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# Minimum Cost Design of Water Distribution Systems

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# Research Report No. 62

# MINIMUM COST DESIGN OF WATER DISTRIBUTION SYSTEMS

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# University of Kentucky Water Resources Institute Lexington, Kentucky

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# ABSTRACT

The objective of this study was to develop the analytical tools and procedures for minimum cost design of water distribution systems. Both analog and digital means of carrying out pressure and flow calculations were developed. As a result of this effort, digital programs for pressure and flow calculations in water distribution systems were written and have been widely distributed to practicing engineers. One procedure is based on a direct solution of the basic system equations using a linearization scheme and has several advantages over conventional techniques such as the Hardy Cross method. These include avoiding the need to initially balance the network and an assured convergence of the procedure.

Using this tool a procedure was developed for selecting pipe diameter which will result in a minimum cost design within the prescribed constraints. The method of steepest ascent and dynamic programming concepts were used to carry out the optimization. This procedure applies to closed loop systems without internal pumping. However, this work provides a basis for extending the concepts to more generalized water distribution systems.

KEY WORDS: water distribution, optimization, piping systems, network design, ecommic efficiency

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#### INTRODUCTION

The objective of the study as outlined in the original proposal was, "to investigate the various functional constraints controlling the design of water distribution systems and develop analytical methods and digital computer routines which can be utilized to design a water distribution system at minimum cost". It was determined early in this investigation that the available means for the hydraulic analysis of water distribution systems did not lend themselves well to a minimum cost analysis. Therefore, a considerable effort was made to develop the analytical tools for pressure and flow calculations which could be incorporated into a minimum cost analysis of water distribution systems.

A promising technique which was investigated was the use of analog simulation for system hydraulics which could be incorporated into a digital-analog scheme to carry out the cost minimization. Techniques for carrying out an analog simulation of pipe system hydraulics on a standard analog computer were developed and reported by the principal investigator (1). It was intended to use the analog computer to model the hydraulics and to use analog to digital conversion and a digital computer to compute the cost. It was felt that with such a model an effective directed search could be undertaken to determine the optimum design of the pipe system. However, it turned out that the necessary equipment for the development of this concept was not available to this project. Since some rather expensive equipment is involved this approach had to be abandoned.

A major effort was devoted to the development of an analytical procedure for hydraulic analysis based on linearization of the basic non-linear system equations. The purpose of developing this approach was that it appeared to offer a method for handling pipe system hydraulics which could be more easily incorporated into a minimum cost study. A scheme which directly solved the basic equation after linearizing the non-linear terms was developed. A publication is available describing this phase of the study (2). This method of hydraulic analysis refereed to as the linear method offered distinct advantages over the conventional Hardy Cross and Newton Raphson methods which are generally used. Therefore, some additional effort was made developing the linear method for generalized situations. As a result of this, a general computer program has been developed and made available to engineers working in this field. Over one hundred and fifty engineering firms have acquired this program.

Finally, using the linear method for hydraulic analysis a computer program has been developed for minimum cost design of closed loop water distribution systems. This work was primarily the effort of C. O. Charles and is documented in his Ph. D. dissertation (3). In this program the method of steepest ascent and concepts of dynamic programming were employed to formulate a procedure which would select the optimum set of pipe diameters for a closed loop system.

#### RESEARCH PROCEDURES

The entire project is concerned with the development of the basic elements of an analytical model to be employed for minimum cost design. This entails the conception, formulation and testing of certain analytical procedures. In most cases either analog or digital computer programs were the end product of this effort. The usual procedures for formulating, debugging and testing computer programs were employed.

# RESULTS

Analog simulation of pipe system hydraulics

This phase of the investigation has been completely documented in Reference 1 and is available through that publication. As previously stated, however, the necessary digital-analog equipment was not available to develop a technique for optimum design using analog simulation.

# Digital programs for the analysis of pipe system hydraulics

A considerable effort was made to develop a digital computer program which would easily handle general water distribution systems in a manner which would lend itself to a minimum cost investigation. This effort resulted in the development of two computer programs. General information pertaining to the development of these programs follows:

The programs will compute steady flow in pipe systems of any arrangement. The system can include pumps, valves, bends, and other minor loss components, storage tanks and source and storage reservoirs. A system of p pipes can be described by the number of junctions, j, the number of closed primary loops, l, and the number of terminal energy points, t, in the system. A junction is simply a point in the system where two or more pipes meet. Any point where flow enters or exits the pipe system is also a junction. A primary loop is a closed loop of pipes in the system which have no other loops within it. A terminal energy point is a point in the system where the fluid energy is known. This is essentially any point where the pressure and the elevation are known. Source or storage reservoirs, pressurized sources, storage tanks and discharge points of known pressure are the most common terminal

energy points. To describe a system the junctions, loops and terminal energy points must be identified. If a terminal energy point and a junction coincide, this point should be identified as a terminal energy point only. If the junctions, loops and terminal energy points are identified with the restriction just stated, the following holds for all pipe systems:

$$p = j + l + t - l$$

(1)

where

p = number of pipes

j = number of junctions

 $\mathcal{L}$  = number of loops

t = number of terminal energy points.

In terms of the unknown discharge in each pipe, a number of continuity and energy equations can be written equaling the number of pipes in the system. For each junction a continuity equation equating the flow into the junction to the flow out is written as:

$$Q_{in} = Q_{out}$$
 (j equations) (2)

For each loop the energy equation can be written as follows

 $\Sigma h_{L} = \Sigma E_{P}$  (*l* equations) (3)

where

 $h_L$  = head loss in each pipe (including minor loss)  $E_P$  = energy put into the liquid by a pump.

If there are no pumps in the loop then the energy equation states that the sum of the head loss around the loop equals zero.

If there are t terminal energy points, t - l energy equations can be written for paths between any two terminal energy points as follows

# $\Delta E = \Sigma h_{L} - \Sigma E_{P} \qquad (t - 1 \text{ equations}) \tag{4}$

where  $\Delta E$  is the energy difference between the two terminal energy points. Any path in the pipe system can be chosen between the points. However, care must be taken to avoid redundant paths. The best method to avoid this difficulty is to either choose all paths starting at one source (like 1-2, 1-3, 1-4, etc.) or to use the previous end point for a path as the starting point for the next path (like 1-2, 2-3, 3-4, etc.). Either of these methods will result in t - 1 equations with no redundant ones.

These junction loop and path equations constitute a set of simultaneous equations equal to the number of pipes in the system which can be solved for the discharge in each pipe. A direct solution of these simultaneous equations is not possible because of the non-linear terms. Two basic methods of solution were considered.

Linear method - For this approach the non-linear terms are linearized giving a set of linear simultaneous equations which can be solved using matrix methods. The linearization is formulated as follows. The line loss is given by:

$$h_{LP} = K_{P}Q^{n}$$
 (5)

where  $K_P$  is a pipe line constant and for the Hazen Williams equation employed in the computer analysis is

$$K_{P} = \frac{4.73 \text{ L}}{\text{C}^{1.852}\text{D}^{4.87}}$$
(6)

Here L = line length in ft, D = line diameter in ft and C is the Hazen Williams roughness coefficient. The discharge Q in eqn. 5 is in cfs and the exponent n = 1.852.

Minor losses are given by a loss coefficient, M, which multiplies the velocity head to give the loss at the component.

This is

$$h_{LM} = M \frac{V^2}{2g}$$
(7)

where V is the mean line velocity and g is the gravitational constant. In terms of the discharge this is

$$h_{LM} = K_M Q^2$$
(8)

where

$$K_{\rm M} = \frac{.02517 \,\,{\rm M}}{{\rm D}^4} \tag{9}$$

The pump head is expressed in two ways.

$$E_{P} = \frac{Z_{P}}{Q}$$
(10)

For this expression the horsepower put into the system by the pump is given as HP and

$$Z_{\rm P} = \frac{550 \text{ HP}}{v} \tag{11}$$

where  $\gamma$  = specific weight of the liquid (#/ft<sup>3</sup>). Alternately the pump head can be expressed as

 $E_{\rm P} = A + BQ + CQ^2 \tag{12}$ 

where A, B, and C are coefficients of a parabolic characteristic curve which defines the pump operation in the vicinity of the operating point. Since this expression is only valid over a specified range it should not be indiscretely employed in an analysis.

The basic energy equation for a loop or a path between terminal energy points is:

$$\Sigma(h_{LP} + h_{LM}) = \Delta E + \Sigma E_{P}$$
(13)

Here  $\Delta E$  is the energy difference between the terminal energy points. This equation can be linearized in terms of a flowrate  $Q_i$  in the vicinity of the solution. This is done as follows

$$h_{LP} = h_{LPi} + \Delta h_{LP} = K_P Q_i^n + n K_P Q_i^{n-1} (Q-Q_i)$$
(14)

$$h_{LM} = h_{LMi} + \Delta h_{LM} = K_M Q_i^2 + 2K_M Q_i (Q_i - Q)$$
(15)

$$E_{\dot{P}} = E_{\dot{P}i} + \Delta E_{P} = \frac{Z_{P}}{Q_{i}} - \frac{Z_{P}}{Q_{j}^{2}} (Q - Q_{i})$$
 (16)

or:

$$E_{P} = A + BQ_{i} + CQ_{i}^{2} + (B + 2CQ_{i})(Q - Q_{i})$$
 (17)

With these substitutions eqn. 13 can be expressed as a linear function of Q as

$$\Sigma (n K_{P}Q_{i}^{n-1} + 2K_{M}Q_{i} + \frac{Z_{P}}{Q_{i}^{2}}) Q =$$

$$\Sigma \left(\frac{2Z_{P}}{Q_{i}} + (n-1) K_{P}Q_{i}^{n} + K_{M}Q_{i}^{2}\right) + \Delta E \qquad (18)$$

For the alternate form of the pump head this equation is

$$\Sigma(\mathbf{n} \mathbf{K}_{\mathbf{P}} \mathbf{Q}_{\mathbf{i}}^{\mathbf{n}-1} + 2\mathbf{K}_{\mathbf{M}} \mathbf{Q}_{\mathbf{i}} - \mathbf{B} - 2\mathbf{C}\mathbf{Q}_{\mathbf{i}})\mathbf{Q} =$$
  
$$\Sigma(\mathbf{A} - \mathbf{C} \mathbf{Q}_{\mathbf{i}}^{2} + (\mathbf{n}-1)\mathbf{K}_{\mathbf{P}} \mathbf{Q}_{\mathbf{i}}^{\mathbf{n}} + \mathbf{K}_{\mathbf{M}} \mathbf{Q}_{\mathbf{i}}^{2}) + \Delta \mathbf{E}$$
(19)

Equation 18 (or 19) is employed to formulate an equation for each loop ( $\Delta E = 0$ ) and t - 1 terminal energy equations which combine with

the j continuity equations to for a set of P simultaneous linear equations in terms of the flowrate in each pipe.

<u>Path method</u> - The same notation previously defined in the description of the linear method is used. The basis of this method is to compute a flow correction  $\triangle Q$  which when added to an initial set of flowrates (which satisfy continuity) will tend to satisfy the energy equation for each path. This is

$$\Sigma(h_{LP} + h_{LM}) = \Delta E + \Sigma E_{P}$$

In terms of the initial flowrate  $\mathbf{Q}_i$  and the flow correction  ${}^{\vartriangle}\mathbf{Q}$  these terms are

$$\mathbf{h}_{\mathrm{LP}} = \mathbf{h}_{\mathrm{LPi}} + \Delta \mathbf{h}_{\mathrm{LP}} = \mathbf{K}_{\mathrm{P}} \mathbf{Q}_{\mathrm{i}}^{n} + \mathbf{n} \mathbf{K}_{\mathrm{P}} \mathbf{Q}_{\mathrm{i}}^{n-1} \Delta \mathbf{Q}$$
(20)

$$h_{LM} = h_{LMi} + \Delta h_{LM} = K_M Q_i^2 + 2K_M Q_i \Delta Q$$
(21)

$$E_{P} = E_{Pi} + \Delta E_{P} = \frac{Z_{P}}{Q_{i}} - \frac{Z_{P}}{Q_{i}^{2}} \Delta Q \qquad (22)$$

 $\mathbf{or}$ 

$$E_{P} = A + BQ_{i} + CQ_{i}^{2} + (B - 2CQ_{i}) \Delta Q$$
 (23)

These can be solved to give a flow correction as

$$\Delta Q = \frac{\Delta E - \Sigma (K_P Q_i^n + K_M Q_i^2 - \frac{Z_P}{Q_i})}{\Sigma (nK_P Q_i^{n-1} + 2K_M Q_i + \frac{Z_P}{Q_i^2})}$$
(24)

 $\mathbf{or}$ 

$$\Delta Q = \frac{\Delta E - \Sigma (K_{P}Q_{i}^{n} + K_{M}Q_{i}^{2} - (A + BQ_{i} + CQ_{i}^{2}))}{\Sigma (nK_{P}Q_{i}^{n-1} + 2K_{M}Q_{i} - B - 2CQ_{i})}$$
(25)

Using either eqn. 24 or eqn. 25 a flow correction is computed for each path and the flowrates of the pipes in that path are corrected by this amount.

For each method the following information must be available before the hydraulic analysis can be made. For each line the length, diameter and the Hazen Williams Roughness coefficient must be known. This latter parameter is available in handbooks and depends on the type and condition of the pipe. Valves, bends, meters, etc. are included in the analysis by determining the minor loss coefficient for the components. The minor loss coefficient is defined as a constant which multiplies the velocity head in the line to give the head loss at that component. In many cases a standard value for this coefficient is given in various references. This coefficient can also be easily determined if discharge-head loss data is available for the component. Several components can be included in a line by summing their minor loss coefficients.

Pumps can be included in two ways. The useful horsepower (or kilowatts) which the pump puts into the system may be specified. Alternately the coefficients of a parabolic characteristic curve may be specified. This curve represents the pump headdischarge relationship as shown below.



In the normal range of pump operation this relationship can be described closely by

$$E_P = A + BQ + CQ^2$$

where A, B, and C are coefficients of the fitted curve. If this representation of the pump is used, however, the solution must yield a discharge in the normal range of operation or the solution will be invalid. This is because the characteristic curve is not valid outside that range.

For each junction the external inflow or outflow is specified and the elevation of the junction is known.

<u>Program description and users options</u> - The programs were developed for use by practicing engineers and were offered to engineers on several bases. Material on the programs has been provided to over 150 engineering firms and individuals. The following brief release provided information to potential users:

Two programs have been developed at the University of Kentucky which will analyze pressure and flow in any pipe system and are available to potential users. These programs are written in FORTRAN IV, G Level and a users guide with source program listings and examples has been prepared. A brief description of the programs follow.

# I Program based on linearized system equations -

This program utilized a new procedure for pipe systems analysis which has several advantages over conventional methods. Because this method simultaneously computes the flow in each pipe, the convergence is very fast (usually 3-4 trials to very high accuracy regardless of size of the system). Also, convergence is assured. Initial flowrates are not assumed and changes in flow system demand only require a change in data pertaining to that demand. However, since matrix methods are employed a computer of sufficient storage must be available to use this method for a system of n pipes approximately (n x (n+35) dimensioned storage locations must be available. The IBM 360-65 computer at the University of Kentucky, for example, will handle systems up to 220 pipes with its present storage capacity and without using additional disc storage. The procedure is fast. A 37 pipe system can be analyzed in about 10 seconds while a 125 pipe system takes 2 minutes and 15 seconds on the University of Kentucky computer.

II Program based on loop and path flow adjustments -

This program is essentially an extension of a loop balancing method similar to the Hardy Cross technique to any type of pipe flow system with pumps, valves, etc. included. It does require as input data initial flowrates which satisfy continuity. In rare cases the Hardy Cross procedure does not produce convergence and this situation could occur. However, for most situations the program produces a fast accurate solution. In addition much less storage is required so a large system can be analyzed with a computer of limited storage capacity.

Basic features of both procedures are:

1. Any piping configurations can be analyzed (closed loop networks, tree systems of combinations).

Flow units of CFS, GPM, MGD or SI units (M<sup>3</sup>/s) can be used.
 Pump, valves and other lossy components, and storage tanks can be included in the pipe system.

4. Pressures and hydraulic grades at indicated points in the system are output in addition to head changes at pumps, valves and in lines.

5. Data preparation for both programs is straightforward and very similar and allows any number of changes in system parameters (pipe sizes, pump characteristics, flow demands, etc.) to be investigated in a single computer run.

The programs are available to interested users in one of the following ways:

1. <u>Attend two day short course at the University of Kentucky</u> announcement attached. This is the best means of gaining the necessary experience for using the programs effectively and is especially recommended for persons not presently using computers for hydraulic analysis. All material and computer source programs are provided for participant. Some post-course consultation and

a post-course laboratory problem chosen by the participant provide additional aid in implementing the programs.

Participate in users course on a correspondence basis. 2. This is primarily for users who have an interest in developing the capability of using the programs but cannot attend a short course. It is desirable that the pariticipant have some background in the use of computers. All material and source programs are provided for the participant in addition to problems. The data for these problems are coded and returned for computer processing. In addition the participant may code and submit data for an additional problem over a period of a year which is of interest to him. Systems of up to 50 pipes will be processed as part of the course (larger systems require a nominal additional charge for computing expenses). Consultation regarding the application of the programs to pipe systems will be provided by phone or mail.

3. Obtain material only. This is primarily for users who are already using a digital computer for hydraulics problems. Users guides, program listings and examples will be sent for both programs.

Complete details of the programs, program listings and examples are provided in the users manuals (4, 5).

Program for optimum design of water distribution networks.

A major effort to develop a programmable procedure for minimum cost design was made. Details of this effort are included in a Ph. D. thesis (3). The salient points of this effort will be covered in this report.

Problem definition - In designing a hydraulic network distribution system, the engineer has not only to meet the demands at particular points in the system, but also should do so within specified constraints and at the least possible cost. For this study the geometrical configuration of the network is prescribed. The cost is a function of diameter and flow and the constraints can be

regarded as of three types: (i) hydraulic (Kirchhoff laws), (ii) pressure and (iii) diameter. There are also different classes of constraints within each type. For example some pressure constraints are of the type that the pressure must be greater than or equal to a minimum while another constraint is that the pressure must not exceed some maximum value. The diameter constraints are normally of two kinds; the first is that no diameter should be less than a certain minimum, the other that the diameters should be available on the market. This becomes necessary since pipe diameters are made commercially in certain discrete sizes. The problem is therefore to find a set of pipe diameters to satisfy all the constraints at the least cost. The method used to do this is a combination of steepest descent (ascent) and dynamic programming.

Since the cost function involves flow in pipes, it is necessary to calculate the flow quickly. Flow is also important in the calculation of pressure since pressure is a function of flow. To compute flow quickly the method of linear analysis which was developed for this purpose is employed.

<u>Problem formulation</u> - Any problem of optimization has essentially two characteristics (1) a cost function and (2) one or more constraints. For the hydraulic network these are described as follows:

(1) <u>Cost function</u> -- The cost function used for network optimum design is divided into two parts: (a) Capital and (b) operation and maintenance costs. For the capital cost the result of the regression analysis performed by Linaweaver and Clark is used. This analysis was carried out on pipe line data for oil, gas and water pipe lines and gives a relationship between the variables, diameter, D, in inches and the capital cost, in dollars, per mile. This relationship is given by

Capital Cost = 1890 D<sup>1.29</sup> per mile

or

Capital Cost =  $0.358^{1.29}$  per foot

The correlation coefficient is 0.98 according to the article.

At the time of this survey the Engineering News Record Construction Cost Index (ENRI) was 877. This enables the capital cost relationship to be updated by the ratio, PRESENT ENRI/877. The procedure described is used herein for the capital cost portion of the cost analysis. Alternate schemes could be used to express the capital cost as some continuous function of pipe diameter.

The capital expenditure is usually incurred at the time of construction of the project, and is paid back over the life of the project. During that time, the value of money is determined by the rate of interest, i% per annum. In order to spread the capital cost evenly over the whole life of the project, it is necessary to multiply the initial cost by a capital recovery factor (crf) where:

$$\operatorname{crf} = \left[\frac{i}{100} \left(1 + \frac{i}{100}\right)^{nn}\right] \left[\left(1 + \frac{i}{100}\right)^{nn} - 1\right]$$

where nn = life of project in years.

Thus in a system of m pipes the annual capital cost is

$$\sum_{i=1}^{m} \quad 0.358 L_i D_i^{1.29} (crf) \quad \frac{(PRESENT ENRI)}{877}$$

where  $L_i = length$  in feet of i-th pipe.

The operation and maintenance portion of the cost function is obtained by first equating the pumping power required for each pipe to that of an equivalent number of kilowatt-hours and then multiplying the number of Kilowatt-hours by the corresponding unit cost. The power utilized in a pipe is related to the corresponding head loss. The Hazen-Williams empirical expression for head loss is used. This expression is  $H_L = KQ^{1.8518}$ 

where 
$$K = \frac{4.77 L(12)^{4.87}}{C^{1.8518} D^{4.87}}$$
 (26)

and C = roughness coefficients.

The power lost in the pipe line is related to  ${\rm H}_{\rm L}$  in the following way:

Power loss =  $\frac{H_L Q \gamma}{550}$  horse power (where Q is in cfs and Y is in the specific weight of the liquid in Lb/ft<sup>3</sup>).

Inserting the head loss equation in this expression gives:

Power loss =  $\frac{KQ^{2.8518}}{550}$  (62.4) horse power (assuming water is the liquid).

The final annual cost due to maintenance and operation in a pipe can be expressed as:

$$KQ^{2.8518} \frac{(62.4)}{550}$$
 (0.746)(365 x 24)c per year

where c = unit cost of electricity in \$ per KwH. Thus the total cost function (R) is

R = 
$$\sum_{i=1}^{m} 0.358 L_i D_i^{1.29} (crf) (Present ENRI/877) +$$

$$\sum_{i=1}^{M} K_{i} Q_{i}^{2.8518} \frac{(62.4)}{550} (0.746)(365 \ge 24)c = R$$
(27)

The factor  $(365 \ge 24)$  assumes that the system is in operation for the whole year. If this is not the case, then this factor can be replaced by the anticipated number of hours in the year that the system will be in operation.

(2) <u>Constraints</u> -- The primary constraint is one which requires the flows to obey basic hydraulic relationships involving continuity of flow at junctions and head losses in the individual loops.

Another type of constraint, dealing with pressures, assumes alternate forms. The system must be designed for a maximum value of pressure which must not be exceeded.

There also may be a minimum pressure required for each junction which is necessary to maintain acceptable system performance. An example of such a minimum is that required by fire-fighting activities for which standards have been developed by National Board of Fire Underwriters (NBFU)(33). It is assumed that though there are variations in minimum pressure, there is none in the maximum.

Acceptable pipe diameters are also constrained. NBFU recommends that the diameter of street mains should not be less than 6-inches. This is again a fire-protection provision where the primary consideration is to obtain an acceptable quantity of water. In addition, diameters available on the commercial market are discrete and not continuous. A 6-inch pipe may be available, while 6.25-inch usually is not. It is therefore necessary that the sizes selected for design must be commercially available.

(3) <u>The mathematical model</u> -- The problem is summarized as follows:

The cost function to be minimized is

$$R = \sum_{i=1}^{m} 0.358 L_i D_i^{1.29} \frac{(\text{Present ENRI})}{877} (\text{crf}) +$$

$$\sum_{i=1}^{m} K_{i} Q_{i}^{2.8518} \frac{(62.4)}{550} (0.746)(365 \times 24)c$$

Subject to the following constraints:

- (i) pressure and flow obey basic hydraulic relationships
- (ii) p ≤ ABMAX where p is pressure and ABMAX is absolute maximum pressure
- (iii) p ≥ ABMIN where ABMIN is the absolute minimum pressure allowed
- (iv)  $P_A \ge P_A$  where  $P_A$  is pressure at junction A and  $P_A$  is the minimum allowed at junction A

- (v)  $D \ge DMIN$  where DIMIN is the absolute minimum size diameter allowed
- (vi)  $D \in (d_1, d_2, d_3, \dots, d_n)$  where  $d_1, d_2, \dots, d_n$  are the available commercial size diameters.

<u>Method of Solution</u> The law of continuity is expressed in terms of flow ( $\Sigma Q_i = 0$ ) while the "head loss" or loop equations ( $\Sigma K_i Q_i^{n} = 0$ ) are functions of flow and pipe properties. The Hazen-Williams line loss expression is used herein. Thus from (26)

$$K = \frac{4.77L(12)^{4.87}}{C^{0.8518}D^{4.87}}$$

which is referred to as the loss coefficient. Flow is given by

$$Q = Av$$
 (28)

where A is the cross-sectional area and v the velocity

It can be demonstrated that the differential dQ is given by

$$dQ = \frac{2Q}{D} dD + A dv$$
 (29)

To preserve continuity, the algebraic sum of the changes in flow at any junction must be zero.

Thus at any junction

$$\Sigma dQ_{4} = 0 \tag{30}$$

and from eqn. (29)

$$\Sigma A_i dv_i = -2\Sigma \frac{Q_i}{D_i} dD_i$$
(31)

It is clear that by considering changes in flow at many junctions, equations involving  $A_i dv_i$  and  $dD_i$  can be obtained. It also follows from the continuity equations that one of these equations would be redundant. The system equations can be expressed in the following form:

$$\begin{bmatrix} \text{Matrix coefficients} \\ \text{are 1, 0, -1} \end{bmatrix} \begin{bmatrix} A_1 dv_1 \\ A_2 dv_2 \\ A_3 dv_3 \\ \vdots \\ \vdots \\ A_m dv_m \end{bmatrix} = \begin{bmatrix} \text{Matrix} \\ \text{coefficients} \\ \text{are } -\frac{2Q}{D} \\ \text{or 0} \end{bmatrix} \begin{bmatrix} dD_1 \\ dD_2 \\ dD_3 \\ \vdots \\ \vdots \\ \vdots \\ dD_m \end{bmatrix}$$
(32)

By considering first order derivatives of the loop equations, another set of equations involving  $A_i dv_i$  and  $dD_i$  can be obtained as follows:

$$H_{L} = KQ^{1.8518}$$

is differentiated to obtain

$$dH_{L} = Q^{1.8518} \frac{dK}{dD} dD + 1.8518 KQ^{0.8518} \frac{\partial Q}{\partial D} dD + 1.8518 KQ^{0.8518} \frac{\partial Q}{\partial V} dD$$

$$+ 1.8518 KQ^{0.8518} \frac{\partial Q}{\partial V} dv \qquad (33)$$
From (29)  $\frac{\partial Q}{\partial D} = \frac{2Q}{D}$  and  $\frac{\partial Q}{\delta V} = A$ 

By substituting in (35)

$$dH_{L} = Q^{1.8518} \frac{dK}{dD} dD + 1.8518 KQ^{0.8518} \frac{2Q}{D} dD$$
$$+ 1.8518 KQ^{0.8518} Adv$$
(34)

From (26) 
$$K = \frac{4.77L(12)^{4.87}}{C^{1.8518}D^{4.87}}$$

By differentiation

$$\frac{dK}{dD} = -\frac{4.87}{D^{5.87}} \frac{4.77L(12)^{4.87}}{C^{1.8518}} = -4.87 \frac{K}{D}$$
(35)

Substitute (35) for  $\frac{dK}{dD}$  in (34), then

$$dH_{L} = -4.87 \frac{KQ^{1.8518}}{D} dD + 3.7036 \frac{KQ}{D} dD$$
$$+ 1.8518 KQ^{0.8518} Adv$$
(36)

Simplifying and combining terms give

$$dH_{L} = -1.1664 \frac{KQ^{1.8518}}{D} dD + 1.8518 KQ^{0.8518} Adv$$
 (37)

These equations of the form  $~^{\Sigma}dH_{L}$  = 0 can be transformed into

1.8518 
$$\sum KQ^{0.8518}(Adv) = 1.1664 \sum_{D}^{K} Q^{0.8518} dD$$
 (38)

or in matrix form:

$$\begin{bmatrix} Matrix Coefficients \\ are either \\ \pm 1.8518 \text{ KQ}^{0.8518} \\ \text{or } 0 \end{bmatrix} \begin{bmatrix} A_1 dv_1 \\ A_2 dv_2 \\ A_3 dv_3 \\ \vdots \\ A_m dv_m \end{bmatrix} = \begin{bmatrix} Matrix coefficients \\ are either \\ \pm 1.1664 \frac{\text{KQ}}{\text{D}}^{1.8518} \\ \text{or } 0 \end{bmatrix} \begin{bmatrix} dD_1 \\ dD_2 \\ dD_3 \\ \vdots \\ \vdots \\ dD_m \end{bmatrix}$$
(39)

By combining all independent equations derived from the basic hydraulic relations, (32) and (39), the following simultaneous equations are obtained.

 $\begin{array}{c} 1 \\ A_2 dv_2 \\ A_3 dv_3 \end{array} \begin{array}{c} \begin{array}{c} \text{coefficients are} \\ \hline D \\ \hline M_2 + m^2 \end{array}$ HEAD LOSS CONTINUETY EQUATIONS EQUATIONS Matrix Coefficients dD  $dD_2$ are  $\pm$  1 or 0 dD3 Matrix Coefficiênts coefficients are  $\pm 1.1664 \frac{KQ}{D} 1.8518$ are ± 1.8518 KQ<sup>0.8518</sup> or 0 0  $dD_m$  $\mathbf{or}$ [<sup>A</sup>m<sup>d</sup> 19 (40)

It is observed that, if m is the number of pipes in the network then the matrices on both sides of equations (40) are of order m x m.

The cost function, as given by eqn. (27) is expressed in terms of the diameter, D, flow Q, and Hazen-Williams' loss coefficients. Q is a function of D and v, K is a function of D. Thus the cost function can be expressed in terms of D and v and the first derivative of the cost function can be put in a form similar to (40).

The cost function previously defined can be written as

$$R = \sum_{i} a_{i} D_{i}^{1.29} + \sum_{i} b_{i} K_{i} Q_{i}^{2.8518}$$

where  $a_i$  and  $b_i$  are constants. Differentiation of R with respect to D's and vi's gives

$$dR = \left(1.29 \Sigma a_{i} D_{i}^{0.29} + \Sigma b_{i} \frac{\delta K_{i}}{\delta D_{i}} Q_{i}^{2.8518}\right) dD_{i}$$

$$+ 2.8518 \Sigma b_{i} K_{i} Q_{i}^{1.8518} \frac{\delta Q_{i}}{\delta D_{i}} dD_{i}$$

$$+ 2.8518 \Sigma b_{i} K_{i} Q_{i}^{1.8518} \frac{\delta Q_{i}}{\delta v_{i}} dv_{i}$$
Since  $\frac{\delta K_{i}}{\delta D_{i}} = -4.87 \frac{K_{i}}{D_{i}} (\text{from 35}) \text{ and from (29)} \frac{\delta Q_{i}}{\delta D_{i}} = \frac{2Q_{i}}{D_{i}}$ 
and  $\frac{\delta Q_{i}}{\delta v_{i}} = A_{i}$  the derivative becomes
$$dR = 1.29 \Sigma a_{i} D_{i}^{0.29} dD_{i} + 0.8336 \Sigma b_{i} \frac{K_{i} Q_{i}^{2.8518}}{D_{i}} dD_{i}$$

$$+ 2.8518 \Sigma b_{i} K_{i} Q_{i}^{-.8518} (A_{i} dv_{i}) \qquad (41)$$

This can be expressed in matrix form as follows:

$$\begin{bmatrix} 1 & e_1 & e_2 & e_3 \dots e_m \end{bmatrix} \begin{bmatrix} dR \\ A_1 dv_1 \\ A_2 dv_2 \\ \vdots \\ \vdots \\ A_m dv_m \end{bmatrix} = \begin{bmatrix} f_1 & f_2 & f_3 \dots f_m \end{bmatrix} \begin{bmatrix} dD_1 \\ dD_2 \\ dD_3 \\ \vdots \\ \vdots \\ dD_m \end{bmatrix}$$
(42)

where  $e_1$ ,  $e_2$ ,  $e_3$ ,...,  $e_m$  are coefficients of  $A_1 dv_1$ ,  $A_2 dv_2$ ,  $A_3 dv_3$ ,...,  $A_m dv_m$  and  $f_1$ ,  $f_2$ ,  $f_3$ ,...,  $f_m$  are coefficients of  $dD_1$ ,  $dD_2$ ,  $dD_3$ ,...,  $dD_m$ .

By combining (40) and (42) the left hand side becomes a matrix of order (m+1)x (m+1) and the right hand one of (m+1)x m. It is also observed that "dR" only occurs once and that it is on the left hand side. Thus one of the columns on this side would have coefficients of

Thus the following sets of equations are obtained



(43)

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Before (43) can be used it is necessary to have values of  $Q_i$  and  $D_i$ . Values of D are assumed and values of Q are calculated. The pressure at the junctions are now computed and, if the pressure constraints are broken, new values of D are assumed and the process repeated until these constraints are satisfied.

In this initial stage, if new values of D are required it has been found convenient to add or subtract a constant increment to each of the diameters. It is usual to add the increments, but if the pressure at any point is greater than ABMAX the increment is subtracted.

After values of D and Q are obtained which are feasible solutions the partial derivatives with respect to diameters and velocities are computed and the matrix coefficients for (43) are not known. From these equations, by manipulating the matrices the following is obtained:

$$\begin{bmatrix} dR \\ A_1 dv_1 \\ A_2 dv_2 \\ A_3 dv_3 \\ \vdots \\ \vdots \\ A_m dv_m \end{bmatrix} = \begin{bmatrix} (m+1)x m \\ matrix \\ \vdots \\ dD_1 \\ dD_2 \\ dD_3 \\ dD_3 \end{bmatrix} - \cdots (44)$$

Since

$$dR = g_1 dD_1 + g_2 dD_2 + g_3 dD_3 + \dots + g_m dD_m$$

(where  $g_1, g_2, \ldots, g_m$  are coefficients of the first row of matrix in (44), then

$$\frac{dD_{1}}{g_{1}} = \frac{dD_{2}}{g_{2}} = \frac{dD_{3}}{g_{3}} = \dots = \frac{dD_{m}}{g_{m}} = \rho$$
(45)

To obtain an optimal value of  $\rho$ , using the method of steepest ascent, it is necessary to solve the equation  $\frac{dR}{d\rho} = 0$ . This is a complex equation and calculating the value of  $\rho$  to satisfy it is very cumbersome and at best approximate. The value assigned to  $\rho$  is a critical factor in using the steepest ascent (descent) method. Too big a value of  $\rho$  oversteps the optimum, too small a value requires too many unnecessary calculations. This makes it difficult to find the optimum value of  $\rho$  and thus a form of direct search technique was developed to determine this value.

Of the coefficients,  $g_1$ ,  $g_2$ ,  $g_3$ ,...,  $g_m$  the one with the biggest absolute value is chosen, say  $g_k$ , and then  $dD_k$  corresponding to  $g_k$  is given a value. The ratio  $dD_k/g_k$  is now known and from (45) all values of  $dD_1$ ,  $dD_2$ ,...,  $dD_m$  are computed. These increments,  $dD_1$ ,  $dD_2$ ,  $dD_3$ ,...,  $dD_m$  may be either positive or negative. If  $g_k$  is negative these increments are first of all algebraically added and if  $g_k$  is positive they are subtracted.

If there is a reduction in cost and no constraints are broken, then the process of algebraically adding or subtracting can be continued, or new partial derivatives computed and the entire procedure so far repeated. If there is no reduction in cost within the specified constraints then the increments are halved and the process of algebraically adding or subtracting is continued. If again there is no reduction, the procedure of halving increments and algebraically adding or subtracting is continued until there is such reduction or the tolerance level is reached. At this stage new matrix coefficients are computed and the whole process is repeated. Thus, for an initid increment size of two inches and tolerance level of half inch, the possible increments likely to be tried after the first are one inch and half-inch.

The process eventually converges to a situation where changes in cost are negligible. At this stage most of the diameters do not satisfy constraint number (vi) that is  $D_i$  ( $d_1$ ,  $d_2$ ,  $d_3$ ,...,  $d_m$ ) and the final step in the procedure is to impose this condition.

If  $D_i$  is the diameter corresponding to  $g_i$  where  $g_i >$ 

any other g and  $D_i$  lies between  $d_p$  and  $d_q$  -- where  $d_p$ and  $d_q$  are members of the class  $(d_1, d_2, d_3, \ldots, d_m)$  and  $D_i$ is not -- then  $D_i$  is increased or decreased to  $d_p$  or  $d_q$  by making  $dD_i = d_p - D_i$  or  $dD_i = d_q - D_i$ . The other dD's are computed and all the diameters are altered proportionately. The cost is computed for both cases when  $dD_i = d_p - D_i$  and  $dD_i = d_q - D_i$ . The cost which is cheaper is noted and the corresponding diameters are chosen.

From now on the diameter for the i-th pipe is fixed and  $dD_i = 0$ . The derivative of cost function with respect to  $D_i$  will not be a term of equation (40), also any derivative with respect to  $D_i$  that is involved in the matrices of (40). The process is repeated until all the diameters are now in an acceptable set.

If, in making incremental changes in diameters, constraint (v) is broken,  $(D \ge DIMIN)$  and if the diameter is not that of the pipe with the largest absolute partial derivative of cost function, the diameter of the pipe breaking this constraint will be assigned the value of DIMIN. The diameter increments for the other pipes will be unaffected. If the pipe breaking the diameter constraint is the one with the largest absolute partial derivative of cost function, then not only is this diameter assigned the value DIMIN, but the changes in the other diameters are proportionately adjusted. If a pressure constraint is broken the incremental changes are reduced proportionately until the pressure constraint is satisfied.

The method just outlined above is dependent on the shape of the cost function to obtain the optimum. If the function is convex then the global optimum is obtained. The pressure constraints, especially (iv) ( $p_A \ge P_A$ ) may have the effect of rendering the function non-convex, i.e., a "hole" in the feasible region. In this case it is advisable to ignore the pressure constraint in the initial stages of the process.

Computer Program - Using the method just described a computer

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# FLOWCHART FOR OPTIMUM DESIGN



FIGURE 1A



# FIGURE 1B

program was written for the optimum design of closed pipe networks. A flowchart depicting the logic of this program is given in Figure 1. A listing of the program is presented in Appendix I. The program is fairly involved and the reader is referred to Reference 5 for the details.

<u>Example</u> - A pipe network of nineteen pipes was analyzed by this program to determine the minimum cost design. Figure 2 shows the geometry of the network. The computer output for this example is presented on the following pages to illustrate the type of information output by the computer. For the given cost information and constraints it would have been highly unlikely that this configuration could have been determined without such an aid as this program. At the present time additional efforts are being made to refine and improve this computer program. When this is completed the program will be made available to potential users.



(to convert flowrate to m<sup>3</sup>/s multiply gpm by 6.309 ×10<sup>-5</sup>)

Pipe Number	Diameter (in.)	Length (ft.)	Roughness
1	12.0	1500	130
2	8.0	1000	130
3	8.0	1200	120
4	8.0	2000	120
5	8.0	2800	120
6	8.0	1100	120
7	8.0	1000	120
8	8.0	2500	120
9	8.0	800	100
10	6.0	1300	100
11	6.0	1000	100
12	10.0	1100	130
13	10.0	1000	130
14	6.0	1800	120
15	6.0	1100	120
16	6.0	1800	120
17	10.0	1200	130
18	6.0	1800	120
19	6.0	1300	120

TABLE I PIPELINE DATA FOR EXAMPLE

# COMPUTER OUTPUT FOR EXAMPLE

MAXIMUM PRESSURE = 150.000 LBS PER SQ IN

MINIMUM PRESSURE= 30.000 LBS PER SQ IN

MINIMUM PRESSURE ALLOWED AT JUNCTION 9 15 50.000 LBS PER SQ IN

SMALLEST DIAMETER ALLOWED= 6.000 INCHES

GREATEST INCREMENTAL CHANGE IN DIAMETER= 6.000 INCHES

WHEN THE LARGEST INCREMENTAL CHANGE IN DIAMETERS IS LESS THAN OR EQUAL TO 0.750 INCHES Such Changes are ignored

TOLERANCE ON FLOW=15.0000 GPM

LIFE OF PROJECT= 50.000 YEARS

RATE OF INTEREST# 5.000 PER ANNUM

COST OF ELECTRICITY=\$ 0.01 PER KILDWATT-HOUR

TOLERANCE ON MONEY#\$ 10.00

BUILD-UP FACTOR= 1.000

ENGINEERING NEWS RECORD INDEX= 877.

#### RESULTS OF OPTIMAL TRIAL

PIPE NO	JUL	ICT I ON	LENGTH	ROUGHNESS	DIAMETER- ORIGINAL	INCHES FINAL	FLOW	PRESSURE A	
1	1	Z	1500.00	130	10.00	6.00	311.71	120,000	123 172
2	2	3	1000.00	130.	14-00	6.00	390.92	123.172	117.600
3	3	4	1200.00	120.	16.00	6.00	286-12	117.600	106-317
4	4	5 ·	2000.00	120.	18.00	18.00	-1106.09	106.317	105.871
5	5	6	2800.00	120.	20.00	20.00	-631.90	105.871	108 346
6	6	7	1100.00	120.	12.00	6.00	498.41	108.346	126.691
7,	7	8	1000.00	120.	24.00	16.00	2431.60	124-691	126-298
8	8	9	2500.00	120.	20.00	20.00	2611.29	126-298	121.777
9	9	1	800.00	100.	18.00	24.00	4638.29	121.777	120.000
10	9	10	1300.00	100.	16.00	6.00	227.00	121.777	123.541
11	10	11	1000.00	150.	14.00	6.00	327.47	123.541	112.683
12	11	12	1100.00	130.	12.00	6.00	205.47	112.683	114-023
13	12	5	1000.00	130.	10.00	6.00	87.99	114-023	105.071
14	10	8	1800.00	120.	12.30	6.00	-179.69	123.541	126.298
15	2	10	1100.00	120.	14.00	6.00	-79+22	123.172	123.541
16	7	11	1800.00	120.	16.00	12.00	-1933.19	124-691	112.683

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17 18 19	3 6 4	11 12 12	1200.00 1800.00 1300.00		130. 120. 120.	18.00 20.00 24.00	6.00 6.00 6.00	104-81 69-68 -107-79	<b>117.600</b> 108.346 106.317	112.68 114.02 114.02	) 3	
		CAPI	TAL COST=\$		12350.43							
		OPTI	MAL COST=\$		13970.22							
ORE USAGE		03 <b>J</b> E	CT CODE=	31512	BYTES, ARRAY	AREA=	119800 BYTES,	TOTAL AREA	VAILABLE= `	170080	BYTES	
DIAGNOSTICS		NU	IMBER OF ERR	ORS=	0, NUM	BER OF	WARNINGS=	0. NUMBER	OF EXTENSE	ONS=	0	
COMPILE TIM	E =	4.4	5 SEC,EXECU	TION	TIME= 716.	00 SEC	WATFIN - VER	SION 1 LEVEL	. 3 MARCH	1971	DATE=	72/344

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# CONCLUSIONS

The linear method developed as a result of this study for the hydraulic analyses of water distribution systems is proving to be a valuable aid to practicing engineers. This is supported by the large number of engineers who are presently using the program.

This method is also a useful tool in the minimum cost study of water distribution systems. The feasibility of developing a routine for minimum cost design also has been established through this study. A working program for closed loop systems has been developed.

A method for solving the hydraulics of water distribution system on a standard analog computer also has been developed. This provides the basic tool for an analog-digital model for optimum network design. However, while this appears to be a very promising technique its practicality is limited. This is because most practicing engineers do not have access to the necessary analog-digital systems.

# PUBLICATIONS

- Wood, D. J., "Analog Analysis of Water Distribution Networks," Transportation Engineering Journal, Proceedings, ASCE, Vol. 97, No. TE2, May 1971, 281-290.
- Wood, D. J., and Charles, C. O., "Hydraulic Network Analyses Using Linear Theory," Proceedings, ASCE, Vol. 98, No. HY7, July 1972, pp. 1157-1170.
- Charles, C. O., "Optimum Design of Hydraulic Networks Using Steepest Descent (Ascent) Method and Dynamic Programming," Ph. D. thesis, University of Kentucky, January 1973 (unpublished).
- 4. Wood, D. J., "Users Guide for Linear Method of Analysis of Water Distribution Systems," Department of Civil Engineering, University of Kentucky, 1973 (unpublished).
- 5. Wood, D. J., "Users Guide for Path Method of Analysis of Water Distribution Systems," Department of Civil Engineering, University of Kentucky, 1973 (unpublished).

# APPENDIX I

# LISTING OF FORTRAN PROGRAM FOR OPTIMUM DESIGN OF CLOSED LOOP SYSTEMS

- IMPLICIT REAL\*9 (A-H,O-Z)
- INTEGER UNITS, TREE (50)
- 4EAL\*8 K(50),L(50),DABS,LIFE,DELEV(50),STAN(2500),SCAN(50) UDIMENSION A150,50).C(50,50).SAVE(2500).CHAIN(2500). IRTUGH(50), DPRE(50), PRESS(50), HT(50), Y(50), PECED(50), D(50), DIA(50), ABDDE(50], DEMAND(50], SD(50], SDELD(50], SQ(50], SDPRE(50), SK(50), 20ELD(50), 3(50), LOOP(50, 50), ML(50), MN(50), JBIGIN(50), JEND(50), LZ(50 3) +NPIPEL501
- READ (5,1000) UNITS,MXX,MMX
- 1000 FORMAT (1615)
- 1700 FORMAT (//18X, 'RATE OF INTEREST=', F6.3, 1X, 'PER ANNUM')
  - 1800 FORMAT (//IBX, WHEN THE LARGEST INCREMENTAL CHANGE IN DIAMETERS IS в 1 LESS THAN OR EQUAL TO \*,1X,F6.3,1X,\*INCHES\* /58X,\*SUCH CHANGES 2ARE IGNORED\*1
- 1900 FORMAT (\*1\*,18%,\*NAXIMUM PRESSURE =\*, F9.3,1%,\*L8S PER SQ IN\*) G
- ZOLO FORMAT (//18x, MINIMUM PRESSURE=", F9.3, 1x, "LBS PER SQ IN") 10
- 2020 FORMAT (//18X. \*MINIMUM PRESSURE ALLOWED AT JUNCTION\*, 13, 1X, \*IS\*, 11 1F9.3,1X,\*LBS PER SQ IN\*1
- 12 2030 FORMAT (//18X, 'TOLERANCE ON FLOW=', F7.4,1X, 'CF5')
- 2031 FORMAT (//18X, 'TOLERANCE ON FLOW=', F7.4,1X, 'GPM') 13
- 2032 FORMAT (//18X, 'TOLERANCE ON FLOW=', F7.4.LX, 'MGD') 14
- 2040 FORMAT [//18X, TOLERANCE ON MONEY=\$', FB.2,1X] 15
- 2050 FORMAT (//18X, "GREATEST INCREMENTAL CHANGE IN DIAMETER=", F6.3.1X, 16 1'INCHES\*}
- 17 2060 FORMAT (//18X, 'SMALLEST DIAMETER ALLOWED=",F6.3,1X,"INCHES")
- 2070 FORMAT (//18X, 'LIFE OF PROJECT=', F7.3, LX, 'YEARS') 19
- 2080 FORMAT (//18X, 'COST OF ELECTRICITY=\$', F5.2, 1X , 'PER KILOWATT-HOUR 19 1\*1
- 20 2090 FURMAT (//18X, \*ENGINEERING NEWS RECURD INDEX=\*, F7.0) 21
  - 1701 FORMAT (//18X, "BUILD-UP FACTOR=", F6.3)
    - IF UNITS=1, FLOW IS IN CFS, IF UNITS=2, FLOW IS IS GPN, IF UNITS=3 FLOW IS c
    - IN MGD. MXX=1 INDICATES THAT UNITS OF PRESSURE AT INLET ARE LBS/IN++2, С
    - C. NOT "FOOT-HEAD OF WATER"
- READ (5,1000) NJ,NP 22
- 23 READ (5,6000) ABMAX, ABMIN, DIMIN, DD1, DD2, DD3, DD4
- С NJ= NO OF JUNCTIONS, NP=NO OF PIPES
  - READ (5,2000) (JBIGIN(I), JEND(I), D(I), ROUGH(I), LII), I=1, NP)
- 25 TOTOEM =0.0
- 26 TOTAL=0.0
- 27 NN≠O

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- 28 KXXK=0
- 1111 FORMAT (1X,1H,10X, "0 U.T.P.U.T") 29
- 30 WRITE (6,1111)
  - WRITE (6,1900) ABMAX
  - WRITE (6,2010) ABMIN
- 32 3000 FORMAT (5F10.4)
- 33
- 34 2000 FORMAT (215, F5.2, 2F10.5) 35
- 00 11 I=1,NJ 36
  - READ (5.3000) PECED(1), DEMAND(1), DELEV(1), DPRE(1), HT(1)
  - PECEDIT) =1.0 INDICATES NODE IS AN INLET. DEMAND (WHETHER INLET OR OUTLET C. С
  - ) IS ALWAYS POSITIVE. DELEV IS ELEVATION IN FEET. DPRE AND HT ARE IN LBS C
  - /IN##2 UNITS, EXCEPT WHEN PECED=1.0 AND MAX .NE. 1 IN WHICH CASE DPRE
  - MAY BE IN . FOOT-HEAD OF WATER. UNITS FOR THAT PARTICULAR INLET
  - IF (PECED([] .NE. 1.0) TOTDEM =TOTDEM+ DEMAND([]
  - IF (PECED(I) .EQ. 1.0) TOTAL =TOTAL + DEMAND(1)
  - IF (PECEDIT) .NE. 1.0) DEMAND(I)=-DEMAND(I)
  - IF (PECED(1) .EQ. 1.0) NN=NN+1
  - IF (HT(1) .NE. 0.0) WRITE (6,2020) 1.HT(1)
- 41 4 Z 11 CONTINUE
- 64 IF (TOTDEM .EQ. TOTAL) GD TO 100

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- 44 WRITE (6,5000) TOTAL, TOTDEM
- 5000 FORMAT(//20X, 'TOTAL='.1X, F10.2,2X, 'TOTDEM='.1X, F10.2,2X, 'INFLOW 45 115 DIFFERENT FROM OUTFLOW!)
- 46 STOP
- 100 00 12 [=1,NP 47
- 48 SD(I)=0.0
- 49 000E(1)=D(1)
  - IF (D(I) LT. DIMIN) DII)=DIMIN
- 51 SCAN([)=0.0
- 52 Q(1)=1.0
- 53 NPIPE(I)=0
  - IF (I .GE. NJ) DEMAND(I)=0.0
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- 55 ML(I)=0
- 56 IF (ML(1) .EQ. 0) MM=1
- 57 IF (ML(I) .EQ. 0) LL=16
- 101 READ (5,1000) (LODP(1,J),J=MM,LL) 58
  - LUOP(I, J) MAY BE POSITIVE OR NEGATIVE ACCORDING TO ASSUMED DIRECTION OF С. C
    - FLOW. (EFFLUX FROM JUNCTION IS POSITIVE, INFLUX IS NEGATIVE). REGARD LAST
    - JUNCTION AS REDUNDANT AND IGNORE IT. FROM I .EQ. NJ THRU I .EQ. NP
    - LOOP(1, J) REFERS TO INDIVIDUAL COMPONENTS OF LOOP EQUATIONS (1.E. SUM OF
  - HEAD LOSSES =0.0) WHILE FROM I .EQ. 1 THRU I .EQ. (NJ-1) LOOP(I,J) REFERS
  - INDIVIDUAL COMPONENTS OF CONTINUITY EQUATIONS C.

59 00 13 J=MM,LL

- IF (LOOP (1,J) .EQ. 0) GO TO 12 60
- 61 13 ML[[]=ML[[]+1
- MM=MM+16 62
- 63 LL=LL+16

GO TO FOL

- С IF J=16 OR MULTIPLE OF 16 SUPPLY A BLANK CARD
- 65 12 CONTINUE
- 66 IF (UNITS .EQ. 1) B=1.
  - IF (UNITS .EQ. 2) 8=.002228
  - IF (UNITS .EQ. 3) B=1.5473
- 68 6000 FORMAT (7F10.4) 69
  - READ(5,9000) RATE, LIFE, CENTS, X, BUILD, ENR
  - 9000 FORMAT (8F10.5)
  - KL=NJ-1
  - READ (5,1000) (TREE(1),1=1,KL)
  - IF (ENR .EQ. 0.0) ENR=877.
  - FAC= {RATE}\*.01\*(1.+{RATE}\*.01)\*\*LIFE/(().+(RATE}\*.01)\*\*LIFE -1.}
  - IF (X .EQ. 0.0) X=1.0
  - IF (BUIED .EQ. 0.0) BUILD=1.0
  - WRITE (6,2060) DIMIN
  - WRITE (6,2050)-001
- 79. WRITE [6,1800] DD2
  - IF (UNITS .EQ. 1) WRITE (6,2030) DD4
  - 1F (UNITS .EQ. 2) WRITE (6,2031) DD4
- 82
- 83 IF (UNITS .EQ. 3) WRITE (6,2032) 004 84
  - WRITE 16,2070) LIFE
  - WRITE (6,1700) RATE
  - WRITE 16,20801 CENTS
  - WRITE (6,20401 DD3
  - WRITE (6,1701) BUILD
  - WRITE (6,2090) ENR
  - N=1
- 91 INN =16
- 141 READ(5,1100) (DIA(1),I=N,INN) 92
- 93 N=N+16
- 94 INN=INN+16
- 95 1100 FORMAT (16F5.2)
  - С IF 16 OR A MULTIPLE OF 16. THEN A BLANK CARD MUST BE INSERTED

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IF (DIA(INN-16) .NE. 0.0) GO TO 141 96 97 N1=0 98 DO 44 [=1.[NN 99 IF (DIA(1) .NE. 0.0) N1=N1+1 100 IF (DIA(1) .EQ. 0.0) GO TO 223 101 44 CONTINUE 223 INN=N1 102 103 N=0 A10=001 104 105 LPL=0 106 CHANGE=0.0 107 EXTRA=0.0 108 CT2=0.0 109 KX=D 110 CT1=0.0 111 KK=0 112 M=1 113 LX×16 IF (MMX .EQ. OJ GO TO 118 114 115 809 READ (5,1000) (MN(1),1=M.LX) 116 LX=LX+16 117 M=M+16 118 IF ((LX-16) .LT. MAX) GD TO 809 119 DO 674 1=1 MMX 120 LM=MN(I) 121 674 NPIPE(LM)=1 122 118 DO 14 1=1,NP 123 IF (D(I) .LT. DIMIN) SCAN(I)+D(I) 124 IF (D(1) .LT. DIMIN) D(1)=DIMIN K([]=4.77\*12.\*\*4.87\*L(])\*(DABS(Q(])))\*\*.8518/(RDUGH(])\*\*1.8518\* 125 10(1)\*\*4.87) IF (UNITS .EQ. 2) K(1)=.000012\*K(1) 126 127 IF (UNITS .EQ. 3) K(I)=2.24416\*K(I) 128 DD 14 J=1,NP 129 14 A(1,J)=0.0 130 602 LL=NJ-1 131 DO 16 1=1.LL 132 LXK=ML(1) 133 00 16 J=1,LXK 134 LN=LOOP(I,J) 135 LM=[ABS(LN] 136 1F (LN .LT. 0) A([,LM)=-1.0 137 IF (LN .GT. 0) A(I,LM)=1.0 138 16 CONTINUE 139 102 DO 17 1=NJ.NP 140 LXK=ML([). 141 00 17 J=1,LXK 142 LN=L00P([.J) LM=IABS(LN) 143 144 A(1,LM)=K(LM) 145 IF (LN .LT. 1) A(T.LM) =-A(T.LM) 146 17 CONTINUE 147 00 18 I=1,NP 148 DO 18 J=1+NP 149 IG= J+(1-1)\*NP 150 18 SAVE(IG) =A(J, [] 151 CALL 4INV (SAVE, NP, DDD, LZ, NN, NP+NP) 152 CALL GMPRD (SAVE, DEMAND, Y, NP, NP, 1, NP\*NP, NP\*1, NP\*1) 153 00 19 I=1.NP 154 1F (KK .E. 0) Q(1)=Y(1) 155 IF (KK .NE. 0) Q(1)=(Q(1)+Y(1))/2.

156 K([]=4.77\*12.\*\*4.87\*L([)\*(DABS(Q([]))\*\*.8518/(ROUGH([)\*\*1.8518\* 10(1)\*\*4.87) 157 IF (UNITS \_EQ. 2) K(I)=.000012=K(I) 158 IF (UNITS .EQ. 3) K(1)=2.24416+K(1) 19 CONTINUE 159 160 KK**≈KK+1** 161 IF (KK -EQ: 1) GU TO 102 162 CT=0.0 163 DO 20 1=1.NP 164 DIFF=DABS(Q(I)-Y(I)) 165 IF (KK .EQ. 5) GO TO 807 166 IF (DIFF GT. 004) GO TO 102 807 CT=CT+ENR\*.358\*D([]\*\*1.29\*L(11/(877.\*8UILD)\*FAC 167 168 CT=CT+ .746/550.\*24.\*365.\*X\*CENTS\*K(1)\*Q(1)\*\*2\*B\*62.4 20 CUNTINUE 169 170 СТЗ=СТ 171 CT4=0.0 172 65Z KK=0 173 125 KL=NJ-1 174 NL #0 175 LK=0 176 DO 21 I=1+KL 177 DO 22 J=1,NJ 176 22 A(I.J)=0.0 179 LM=IABS(TREE(I)) 180 LN=JBIGIN(LM) 181 LO=JEND(LM) 182 IF (LN .EQ. LK) GO TO 113 183 IF (LO .EQ. LK) GO TO 113 184 IF (PECED(LN) .EQ. 1.0) NL=NL+1 185 IF (PECEDILN) .EQ. 1.0) LK=LN 186 IF (PECED(LO) .EQ. 1.0) NL=NL+1 187 IF (PECED(10) .EQ. 1.0) LK=LO IF (PECEDILN) .NE. 1.0 .AND. PECEDILD) .NE. 1.0) GO TO 113 188 189 IF LINL+KLA .GT. NJA GO TO 801 190 DD 23 J=1,NJ 23 A((KL+NL), J)=0.0 191 113 IF (PECED(LN) .EQ. 1.0) A((NL+KL),LN)=1.0 192 IF (PECED(LO) .EQ. 1.0) A((NL+KL),LO)=1.0 193 194 IF (PECEDILN) .EQ. 1.0 .AND. MXX .EQ. 1) PRESSINL+KL)=DPREILN)\* 1144./62.4+DELEV(LN) 195 IF (PECED(LO) .EQ. 1.0 .AND. MXX'.EQ. 1) PRESS(NL+KL)=DPRE(LO)\* 1144./62.4+DELEV(LD) 196 IF (PECED(LN) .EQ. 1.0 .AND. MXX .NE.1) PRESS(NL+KL)=DPRE(LN)+ 1DELEVILN) IF (PECED(LO) .EQ. 1.0 .AND. MXX .NE.1) PRESS(NL+KL)=OPRE(LO)+ 197 IDELEV(LO) IF (PECED(LN) .NE. 1.0) A(1,LN)=1.0 198 199 1F (PECED(LO) .NE. 1.0) A(1,LO)=-1.0 200 801 IF ((NL+KL) .GT. NJ) A(1,LN)=1.0 IF ((NL+KL) .GT. NJ) A(1,LO)=-1.0 201 PRESS(I)=K(LM)+Q(LM) 202 IF (TREE(1) .LT. 0) PRESS(I)=-PRESS(I) IF ((NL+KL) .GT. NJ) GO TO 21 203 204 IF (PECED(LN) .EQ. 1.0) PRESS(I)=PRESS(I)-PRESS(NL+KL) 205 IF (PECED(LO) .EQ. 1.0) PRESS(I)=PRESS(1)+PRESS(NL+KL) 206 207 21 CUNTINUE DD 24 I=1.NJ 208 209 DO 24 J=1+NJ IG=J+(1-1)\*NJ 210 211 24 SAVE(IG)=A(J,I)

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CALL MINV (SAVE,NJ,DDD,LZ,MN,NJ\*NJ) 212 CALL GMPRD (SAVE, PRESS, Y, NJ, NJ, 1, NJ\*NJ, NJ\*1, NJ\*L) 213 676 00 25 1=1,NJ 214 DPRE(I)=(Y(I)-DELEV(I))\*62.4/144. 215 IF (KX .NE. 0) GO TO 104 IF (DPRE(I) .GT. ABMAX) GO TO 105 216 217 IF (DPRE(1) .LT. ABMIN) GO TO 105 218 IF (DPRE(1) .LT. HT(1) .AND. HT(1) .NE. 0.0) GO TO 105 219 GO TO 25 220 105 IF (DUL .LE. 002) GO TO 673 221 DO 26 J=1,NP 222 D(J)=0(J)+DD1 223 IF (DPRE(I) .GT. ABMAX) D(J)=D(J)-2\*(DDI) 224 26 CONTINUE 225 226 GO TO 118 104 IF (DPRE(I) .GT. ABMAX) GO TO 106 227 IF (DPRE(I) .LT. ABMIN) GO TO 106 228 IF (DPRE(I) .LT. HT(I) .AND. HT(I) .NE. 0.0) GO TO 106 229 230 GO TO 25 106 DIFF=DABS(D(N)-SD(N)) 231 IF (DIFF .LE. .DOD1 .AND. CT4 .NE. 0.0) CT3=CT4 232 IF (DIFF .LE. .0001) GO TO 208 **23**3 DIFF=DABS(DELD(N)-EXTRA) 234 IF (DIFF .LE. .0001) DELD(N)=EXTRA 235 IF (DELDIN) .NE. EXTRA) GO TO 671 236 DIFF=DASSIDELD(N)-0.0) Z37 IF (DIFF .LE. .0001) GO TO 671 238 661 DD 207 J=1.NP 239 D(J)=D(J)-DELD(J)+SDELD(J) 240 0(J)=SQ(J) 241 207 CONTINUE 242 NPIPE(N)=1 243 CHANGE=0.0 244 EXTRA=0.0 245 246 KX=0 247 GO TO 118 208 DO 209 J=1.NP 248 249 SD( J)=0.0 209 D(J)=D(J)+DELD(J)-SDELD(J) 250 NPIPE(N)=1 251 252 CHANGE=0.0 EXTRA=0.0 253 . 254 KX=0 GO TO 118 255 671 CT4=0.0 256 GO TO 205 257 25 CONTINUE 256 DIFF=DABS(EXTRA-0.0) 259 IF (DIFF .LE. .0001) EXTRA=0.0 260 1F (EXTRA .EQ. 0.0) GO TO 669 261 DIFF= DASS(DELD(N)-EXTRA) 262 IF (DIFF .LE. .0001) DELD(N)=EXTRA 263 IF (DELD(N) .EQ. EXTRA) NPIPE(N)=L 264 265 659 KX=KX+1 1F (CT4 .EQ. 0.0) GO TO 654 266 660 IF (CT1 .NE. 0.0 .AND. CT1 .GT. CT4) CT3=CT4 654 IF (CT1 .NE. 0.0 .AND. CT3 .EQ. CT4) GO TO 139 267 26.8 IF (CT1 .NE. 0.0 .AND. CT1 .LT. CT4) CT3=CT1 269 653 IF (CT1 .NE. 0.0 .AND. CT3 .EQ. CT1) GO TO 139 270 IF (CT4 .EQ. 0.0) GO TO 646 271 IF ICT3 .GT. CT4 \_AND. DD1 .LE. DD21 CT3=CT4 272

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273 646 IF (KX .EQ. 1) GO TO 139 IF ICT3 .GT. CT4 .AND. CT4 .NE. 0.01 CT3=CT4 274 275 1F (CT3 .NE. CT4) GO TO 139 1F (001 .NE. AID) 001=001/2. 276 277 DIFF=DABS(CT2-CT3) 278 IF IDIFF .LE. DD3 .AND. DD1 .LE. DD2) GO TO 673 279 IF (DD1 .LE. DD2) GO TO 139 28 C 139 NM=NP+1 281 K X K X = O 282 KX=1 001=410 283 284 EXTRA=0.0 Z85 CHANGE=0.0 286 CT1=0.0 267 LPL=0 268 CT2=CT3 289 NNN=0 290 N=1 291 00 34 1=1.NM IF (I .E. NM) GO TO 666 292 293 SD([)=0\_0 TE (NPIPE(I) .EQ. 1) NNN+NNN+1 294 295 IF (NP[PE(N] .EQ. 1] N=N+1 296 NPP=NP-1 297 IF (NNN .GT. NPP/2) OD1=AID/2. 298 IF (001 .LE. DD2) 001=AID 299 IF (NNN .EQ. NP) GO TO 137 30.0 666 DO 35 J=1.NM A(1,J)=0.0 301 302 C(I,J)=0.0 303 35 CONTINUE 304 34 CONTINUE 305 A(1,1)#1.0 306 00 32 [=1,NM 307 00 33 J=1,NM IF (I .NE. 11 GO TO 121 308 309 IF (J .EQ. 1) GO TO 142 A([,J]=.746/550\_#24.#365\_#X#CENTS#2.8518#K(J-1)#Q(J-1)#B#62.4 310 1+(-1.) 311 IF (J \_EQ. NM) GO TO 33 142 C([,J)=ENR+.358+1.29+ D(J)++.29+L(J)/(877.+BUILD)+FAC+.746+24.+365 312 1.\*X\*CENTS\*.8336\*Q(J)\*DA8S(Q(J))\*K(J)\*62.4/550.\*B/D(J) 313 IF (NPIPE(J) .EQ. 1) C(I,J)=0.0 314 GO TO 33 315 121 IF (J .EQ. 1) GO TO 33 LN=1ABS(LOOP((I-1),(J-1))) 316 317 IF (LOOP(([-1),(J-1)))120,32,122 120 IF (I .LE. NJ) A(I,(LN+1))=-1.0 318 319 IF (I .LE. NJ) C(I.LN) =2.+Q(LN)/D(LN) IF (NPIPE(LN) .EQ. 1) G(I,LN)=0.0 320 321 IF (I .GT. NJ) A(I,(LN+1))=-1.8518\*K(LN) 322 IF (Q(LN) .LT. 0.0) A([,(LN+1))=-A([,(LN+1)) 323 IF (1 .GT. NJ) C(1,LN)=-1.1664\*K(LN)\*Q(LN)/D(LN) IF (NPIPE(LN) .EQ. 1) C(I,LN)=0.0 324 325 GO TO 33 326 122 IF (I .LE. NJ) A(I,(LN+1))=1.0 1F (1 .LE. NJ) C(1.LN)=-2.0+Q(LN)/D(LN) 327 32.6 IF (NPIPE(LN) .EQ. 1) C(1,LN)=0.0 IF (| .GT. NJ) A([,(LN+1))=1.8518\*K(LN) 329 IF (Q(LN) .LT. 0.0) A(1,(LN+1))=-A(1,(LN+1)) 330 IF ([ .GT. NJ) C(I,LN)=1.1664\*K(LN)\*Q(LN)/D(LN) 331

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IF (NP1PE(LN) .EQ. 1) C([.LN)=0.0 332 **33 CONTINUE** £ ל E 32 CONTINUE 334 DO 59 1=1.NM 335 DD 60 J=1.NM 336 337 IG=J+(1-1)\*NM SAVE(IG)=A(J.I) 338 IF (I .E. NM) GO TO 60 339 IG=J+(1-1)\*NM 34.0 CHAIN(IG)=C(J,I) 341 SCAN(I)=0.0 342 60 CONTINUE 343 59 CONTINUE 344 CALL MINY ISAVE, NM, DDD, LZ, MN, NM\*NMI 345 CALL GMPRD (SAVE, CHAIN, STAN, NM, NM, NP, NM\*NM, NM\*NP, NM\*NP) 346 347 N = 1 348 00 36 I=1,NP IF (NPIPE(N) .EQ. 1) N=N+1 349 NT=1+(N-1)\*NM 350 NK=1+(1-1)\*NM 351 IF (DABS(STAN(NK)) .GT. DABS(STAN(NT)))N=1 352 NT=1+(N-1)+NH 353 C(1,1)=STAN(NK) 354 355 36 CONTINUE SCAN(N)=0.0 356 IF (C(1+N) .GT..O .AND. KXKX .EQ. 0) LODK=1 357 IF (C(L,N) .LT..O .AND. KXKX .EQ. 0) LODK=0 356 1F (DD1 .GT. 002) GO TO 123 359 673 DD 672 J=1.INN 360 DIFF=DABS(D(N)-DIA(J)) 361 IF (DIFF .LE. .0001) D(NI=DIA(J) 362 IF (DIA(J) .LT. D(N)) GD TD 672 363 IF (KX .EQ. 0) DELD(N)=DIA(J+1)-D(N) 364 IF (D(N) .EQ. DIA(J)) NPIPE(N)=1 365 IF (D(N) .EQ. DIA(J)) GO TO 118 366 LPL=0 367 IF (D(N) .LE. DIMIN) DELD(N)=DIMIN-D(N) 368 IF (D(N) .GT. DIMIN) DELD(N)=DIA(J-1)-D(N) 369 EXTRA=DIA(J)-D(N) 370 644 CHANGE=DELD(N) 371 L00K=3 372 [F (KX .EQ. 0) KX=1 373 374 GO TO 668 375 672 CONTINUE 123 IF (001 .GT. DD2) DELD(N)=001 376 NPP=NP-1 377 668 NNN#0 378 DIFF=DABS(CHANGE-0.0) 379 IF (DIFF .LE. .0001) CHANGE=0.0 380 DIFF=DABS(C(1,N)-0.0) 381 1F (DIFF .LE. .0001) C(L.N)=0.0 382 DO 37 1=1+NP 383 IF (NPIPE(I) .EQ. 1) NNN=NNN+1 384 IF (C(1-N) .EQ. 0.0) DD1=0D2 385 IF (C(1,N) .EQ. 0.0) GO TO 651 386 DELD(1)=DELD(N)+C(1,1)/C(1,N) 387 IF ILOOK .EQ. 1) DELD(1)=-OELD(1) 388 IF (SCAN(1) .EQ. 0.0) D(1)=D(1)+DELD(1) 389 390 M=O IF (SCANIL) .NE. 0.0 .ANC. D(1) .EQ. DIMIN) M=1 391 IF (D(1) .EQ. DIMIN .AND. SCAN(1) .NE. 0.0) D(1)=SCAN(1)+DELD(1) 39.2

393 IF (M -EQ. I) GO TO 804 394 IF (SCAN(1) .NE. 0.0 .AND. D(1) .NE. DIMIN) D(1)+DELD(1) 395 804 IF (D(1) .GE. DIMIN) GO TO 37 396 LPL=1 397 IF (LPL .NE. N) SCAN(I)=D(I) 398 IF (LPL .NE. N) GO TO 37 D(LPL)=D(LPL)-DELD(LPL) 399 400 DELO(LPL)=DIMIN-D(LPL) 401 00 67J J=1,NP 40.2 IF (J .LT. LPL) U(J)=D(J)-DELD(J) 403 DELO(J)=DELD(LPL)+C(1,J)/C(1,LPL) 404 DLJ1=DLJ1+DELD(J) D(LPL)=DIMIN 405 406 670 CONTINUE 407 IF (DELD(N) .EQ. 0.0) 001=002 408 IF (DELD(N) .E4. 0.0) GD TO 651. 469 GO TU 127 410 37 CONTINUE 411 GO TO 127 412 127 KKK=0 413 00 78 1=1.NP 414 IF (D(I) .LT. DIMIN) SCAN(I)=D(I) IF (D(I) .LT. DIMIN) D(I)=DIMIN 415 416 K(1)=4.77\*12.\*\*4.87\*L(1)\*10ABS(Q(I)))\*\*.8518/(ROUGH(1)\*\*1.8518\* 10(1)\*\*4.87) 417 IF (UNITS .EQ. 2) KII)=.000012\*K(1) 418 IF (UNITS .EQ. 3) K(I)=2.24416\*K(I) 419 IF (NNN .EQ. NPP .AND. I .NE. N) DELD(I)=0.0 420 00 78 J=1.NP 4Z I 78 A([+])=0.0 422 DO 79 1=1.NP 423 LXK=ML(1) 424 00 65 J=1,LXK 425 LM=IA8S(LOOP(I,J)) 426 ALE.LMJ=L.O. 427 1F (LOOP(1,J) .LT. 0) A(1,LM)=-1.0 428 65 CONTINUE 429 79 CONTINUE 144 DO 82 I=NJ, NP 430 431 LP=ML(I) 432 DO 83 J=1.LP 433 LM=IABS(LOOP(1,J)) 434 A(I,LM) = K(LM)IF (LOOP(I,J) .LT. O)A(I,LM)=-K(LM) 435 436 83 CONTINUE 437 **B2 CONTINUE** 438 DO 84 I=1,NP 439 DO 84 J=1,NP 440 IG= J+(I-1)\*NP 441 84 SAVE(IG)=A(J.I) 442 CALL MINV (SAVE.NP, DOD, LZ, MN, NP+NP) 443 CALL GMPRD (SAVE, DEMAND, Y, NP, NP, 1, NP+NP, NP+1, NP+1) 444 DO 85 I=1,NP 445 IF (KKK .EQ. 0) ((1)=Y(() 446 IF (KKK .NE. 0) Q([)=(Q(])+Y([))/2. 447 K(1)=4.77=12.\*\*4.87+L(1)\*(DABS(Q(1)))\*\*.8518/(ROUGH(1)\*\*1.8518\* 10(I)\*\*4.87) IF (UNITS .EQ. 2) K(1)=.000012\*K(1) 448 449 IF (UNITS .EQ. 3) K(1)=2.24416\*K(1) 450 **35 CONTINUE** 451 KKK=KKK+1

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452 IF (KKK .Eu. 1) GO TO 144 453 IF (001 .LE. 002) GO TO 713 IF (CT4 .EQ. 0.0) GO TO 713 454 455 IF (KX .NE. 1 .AND. ET3 .GT. CT4) CT3=CT4 456 713 CT=0.0 457 00 86 1=1+NP 45 B DIFF =DABS(Q(I)-Y(I)) 459 IF (KKK .EQ. 5) GO TO 808 460 IF (DIFF .GT. DD4) GO TO 144 838 CT=CT+ENR\*.358\*D(1)\*\*1.29\*L(1)/(877.\*BUILU)\*FAC 461 462 CT=CT+ .746/550.+24.+365.+X+CENTS+K(1)+Q(1)++2+8+62.4 46.3 36 CONTINUE 464 CT4≖CT 465 IF (LDDK .EQ. 3) GO TO 667 466 1F (DD1 -GT. DD2) GO TO 212 467 667 DIFF=DABS(CHANGE-0.0) 46 B IF (DIFF .LE. .0001) GD TO 212 DIFF=DABS(CHANGE-DELD(N)) 469 470 IF (DIFF .LE. .0001) CHANGE-DELD(N) 471 IF (DELD(N) .NE. CHANGE) GO TO 212 472 663 CT1=CT4 473 DO 206 I=1.NP 474 SDELD(1)=DELD(1) 475 SD(1)=D(1) 476 D(1)=D(1)-DELD(1) 477 SQ(1)=Q(1) 478 SK(I)=K(I) 479 206 CONTINUE 460 DELD(N)=EXTRA 481 LPL±0 482 GO TO 668 212 DIFF=DABS(EXTRA-0.0) 483 484 IF (DIFF .LE. .0001) EXTRA=0.0 IF (EXTRA .NE. 0.0 .AND. GT4 .LT. CT11 GD TO 125 485 486 IF (EXTRA .NE. 0.0) GO TO 655 487 IF (KX .EQ. 1 .AND. DD1 .LE. DO2) GO TO 673. 488 662 DIFF=DABS(EXTRA-DELD(N)) IF (DIFF .LE. .0001) EXTRA=DELD(N) 489 490 IF (DELD(N) .EQ. EXTRA .AND. CT4 .LT. CT1) GO TO 125 IF (DELD(N) .NE. EXTRA) GO TO 718 491 49Z DIFF=DABS(DELD(N)-0.0) IF (DIFF .LE. .0001) GO TO 659 493 494 651 IF (LOOK .NE. 3) LOOK=LOOK-1 495 IF (LOOK .LT. 0) LOOK=1 496 KXKX=KXKX+1 497 IF (KXKX .EQ. 1 .AND. CT2 .LE. CT3) DD1=1.5+DD2 IF (KXKX .EQ. 1 .AND. CT2 .LE. CT3) GOTO 123 498 499 IF (CT1 -EQ. 0.0 .AND. CHANGE .EQ. 0.0) GO TO 673 50 C 655 DO 211 J=1.NP 501 D(J)=D(J)-DELD(J)+SDELD(J) 50 Z (L)02={L)0 503 0(1)=50(1) 504 KEJ)=SK(J) 505 211 CONTINUE 506 DIFF=JABS(EXTRA-DELD(N)) 507 IF (DIFF .LE. .0001) EXTRA=DELD(N) 508 IF (DELD(N) .EQ. EXTRA) GO TO 125 509 718 IF (CT3 .GT. CT4) GO TO 125 510 205 00 203 I=1,NP 511 1F (SCAN(1) \_EQ. 0.0) D(1)=D(1)=DELD(1) 512 IF (SCAN(I) .NE. 0.0 .AND. D(I) .EQ. DIMIN) M=1

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513 IF (D(1) .EQ. DIMIN .AND. SCAN(1) .NE. 0.0) D(1)=SCAN(1)-DELD(1) 514 IF (M .F4. 1) 50 TO 203 515 IF (SCAN(I) .NE. 0.0 .AND. D(I) .NE. DIMINJ D(I)=D(I)-DELD(I) 515 203 CONTINUE 517 001=001/2. 513 IF (KX .Ew. 1 .AND. DOI .LE. DD2) GU TO 651 519 DIFF=DABS(CT2-CT3) IF (D01 .LE. D02 .AND. DIFF .LE. D03) GO TO 673 520 IF (DDL .LE. DD2) GO TO 118 521 522 659 DIFF=0ABS(EXTRA-0.0) 523 IF (DIFF .LE. .0001) EXTRA=0.0 IF (EXTRA .NE. 0.0) GO TO 667 524 525 IF (KK .EQ. 2) GO TO 139 526 GO TO 123 527 137 CT=0.0 528 1200 FORMAT (//40X, "RESULTS OF OPTIMAL TRIAL!) 529 WRITE (6+1200) 530 WRITE (6,1500) 531 15000FORMAT (//,1X, 'PIPE ND',5X, 'JUNCTION', SX, 'LENGTH', 5X, 'ROUGHNESS', 14X, "DIAMETER-INCHES", 7X, "FLOW", 7X, "PRESSURE AT JUNCTIONS", / 50X, 2"URIGINAL', 3X, "FINAL', 18X, "BEGIN', 8X, "END"] 53 Z CT=0.0 CP=0.0 533 534 00 62 1=1,NP 535 CT=CT+ENR\*.358\*D(1)\*\*1.29\*L(1)/(877.\*BUILD)\*FAC 536 CP=CP+ENR\*.358\*D(1)\*\*1.29\*L(1)/(877.\*BUILD)\*FAC 537 CT=CT+ .746/550.\*24.\*365.\*X\*CENTS\*K([]\*Q([)\*\*2\*B\*62.4 53 8 LN=JB[G[N(1) 539 LG=JEND(I) WRITE (6.1300) [,LN,LG,L(1),ROUGH(1),BODE(1),D(1),Q(1),DPRE(LN), 540 10PRE(LG) 541 62 CONTINUE 542 WRITE (6,1600) CP 543 WRITE (6:1400) CT3 544 1600 FORMAT (//18X, 'CAPITAL COST=\$', F15.2.1X) 545 1400 FORMAT (//18X, 'OPTIMAL COST=\$\*, F15.2, 1X) 546 1300 FORMAT (3x,13,5x,13,2x,13,3x,F9.2,5x,F6.0,8x,F5.2,6x,F5.2,5x,F9.2, 1 3X,F3\_3,3X,F8\_31 547 RETURN 54.9 END C MINV С MINV С MENV SUBROUTINE MINV C. MINV C MINV PURPOSE C MINV 60 INVERT & MATRIX MENV 70 MINV USAGE C MINV 90 CALL MINV(A,N,D,L,M) C MENV 100 c MINV 140 DESCRIPTION OF PARAMETERS C. **MINV 120** A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY MINV 130 £ RESULTANT INVERSE. Ċ MINV 140 N - ORDER OF MATRIX A MINV 150 D - RESULTANT DETERMINANT ٢ MINV 160 С L - WORK VECTOR OF LENGTH N MINV 170 M - WORK VECTUR OF LENGTH N С MINV 180 C MINV 190 C REMARKS MINV 200 MATRIX A MUST BE A GENERAL MATRIX t MINV 210

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576			MINA BIO
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581		18 IP=N*[T+1]	1111 070
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	c		MINV1000
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50.7			MINV1020
24.0		KETURN	MINV1030
591		48 DO 55 I=l.N	MINV1040
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593		50 IKENK+T	711471020
604			M1NV1060
274			MINV1070
595		55 CUNTINUE	MINV1080
	c		MINULOGO
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	Ľ		MINV1110
596		DC 66 I=1,N	
597		[K=NK+[	MINUTION
598			WINATT20
560			MINV1140
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000			MENVI160
601			MINULIZO
602		IF(I+K) 60.65.60	HINVIIIU
603		60 1E1 (+K) 62.65.62	MINVEIGO
60.6			MINV1190
202		OZ KJAJJETK	HINVI200
000		ALIJJ≄HOLD*A(KJ)+ALIJ)	MINV1210
606		65 CONTINUE	41441220
607		66 CONTINUE	MINVLZZU
	C	-	
	ř	DIVIDE ADV AV ATVAT	MINV1230
	2	NAME FOR BE PLADI	MINV1240
	ι.		MINVIZEO
608		K J=K-N	1111112 JU
609		00 75 Jal-N	#TNA1560
410			MINV1270
411			MINV1280
011		IF(J-K) 70,75,70	N INVI 200
612		70 A(KJ)=A(KJ)/BIGA	11 1157 12 7U
613		75 CONTINUE	D1041300
	C		MINVIJO
	č		OSELVNIM -
	4	PRODUCT OF PRVOTS	MINVING
	C		M [ MUL 1 / O
614		Ð≍D≉BIGA	191 JUL 200
	C .		MINV1350
	č		#[NV1360
	5	NEFLAGE PLVDI BY RECIPROGAL	N[NV1370
	L		-MINV1380
615		AIKK}=1.07HIGA	MINVIADO

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ממערכים איז איירייבין אבע איירייבער אבער אייריאנע אייריאנעראיי איזעראיינארא אייריאנעראיי

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- 1. (C. (C. C. C. ))

C         FINAL ROW AND CULUNN INTERCHANGE         MINV1420           617         KN         MINV1430           618         100 R=1(K-1)         MINV14430           619         105 N=1(K)         MINV14430           619         105 I=L(K)         MINV14430           620         105 I=L(K)         MINV14430           621         106 Ju-m4(K-1)         MINV14430           622         108 Ju-m4(K-1)         MINV14430           623         Ju-m4(K-1)         MINV14430           624         UO 113 J=L,N         MINV14430           625         JK-304-J         MINV1540           626         MICD=ALJKI         MINV1540           627         JI=AR+J         MINV1540           628         IJ =AR+J         MINV1540           629         100 ALJI) =MUD         MINV1540           621         IALK) = ALJII         MINV1540           623         IP (J=K) ID (JO (J25         MINV1540           634         IP (J=K) IN (MIN)         MINV1540           635         HOLD=ALKII         MINV1540           636         IJ =K/L         MINV1640           637         ALJII = HOLD         MINV1640           6	616	_	80	CONTINUE	MINV1400
C         FINAL RUM AND CULUMN INTERCHANGE         MINV1420           C         KAN         MINV1440         MINV1440           C         IOD N=(K-1)         MINV1440         MINV1440           C         IOD N=(K-1)         MINV1440         MINV1440           C         IOD N=(K-1)         MINV1440         MINV1440           C         IOD JUNTK-1)         MINV1440         MINV1440           C         IOD JUNTK-1)         MINV1440         MINV1440           C         MINV1440         MINV1440         MINV1440           C         MINV1440         MINV1440         MINV1440           C         MINV1440         MINV1440         MINV1440           C         MINV1440         MINV1450         MINV1450           C         MINV1440         MINV1500         MINV1500           C         MINV1500		c			MINVL410
11     KN     NINV1430       100     N=1K-11     NINV14430       101     IF(K) 150.150.105     NINV14450       102     I05     I=(K) 150.120.120.108     NINV14450       101     I=(I-K) 120.120.108     NINV14490       102     I06     J=NK-1-1     NINV1490       103     J=NK-1-1     NINV1490       104     J=NK-1-1     NINV1490       105     J=NK-1-1     NINV1490       106     J=NK-1     NINV1490       107     J=NK-1     NINV1490       108     J=NK-1     NINV1490       109     J=NK-1     NINV1510       100     NINV1510     NINV1520       110     AIJK1AIJI1     NINV1520       110     AIJK1AIJI1     NINV1520       11-KI-K-L     NINV1520     NINV1520       11-KI-K-L     NINV1520     NINV1520       11-KI-K-L     NINV1520     NINV1520       11-KI-K-L     NINV1520     NINV1540       631     IAJI1-HOLD     NINV1540       635     IAJI1-HOLD     NINV1540       636     IAJI1-HOLD     NINV1540       637     AILTI-K-L     NINV1540       638     IAJI1-HOLD     NINV1640       639     GO TO 100		č		FINAL ROW AND CULUMN INTERCHANGE	M1NV1420
100         164         100         164 <td>617</td> <td>Č</td> <td></td> <td></td> <td>MENV1430</td>	617	Č			MENV1430
100       F(R) 1 (50, 100       MINV1460         620       105 1 + (1K)       MINV1470         621       105 1 + (1K)       MINV1470         622       108 Ju+nK(1-1)       MINV1480         623       JR=N+(1-1)       MINV1470         624       JD 110 J=1.N       MINV1500         625       JK=J0+J       MINV1500         626       HOLD-a(JK)       MINV1500         627       JI=JR+J       MINV1500         628       ALJK)+-A(JT)       MINV1500         629       110 A(JL) = HOLD       MINV1500         621       I2 K I=K-N       MINV1500         623       JD 130 T=1,N       MINV1500         631       JD (JO,100,125       MINV1500         632       L3 K I=K-N       MINV1500         633       D0 130 T=1,N       MINV1500         641       GO TO 100 4       MINV1500         643       L3 A(JL) = HOLD       MINV1500         644       ENO       MINV1500         645       GO TO 100 4       MINV1500         646       ENO       GMR 20         647       ENO       GMR 20         648       L3 A(JL) = HOLD       GMR 20	61.6		1.50		MINV1440
220         105         1-11×1         101/101/00         MINV1470           221         17(1-K)         120+120+120+120         MINV1470           223         J&R=NR+1-11         MINV1470           224         UD         110         J=1N           225         J&K=100+1         MINV1500           226         UD         110         J=1N           226         HQLDaA(JK)         MINV1500         MINV1500           226         A(JK)=-A(JI)         MINV1500         MINV1500           226         A(JK)=-A(JI)         MINV1500         MINV1500           227         125         KI=K-N         MINV1500           228         IA(JI)=HOLD         MINV1500         MINV1500           231         17(J-K)         MINV1500         MINV1500           233         D0 130 T=LN         MINV1500         MINV1500           234         MOLD-AtK11         MINV1500         MINV1600           235         MIQLTAKK1         MINV1600         MINV1600           236         D0 130 T=LN         MINV1600         MINV1600           236         GO TO 100         MINV1600         MINV1600           236         GO TO 100         MINV1	61.9			TECK 150.150.105	MINV1450
221       Implicient       Implicient       Implicient         221       IDB       Jummet I-11       Implicient       Implicient         233       Jammet I-11       Implicient       Implicient       Implicient         244       UD 120 Jain       Implicient       Implicient       Implicient         254       Jummet I-11       Implicient       Implicient       Implicient         255       Jummet I-11       Implicient       Implicient       Implicient         254       Jummet I-11       Implicient       Implicient       Implicient         255       Jummet I-11       Implicient       Implicient       Implicient         256       Jumet I-11       Implicient       Implicient       Implicient         256       Jumet I-11       Implicient       Implicient       Implicient         256       Jumet I-11       Implicient       Implicient       Implicient         257       Jumet I-11       Implicient       Implicient       Implicient         258       Jumet I-11       Implicient       Implicient       Implicient         259       D0 Jao Jaon       Implicient       Implicient       Implicient         250       Althi-11       Implicien	620		105		MINV1460
222         108         juiner(k-1)         MINU 1490           623         JR=Net[-1]         MINU 1490           624         UD 110 J=1.N         MINU 1510           625         JULD J=1.N         MINU 1510           626         HOLDFA(JK)         MINU 1510           627         JI=3R+J         MINU 1500           628         ALJK)=-ALJI         MINU 1500           629         10 ALJI = HULD         MINU 1500           630         120 J=MIKI         MINU 1500           631         IF(J=K)         MINU 1500           632         125 KI=K=N         MINU 1500           633         D0 130 I=LN         MINU 1500           634         KI=K+N         MINU 1500           635         HOLD-AIKII         MINU 1600           636         JI=KI-K+J         MINU 1600           637         AKII=-AIJI         MINU 1600           638         GO TO 100         MINU 1600           640         I50 RETURN         MINU 1600           641         END         MINU 1600           642         C         MINU 1600           643         GO TO 1000         MINU 1600           644         C <td>621</td> <td></td> <td></td> <td>IF(I-K) 120,120,108</td> <td>MINVLATO</td>	621			IF(I-K) 120,120,108	MINVLATO
623         MINU1500           624         UD 110 Jel, N           625         JK=J0+J           626         HOLD=ALJAL           627         JI=JR+J           628         MINU1520           627         JI=JR+J           628         ALJAL           629         10 ALJI           629         110 ALJI           620         120 J=M(K)           621         JF(J-K) 100,100,125           623         12 KI=K+M           634         KI=KI=M           635         DI 30 I=L, N           636         JI=KI           637         ALKI=K+J           638         10 ALJI = HOLO           639         GO TO 100           641         END           642         END           643         C           7         GMRR 10           7         GMRR 10           641         END           642         END           643         C           7         GMRR 10           7         GMRR 10           7         GMRR 10           7         GMRR 10           7	62.2		108	J = N + (K - 1)	MINV1480
624         UD 110 JEI,N         MINUISIO           625         JK-JOAJ         MINUISIO           626         HOLDEALJAN         MINUISIO           627         JI-3/8-J         MINUISIO           628         ALAKIALJIN         MINUISIO           629         ID ALLI =HOLD         MINUISIO           630         120 J=MIK         MINUISIO           631         IF(J-K) IDO.100.125         MINUISIO           633         DD 130 I=LN         MINUISIO           634         KI=KIN         MINUISIO           635         DO 130 I=LN         MINUISIO           636         JI-KI-K-J         MINUISIO           637         ALKII         MINUISIO           638         IJO ALJI         MINUISIO           639         GO TO 100         MINUISIO           640         IS RETURN         MINUISIO           641         ENO         MINUISIO           642         C         GMPR 30           643         GARPR 40         GMPR 10           644         ENO         MINUISIO           645         GURADEA         MATRIX           646         ENO         GMPR 10           <	623			JR = N + (1 - 1)	MINV1600
625         JK-J0+J         HINVISSO           626         HOLD=ALJAI         HINVISSO           627         JI=JR+J         HINVISSO           628         ALJAI-ALJI         HINVISSO           629         110 ALJI=HOLD         HINVISSO           621         12 ALJI=HOLD         HINVISSO           630         120 J=M(K)         HINVISSO           631         16 (J-K) 100,100,125         HINVISSO           632         12 KI=K+N         HINVISSO           633         D0 130 I=L,N         HINVISSO           634         KI=KI+N         HINVISSO           635         HOLD=ALKII         HINVISSO           636         13 HINVISSO         HINVISSO           637         ALKI=K+H         HINVISSO           638         130 ALJI=HOLO         HINVISSO           641         END         HINVISSO           642         GD 100         GMRR 30           643         US REQUTINE GNPRD         GMRR 40           644         END         GMRR 100           7         MALTPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL GHRR 100           7         GUSAGE         GMRR 100           7         C	624			DO 110 J=1.N	MINVISIO
626         HOLD=ALJA)         HINUSSO           627         JI=JR*J         HINUSSO           628         ALJK)=-ALJI)         HINUSSO           629         ILO         ALJK)         HINUSSO           630         IZO         J=M(K)         HINUSSO           631         IF(J=K)         HOLD         HINUSSO           632         IZO         J=M(K)         HINUSSO           633         DO <iso< td="">         HINUSSO         HINUSSO           634         IF(J=K)         HINUSSO         HINUSSO           635         LESKIR         HINUSSO         HINUSSO           636         JIANIA         HINUSSO         HINUSSO           637         GO ISO ICO         HINUSSO         HINUSSO           638         JIANIA         HINUSSO         HINUSSO           639         GO IO IOO         HINUSSO         HINUSSO           640         ISO RETURN         HINUSSO         HINUSSO           6         C         GMPR 10         GMPR 30           6         GMPR 10         GMPR 40         GMPR 40           7         C         GMRP 10         GMPR 10           7         SUBROUTINE GMPRD</iso<>	62.5			JK=JQ+3	H[NV]520
627         JI=JR+J         HINUISSO           628         ALJKI=ALJI)         HINUISSO           629         110 ALJI=HOLD         HINUISSO           630         120 J=M(K)         HINUISSO           631         IFFIJ=KI 100,100,125         HINUISSO           632         125 KI=K=N         HINUISSO           633         D0 130 I=I,N         HINUISSO           634         D0 130 I=I,N         HINUISSO           635         HOLD-ALKII         HINUISSO           636         JI=KI=KI+N         HINUISSO           637         ALKII=ALJI)         HINUISSO           638         130 ALJI =HOLD         HINUISSO           640         ISO RETURN         HINUISSO           641         END         HINUISSO           642         END         GMPR 30           641         END         GMPR 30           641         END         GMPR 30           642         END         GMPR 30           643         G G TO 100         GMPR 30           644         END         GMPR 30           65         GMPR 30         GMPR 30           66         GMPR 100         GMPR 100	626			HOLD=A(JK)	MINV1530
628         AIJK)=-A(JI)         MINU1550           630         120 J=MLQ         MINU1550           631         IF(J=K) 100,100,125         MINU1550           632         125 K]=K=Y         MINU1560           633         D0 130 T=T,N         MINU1560           634         K]=K+N         MINU1600           635         H0LD=AIKI         MINU1660           636         JI=KI-K+J         MINU1660           637         AKI=KIN         MINU1660           638         I30 AJJI=HDLD         MINU1660           639         GO TO 100         MINU1660           640         IS RETURN         MINU1660           641         END         MINU1660           C         SUBROUTINE GMPRD         GMPR 30           C         SUBROUTINE GMPRD         GMPR 40           C         WULTIPLY TWD GENERAL MATRICES TO FORM A RESULTANT GENERAL GMPR 70           C         RATRIX         GMPR 10           C         C         GMPR 10	627			L + At = I L	MINV1540
629         110 A(31) = HOLD         NINU1550           630         120 J=M(K)         NINU1570           631         125 K1=K-N         NINU1570           632         125 K1=K-N         MINU1580           633         00 130 1=1.N         MINU1580           634         K1=K1+N         MINU1630           635         JJ=K1-K+J         MINU1630           636         J30 A(J) = HOLD         MINU1630           637         A(K1)=-A(J)         MINU1630           638         GO TO 100         MINU1650           641         END         MINU1650           641         END         MINU1660           641         END         GMPR 30           641         END         GMPR 30           65         JURPOSE         GMPR 30           66         VURPOSE         GMPR 30           67         MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL GMPR 100           68         C         SUBROUTINE GMPRD         GMPR 30           67         C         SUBROUTINE GMPRD         GMPR 100           68         C         MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL GMPR 100           61         C         SUBROUTINE	628			A[JK}=-A[J])	MINV1550
630       120 J=M(K)       MINV1570         631       IF(J=K) 100,100,125       MINV1580         632       125 K1=K=N       MINV1580         633       D0 130 T=1,N       MINV1570         634       K1=K1=N       MINV1610         635       H0LD=A1K11       MINV1620         636       J1=K1=K+J       MINV1630         637       A1K11=A1J1       MINV1660         638       130 A1J11 = H0LD       MINV1660         639       G0 T0 100       MINV1660         640       ISO RETURN       MINV1660         641       END       MINV1660         C       SUBROUTINE GMPRD       GMPR 30         C       SUBROUTINE GMPRD       GMPR 400         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL       GMPR 50         C       WATRIX       GMPR 100       GMPR 100         C       USAGE       GMPR 100       GMPR 100         C       CALL GMPRD(A, B, R, N, H, L)       GMPR 100       GMPR 100         C       CALL GMPRD(A, B, R, N, H, L)       GMPR 100       GMPR 100         C       CALL GMPRD(A, B, R, N, H, L)       GMPR 100       GMPR 100         C       CALL GMPRD(A, B, R, N, H, L	629		110	A(JI) =HOLD	MINV1560
631         IF(J=K) 100,100,125         HINVISSO           632         L25 KI=K-N         HINVISSO           633         D0 130 T=1,N         HINVISSO           634         KI=KI=N         HINVISSO           635         HOLD=AIKII         HINVISSO           636         JI=KI=K+J         HINVISSO           637         AKIT=A-K+J         HINVISSO           638         GO TO 100         HINVISSO           640         ISO RETURN         HINVISSO           641         END         HINVISSO           C         SUBROUTINE GMPRD         GMPR 30           C         SUBROUTINE GMPRD         GMPR 40           C         MATRIX         GMPR 40           C         SUBROUTINE GMPRD         GMPR 40           C         MATRIX         GMPR 40           C         SUBROUTINE GMPRD         GMPR 40           C         USAGE         GMPR 100           C         CALL GMPRD(A, 6, R, N, H, L)         GMPR 102           C         CALL GMPRD(A, 6, R, N, H, L)         GMPR 100           C         CALL GMPRD(A, 6, R, N, H, L)         GMPR 100           C         CALL GMPRD(A, 6, R, N, H, L)         GMPR 100	630		120	J=M [K]	<b>MINV1570</b>
632       125 Kirk-N       MINVISOD         633       DD 130 Itl,N       MINVI600         634       KIEKIAN       MINVI600         635       HOLD=AIKII       MINVI600         636       JIKII-ALJI       MINVI600         637       AIKII-ALJI       MINVI600         638       I30 AJJI HOLD       MINVI600         639       GO TO 100       MINVI600         640       ISO RETURN       MINVI600         641       END       MINVI600         642       C       GMPR         643       C       GMPR         644       END       MINVI600         645       C       MINVI600         646       C       GMPR         647       C       GMPR         648       C       GMPR         649       C       GMPR         640       C       GMPR         641       END       MINVI600         642       C       GMPR         643       C       GMPR         644       C       GMPR         655       GMPR       GMPR         66       GMPR       GMPR	631			IF(J-K) 100,100,125	MINV1580
633       D0 130 T=1,N       MINV1600         634       KI=KI+N       MINV1610         635       H0LD=AIKI)       MINV1620         636       JI=XI-K-J       MINV1620         637       AIKI]=-AIJI       MINV1620         638       130 AIJI =H0LD       MINV1640         639       GO TO 100       MINV1660         640       ESO RETURN       MINV1660         641       END       MINV1660         C       SUBROUTINE GNPRD       GMPR 30         C       SUBROUTINE GNPRD       GMPR 40         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL       GMPR 60         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL       GMPR 10         C       MATRIX       GMPR 10       GMPR 10         C       USAGE.       GMPR 100       GMPR 110         C       C ALL GHPRD(A, &, R, N, M, L)       GMPR 110       GMPR 110         C       USAGE.       GMPR 100       GMPR 100         C       C ALL GHPRD(A, &, B, R, N, M, L)       GMPR 110       GMPR 120         C       USAGE.       GMPR 110       GMPR 120         C       USAGE.       GMPR 110       GMPR 120	632		125	KI=K-N	MINV1590
634       KI=KI+N       MINV1650         635       HOLD-AIKIN       MINV1620         636       JI=KI-K+J       MINV1630         637       AIKIN=-AIJIN       MINV1630         638       130 AIJN=HOLD       MINV1650         639       GO TO 100       MINV1660         640       ISO RETURN       MINV1660         641       END       GMPR         C       SUBROUTINE GNPRD       GMPR         C       SUBROUTINE GNPRD       GMPR         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL       GMPR         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL       GMPR         C       USAGE       GMPR 100         C       USAGE       GMPR 100         C       USAGE       GMPR 110         C       DESCRIPTION OF PARAMETERS       GMPR 140         C       B - NAME OF FIRST IMPUT MATRIX       GMPR 140         C       B - NAME OF FURST INPUT MATRIX       GMPR 160         C       A - NAME OF FURST INPUT MATRIX       GMPR 160         C       A - NAME OF FURST INPUT MATRIX       GMPR 160         C       A - NAME OF FURST INPUT MATRIX       GMPR 160         C       <	633			DO 130 T=1,N	MINV1600
635       HILD-AIKII       HINV1620         636       JI=KII-K+J       HINV1630         637       AKKII=-AIJI       HINV1630         638       130 AIJI = HOLD       HINV1650         639       GO TO 100       HINV1660         640       150 RETURN       HINV1660         641       END       HINV1660         C       GMPR       GMPR         C       SUBROUTINE GMPRD       GMPR         C       SUBROUTINE GMPRD       GMPR         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL GMPR TO         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL GMPR TO         C       MATRIX       GMPR 80         C       USAGE       GMPR 100         C       C       GMPR 100         C       DESCRIPTION OF PARAMETERS       GMPR 130         C       DESCRIPTION OF PARAMETERS       GMPR 150         C       DESCRIPTION OF PARAMETERS       GMPR 150         C       DESCRIPTION OF PARAMETERS       GMPR 150         C       A - NAME OF FUSTIN PUT MATRIX       GMPR 150         C       DESCRIPTION OF PARAMETERS       GMPR 150         C       A - NAME OF COLUMNS IN A AND RDWS IN B	634			KI=KI+N	MINV1610
636       J1=K1-K+J       MINV1630         637       A(K1)=-A(J))       MINV1660         638       130       A(J)       HINV1660         639       GO TO 100       MINV1660         640       150       RETURN       MINV1660         641       END       MINV1660         641       END       GMPR       20         641       END       GMPR       20         642       C       SUBROUTINE GMPRD       GMPR       30         643       G       PURPOSE       GMPR       50         644       C       MATRIX       GMPR       50         645       C       SUBROUTINE GMPRD       GMPR       50         646       C       MATRIX       GMPR       50         647       GUSAGE       GMPR       50       GMPR       50         648       GUSAGE       GMPR       50       GMPR       50         649       C       USAGE       GMPR       60       MPR       50         649       C       USAGE       GMPR       60       MPR       100         649       SECOND INPUT MATRIX       GMPR       60       GMPR	635			HOLD=A(KI)	MINV1620
637       A(K []=-A[J])       MINU[640         638       130 A(J]]=H0LD       MINU[650         639       GO TO 100       MINU[650         640       150 RETURN       MINU[660         641       END       MINU[660         C       GMPR       10         C       GMPR       10         C       GMPR       10         C       SUBROUTINE GMPRD       GMPR         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL       GMPR         C       MATRIX       GMPR       60         C       USAGE       GMPR 100       GMPR 100         C       C       GMPR 10       GMPR 10         C       MATRIX       GMPR 10       GMPR 10         C       USAGE       GMPR 100       GMPR 10         C       DESCRIPTION OF PARAMETERS       GMPR 110         C       DESCRIPTION OF PARAMETERS       GMPR 130         C       A - NAME OF SECOND INPUT MATRIX       GMPR 150         C       N = NAME OF SECOND INPUT MATRIX       GMPR 150         C       R - NAME OF COLUMNS IN A AND ROWS IN B       GMPR 160         C       N = NUMBER OF COLUMNS IN A AND ROWS IN B       GMPR 170	636			JI *KI-K+J	MINV1630
B3B       130 AUJI) = HULD       MIAV1650         G39       GO TO LDO       MIAV1660         640       150 RETURN       MIAV1660         641       END       MIAV1670         641       END       GMPR         C       GMPR       10         C       GMPR       10         C       SUBROUTINE GMPRD       GMPR         C       SUBROUTINE GMPRD       GMPR         C       SUBROUTINE GMPRD       GMPR         C       MATRIX       GMPR         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL       GMPR         C       MATRIX       GMPR       60         C       MATRIX       GMPR 100       GMPR 100         C       CALL GMPRD(A, B, R, N, M, L)       GMPR 100       GMPR 100         C       USAGE       GMPR 110       GMPR 110         C       DESCRIPTION OF PARAMETERS       GMPR 120       GMPR 110         C       DESCRIPTION OF PARAMETERS       GMPR 160       GMPR 150         C       A - NAME OF FIRST INPUT MATRIX       GMPR 150       GMPR 160         C       R - NAME OF COLUMNS IN A AND RDWS IN B       GMPR 160       GMPR 160         C <td< td=""><td>637</td><td></td><td></td><td>A(KI) = -A(JI)</td><td>MINV1640</td></td<>	637			A(KI) = -A(JI)	MINV1640
640       150 RETURN       MINV1660         641       END       GHPR         641       END       GHPR         641       END       GHPR         641       END       GHPR         642       END       GHPR         643       END       GHPR         644       END       GHPR         645       SUBROUTINE GHPRD       GHPR         645       C       SUBROUTINE GHPRD         645       GHPR       GHPR         645       C       PURPOSE       GHPR         645       GHPR       GHPR       GHPR         645       C       MATRIX       GHPR         645       C       USAGE       GHPR 100         645       C       GHPR 100       GHPR 100         645       C       GHPR 110       GHPR 110         645       C       GHPR 120       GHPR 120         7       GHPR 120       GHPR 120       GHPR 120         7       DESCPIPTION OF PARAMETERS       GHPR 120         7       DESCPIPTION OF PARAMETERS       GHPR 120         7       DESCPIPTION OF PARAMETERS       GHPR 120         7       NUM	638		130	A(JI) = HOLD	MINV1650
641       ISU RETURN       MINU1670         641       END       MINU1680         C       GMPR       10         C       SUBROUTINE GMPRD       GMPR         C       PURPOSE       GMPR         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL       GMPR         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL       GMPR         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL       GMPR         C       WSAGE       GMPR       60         C       USAGE       GMPR       60         C       C       ALL MARDICES       GMPR       60         C       DESCRIPTION OF PARAMETERS       GMPR       60         C       AL MARE OF COLUMN	639				MINV1660
B#1       END       MINV1680         GHPR       10         C	640		150	RETORN	MINV1670
GMPR 10 GMPR 20 GMPR 20 GMPR 30 GMPR 40 GMPR 40 GMPR 50 GMPR 50 GMPR 60 MULTIPLY TWO GENERAL MATRICES TO FORM A RESULTANT GENERAL GMPR 70 MATRIX GMPR 100 C MATRIX GMPR 100 C CALL GMPRO(A, 8, R, N, M, L) GMPR 100 C CALL GMPRO(A, 8, R, N, M, L) GMPR 100 C CALL GMPRO(A, 8, R, N, M, L) GMPR 100 C DESCRIPTION OF PARAMETERS GMPR 120 C DESCRIPTION OF PARAMETERS GMPR 140 C MARE 0F FIRST INPUT MATRIX GMPR 140 C M - NAME 0F SECOND INPUT MATRIX GMPR 150 C R - NAME 0F SECOND INPUT MATRIX GMPR 150 C R - NAME 0F GOLUMNS IN A AND RDWS IN B C L - NUMBER OF COLUMNS IN A AND RDWS IN B C L - NUMBER OF COLUMNS IN A M - NUMBER OF COLUMNS IN A C MPR 120 C ALL MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A GMPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX B C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX B C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX B C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX A CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX A CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX A CANNOT DE IN THE SAME LOCATION AS MATRIX A C MPR 220 C MATRIX A CANNOT DE IN THE SAME LOCATION AS MATRI	041	r		END	MENV1680
C       SUBROUTINE GNPRD       GNPR 30         C       SUBROUTINE GNPRD       GNPR 40         C       PURPOSE       GNPR 50         C       MULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL GNPR 70         MATRIX       GNPR 80         C       USAGE       GNPR 90         C       USAGE       GNPR 100         C       CALL GNPRD(A, B, R, N, M, L)       GNPR 120         C       CALL GNPRD(A, B, R, N, M, L)       GNPR 120         C       DESCRIPTION OF PARAMETERS       GNPR 140         C       DESCRIPTION OF PARAMETERS       GNPR 150         C       A - NAME OF FIRST INPUT MATRIX       GNPR 150         C       N - NUMBER OF SECOND INPUT MATRIX       GNPR 150         C       R - NAME OF FIRST INPUT MATRIX       GNPR 150         C       N - NUMBER OF COLUMNS IN A AND RDWS IN B       GNPR 160         C       M - NUMBER OF COLUMNS IN A AND RDWS IN B       GNPR 210         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GNPR 220         C       ALL MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A       GNPR 220         C       MATRIX R CANNOT DE IN THE SAME LOCATION AS MATRIX A       GNPR 230         C       UF MATRIX R CANNOT DE IN THE SAME LOCATION AS MA		ř			GMPR 10
CSUBROUTINE GMPRDGMPR 40CGMPR 50GMPR 60CPURPOSEGMPR 60CMULTIPLY TWD GENERAL MATRICES TO FURM A RESULTANT GENERALGMPR 60CMATRIXGMPR 100CCALL GMPRD(A, B, R, N, M, L)GMPR 100CCALL GMPRD(A, B, R, N, M, L)GMPR 100CCALL GMPRD(A, B, R, N, M, L)GMPR 110CDESCRIPTION OF PARAMETERSGMPR 120CDESCRIPTION OF PARAMETERSGMPR 120CDESCRIPTION OF PARAMETERSGMPR 140CA - NAME OF FIRST INPUT MATRIXGMPR 150CR - NAME OF OUTPUT MATRIXGMPR 160CN - NUMBER OF ROWS IN AGMPR 150CN - NUMBER OF COLUMNS IN A AND ROWS IN BGMPR 120CCALL MATRICES MUST RE STUPED AS GENERAL MATRICESGMPR 220CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 230CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 240CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 240CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 240CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 240CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 240CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 240CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 240CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 240CMATRIX R CANN		ř		***************************************	GMPR 20
GAPR		ř		SUBROUTING CMORD	GMPR 30
CPURPOSEGMPRSUCMULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERALGMPR60CMATRIXGMPR80CUSAGEGMPR90CUSAGEGMPR100CCCALLGMPR140GMPRCUSAGEGMPR100CCCALLGMPR140CDESCRIPTION OF PARAMETERSGMPR 120CDESCRIPTION OF PARAMETERSGMPR 140CA - NAME OF FIRST INPUT MATRIXGMPR 140CB - NAME OF SECOND INPUT MATRIXGMPR 140CB - NAME OF SECOND INPUT MATRIXGMPR 140CCN - NUMBER OF ROWS IN AGMPR 150CN - NUMBER OF ROWS IN A AND ROWS IN BGMPR 170CM - NUMBER OF COLUMNS IN A AND ROWS IN BGMPR 180CL - NÚMBER UF COLUMNS IN A AND ROWS IN BGMPR 220CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 230CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 240CMATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX AGMPR 240COF MATRIX 3GMPR 250GMPR 260COF MATRIX 3GMPR 260GMPR 260CNUMEGMPR 260GMPR 260CNUMEGMPR 150GMPR 300CNUMEGMPR 150GMPR 300CTHE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		č			GRPK 40
C       HULTIPLY TWO GENERAL MATRICES TO FURM A RESULTANT GENERAL MATRIX       GMPR TO GMPR 100         C       MATRIX       GMPR 100         C       CALL GMPRD(A, B, R, N, M, L)       GMPR 100         C       CALL GMPRD(A, B, R, N, M, L)       GMPR 100         C       DESCRIPTION OF PARAMETERS       GMPR 110         C       DESCRIPTION OF PARAMETERS       GMPR 130         C       DESCRIPTION OF PARAMETERS       GMPR 140         C       B - NAME OF FIRST INPUT MATRIX       GMPR 140         C       B - NAME OF OUTPUT MATRIX       GMPR 150         C       N - NAME OF OUTPUT MATRIX       GMPR 160         C       N - NUMBER OF COLUMNS IN A       GMPR 170         C       M - NUMBER OF COLUMNS IN A AND RDWS IN B       GMPR 180         C       L - NUMBER UF COLUMNS IN B       GMPR 220         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 240         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 250         C       UF MATRIX 3       GMPR 250       GMPR 250         C       UF MATRIX 3		č		PURPOSE	GMPK 50
C       MATRIX       GMPR 80         C       USAGE.       GMPR 90         C       CALL GMPRD(A, B, R, N, M, L)       GMPR 100         C       CALL GMPRD(A, B, R, N, M, L)       GMPR 110         C       DESCRIPTION OF PARAMETERS       GMPR 120         C       DESCRIPTION OF PARAMETERS       GMPR 130         C       A - NAME OF FIRST INPUT MATRIX       GMPR 140         C       B - NAME OF SECOND INPUT MATRIX       GMPR 140         C       B - NAME OF SECOND INPUT MATRIX       GMPR 150         C       N - NUMBER OF ROWS IN A       GMPR 160         C       N - NUMBER OF COLUMNS IN A AND ROWS IN B       GMPR 160         C       N - NUMBER OF COLUMNS IN A       GMPR 120         C       A - NUMBER UF COLUMNS IN B       GMPR 200         C       PEMARKS       GMPR 200         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 230         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 240         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 240 <t< td=""><td></td><td>ċ</td><td></td><td>MULTIPLY TWO GENERAL MATRICES TO FORM A RESULTANT GENERAL</td><td>CMPR TO</td></t<>		ċ		MULTIPLY TWO GENERAL MATRICES TO FORM A RESULTANT GENERAL	CMPR TO
C       USAGE.       GMPR 90         C       USAGE.       GMPR 100         C       CALL GMPRD(A, B, R, N, M, L)       GMPR 100         C       DESCRIPTION OF PARAMETERS       GMPR 120         C       DESCRIPTION OF PARAMETERS       GMPR 130         C       A - NAME OF FIRST INPUT MATRIX       GMPR 130         C       B - NAME OF SECOND INPUT MATRIX       GMPR 150         C       B - NAME OF SUCTON INPUT MATRIX       GMPR 150         C       R - NAME OF COLUMNS IN A AND RDWS IN B       GMPR 160         C       N - NUMBER OF COLUMNS IN A AND RDWS IN B       GMPR 190         C       L - NÜMBER UF COLUMNS IN A B       GMPR 210         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 250         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 250         C       UF MATRIX 3       GMPR 250       GMPR 250         C       UF MATRIX 3       GMPR 260       GMPR 260         C       UF MATRIX 3       GMPR 260       GMPR 260         C       UF MATRIX 3       GMPR 260 <td></td> <td>с</td> <td></td> <td>MATRIX</td> <td></td>		с		MATRIX	
C       USAGE.       GMPR 100         C       CALL GMPRD(A, B, R, N, M, L)       GMPR 100         C       CALL GMPRD(A, B, R, N, M, L)       GMPR 110         C       DESCRIPTION OF PARAMETERS       GMPR 120         C       DESCRIPTION OF PARAMETERS       GMPR 130         C       A - NAME OF FIRST INPUT MATRIX       GMPR 140         C       B - NAME OF SECOND INPUT MATRIX       GMPR 160         C       R - NAME OF OUTPUT MATRIX       GMPR 160         C       N - NUMBER OF COLUMNS IN A       GMPR 170         C       N - NUMBER OF COLUMNS IN A AND RDWS IN B       GMPR 190         C       L - NUMBER OF COLUMNS IN B       GMPR 200         C       PEMARKS       GMPR 220         C       ALL MATRICES MUST BE STUPED AS GENERAL MATRICES       GMPR 220         C       ALL MATRICES MUST BE STUPED AS GENERAL MATRICES       GMPR 220         C       ALL MATRICES MUST BE IN THE SAME LOCATION AS MATRIX A       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 240         C       UF MATRIX 3       GMPR 250       GMPR 250         C       UF MATRIX 3       GMPR 260       GMPR 260         C       UF MATRIX 3       GMPR 260       GMPR 260		Ċ			CMDD DO
C       CALL GMPRD(A,B,R,N,M,L)       GMPR 110         C       DESCRIPTION OF PARAMETERS       GMPR 120         C       DESCRIPTION OF PARAMETERS       GMPR 130         C       A - NAME OF FIRST INPUT MATRIX       GMPR 140         C       B - NAME OF SECOND INPUT MATRIX       GMPR 140         C       B - NAME OF OUTPUT MATRIX       GMPR 150         C       N - NUMBER OF OUTPUT MATRIX       GMPR 160         C       N - NUMBER OF ROWS IN A       GMPR 170         C       N - NUMBER OF COLUMNS IN A AND ROWS IN B       GMPR 170         C       N - NUMBER OF COLUMNS IN A       GMPR 200         C       L - NÚMBER UF COLUMNS IN B       GMPR 200         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 210         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 240         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 240         C		Ċ.		USAGE.	GMPR 100
C       DESCRIPTION OF PARAMETERS       GMPR 120         C       DESCRIPTION OF PARAMETERS       GMPR 130         C       A - NAME OF FIRST INPUT MATRIX       GMPR 140         C       B - NAME OF SECOND INPUT MATRIX       GMPR 150         C       B - NAME OF SECOND INPUT MATRIX       GMPR 160         C       N - NUMBER OF ROWS IN A       GMPR 160         C       N - NUMBER OF ROWS IN A       GMPR 180         C       N - NUMBER OF COLUMNS IN A AND ROWS IN B       GMPR 190         C       L - NÚMBER UF COLUMNS IN A AND ROWS IN B       GMPR 210         C       PEMARKS       GMPR 210         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 220         C       NUMBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER DF ROUGMPR 250       GMPR 260         C       UF MATRIX 3       GMPR 280       GMPR 280         C       SUBROJTINES AND FUNCTION SJBPRDGRAMS REQUIRED       GMPR 280         C       NUME       GMPR 280       GMPR 290		С		CALL GMPRD(A,8,R,N,M,L)	GMPR 110
C       DESCRIPTION OF PARAMETERS       GMPR 130         C       A - NAME OF FIRST INPUT MATRIX       GMPR 140         C       B - NAME OF SECOND INPUT MATRIX       GMPR 150         C       R - NAME OF SECOND INPUT MATRIX       GMPR 160         C       N - NUMBER OF QUIPUT MATRIX       GMPR 160         C       N - NUMBER OF COLUMNS IN A AND RDWS IN B       GMPR 180         C       L - NUMBER OF COLUMNS IN A AND RDWS IN B       GMPR 190         C       L - NUMBER UF COLUMNS IN B       GMPR 210         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 240         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 250         C       UF MATRIX 3       GMPR 250       GMPR 250         C       UF MATRIX 3       GMPR 260       GMPR 260         C       SUBRUJTINES AND FUNCTION SJBPRDGRAMS REQUIRED       GMPR 260         C       NUNE       GMPR 260       GMPR 260         C       NUNE       GMPR 300       GMPR 300         C       NUNE       GMPR 310       GMPR 320 <t< td=""><td></td><td>С</td><td></td><td></td><td>GMPR 120</td></t<>		С			GMPR 120
C       A - NAME OF FIRST INPUT MATRIX       GMPR 140         C       B - NAME OF SECOND INPUT MATRIX       GMPR 150         C       R - NAME OF OUTPUT MATRIX       GMPR 160         C       R - NAME OF OUTPUT MATRIX       GMPR 160         C       N - NUMBER OF ROWS IN A       GMPR 170         C       M - NUMBER OF COLUMNS IN A       GMPR 170         C       M - NUMBER OF COLUMNS IN A       GMPR 180         C       L - NÜMBER UF COLUMNS IN B       GMPR 200         C       PEMARKS       GMPR 220         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 240         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 250         C       UF MATRIX 3       GMPR 250       GMPR 250         C       UF MATRIX 3       GMPR 260       GMPR 260         C       USRUJTINES AND FUNCTION SJ&BPRDGRAMS REQUIRED       GMPR 300         C       NUNE       GMPR 300       GMPR 300         C       NUNE       GMPR 15 PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		C		DESCRIPTION OF PARAMETERS	GMPR 130
C       B       NAME OF SECOND INPUT MATRIX       GMPR 150         C       R       NAME OF QUIPUT MATRIX       GMPR 160         C       N       NUMBER OF ROWS IN A       GMPR 170         C       M       NUMBER OF COLUMNS IN A AND ROWS IN B       GMPR 180         C       M       NUMBER OF COLUMNS IN A AND ROWS IN B       GMPR 190         C       M       NUMBER OF COLUMNS IN B       GMPR 200         C       L       NUMBER UF COLUMNS IN B       GMPR 210         C       PEMARKS       GMPR 210       GMPR 210         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 240         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 250         C       UF MATRIX A CANNOT BE IN THE SAME LOCATION AS MATRIX B       GMPR 250         C       UF MATRIX A CANNOT BE IN THE SAME LOCATION AS MATRIX B       GMPR 250         C       UF MATRIX A CANNOT BE IN THE SAME LOCATION AS MATRIX B       GMPR 260         C       UF MATRIX A       GMUST GMPR 260       GMPR 260         C       UF MATRIX B       GMPR 260		С		A - NAME OF FIRST INPUT MATRIX	GMPR 140
C       R - NAME DF OUTPUT MATRIX       GMPR 160         C       N - NUMBER OF ROWS IN A       GMPR 170         C       M - NUMBER OF COLUMNS IN A AND ROWS IN B       GMPR 180         C       L - NÚMBER UF COLUMNS IN A AND ROWS IN B       GMPR 190         C       L - NÚMBER UF COLUMNS IN B       GMPR 200         C       PEMARKS       GMPR 210         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 220         C       NUMBER OF COLUMNS OF MATRIX A NUST BE EQUAL TO NUMBER DF ROWGMPR 250       GMPR 260         C       OF MATRIX 3       GMPR 260       GMPR 260         C       SUBROJTINES AND FUNCTION SJBPRDGRAMS REQUIRED       GMPR 260         C       NUME       GMPR 260       GMPR 260         C       NUME       GMPR 280       GMPR 300         C       NUME       GMPR 310       GMPR 320         C       MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320       GMPR 320		C		B - NAME OF SECOND INPUT MATRIX	GMPR 150
C       N - NUMBER OF ROWS IN A       GMPR 170         C       M - NUMBER OF COLUMNS IN A AND ROWS IN B       GMPR 180         C       L - NUMBER OF COLUMNS IN A AND ROWS IN B       GMPR 190         C       L - NUMBER OF COLUMNS IN B       GMPR 200         C       PEMARKS       GMPR 200         C       ALL MATRICES MUST BE STUPED AS GENERAL MATRICES       GMPR 210         C       ALL MATRICES MUST BE STUPED AS GENERAL MATRICES       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 250         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX B       GMPR 250         C       UF MATRIX 3       GMPR 250         C       UF MATRIX 3       GMPR 260         C       UF MATRIX 3       GMPR 260         C       SUBRUJTINES AND FUNCTION SJBPRDGRAMS REQUIRED       GMPR 260         C       NUME       GMPR 300         C       NUME       GMPR 300         C       THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		С		R - NAME DF OUTPUT MATRIX	GMPR 160
C       M - NUMBER OF COLUMNS IN A AND RDWS IN B       GMPR 180         C       L - NÚMBER UF COLUMNS IN B       GMPR 190         C       PEMARKS       GMPR 200         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 210         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX B       GMPR 240         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX B       GMPR 240         C       MATRIX A CANNOT BE IN THE SAME LOCATION AS MATRIX B       GMPR 240         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX B       GMPR 240         C       MUBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROWGMPR 250       GMPR 260         C       UF MATRIX 3       GMPR 260       GMPR 260         C       UF MATRIX 3       GMPR 260       GMPR 260         C       SUBRUJTINES AND FUNCTION SJEPROGRAMS REQUIRED       GMPR 290         C       NUNE       GMPR 300       GMPR 300         C       NUNE       GMPR 310       GMPR 320         C       THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		C		N ~ NUMBER OF ROWS IN A	GMPR 170
C L - NOMBER OF CULUMNS IN B GMPR 190 C GMPR 200 C PEMARKS GMPR 210 C ALL MATRICES MUST BE STUPED AS GENERAL MATRICES GMPR 220 C MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A GMPR 230 C MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A GMPR 240 C MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A GMPR 240 C MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A GMPR 240 C MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A GMPR 240 C MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A GMPR 240 C MUMBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROWGMPR 250 C UF MATRIX 3 C SUBROJTINES AND FUNCTION SJBPRDGRAMS REQUIRED GMPR 260 C NUME C NUME C METHOD C METHOD C THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		Ľ.		M - NUMBER OF COLUMNS IN A AND ROWS IN B	GMPR 180
C PEMARKS GMPR 200 C PEMARKS GMPR 210 C ALL MATRICES MUST BE STUPED AS GENERAL MATRICES GMPR 220 C MATRIX & CANNOT BE IN THE SAME LOCATION AS MATRIX A GMPR 220 C MATRIX & CANNOT BE IN THE SAME LOCATION AS MATRIX A GMPR 240 C NUMBER OF COLUMNS OF MATRIX A NUST BE EQUAL TO NUMBER DF ROMGMPR 250 C DF MATRIX 3 C SUBROJTINES AND FUNCTION SJBPRDGRAMS REQUIRED GMPR 260 C NUME GMPR 290 C NUME GMPR 300 C METHOD C THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		Ę		L - NUMBER OF CULUMNS IN B	GMPR 190
C       MEMARS       GMPR 210         C       ALL MATRICES MUST RE STUPED AS GENERAL MATRICES       GMPR 220         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 230         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 240         C       MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A       GMPR 250         C       MUBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROWGMPR 250       GMPR 250         C       OF MATRIX 3       GMPR 260       GMPR 260         C       SUBROJTINES AND FUNCTION SJEPROGRAMS REQUIRED       GMPR 260         C       NUME       GMPR 290       GMPR 300         C       NUNE       GMPR 310       GMPR 320         C       THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320       C		L r			GMPR 200
C MATRIX & CANNOT BE IN THE SAME LOCATION AS MATRIX A GMPR 230 C MATRIX & CANNOT BE IN THE SAME LOCATION AS MATRIX A GMPR 230 C MUMBER OF COLUMNS OF MATRIX A MOST BE EQUAL TO NUMBER OF ROWGMPR 250 C OF MATRIX 3 C SUBROJTINES AND FUNCTION SJBPRDGRAMS REQUIRED GMPR 260 C SUBROJTINES AND FUNCTION SJBPRDGRAMS REQUIRED GMPR 260 C GMPR 260 C MONE GMPR 300 C METHOD GMPR 310 C THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		č		MEMARKS	GMPR 210
C MATRIX & CANNOT DE IN THE SAME LOCATION AS MATRIX & GMPR 230 C NUMBER OF COLUMNS OF MATRIX A MUST DE EQUAL TO NUMBER OF ROWGMPR 250 C OF MATRIX 3 GMPR 260 C SUBROJTINES AND FUNCTION SJBPRDGRAMS REQUIRED GMPR 260 C NUME GMPR 290 C NUME GMPR 300 C METHOD GMPR 310 C THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		ř		MATEIX 2 CANNOT RE IN THE CAME DOALTAN AS MATRICES	GMPR 220
C NUMBER OF COLUMNS OF MATRIX & NUST GE EQUAL TO NUMBER OF ROMGMPR 250 C OF MATRIX 3 GMPR 260 C SUBROJIINES AND FUNCTION SJBPRDGRAMS REQUIRED GMPR 260 C NUME GMPR 290 C NUME GMPR 300 C METHOD GMPR 310 C THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		č		MATRIX & CANNOT BE IN THE SAME LOCATION AS MATRIX A	GMPK 230 CMOD 340
C DE MATRIX 3 GMPR 260 C SUBROJTINES AND FUNCTION SJBPRDGRAMS REQUIRED GMPR 260 C NUNE GMPR 260 C NUNE GMPR 260 C GMPR 26		ē		NUMBER OF COLUMNS OF MATRIX A MUST BE EDUAL TO NUMBER OF D	00000 290 196898 250
C GMPR 270 C SUBRUITINES AND FUNCTION SUBPROGRAMS REQUIRED GMPR 280 C NUME GMPR 290 C GMPR 300 C METHOD GMPR 310 C THE M BY E MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		c		UF MATRIX 3	GMPR 260
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED GMPR 280 C NUNE GMPR 290 C GMPR 300 C METHOD GMPR 310 C THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		Ĉ			GMPR 270
C NUNE GMPR 290 C GMPR 300 C METHOD GMPR 310 C THE M BY E MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		C		SUBRUITINES AND FUNCTION SUBPROGRAMS REQUIRED	GMPR 260
C GMPR 300 C METHOD GMPR 310 C THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		С		NUNE	GMPR 290
C METHOD GMPR 310 C THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		С			GMPR 300
C THE M BY E MATRIX & IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320		C		METHOD	GMPR 310
		c		THE M BY L MATRIX & IS PREMULTIPLIED BY THE N BY M MATRIX A	A GMPR 320

	C	AND THE RESULT IS STORED IN THE N BY L MATRIX R.	GMPR	330
	Ć		GMPR	340
	Ĺ		GMPR	350
	Ĺ		GMPR	360
642		SUBROUTINE GMPRD (A.B.R.N.M.L.NTM.MTL.NTL)		
643		DIMENSION A(NTM), B(MTL), R(NTL)		
644		DOUBLE PRECISION A.B.R		
	C			
645		[R=0	GMPR	390
646		I K = - M	GMPR	400
647		DD 10 K=1.1	GMPR	410
64.8			GMPR	420
649			GMPR	430
450			GMPR	440
451			GMPR	450
001		J = J = J	GMPR	460
69Z			GMPR	470
000			GMPR	480
654		DD 10 I=1,M	GMPR	490
655		* N+IL=IL	GNDP	500
656		IB=IB+1	CMDR	510
657	10	R(IR)=R(IR)+A(JI)+8(IB)	CMPP	510
658		RETURN	CHOD	520
659		END	GMPR	220
			GMPR	>40

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