocation problem (DBAP) and its formulation are presented by Imai *et al* [6], in which the berth space is represented as a finite set of berthing points. This problem is characterized by the dynamic arrival of vessels to these berthing points. It is an extension of the static berth allocation problem (SBAP) studied by Imai *et al* [7], which considers the case where all ships are assumed to arrive at the port before the earliest available time of the berths. The objective function minimizes the sum of waiting time for the availability of the berth assigned to each ship, plus the handling time it spends at the berth. Imai *et al* [6] proposed a Lagrangian relaxation for the DBAP, where the subproblem was an assignment problem. Imai *et al* [8] considered service priorities of vessels in DBAP and a genetic-algorithm-based heuristic is developed to solve the problem. Cordeau *et al* [9] focused on the BAP of discrete berth layout with time windows with respect to berth availability, and two mathematical models were proposed to minimize the sum of service times of vessels.

Hansen *et al* [5] considered dynamic arrival of vessels and discrete berth layout in their study. The objective function included handling time cost, waiting time cost, earliness benefit and lateness cost and is referred to as MCBAP. They proposed an MILP; however, they did not present the computational efficiency of their proposed model. The authors also proposed multi-start variable neighbourhood descent (MVND) and variable neighbourhood search (VNS) heuristics to solve the MCBAP instances. In view of the computational complexity associated with MILP models in the context of discrete BAPs, many researchers concentrated on the development of heuristics and meta-heuristics [10, 11], apart from Hansen *et al* [5]. Liang *et al* [10] proposed a non-linear mixed integer mathematical model for discrete berth layout with the objective of minimizing the sum of handling time, waiting time and tardiness of vessels. Kovač *et al* [11] proposed meta-heuristics for the MCBAP with hybrid berth layout.

The discrete BAP is an NP-hard problem [12, 13], and hence heuristic procedures are proposed to solve the large-sized problems. A survey on meta-heuristic approaches for the BAP is given by Kovač [14]. Many studies combined the discrete berth allocation with quay crane scheduling (e.g., see [15, 16]). Berth planning decisions associated with vessels having bulk materials differ from those of container vessels. The studies of Ernst *et al* [17] and Pratap *et al* [18] dealt with the BAP in bulk ports under various operating conditions.

Based on the review of literature, it is seen that many studies have adopted complex binary variable structures in the mathematical formulations related to logistics (e.g., [19, 20]) and also in the mathematical formulations of the BAP in maritime logistics (especially the work by Hansen *et al* [5]). The MILP model proposed by Hansen *et al* [5] was not able to solve a sample problem instance of size 5 berths and 10 ships optimally in a reasonable time, and this limitation was specifically reported in section 2 of Hansen *et al* [5]. This observation by Hansen *et al* [5] has provided us the motivation for developing computationally superior MILP models for the MCBAP. Hence, based on this research gap, we propose three new computationally efficient MILP models and compare to the existing model by Hansen *et al* [5]; see [5] for more insight into the MCBAP. The main purpose of the development of computationally efficient MILP models is that MILP models provide optimal solutions for medium-sized problem instances and for some large-sized problem instances, and can be used to evaluate the quality of heuristic/meta-heuristic solutions, thereby contributing to the literature on operations research (OR) models.

The contributions of this study are three-fold.

(i) Three new computationally efficient MILP models for the MCBAP, which are computationally superior to the existing model by Hansen *et al* [5], are proposed.
(ii) The computational performance of the three new MILP models and the MILP model by Hansen *et al* [5] is evaluated.

(iii) It is found that one of the assumptions (in terms of the cost parameters at a container terminal) considered in the work of Hansen *et al* [5] need not be true in all cases. The proposed models do not have such an assumption and are more generic. We therefore suggest modifications in the existing model [5], thereby making the model of Hansen *et al* [5] more realistic by relaxing such an assumption.

3. Problem description

The study of MCBAP in a container terminal is presented in this section. A set of berths B, indexed by $j = 1, 2, \ldots, J$, is available at a container terminal for receiving a set of ships/vessels V, indexed by $i = 1, 2, \ldots, N$. The berths are assumed to be discrete and can handle only one vessel at a time. The vessels arriving in a container terminal have the following set of deterministic attributes: expected arrival time (A_i), scheduled departure time (d_i) and handling time ($t_{i,j}$). This set of attributes is communicated to the container terminal operator. The handling time ($t_{i,j}$) of a vessel i varies according to the berth j to which it is assigned. This problem can be termed as DBAP as the planning activity is carried out even before the arrival of vessels, by considering their expected arrival times. The earliest time from which each berth is available (S_j) is also taken into account by the terminal operator for planning.

The objective function of this problem includes the ship-dependent waiting cost α_i per unit time, the benefit β_i per unit time associated with an early service completion of vessel i before its scheduled departure time (d_i), the ship-dependent lateness cost γ_i per unit time and the cost of handling vessel i at berth j ($c_{i,j}$). Based on the data available, a berth plan is derived by the terminal operator. The problem necessarily aims at determining the berth to which each vessel is to be assigned, the start time and end time of handling of the vessels. The discrete berth planning system pertains to two types of decisions associated with berth allocation. One is the assignment of each vessel to one of the berths and the other deals with sequencing of vessels assigned to a specific berth.

4. MILPs

The MCBAP problem under study is modelled in this paper using three newly proposed MILP models. These models are developed and presented in this section. The following assumptions are made as a part of the proposed formulations.

- Each berth can serve only one vessel at a given time.
- Physical restrictions at the terminal (e.g., water depth) are not considered.
- Handling time of a vessel depends on the berth to which it is assigned; berths are non-identical with respect to the associated vessel-handling infrastructure.
- There is no service interruption during vessel handling activities.
- Information on vessel arrival in the container terminal during the planning period is known in advance. As deterministic scenario is considered, parameters are known a priori.

Parameters

S_j time from which berth j is available for berthing a vessel
A_i arrival time of vessel i
d_i scheduled departure of vessel i
$t_{i,j}$ handling time of vessel i on berth j
α_i cost associated with per unit waiting time of vessel i
γ_i cost associated with per unit time of tardiness of vessel i
β_i benefit associated with per unit time of early service completion of vessel i
$c_{i,j}$ cost associated with handling of vessel i on berth j
V set of vessels $\{1, 2, \ldots, N\}$
B set of berths $\{1, \ldots, J\}$
G a sufficiently large positive integer, where

$$G \geq \sum_{i=1}^{N} \max_j(t_{i,j}) + \max\left\{\max_i(A_i); \max_j(S_j)\right\}$$

Decision variables

f_i a continuous variable that represents the time at which the service on vessel i ends at its allotted berth
s_i a continuous variable that represents the time at which vessel i begins to be handled at its allotted berth
l_i a continuous variable that represents the tardiness of ship i from its scheduled departure
w_i a continuous variable that represents the waiting time of ship i for getting berthed
e_i a continuous variable that represents the earliness of ship i from its scheduled departure
$f'_{i,j,k}$ a continuous variable that represents the time at which service on ship i ends when processed on berth j at position k in the sequence of vessels served on that berth

4.1 *MILP model 1 (MCBAP1)*

We propose a new and computationally efficient MILP for the BAP under study by bringing in three-dimensional precedence-based binary variables ($y_{i',i,j}$) to represent the sequence of vessels at a given berth, and two-dimensional binary variables ($\lambda_{i,j}$) to represent the allocation of vessels to a berth. This formulation is referred to as MCBAP1, or simply M1.

Binary variables

- Del_j a binary variable that takes the value 1 if berth j is engaged during the planning horizon; 0 otherwise
- $\lambda_{i,j}$ a binary variable that takes the value 1 if vessel i is allocated to berth j; 0 otherwise
- $y_{i',i,j}$ a binary variable that takes the value 1 if vessel i is handled immediately after vessel i' at berth j; 0 otherwise
- $z'_{N+j,i}$ a binary variable that takes the value 1 if vessel i is handled as the first vessel at berth j; 0 otherwise; $N+j$ is a fictitious vessel indicating the start of the sequence of vessels at berth j
- $z''_{i,N+J+j}$ a binary variable that takes the value 1 if vessel i is handled as the last vessel at berth j; 0 otherwise; $N+J+j$ is a fictitious vessel indicating the end of the sequence of vessels at berth j
- ψ_i A binary variable that restricts the simultaneous computation of earliness and lateness of vessel i

Objective function

$$Min\ Z = \sum_{i=1}^{N}\alpha_i w_i + \sum_{i=1}^{N}\sum_{j=1}^{J} c_{i,j}\lambda_{i,j} - \sum_{i=1}^{N}\beta_i e_i + \sum_{i=1}^{N}\gamma_i l_i \tag{1}$$

subject to

$$\sum_{j=1}^{J}\lambda_{i,j} = 1 \quad \forall i \in V \tag{2}$$

$$\sum_{i'=1,\ i'\neq i}^{N} y_{i',i,j} + z'_{N+j,i} = \lambda_{i,j} \quad \forall i \in V, \quad \forall j \in B \tag{3}$$

$$\sum_{i'=1,\ i'\neq i}^{N} y_{i,i',j} + z''_{i,N+J+j} = \lambda_{i,j} \quad \forall i \in V,\ \forall j \in B \tag{4}$$

$$\sum_{i=1}^{N} z'_{N+j,i} \le Del_j \quad \forall j \in B \tag{5}$$

$$\sum_{i=1}^{N} z''_{i,N+J+j} \le Del_j \quad \forall j \in B \tag{6}$$

$$\sum_{i=1}^{N}\lambda_{i,j} \le N \times Del_j \quad \forall j \in B \tag{7}$$

$$\sum_{i=1}^{N}\lambda_{i,j} \ge Del_j \quad \forall j \in B \tag{8}$$

$$\sum_{i=1}^{N}\lambda_{i,j} \le N\sum_{i=1}^{N} z'_{N+j,i} \quad \forall j \in B \tag{9}$$

$$\sum_{i=1}^{N}\lambda_{i,j} \le N\sum_{i=1}^{N} z''_{i,N+J+j} \quad \forall j \in B \tag{10}$$

$$s_i \ge f_{i'} - G\left(1 - \sum_{j=1}^{J} y_{i',i,j}\right) \quad \forall i \in V,\ i' \in V \text{ and } i' \neq i \tag{11}$$

$$s_i \ge A_i \quad \forall i \in V \tag{12}$$

$$s_i \ge \sum_{j=1}^{J} S_j\lambda_{i,j} \quad \forall i \in V \tag{13}$$

$$f_i = s_i + \sum_{j=1}^{J} t_{i,j}\lambda_{i,j} \quad \forall i \in V \tag{14}$$

$$l_i - e_i = f_i - d_i \quad \forall i \in V \tag{15}$$

$$w_i \ge s_i - A_i \quad \forall i \in V \tag{16}$$

$$l_i \le G \times \psi_i \quad \forall i \in V \tag{17}$$

$$e_i \le G(1 - \psi_i) \quad \forall i \in V \tag{18}$$

$$\begin{aligned} &y_{i',i,j} \in \{0,1\}\ \ \forall j \in B,\ \forall i \in V,\\ &\forall i' \in V \text{ and } i' \neq i;\ \lambda_{i,j} \in \{0,1\};\\ &z'_{N+j,i} \in \{0,1\};\ z''_{i,N+J+j} \in \{0,1\}\ \forall j \in B,\\ &\forall i \in V;\ \psi_i \in \{0,1\}\ \forall i \in V\\ &\text{and all other variables are } \ge 0. \end{aligned} \tag{19}$$

Objective function (1) minimizes the sum of costs associated with the waiting time and tardiness of vessels, and operating costs of handling vessels. The early service completion of vessel handling attracts benefit and is included as a part of the objective function. Constraint (2) states that a vessel can be assigned to at most only one berth. Equations (3) and (4) ensure that each vessel can have at most one predecessor and one successor (including the fictitious vessels at its respective berth) in the sequence. Constraints (5) and (6) ensure that each berth, if engaged in the planning horizon, can have at most one fictitious start vessel ($N+j$) and one fictitious end vessel ($N+J+j$). Constraints (7) and (8) ensure that if at least one vessel is assigned to berth j, then the berth is active, and that a berth is inactive if no vessel is assigned to it. Constraints (9) and (10) ensure the allocation of fictitious vessels to a berth if any of the vessels is assigned to that berth. Constraint (11) states that the start time of a vessel is greater than or equal to the completion time of its predecessor in the sequence. Constraint (12) makes the start time of a vessel greater than or equal to its arrival time. Constraint (13) makes the start time of a vessel assigned to a berth to be greater than or equal to the time at which the berth is available for service. Equation (14) calculates the completion time of a vessel as the sum of the respective start time and its handling time. Expression (15) determines the earliness and lateness of all vessels. Expression (16) determines the waiting time for each vessel especially when earliness reward is greater than the tardiness penalty with respect to a given vessel. Constraints (17) and (18) restrict the simultaneous computation of earliness and lateness of a vessel. Finally, expression (19) defines all the binary variables and continuous variables. Note that the binary variable ψ_i is introduced because service completion can be either early or late but not both for a vessel.

4.2 *MILP model 2 (MCBAP2)*

Our second MILP model for the MCBAP is presented in this section. A two-dimensional precedence-based binary variable ($z_{i',i}$) is used in this formulation to represent the sequence of vessels and a two-dimensional binary variable ($\lambda_{i,j}$) is used to represent the allocation of a vessel to a berth. This formulation is referred to as MCBAP2, or simply M2.

Parameters

Apart from the parameters presented earlier, the following parameters are used in MCBAP2.

$\lambda_{N+j,j}$ a variable that is given a value of 1 to indicate that the fictitious vessel $N+j$ is allocated to berth j; $\lambda_{N+j,j}=1 \quad \forall j \in B$

$\lambda_{N+J+j,j}$ a variable that is given a value of 1 to indicate that the fictitious vessel $N+J+j$ is allocated to berth j; $\lambda_{N+J+j,j}=1 \quad \forall j \in B$

Binary variables

Apart from the binary variables, Del_j, $\lambda_{i,j}$, $z'_{N+j,i}$, $z''_{i,N+J+j}$ and ψ_i, presented in MCBAP1, the following binary variable is used in MCBAP2.

$z_{i',i}$ a binary variable that takes the value 1 if vessels i and i' are allocated to berth j and vessel i is handled immediately after vessel i' at berth j; 0 otherwise; this binary variable necessarily equals 0 if vessels i and i' are allotted to different berths, and hence the number of $z_{i',i}$ active variables is reduced

Objective function

$$Min\ Z = \sum_{i=1}^{N} \alpha_i w_i + \sum_{i=1}^{N}\sum_{j=1}^{J} c_{i,j}\lambda_{i,j} - \sum_{i=1}^{N} \beta_i e_i + \sum_{i=1}^{N} \gamma_i l_i \tag{20}$$

subject to

$$\sum_{j=1}^{J} \lambda_{i,j} = 1 \quad \forall i \in V \tag{21}$$

$$\lambda_{i,j} - \lambda_{i',j} \le 1 - z_{i,i'} \quad \forall j \in B,\ \forall i \in V,\ i' \in V \text{ and } i' \ne i \tag{22}$$

$$\lambda_{N+j,j} - \lambda_{i,j} \le 1 - z'_{N+j,i} \quad \forall j \in B,\ \forall i \in V \tag{23}$$

$$\lambda_{N+J+j,j} - \lambda_{i,j} \le 1 - z''_{i,N+J+j} \quad \forall j \in B,\ \forall i \in V \tag{24}$$

$$\sum_{i'=1, i'\ne i}^{N} z_{i,i'} + \sum_{j=1}^{J} z''_{i,N+J+j} = 1 \quad \forall i \in V \tag{25}$$

$$\sum_{i'=1, i'\ne i}^{N} z_{i',i} + \sum_{j=1}^{J} z'_{N+j,i} = 1 \quad \forall i \in V \tag{26}$$

$$z_{i,i'} + z_{i',i} \le 1 \quad i = 1,2,\ldots,N-1 \text{ and } i' = i+1, i+2,\ldots,N \tag{27}$$

$$\sum_{i=1}^{N} z'_{N+j,i} = Del_j \quad \forall j \in B \tag{28}$$

$$\sum_{i=1}^{N} z'_{N+j,i} \le \sum_{i=1}^{N} \lambda_{i,j} \quad \forall j \in B \tag{29}$$

$$\sum_{i=1}^{N} \lambda_{i,j} \le N \sum_{i=1}^{N} z'_{N+j,i} \quad \forall j \in B \tag{30}$$

$$\sum_{i=1}^{N} z''_{i,N+J+j} = Del_j \quad \forall j \in B \tag{31}$$

$$\sum_{i=1}^{N} z''_{i,N+J+j} \le \sum_{i=1}^{N} \lambda_{i,j} \quad \forall j \in B \tag{32}$$

$$\sum_{i=1}^{N} \lambda_{i,j} \le N \sum_{i=1}^{N} z''_{i,N+J+j} \quad \forall j \in B \tag{33}$$

$$Del_j \ge \lambda_{i,j} \quad \forall i \in V,\ j \in B \tag{34}$$

$$f_i \ge A_i + \sum_{j=1}^{J} t_{i,j}\lambda_{i,j} \quad \forall i \in V \tag{35}$$

$$f_i \ge f_{i'} + \sum_{j=1}^{J} t_{i,j}\lambda_{i,j} - G\left(1 - z_{i',i}\right) \quad \forall i \in V,\ i' \in V \text{ and } i' \ne i \tag{36}$$

$$f_i \ge S_j\lambda_{i,j} + t_{i,j}\lambda_{i,j} - G\left(1 - z'_{N+j,i}\right) \quad \forall i \in V,\ j \in B \tag{37}$$

$$l_i - e_i = f_i - d_i \quad \forall i \in V \tag{38}$$

$$w_i \geq f_i - \sum_{j=1}^{J} t_{i,j}\lambda_{i,j} - A_i \quad \forall i \in V \tag{39}$$

$$l_i \leq G \times \psi_i \quad \forall i \in V \tag{40}$$

$$e_i \leq G(1-\psi_i) \quad \forall i \in V \tag{41}$$

$$\begin{aligned} & z_{i',i} \in \{0,1\} \ \forall i \in V,\ \forall i' \in V \text{ and } i' \neq i, \\ & \psi_i \in \{0,1\}\ \forall i \in V, \\ & z'_{N+j,i} \in \{0,1\};\ z''_{i,N+J+j} \in \{0,1\};\ \lambda_{i,j} \in \{0,1\} \\ & \forall j \in B,\ \forall i \in V, \\ & \text{and all other variables are } \geq 0. \end{aligned} \tag{42}$$

The objective function (20) is similar to the previous formulation MCBAP1. Constraint (21) directs the assignment of a vessel to not more than one berth. Constraint (22) states that $z_{i',i}$ equals zero when vessels i and i' are allotted to different berths. In addition, when vessels i and i' are allotted to berth j, $z_{i',i}$ is either 0 or 1, depending, respectively, on whether immediate precedence is absent or present. Constraint (23) states that $z'_{N+j,i}$ equals zero if vessel i is not allotted to berth j. In addition $z'_{N+j,i}$ is either 1 or 0 depending, respectively, on whether vessel i is the first in the sequence or not. Similarly constraint (24) states that $z'_{i,N+J+j}$ equals zero if vessel i is not allotted to berth j. In addition $z'_{i,N+J+j}$ is either 1 or 0 depending on whether vessel i is the last in the sequence or not. Constraint (25) is to make sure that there is exactly one successor for vessel i, including the fictitious vessel $N+j$. Constraint (26) makes exactly one predecessor for vessel i, including the fictitious vessel $N+J+j$. A vessel can have only one type of sequence relationship with another vessel. Either a vessel can precede or succeed another vessel. This condition is enforced by constraint set (27). Constraints (28)–(30) ensure that fictitious vessel $N+j$ with respect to berth j is active only if at least one vessel is assigned to berth j. Constraints (31)–(33) ensure that fictitious vessel $N+J+j$ with respect to berth j is active only if at least one vessel is assigned to berth j. Constraint (34) introduces a berth-oriented binary variable to take the value of 1, if at least one vessel is assigned to a berth. The variables f_i, w_i, e_i and l_i are calculated in a similar way as those in MCBAP1 and are given as constraints (35)–(41). Finally expression (42) defines all the binary variables and continuous variables.

4.3 *MILP model 3 (MCBAP3)*

Our third MILP model for the MCBAP is presented in this section. A three-dimensional position-based binary variable ($x_{i,j,k}$) is used in this formulation to represent the allocation of vessels to a berth and its sequence. This formulation is referred to as MCBAP3, or simply M3.

Binary variable

$x_{i,j,k}$ a binary variable that takes the value 1 if ship i is processed on berth j at position k in the sequence of vessels served on berth j; 0 otherwise

Objective function

$$Min\ Z = \sum_{i=1}^{N}\alpha_i w_i + \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{N} c_{i,j}x_{i,j,k} - \sum_{i=1}^{N}\beta_i e_i + \sum_{i=1}^{N}\gamma_i l_i \tag{43}$$

subject to

$$\sum_{j=1}^{J}\sum_{k=1}^{N} x_{i,j,k} = 1 \quad \forall i \in V \tag{44}$$

$$\sum_{i=1}^{N} x_{i,j,k} \leq 1 \quad \forall j \in B, k = 1,2,\ldots,N \tag{45}$$

$$\sum_{i=1}^{N} x_{i,j,k} \geq \sum_{i=1}^{N} x_{i,j,k+1} \quad \forall j \in B \text{ and } k = 1,2,\ldots,(N-1) \tag{46}$$

$$f'_{i,j,k} \leq G \times x_{i,j,k} \quad \forall i \in V,\ \forall j \in B \text{ and } k = 1,2,\ldots,N \tag{47}$$

$$f'_{i,j,k} \geq \sum_{i'=1, i'\neq i}^{N} f'_{i',j,k-1} + t_{i,j} - G\big(1 - x_{i,j,k}\big)\ \forall i \in V,\ \forall j \in B \text{ and } k = 2,\ldots,N \tag{48}$$

$$f_i = \sum_{j=1}^{J}\sum_{k=1}^{N} f'_{i,j,k} \quad \forall i \in V \tag{49}$$

$$f_i \geq A_i + \sum_{j=1}^{J}\left(t_{i,j} \times \sum_{k=1}^{N} x_{i,j,k}\right) \quad \forall i \in V \tag{50}$$

$$f_i \geq \left(\sum_{j=1}^{J}\left(S_j \times \sum_{k=1}^{N} x_{i,j,k}\right)\right) + \sum_{j=1}^{J}\left(t_{i,j} \times \sum_{k=1}^{N} x_{i,j,k}\right) \quad \forall\ i \in V \tag{51}$$

$$l_i - e_i = f_i - d_i \quad \forall i \in V \tag{52}$$

$$w_i \geq f_i - \sum_{j=1}^{J}\left(t_{i,j} \times \sum_{k=1}^{N} x_{i,j,k}\right) - A_i \quad \forall i \in V \tag{53}$$

$$l_i \leq G \times \psi_i \quad \forall i \in V \tag{54}$$

$$e_i \leq G(1-\psi_i) \quad \forall i \in V \tag{55}$$

$$\begin{aligned} & x_{i,j,k} \in \{0,1\},\ \psi_i \in \{0,1\},\ \forall i \in V, \\ & \forall j \in B \text{ and } k = 1,2\ldots,N; \\ & \text{and all other variables are } \geq 0. \end{aligned} \tag{56}$$

The objective function (43) is similar to the previous formulations. In the constraints, Eq. (44) states that a vessel can be assigned only to a single berth and in only one position. Capacity constraint (45) states that a given position on a berth can accommodate at most one vessel. Compression constraint (46) restricts the occurrence of any position left unoccupied (in between the occupied positions in the sequence of vessels) with respect to a berth. Constraint (47) sets $f'_{i,j,k}$ equal to zero, where vessel i is not assigned to berth j in position k of the sequence of vessels at berth j. Constraint (48) defines the completion time of a vessel handled at a specific position on a berth, being related to the sum of its processing time and the completion time of the vessel preceding immediately. Constraint (49) defines the completion time of a vessel. The variables f_i, w_i, e_i and l_i are calculated in a similar way as that in M1, and are given as constraints (50)–(55). Finally, expression (56) defines all the binary variables and continuous variables.

4.4 *A comparison of the MILP formulations under evaluation*

Table 1 shows the number of binary variables and the number of constraints of the MILP formulations for all the proposed models as well as the benchmark model proposed by Hansen *et al* [5]. In the rest of this paper, the benchmark model is referred to as M4. The non-negative condition imposed on variables is not considered in determining the number of constraints. The comparison is made based on the parameters N and J, where N and J denote the number of vessels (i.e., $|V|$) and the number of berths (i.e., $|B|$), respectively. It is noteworthy that the proposed models have less number of constraints and comparable number of binary variables in comparison to the existing model, thereby leading to more computational efficiency.

The proposed three models have three different approaches in terms of defining the key binary variables related to the sequence of processing vessels on a given berth. The associated novelties and contributions are presented here.

- MCBAP1 defines a three-dimensional binary variable $y_{i',i,j}$ that is related to the precedence of vessel i' with respect to vessel i on berth j, if they are allotted to the same berth. The allocation of vessel i to berth j is indicated by the binary variable $\lambda_{i,j}$. In addition, we introduce fictitious vessels with respect to every berth j, denoted by $N+j$ and $N+J+j$, respectively, to denote the start and finish of sequence of vessels serviced by berth j. Constraints such as (3)–(6) and (9)–(11) make our precedence-based MILP formulation unique and distinct, and hence computationally superior.
- MCBAP2 also proposes a precedence-based binary variable $z_{i',i}$ with respect to vessels i' and i; however, this binary variable ($z_{i',i}$) is unique and proposed for the first time in the context of MCBAP. This binary variable reduces the dimensional complexity of MCBAP2. However, this reduction in the dimensional complexity is associated with an increase in the number of constraints (see table 1 for details).
- MCBAP3 makes use of a position-based binary variable $(x_{i,j,k})$ that is related to the possible placement of vessel i in position k of the sequence of vessels allotted to berth j. While the model by Hansen *et al* [5] also presents a similar position-based binary variable, our MCBAP3 results in a less number of total constraints due to the use of constraints (52)–(55). They make our proposed MCBAP3 computationally superior to the formulation by Hansen *et al* [5].

Table 1. A comparison of the sizes of MILP formulations under evaluation.

MILP models	Number of binary variables	Number of constraints
M1	$N^2J + 2NJ + N + J$	$N^2 + 2NJ + 7N + 6J$
M2	$N^2 + 3NJ + J$	$N^2(J + 3/2) + 3NJ + (13/2)N + 6J$
M3	$N^2J + N$	$2N^2J + NJ + 8N - J$
M4	$N^2J + NJ$	$3N^2J + 4NJ + N$

5. Modifications in the existing model of Hansen *et al* [5]

5.1 *Redefining the Big M values*

Hansen *et al* [5] defined four Big M coefficients in their formulation, denoted by M_{ik}, M'_{ik}, M''_{ik} and M'''_{ik}. Our computational experiments have identified the inadequacy of the Big M coefficients in their formulation. For some problems, Big M calculated is too small such that the optimum solution is not achieved. Hence, this study redefines the Big-M coefficients as follows:

M_{ik} is the sum of the k largest t_{ij}, plus the largest of the maximum arrival time of the ships and the time when berth i becomes available

M'_{ik} is equal to the largest α_j multiplied by M_{ik}

M''_{ik} is equal to the largest β_j multiplied by M_{ik}

M'''_{ik} is equal to the largest γ_j multiplied by M_{ik}

It is to be noted that in the existing formulation, index i denotes the berths, index j denotes the vessels and index k denotes the position in the sequence of vessels at a berth. A sample problem instance that enforces the redefinition of Big M values is given in the link <http://dx.doi.org/10.17632/z4sh59w8sw.2#file-ebccb3a9-6076-44fe-ab40-2e9070b7527b>.

5.2 *Modification of the relation between earliness premium and lateness penalty*

In the existing model, Hansen *et al* [5] assumed that the ship-dependent premium per unit of earliness time of service is less than the ship-dependent penalty per unit of time of lateness of service (i.e., $\beta_i \leq \gamma_i$). It need not be true in all cases. Our proposed formulations do not have such an assumption and are more realistic. The following additional constraints are proposed in the existing model for overcoming such a restriction:

$$l_{ik} \leq G \times \psi_{ik} \quad \forall i \in B,\ k = 1, 2, \ldots N \tag{57}$$

$$e_{ik} \leq G(1 - \psi_{ik}) \quad \forall i \in B,\ k = 1, 2, \ldots N \text{ and} \tag{58}$$

$$\psi_{ik} \in \{0, 1\} \quad \forall i \in B,\ k = 1, 2\ldots, N \tag{59}$$

Constraints (57) and (58), and the binary variable ψ_i, are introduced to prevent the simultaneous presence or computation of earliness and tardiness of vessels when $\beta_i \leq \gamma_i$. We also tested the performance of the M4 with additional constraints (57)–(59), and could not observe any notable difference in the computational performance in terms of the average CPU time.

6. Computational experimentation

The computational performance of the proposed MILP models is evaluated using a large number of randomly generated problem instances with different sizes with respect to various numbers of vessels and berths and by varying parameters, following the existing literature. Since benchmark instances are not available in the literature, problems are generated randomly as suggested by earlier researchers, especially by Hansen and Oguz [21] and Hansen *et al* [5]. We first explain how the problem instances are generated, and then present the computational performance evaluation of the various models.

6.1 *Generation of problem instances*

Similar to the study of Hansen *et al* [5], two sets of problem instances are generated for studying the MCBAP problem with three different number of berths $|B| \in \{5, 10, 20\}$. The first set represents the extended version of instances from Hansen and Oguz [21] and Imai *et al* [6], and is now described. The arrival time of the vessel (A_i) is an integer value obtained from a uniform distribution in the range of [1, (7000/60) × ($|V|/|B|$)] where $|B|$ denotes the number of berths and $|V|$ denotes the number of vessels. Handling time of vessel i on berth j is obtained by $t_{i,j} = (2u_{ij} + 1.5) \times 2000/60$, where u_{ij} is a uniformly distributed random number between 0 and 1. Earliest available time of the berths (S_j) is set for each instance equal to a given fraction fr of the time interval between the first and last arriving ships. Imai *et al* [6] defined four different instance groups (IGs) with fr equal to $1/2, 3/5, 5/8$ and $7/8$, and that are indexed by 1, 2, 3 and 4, respectively, as shown in tables 2 and 3. The DBAP with all the ships arriving before the earliest available time of the berths (S_j) reduces to the SBAP, and hence such occurrence is restricted by considering $fr < 1$. The per-unit cost associated with waiting time of vessel i (α_i) is an integer number generated randomly from a uniform distribution in the interval [1, 10]. The values of γ_i and β_i for vessels are all set equal to 1. Due date d_i is set to $A_i + (T_{max} - T_{min}) \times r$, where T_{max} and T_{min} are maximum and minimum elements of matrix $(t_{i,j})$, respectively, and r is a random number from the interval [0, 1]. Costs for handling vessels are generated as $c_{i,j} = 10 \times t_{i,j} + r \times t_{i,j}$, where r is a random number in the interval [0, 1]. The first set of instances contains the generated problem instances for berth size $|B|$ = {5, 10} and number of vessels $|V|$ = {10, 15, 20, 25, 30, 35, 40, 45, 50} with varying parameters of S_j (i.e., $fr = \{1/2, 3/5, 5/8, 7/8\}$). For each combination of $|V|$ and S_j, three instances are generated. Therefore, a total of 216 problem instances are thus generated in the first set.

The second set of generated problem instances is now described. Availability times of the berths (S_j), arrival times of the vessels (A_i) and handling times of the vessels $(t_{i,j})$ are integer numbers generated from a uniform distribution in the intervals [0, 10], [10, 20] and [1, 11], respectively, for berths and vessels. All other input values are generated as in the case of the first set of instances. The second set of instances contains the generated problem instances for berth size $|B|$ = 20 and number of vessels $|V|$ = {20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150}. For each $|V|$, five instances are generated. Therefore a total of 70 instances are thus generated. All the details of problem instances generated for this study are given in the link <http://dx.doi.org/10.17632/z4sh59w8sw.2>.

All the proposed MILP models and the benchmark model (M4) are implemented using IBM Cplex 12.7.1 with default settings, and on a system with 64-bit Intel i5 3.2 GHz Processor and 8.0 GB of RAM. A time limit of 3600 s is imposed for running the MILP models, similar to most previous researchers. The randomly generated problem instances in the first set are tested with all the MILP models to study the computational efficiency of the proposed formulations. Further, the randomly generated problem instances in the second set are tested with MCBAP1 to study the computational efficiency/robustness of the proposed model MCBAP1 in large-sized problem instances.

6.2 *Computational performance evaluation of MILP models*

The performance analysis of the proposed formulations (M1, M2 and M3) and the existing formulation by Hansen

Table 2. Computational performance of MILP models (B = 5).

		Average optimality gap[a] (%)				Average CPU time[b] (s)				Number of optimal solutions[c]			
$\|B\|\times\|V\|$	IG	M1	M2	M3	M4	M1	M2	M3	M4	M1	M2	M3	M4
5×10	1	0	0	0	9.82	1.18	7.74	32.09	1628	3	3	3	1
	2	0	0	0	18.73	0.94	7.33	20.35	–	3	3	3	0
	3	0	0	0	34.41	1.41	20.39	45.60	2730	3	3	3	1
	4	0	0	0	35.39	5.72	89.31	126.69	–	3	3	3	0
5×15	1	0	3.05	10.87	100	623.1	2201	–	–	3	1	0	0
	2	2.26	5.68	9.43	100	151.1	945.6	–	–	2	2	0	0
	3	2.65	14.94	12.44	100	647.1	–	–	–	1	0	0	0
	4	6.79	31.93	12.57	100	–	–	–	–	0	0	0	0
5×20	1	10.08	18.58	16.85	100	–	–	–	–	0	0	0	0
	2	11.12	35.75	16.86	100	–	–	–	–	0	0	0	0
	3	9.53	32.07	14.91	100	–	–	–	–	0	0	0	0
	4	11.67	54.53	15.61	100	–	–	–	–	0	0	0	0
5×25	1	15.46	37.78	20.44	100	–	–	–	–	0	0	0	0
	2	15.99	48.24	20.02	100	–	–	–	–	0	0	0	0
	3	15.04	49.46	18.88	100	–	–	–	–	0	0	0	0
	4	16.87	68.38	19.54	100	–	–	–	–	0	0	0	0
5×30	1	16.46	44.49	20.15	100	–	–	–	–	0	0	0	0
	2	17.94	54.00	21.28	100	–	–	–	–	0	0	0	0
	3	19.77	57.15	22.52	100	–	–	–	–	0	0	0	0
	4	18.75	72.97	20.69	100	–	–	–	–	0	0	0	0
5×35	1	20.47	52.33	24.09	100	–	–	–	–	0	0	0	0
	2	20.30	60.13	22.12	100	–	–	–	–	0	0	0	0
	3	22.38	63.25	25.21	100	–	–	–	–	0	0	0	0
	4	19.51	77.08	21.16	100	–	–	–	–	0	0	0	0
5×40	1	23.24	63.19	25.64	100	–	–	–	–	0	0	0	0
	2	23.79	67.79	25.38	100	–	–	–	–	0	0	0	0
	3	23.66	69.37	25.32	100	–	–	–	–	0	0	0	0
	4	22.85	81.58	23.81	100	–	–	–	–	0	0	0	0
5×45	1	23.42	62.08	24.59	100	–	–	–	–	0	0	0	0
	2	26.71	71.34	28.45	100	–	–	–	–	0	0	0	0
	3	26.78	71.65	28.17	100	–	–	–	–	0	0	0	0
	4	23.44	81.25	24.43	100	–	–	–	–	0	0	0	0
5×50	1	25.91	68.15	27.71	100	–	–	–	–	0	0	0	0
	2	26.64	74.58	28.38	100	–	–	–	–	0	0	0	0
	3	27.38	74.69	29.03	100	–	–	–	–	0	0	0	0
	4	24.06	84.47	25.09	100	–	–	–	–	0	0	0	0

[a]Optimality gap (%) is computed as 100 × (objective function value – current MILP best bound)/ objective function value, and is reported by the solver during termination.

[b]Average CPU time taken for problem instances where optimum solutions have been reached within 3600 s.

[c]Number of problem instances solved optimally within 3600 s in a set of 3 problem instances with similar parameters.

et al [5], referred to as M4 in this paper, for the MCBAP is presented in this section. For each problem instance and for each MILP formulation, the objective function value and the computational time within which the optimal solution is obtained and the optimality gap for the instances that could not be solved within 3600 s are observed. The formulations are compared based on the average computational time taken to solve the problems optimally, based on the average percentage optimality gap and based on the number of test instances solved to optimality within the given time limit of 3600 s for the problem instances with similar parameters (i.e., for the given set of $|B|$, $|V|$ and fr in tables 2 and 3 and for the given set of $|B|$ and $|V|$ in table 4). The experimental results are compiled and presented in tables 2–4. Table 3 in this paper follows the same settings as in table 3 of Hansen *et al* [5], used for the heuristics evaluation in that paper. Similarly table 4 follows similar settings as in table 4 of the paper just cited. In tables 2–4, the average CPU time is shown as '–' when none of the problem instances converges to optimality in the given time limit of 3600 s. With respect to tables 2 and 3, when, for less than 3 problem instances, the optimal solution has been obtained, the average optimality gap over 3 problem instances (when executed up to 3600 s) is computed and reported. The CPU times corresponding to problem instances that attained optimal solutions (within the time limit of 3600 s) are averaged and

Table 3. Computational performance of MILP models (B = 10).

		Average optimality gap (%)				Average CPU time (s)				Number of optimal solutions			
$\lvert B\rvert \times \lvert V\rvert$	IG[a]	M1	M2	M3	M4	M1	M2	M3	M4	M1	M2	M3	M4
10×10	1	0	0	0	12.53	0.39	5.60	4.22	–	3	3	3	0
	2	0	0	0	9.52	0.50	3.92	1.45	2259	3	3	3	1
	3	0	0	0	17.01	0.33	6.78	2.51	–	3	3	3	0
	4	0	0	0	26.94	0.43	9.31	0.70	–	3	3	3	0
10×15	1	0	0	1.28	77.44	6.09	487	1406	–	3	3	2	0
	2	0	0	3.26	82.15	16.12	1164	–	–	3	3	0	0
	3	0	2.90	3.07	93.42	23.81	602	2184	–	3	1	1	0
	4	0	7.22	3.55	93.05	34.23	–	–	–	3	0	0	0
10×20	1	0	4.51	5.21	100	866	509	560	–	3	1	1	0
	2	0	13.17	5.05	100	98.61	–	–	–	3	0	0	0
	3	0.34	17.86	6.14	100	310	–	–	–	2	0	0	0
	4	4.93	39.31	9.38	100	–	–	–	–	0	0	0	0
10×25	1	6.80	19.93	11.60	100	–	–	–	–	0	0	0	0
	2	7.33	28.93	12.13	100	–	–	–	–	0	0	0	0
	3	6.79	30.06	11.73	100	–	–	–	–	0	0	0	0
	4	9.82	51.57	14.01	100	–	–	–	–	0	0	0	0
10×30	1	7.93	30.31	11.72	100	–	–	–	–	0	0	0	0
	2	6.99	35.54	10.79	100	–	–	–	–	0	0	0	0
	3	7.73	37.77	11.34	100	–	–	–	–	0	0	0	0
	4	10.34	59.25	13.80	100	–	–	–	–	0	0	0	0
10×35	1	12.92	37.93	15.92	100	–	–	–	–	0	0	0	0
	2	12.96	44.21	16.44	100	–	–	–	–	0	0	0	0
	3	15.35	48.83	18.13	100	–	–	–	–	0	0	0	0
	4	14.26	64.07	17.19	100	–	–	–	–	0	0	0	0
10×40	1	11.88	45.07	15.13	100	–	–	–	–	0	0	0	0
	2	12.84	53.46	15.30	100	–	–	–	–	0	0	0	0
	3	14.72	51.73	16.88	100	–	–	–	–	0	0	0	0
	4	15.06	68.03	17.11	100	–	–	–	–	0	0	0	0
10×45	1	15.35	49.98	18.23	100	–	–	–	–	0	0	0	0
	2	17.91	58.66	21.29	100	–	–	–	–	0	0	0	0
	3	18.68	59.03	21.49	100	–	–	–	–	0	0	0	0
	4	17.29	70.90	18.70	100	–	–	–	–	0	0	0	0
10×50	1	15.94	56.32	19.00	100	–	–	–	–	0	0	0	0
	2	16.65	63.02	20.02	100	–	–	–	–	0	0	0	0
	3	18.29	63.30	21.53	100	–	–	–	–	0	0	0	0
	4	18.42	75.62	20.13	100	–	–	–	–	0	0	0	0

[a]IG denotes instance group (see section 6.1).

reported. Similarly, in table 4, 5 problem instances are considered in each row for the computation of average optimality gap and average CPU time.

In the first set of instances (i.e., $|B| = \{5, 10\}$) for number of vessels $|V| = 10$, all the proposed models are capable of finding the optimum solution in all the test instances within a few seconds, where M4 is able to find optimality in not more than 2 out of 12 test instances. When the problem size is increased (i.e., $|V|$ =15), all the proposed models have either attained optimality or been terminated with a small optimality gap after the given execution time limit, where M4 started showing large optimality gap in all the test instances. For other test instances with the number of vessels more than 20, all the proposed models maintain their computational performance with less optimality gap, compared with the existing model (M4). Within the proposed models it is observed that M1 always shows better computational performance than those by M2 and M3. In addition to the size of vessels, the parameter *fr* also affects the computational performance of all the models. Lower the value of the fraction (*fr*), test instances are relatively easy to solve for all models (see tables 2 and 3).

In the second set of instances (i.e., $|B|$=20) for number of vessels $|V|$ = 20, 30, 40, 50, 60 70, 80 and 90 the proposed model MCBAP1 is capable of finding the optimum solution in all the test instances within the given time for execution, where M4 is unable to reach optimality in any of the test instances. When the problem size is increased (i.e., $|V|$ =100 and 110), the proposed model M1 has either attained optimality or been terminated with a small optimality gap.

Table 4. Computational performance of MILP models (B = 20).

	Optimality gap (%)		Average CPU time (s)		Number of optimal solutions[a]	
$\lvert B\rvert \times \lvert V\rvert$	M1	M4	M1	M4	M1	M4
20×20	0	100	0.34	–	5	0
20×30	0	100	0.66	–	5	0
20×40	0	100	1.72	–	5	0
20×50	0	100	14.32	–	5	0
20×60	0	100	45.31	–	5	0
20×70	0	100	70.53	–	5	0
20×80	0	100	231	–	5	0
20×90	0	100	494	–	5	0
20×100	1.22	100	1073	–	2	0
20×110	5.89	100	–	–	0	0
20×120	9.17	100	–	–	0	0
20×130	14.16	100	–	–	0	0
20×140	17.04	100	–	–	0	0
20×150	24.2	100	–	–	0	0

[a]Number of problem instances solved optimally within 3600 s in a set of 5 problem instances with similar parameters.

For other test instances with number of vessels more than 110, the proposed model M1 maintains its superior computational performance with less optimality gap.

6.3 *Overall findings*

- Our experimental findings reveal the robustness of the computational performance of the proposed model, MCBAP1, in the BAP under study. The MCBAP1 model maintains its superiority of computational efficiency in all problem sizes, especially in relation to the existing formulation.
- The MCBAP2 and MCBAP3 perform better than the existing formulation in all the test instances.
- Problem instances generated with higher values of fr are found to be difficult to solve for all models, resulting in large computational times or large optimality gaps, due to relatively tight due-date settings.
- Following the studies of Hansen *et al* [5], two different sets of data are generated with different characteristics. In both sets our proposed model MCBAP1 maintains its computational efficiency. The second set of instances is found to yield optimal solutions for more problem instances compared with the first set of instances because of the relatively larger values of $Slack_i/max_j\{t_{ij}\}$ associated with each vessel, where $Slack_i = d_i - max\{A_i, min_j\{S_j\}\}$.

7. Managerial insights and implications

The proposed models can be used to obtain optimal solutions/lower bounds on the optimal solutions, which can be used to evaluate the quality of heuristic solutions for the medium-sized/ large-sized problems. It is to be noted that our proposed MILP models M1 and M3 yield solutions with an optimality gap of less than 30% in worst case and often much better, implying that we can obtain lower bounds on optimal solutions even for the large-sized problems reported in tables 2–4. Such lower bounds can be used for evaluating heuristic solutions for the large-sized problem instances in an absolute manner. It is also noteworthy that the existing model [5] is associated with an optimality gap of 100% in most of the problem instances. Moreover, the proposed models yield upper bounds observed by the optimality gap of less than 100%. Such upper bound solutions, reported by the solver using the proposed MILP models, can be used to heuristically schedule vessels in large-sized real-life container terminals.

8. Conclusions

In this paper, three new MILP models are formulated by considering the problem of dynamic berth allocation with minimum cost objective (called MCBAP) in maritime logistics. The computational performance of the proposed MILP formulations is measured in terms of computational time required to attain optimality, optimality gap and the number of optimum solutions obtained within the given time limit, and compared to the existing formulation. Exhaustive computational experiments have been conducted to analyse the effect of problem size on the performance of MILP models. All the proposed models perform better than the benchmark model. The comparison of the proposed MILP models based on the experimental results indicates that the proposed formulation, MCBAP1, performs the best and is robust with increase in number of vessels as well as increase in number of berths with huge savings in computational cost. Using this formulation, we can achieve optimality or near-optimality in most of the problem instances, within the time limit of 3600 s. Future work can look at the development of efficient lower bounds in order to evaluate heuristic solutions in the large-sized problem instances.

Acknowledgement

The authors are thankful to the reviewer and the Editor for their constructive suggestions and comments to improve the initial version of our manuscript.

References

[1] UNCTAD (United Nations Conference on Trade and Development) 2018 *Review of maritime transport 2018*

[2] Wang F and Lim A 2007 A stochastic beam search for the berth allocation problem. *Decis. Support Syst.* 42: 2186–2196

[3] Bierwirth C and Meisel F 2010 A survey of berth allocation and quay crane scheduling problems in container terminals. *Eur. J. Oper. Res.* 202: 615–627

[4] Bierwirth C and Meisel F 2015 A follow-up survey of berth allocation and quay crane scheduling problems in container terminals. *Eur. J. Oper. Res.* 244: 675–689

[5] Hansen P, Oğuz C and Mladenović N 2008 Variable neighborhood search for minimum cost berth allocation. *Eur. J. Oper. Res.* 191: 636–649

[6] Imai A, Nishimura E and Papadimitriou S 2001 The dynamic berth allocation problem for a container port. *Transp. Res. Part B* 35: 401–417

[7] Imai A, Nagaiwa K and Tat C W 1997 Efficient planning of berth allocation for container terminals in Asia. *J. Adv. Transp.* 31: 75–94

[8] Imai A, Nishimura E and Papadimitriou S 2003 Berth allocation with service priority. *Transp. Res. Part B* 37: 437–457

[9] Cordeau J F, Laporte G, Legato P and Moccia L 2005 Models and tabu search heuristics for the berth-allocation problem. *Transp. Sci.* 39: 526–538

[10] Liang C, Huang Y and Yang Y 2009 A quay crane dynamic scheduling problem by hybrid evolutionary algorithm for berth allocation planning. *Comput. Ind. Eng.* 56: 1021–1028

[11] Kovač N, Stanimirović Z and Davidović T 2018 Metaheuristic approaches for the minimum cost hybrid berth allocation problem. In: Konstantopoulos C and Pantziou G (Eds.) *Modeling, Computing and Data Handling Methodologies for Maritime Transportation*. Cham: Springer, pp. 1–47

[12] Lim A 1998 The berth planning problem. *Oper. Res. Lett.* 22: 105–110

[13] Golias M M, Boile M and Theofanis S 2010 A lamda-optimal based heuristic for the berth scheduling problem. *Transp. Res. Part C: Emerg. Technol.* 18: 794–806

[14] Kovač N 2017 Metaheuristic approaches for the berth allocation problem. *Yugosl. J. Oper. Res.* 27: 265–289

[15] Park Y and Kim K H 2003 A scheduling method for berth and quay cranes. *OR Spectr.* 25: 1–23

[16] Imai A 2008 The simultaneous berth and quay crane allocation problem. *Transp. Res. Part E* 44: 900–920

[17] Ernst A T, Oğuz C, Singh G and Taherkhani G 2017 Mathematical models for the berth allocation problem in dry bulk terminals. *J. Sched.* 20: 459–473

[18] Pratap S, Nayak A, Kumar A, Cheikhrouhou N and Tiwari M K 2017 An integrated decision support system for berth and ship unloader allocation in bulk material handling port. *Comput. Ind. Eng.* 106: 386–399

[19] Anoop K P, Panicker V V, Narayanan M and Kumar C T S 2018 A mathematical model and solution methods for rail freight transportation planning in an Indian food grain supply chain. *Sādhanā* 43: 1–20

[20] Madankumar S and Rajendran C 2019 A mixed integer linear programming model for the vehicle routing problem with simultaneous delivery and pickup by heterogeneous vehicles, and constrained by time windows. *Sādhanā* 44: 1–14

[21] Hansen P and Oguz C 2003 *A note on formulations of static and dynamic berth allocation problems*. Les Cah du GERAD G-2003-30 HEC Montréal