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Minimum Cost Design of Water Distribution Systems

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MINIMUM COST DESIGN OF WATER DISTRIBUTION SYSTEMS

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ABSTRACT

The objective of this study was to develop the analytical tools and procedures for minimum cost design of water distribution systems. Both analog and digital means of carrying out pressure and flow calculations were developed. As a result of this effort, digital programs for pressure and flow calculations in water distribution systems were written and have been widely distributed to practicing engineers. One procedure is based on a direct solution of the basic system equations using a linearization scheme and has several advantages over conventional techniques such as the Hardy Cross method. These include avoiding the need to initially balance the network and an assured convergence of the procedure.

Using this tool a procedure was developed for selecting pipe diameter which will result in a minimum cost design within the prescribed constraints. The method of steepest ascent and dynamic programming concepts were used to carry out the optimization. This procedure applies to closed loop systems without internal pumping. However, this work provides a basis for extending the concepts to more generalized water distribution systems.

KEY WORDS: water distribution, optimization, piping systems, network design, ecomomic efficiency

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INTRODUCTION

The objective of the study as outlined in the original proposal was, "to investigate the various functional constraints controlling the design of water distribution systems and develop analytical methods and digital computer routines which can be utilized to design a water distribution system at minimum cost". It was determined early in this investigation that the available means for the hydraulic analysis of water distribution systems did not lend themselves well to a minimum cost analysis. Therefore, a considerable effort was made to develop the analytical tools for pressure and flow calculations which could be incorporated into a minimum cost analysis of water distribution systems.

A promising technique which was investigated was the use of analog simulation for system hydraulics which could be incorporated into a digital-analog scheme to carry out the cost minimization. Techniques for carrying out an analog simulation of pipe system hydraulics on a standard analog computer were developed and reported by the principal investigator (1). It was intended to use the analog computer to model the hydraulics and to use analog to digital conversion and a digital computer to compute the cost. It was felt that with such a model an effective directed search could be undertaken to determine the optimum design of the pipe system. However, it turned out that the necessary equipment for the development of this concept was not available to this project. Since some rather expensive equipment is involved this approach had to be abandoned.

A major effort was devoted to the development of an analytical procedure for hydraulic analysis based on linearization of the basic non-linear system equations. The purpose of developing this approach was that it appeared to offer a method for handling pipe system hydraulics which could be more easily incorporated into a minimum cost study. A scheme which directly solved the basic equation after linearizing the non-linear terms was developed. A publication is available describing this phase of the study (2). This method of hydraulic analysis refereed to as the linear method offered distinct advantages over the conventional Hardy Cross and Newton Raphson methods which are generally used. Therefore, some additional effort was made developing the linear method for generalized situations. As a result of this, a general computer program has been developed and made available to engineers working in this field. Over one hundred and fifty engineering firms have acquired this program.

Finally, using the linear method for hydraulic analysis a computer program has been developed for minimum cost design of closed loop water distribution systems. This work was primarily the effort of C. O. Charles and is documented in his Ph. D. dissertation (3). In this program the method of steepest ascent and concepts of dynamic programming were employed to formulate a procedure which would select the optimum set of pipe diameters for a closed loop system.

RESEARCH PROCEDURES

The entire project is concerned with the development of the basic elements of an analytical model to be employed for minimum cost design. This entails the conception, formulation and testing of certain analytical procedures. In most cases either analog or digital computer programs were the end product of this effort. The usual procedures for formulating, debugging and testing computer programs were employed.

RESULTS

Analog simulation of pipe system hydraulics

This phase of the investigation has been completely documented in Reference 1 and is available through that publication. As previously stated, however, the necessary digital-analog equipment was not available to develop a technique for optimum design using analog simulation.

Digital programs for the analysis of pipe system hydraulics

A considerable effort was made to develop a digital computer program which would easily handle general water distribution systems in a manner which would lend itself to a minimum cost investigation. This effort resulted in the development of two computer programs. General information pertaining to the development of these programs follows:

The programs will compute steady flow in pipe systems of any arrangement. The system can include pumps, valves, bends, and other minor loss components, storage tanks and source and storage reservoirs. A system of p pipes can be described by the number of junctions, j, the number of closed primary loops, ℓ, and the number of terminal energy points, t, in the system. A junction is simply a point in the system where two or more pipes meet. Any point where flow enters or exits the pipe system is also a junction. A primary loop is a closed loop of pipes in the system which have no other loops within it. A terminal energy point is a point in the system where the fluid energy is known. This is essentially any point where the pressure and the elevation are known. Source or storage reservoirs, pressurized sources, storage tanks and discharge points of known pressure are the most common terminal energy points. To describe a system the junctions, loops and terminal energy points must be identified. If a terminal energy point and a junction coincide, this point should be identified as a terminal energy point only. If the junctions, loops and terminal energy points are identified with the restriction just stated, the following holds for all pipe systems:

$$p = j + \ell + t - 1 \tag{1}$$

where

p = number of pipes
j = number of junctions
ℓ = number of loops
t = number of terminal energy points.

In terms of the unknown discharge in each pipe, a number of continuity and energy equations can be written equaling the number of pipes in the system. For each junction a continuity equation equating the flow into the junction to the flow out is written as:

$$Q_{in} = Q_{out} \qquad \text{(j equations)} \tag{2}$$

For each loop the energy equation can be written as follows

$$\Sigma h_L = \Sigma E_P \qquad (\ell \text{ equations}) \tag{3}$$

where

h_L = head loss in each pipe (including minor loss)
E_P = energy put into the liquid by a pump.

If there are no pumps in the loop then the energy equation states that the sum of the head loss around the loop equals zero.

If there are t terminal energy points, t - 1 energy equations can be written for paths between any two terminal energy points as follows

$$\Delta E = \Sigma h_L - \Sigma E_P \qquad \text{(t - 1 equations)} \qquad (4)$$

where ΔE is the energy difference between the two terminal energy points. Any path in the pipe system can be chosen between the points. However, care must be taken to avoid redundant paths. The best method to avoid this difficulty is to either choose all paths starting at one source (like 1-2, 1-3, 1-4, etc.) or to use the previous end point for a path as the starting point for the next path (like 1-2, 2-3, 3-4, etc.). Either of these methods will result in t - 1 equations with no redundant ones.

These junction loop and path equations constitute a set of simultaneous equations equal to the number of pipes in the system which can be solved for the discharge in each pipe. A direct solution of these simultaneous equations is not possible because of the non-linear terms. Two basic methods of solution were considered.

Linear method - For this approach the non-linear terms are linearized giving a set of linear simultaneous equations which can be solved using matrix methods. The linearization is formulated as follows. The line loss is given by:

$$h_{LP} = K_P Q^n \qquad (5)$$

where K_P is a pipe line constant and for the Hazen Williams equation employed in the computer analysis is

$$K_P = \frac{4.73\ L}{C^{1.852} D^{4.87}} \qquad (6)$$

Here L = line length in ft, D = line diameter in ft and C is the Hazen Williams roughness coefficient. The discharge Q in eqn. 5 is in cfs and the exponent n = 1.852.

Minor losses are given by a loss coefficient, M, which multiplies the velocity head to give the loss at the component.

This is

$$h_{LM} = M\frac{V^2}{2g} \tag{7}$$

where V is the mean line velocity and g is the gravitational constant. In terms of the discharge this is

$$h_{LM} = K_M Q^2 \tag{8}$$

where

$$K_M = \frac{.02517\ M}{D^4} \tag{9}$$

The pump head is expressed in two ways.

$$E_P = \frac{Z_P}{Q} \tag{10}$$

For this expression the horsepower put into the system by the pump is given as HP and

$$Z_P = \frac{550\ HP}{\gamma} \tag{11}$$

where γ = specific weight of the liquid ($\#/ft^3$). Alternately the pump head can be expressed as

$$E_P = A + BQ + CQ^2 \tag{12}$$

where A, B, and C are coefficients of a parabolic characteristic curve which defines the pump operation in the vicinity of the operating point. Since this expression is only valid over a specified range it should not be indiscretely employed in an analysis.

The basic energy equation for a loop or a path between terminal energy points is:

$$\Sigma(h_{LP} + h_{LM}) = \Delta E + \Sigma E_P \tag{13}$$

Here ΔE is the energy difference between the terminal energy points. This equation can be linearized in terms of a flowrate Q_i in the vicinity of the solution. This is done as follows

$$h_{LP} = h_{LPi} + \Delta h_{LP} = K_P Q_i^{\,n} + nK_P Q_i^{\,n-1}(Q-Q_i) \tag{14}$$

$$h_{LM} = h_{LMi} + \Delta h_{LM} = K_M Q_i^{\,2} + 2K_M Q_i(Q_i - Q) \tag{15}$$

$$E_P = E_{Pi} + \Delta E_P = \frac{Z_P}{Q_i} - \frac{Z_P}{Q_i^{\,2}}(Q - Q_i) \tag{16}$$

or:

$$E_P = A + BQ_i + CQ_i^{\,2} + (B + 2CQ_i)(Q - Q_i) \tag{17}$$

With these substitutions eqn. 13 can be expressed as a linear function of Q as

$$\Sigma\left(n\, K_P Q_i^{\,n-1} + 2K_M Q_i + \frac{Z_P}{Q_i^{\,2}}\right) Q =$$

$$\Sigma\left(\frac{2Z_P}{Q_i} + (n-1)\, K_P Q_i^{\,n} + K_M Q_i^{\,2}\right) + \Delta E \tag{18}$$

For the alternate form of the pump head this equation is

$$\Sigma(n\, K_P Q_i^{\,n-1} + 2K_M\, Q_i - B - 2CQ_i)Q =$$

$$\Sigma(A - CQ_i^{\,2} + (n-1)K_P Q_i^{\,n} + K_M Q_i^{\,2}) + \Delta E \tag{19}$$

Equation 18 (or 19) is employed to formulate an equation for each loop ($\Delta E = 0$) and t - 1 terminal energy equations which combine with the j continuity equations to for a set of P simultaneous linear equations in terms of the flowrate in each pipe.

Path method - The same notation previously defined in the description of the linear method is used. The basis of this method is to compute a flow correction ΔQ which when added to an initial set of flowrates (which satisfy continuity) will tend to satisfy the energy equation for each path. This is

$$\Sigma(h_{LP} + h_{LM}) = \Delta E + \Sigma E_P$$

In terms of the initial flowrate Q_i and the flow correction ΔQ these terms are

$$h_{LP} = h_{LPi} + \Delta h_{LP} = K_P Q_i^{\,n} + nK_P Q_i^{\,n-1} \Delta Q \tag{20}$$

$$h_{LM} = h_{LMi} + \Delta h_{LM} = K_M Q_i^{\,2} + 2K_M Q_i \Delta Q \tag{21}$$

$$E_P = E_{Pi} + \Delta E_P = \frac{Z_P}{Q_i} - \frac{Z_P}{Q_i^{\,2}} \Delta Q \tag{22}$$

or

$$E_P = A + BQ_i + CQ_i^{\,2} + (B - 2CQ_i)\Delta Q \tag{23}$$

These can be solved to give a flow correction as

$$\Delta Q = \frac{\Delta E - \Sigma(K_P Q_i^{\,n} + K_M Q_i^{\,2} - \frac{Z_P}{Q_i}}{\Sigma(nK_P Q_i^{\,n-1} + 2K_M Q_i + \frac{Z_P}{Q_i^{\,2}})} \tag{24}$$

or

$$\Delta Q = \frac{\Delta E - \Sigma(K_P Q_i^{\,n} + K_M Q_i^{\,2} - (A + BQ_i + CQ_i^{\,2}))}{\Sigma(nK_P Q_i^{\,n-1} + 2K_M Q_i - B - 2CQ_i)} \tag{25}$$

Using either eqn. 24 or eqn. 25 a flow correction is computed for each path and the flowrates of the pipes in that path are corrected by this amount.

For each method the following information must be available before the hydraulic analysis can be made. For each line the length, diameter and the Hazen Williams Roughness coefficient must be known. This latter parameter is available in handbooks and depends on the type and condition of the pipe. Valves, bends, meters, etc. are included in the analysis by determining the minor loss coefficient for the components. The minor loss coefficient is defined as a constant which multiplies the velocity head in the line to give the head loss at that component. In many cases a standard value for this coefficient is given in various references. This coefficient can also be easily determined if discharge-head loss data is available for the component. Several components can be included in a line by summing their minor loss coefficients.

Pumps can be included in two ways. The useful horsepower (or kilowatts) which the pump puts into the system may be specified. Alternately the coefficients of a parabolic characteristic curve may be specified. This curve represents the pump head-discharge relationship as shown below.

In the normal range of pump operation this relationship can be described closely by

$$E_P = A + BQ + CQ^2$$

where A, B, and C are coefficients of the fitted curve. If this representation of the pump is used, however, the solution must yield a discharge in the normal range of operation or the solution will be invalid. This is because the characteristic curve is not valid outside that range.

For each junction the external inflow or outflow is specified and the elevation of the junction is known.

Program description and users options - The programs were developed for use by practicing engineers and were offered to engineers on several bases. Material on the programs has been provided to over 150 engineering firms and individuals. The following brief release provided information to potential users:

Two programs have been developed at the University of Kentucky which will analyze pressure and flow in any pipe system and are available to potential users. These programs are written in FORTRAN IV, G Level and a users guide with source program listings and examples has been prepared. A brief description of the programs follow.

I Program based on linearized system equations -

This program utilized a new procedure for pipe systems analysis which has several advantages over conventional methods. Because this method simultaneously computes the flow in each pipe, the convergence is very fast (usually 3-4 trials to very high accuracy regardless of size of the system). Also, convergence is assured. Initial flowrates are not assumed and changes in flow system demand only require a change in data pertaining to that demand. However, since matrix methods are employed a computer of sufficient storage must be available to use this method for a system of n pipes approximately (n x (n+35) dimensioned storage locations must be available. The IBM 360-65 computer at the University of Kentucky, for example, will handle systems up to 220 pipes with its present storage capacity and without using additional disc storage. The procedure is fast. A 37 pipe system can be analyzed in about 10 seconds while a 125 pipe system takes 2 minutes and 15 seconds on the University of Kentucky computer.

II Program based on loop and path flow adjustments -

This program is essentially an extension of a loop balancing method similar to the Hardy Cross technique to any type of pipe flow system with pumps, valves, etc. included. It does require as input data initial flowrates which satisfy continuity. In rare cases the Hardy Cross procedure does not produce convergence and this situation could occur. However, for most situations the program produces a fast accurate solution. In addition much less storage is required so a large system can be analyzed with a computer of limited storage capacity.

Basic features of both procedures are:

1. Any piping configurations can be analyzed (closed loop networks, tree systems of combinations).
2. Flow units of CFS, GPM, MGD or SI units (M^3/s) can be used.
3. Pump, valves and other lossy components, and storage tanks can be included in the pipe system.
4. Pressures and hydraulic grades at indicated points in the system are output in addition to head changes at pumps, valves and in lines.
5. Data preparation for both programs is straightforward and very similar and allows any number of changes in system parameters (pipe sizes, pump characteristics, flow demands, etc.) to be investigated in a single computer run.

The programs are available to interested users in one of the following ways:

1. Attend two day short course at the University of Kentucky - announcement attached. This is the best means of gaining the necessary experience for using the programs effectively and is especially recommended for persons not presently using computers for hydraulic analysis. All material and computer source programs are provided for participant. Some post-course consultation and

a post-course laboratory problem chosen by the participant provide additional aid in implementing the programs.

2. Participate in users course on a correspondence basis. This is primarily for users who have an interest in developing the capability of using the programs but cannot attend a short course. It is desirable that the pariticipant have some background in the use of computers. All material and source programs are provided for the participant in addition to problems. The data for these problems are coded and returned for computer processing. In addition the participant may code and submit data for an additional problem over a period of a year which is of interest to him. Systems of up to 50 pipes will be processed as part of the course (larger systems require a nominal additional charge for computing expenses). Consultation regarding the application of the programs to pipe systems will be provided by phone or mail.

3. Obtain material only. This is primarily for users who are already using a digital computer for hydraulics problems. Users guides, program listings and examples will be sent for both programs.

Complete details of the programs, program listings and examples are provided in the users manuals (4, 5).

Program for optimum design of water distribution networks.

A major effort to develop a programmable procedure for minimum cost design was made. Details of this effort are included in a Ph. D. thesis (3). The salient points of this effort will be covered in this report.

Problem definition - In designing a hydraulic network distribution system, the engineer has not only to meet the demands at particular points in the system, but also should do so within specified constraints and at the least possible cost. For this study the geometrical configuration of the network is prescribed. The cost is a function of diameter and flow and the constraints can be regarded as of three types: (i) hydraulic (Kirchhoff laws), (ii) pressure and (iii) diameter. There are also different classes of constraints within each type. For example some pressure constraints are of the type that the pressure must be greater than or equal to a minimum while another constraint is that the pressure must not exceed some maximum value. The diameter constraints are normally of two kinds; the first is that no diameter should be less than a certain minimum, the other that the diameters should be available on the market. This becomes necessary since pipe diameters are made commercially in certain discrete sizes. The problem is therefore to find a set of pipe diameters to satisfy all the constraints at the least cost. The method used to do this is a combination of steepest descent (ascent) and dynamic programming.

Since the cost function involves flow in pipes, it is necessary to calculate the flow quickly. Flow is also important in the calculation of pressure since pressure is a function of flow. To compute flow quickly the method of linear analysis which was developed for this purpose is employed.

Problem formulation - Any problem of optimization has essentially two characteristics (1) a cost function and (2) one or more constraints. For the hydraulic network these are described as follows:

(1) Cost function -- The cost function used for network optimum design is divided into two parts: (a) Capital and (b) operation and maintenance costs. For the capital cost the result of the regression analysis performed by Linaweaver and Clark is used. This analysis was carried out on pipe line data for oil, gas and water pipe lines and gives a relationship between the variables, diameter, D, in inches and the capital cost, in dollars, per mile. This relationship is given by

$$\text{Capital Cost} = 1890\ D^{1.29} \text{ per mile}$$

or

$$\text{Capital Cost} = 0.358^{1.29} \text{ per foot}$$

The correlation coefficient is 0.98 according to the article.

At the time of this survey the Engineering News Record Construction Cost Index (ENRI) was 877. This enables the capital cost relationship to be updated by the ratio, PRESENT ENRI/877. The procedure described is used herein for the capital cost portion of the cost analysis. Alternate schemes could be used to express the capital cost as some continuous function of pipe diameter.

The capital expenditure is usually incurred at the time of construction of the project, and is paid back over the life of the project. During that time, the value of money is determined by the rate of interest, i% per annum. In order to spread the capital cost evenly over the whole life of the project, it is necessary to multiply the initial cost by a capital recovery factor (crf) where:

$$crf = \left[\frac{i}{100}\left(1 + \frac{i}{100}\right)^{nn}\right] \Bigg/ \left[\left(1 + \frac{i}{100}\right)^{nn} - 1\right]$$

where nn = life of project in years.

Thus in a system of m pipes the annual capital cost is

$$\sum_{i=1}^{m} 0.358\, L_i D_i^{1.29}\,(crf)\,\frac{(\text{PRESENT ENRI})}{877}$$

where L_i = length in feet of i-th pipe.

The operation and maintenance portion of the cost function is obtained by first equating the pumping power required for each pipe to that of an equivalent number of kilowatt-hours and then multiplying the number of Kilowatt-hours by the corresponding unit cost. The power utilized in a pipe is related to the corresponding head loss. The Hazen-Williams empirical expression for head loss is used. This expression is $H_L = KQ^{1.8518}$

where $$K = \frac{4.77L(12)^{4.87}}{C^{1.8518}D^{4.87}} \qquad (26)$$

and C = roughness coefficients.

The power lost in the pipe line is related to H_L in the following way:

Power loss = $\frac{H_L Q \gamma}{550}$ horse power (where Q is in cfs and γ is in the specific weight of the liquid in Lb/ft^3).

Inserting the head loss equation in this expression gives:

Power loss = $\frac{KQ^{2.8518}}{550}$ (62.4) horse power (assuming water is the liquid).

The final annual cost due to maintenance and operation in a pipe can be expressed as:

$$KQ^{2.8518} \frac{(62.4)}{550} (0.746)(365 \times 24)c \text{ per year}$$

where c = unit cost of electricity in \$ per KwH. Thus the total cost function (R) is

$$R = \sum_{i-1}^{m} 0.358\, L_i D_i^{1.29} (crf)(\text{Present ENRI}/877) +$$

$$\sum_{i=1}^{m} K_i Q_i^{2.8518} \frac{(62.4)}{550} (0.746)(365 \times 24)c = R \qquad (27)$$

The factor (365 x 24) assumes that the system is in operation for the whole year. If this is not the case, then this factor can be replaced by the anticipated number of hours in the year that the system will be in operation.

(2) Constraints -- The primary constraint is one which requires the flows to obey basic hydraulic relationships involving continuity of flow at junctions and head losses in the individual loops.

Another type of constraint, dealing with pressures, assumes alternate forms. The system must be designed for a maximum value of pressure which must not be exceeded.

There also may be a minimum pressure required for each junction which is necessary to maintain acceptable system performance. An example of such a minimum is that required by fire-fighting activities for which standards have been developed by National Board of Fire Underwriters (NBFU)(33). It is assumed that though there are variations in minimum pressure, there is none in the maximum.

Acceptable pipe diameters are also constrained. NBFU recommends that the diameter of street mains should not be less than 6-inches. This is again a fire-protection provision where the primary consideration is to obtain an acceptable quantity of water. In addition, diameters available on the commercial market are discrete and not continuous. A 6-inch pipe may be available, while 6.25-inch usually is not. It is therefore necessary that the sizes selected for design must be commercially available.

(3) The mathematical model -- The problem is summarized as follows:

The cost function to be minimized is

$$R = \sum_{i=1}^{m} 0.358\, L_i D_i^{1.29} \frac{(\text{Present ENRI})}{877} (\text{crf}) +$$

$$\sum_{i=1}^{m} K_i Q_i^{2.8518} \frac{(62.4)}{550} (0.746)(365 \times 24) c$$

Subject to the following constraints:

(i) pressure and flow obey basic hydraulic relationships

(ii) $p \leq$ ABMAX where p is pressure and ABMAX is absolute maximum pressure

(iii) $p \geq$ ABMIN where ABMIN is the absolute minimum pressure allowed

(iv) $p_A \geq P_A$ where p_A is pressure at junction A and P_A is the minimum allowed at junction A

(v) $D \geq DMIN$ where DIMIN is the absolute minimum size diameter allowed

(vi) $D \epsilon (d_1, d_2, d_3, \ldots, d_n)$ where d_1, d_2, ..., d_n are the available commercial size diameters.

Method of Solution The law of continuity is expressed in terms of flow ($\Sigma Q_i = 0$) while the "head loss" or loop equations ($\Sigma K_i Q_i^n = 0$) are functions of flow and pipe properties. The Hazen-Williams line loss expression is used herein. Thus from (26)

$$K = \frac{4.77L(12)^{4.87}}{C^{0.8518} D^{4.87}}$$

which is referred to as the loss coefficient. Flow is given by

$$Q = Av \qquad (28)$$

where A is the cross-sectional area and v the velocity

It can be demonstrated that the differential dQ is given by

$$dQ = \frac{2Q}{D} dD + A\, dv \qquad (29)$$

To preserve continuity, the algebraic sum of the changes in flow at any junction must be zero.

Thus at any junction

$$\Sigma\, dQ_i = 0 \qquad (30)$$

and from eqn. (29)

$$\Sigma A_i dv_i = -2 \Sigma \frac{Q_i}{D_i} dD_i \qquad (31)$$

It is clear that by considering changes in flow at many junctions, equations involving $A_i dv_i$ and dD_i can be obtained. It also follows from the continuity equations that one of these equations would be redundant. The system equations can be expressed in the following form:

$$
\begin{bmatrix} \text{Matrix coefficients} \\ \text{are } 1, 0, -1 \\ \\ \\ \\ \\ \\ \end{bmatrix}
\begin{bmatrix} A_1 dv_1 \\ A_2 dv_2 \\ A_3 dv_3 \\ \cdot \\ \cdot \\ \cdot \\ A_m dv_m \end{bmatrix}
=
\begin{bmatrix} \text{Matrix} \\ \text{coefficients} \\ \text{are } -\frac{2Q}{D} \\ \text{or } 0 \\ \\ \\ \\ \end{bmatrix}
\begin{bmatrix} dD_1 \\ dD_2 \\ dD_3 \\ \cdot \\ \cdot \\ \cdot \\ dD_m \end{bmatrix}
\qquad (32)
$$

By considering first order derivatives of the loop equations, another set of equations involving $A_i dv_i$ and dD_i can be obtained as follows:

$$H_L = KQ^{1.8518}$$

is differentiated to obtain

$$dH_L = Q^{1.8518} \frac{dK}{dD} dD + 1.8518\ KQ^{0.8518} \frac{\partial Q}{\partial D} dD$$

$$+ 1.8518\ KQ^{0.8518} \frac{\partial Q}{\partial v} dv \qquad (33)$$

From (29) $\frac{\partial Q}{\partial D} = \frac{2Q}{D}$ and $\frac{\partial Q}{\partial v} = A$

By substituting in (35)

$$dH_L = Q^{1.8518} \frac{dK}{dD} dD + 1.8518\ KQ^{0.8518} \frac{2Q}{D} dD$$

$$+ 1.8518\ KQ^{0.8518} Adv \qquad (34)$$

From (26) $$K = \frac{4.77L(12)^{4.87}}{C^{1.8518} D^{4.87}}$$

By differentiation

$$\frac{dK}{dD} = -\frac{4.87}{D^{5.87}} \frac{4.77L(12)^{4.87}}{C^{1.8518}} = -4.87 \frac{K}{D} \qquad (35)$$

Substitute (35) for $\frac{dK}{dD}$ in (34), then

$$dH_L = -4.87 \frac{KQ^{1.8518}}{D} dD + 3.7036 \frac{KQ}{D} dD + 1.8518 KQ^{0.8518} Adv \quad (36)$$

Simplifying and combining terms give

$$dH_L = -1.1664 \frac{KQ^{1.8518}}{D} dD + 1.8518 KQ^{0.8518} Adv \quad (37)$$

These equations of the form $\Sigma dH_L = 0$ can be transformed into

$$1.8518 \sum KQ^{0.8518}(Adv) = 1.1664 \sum \frac{K}{D} Q^{0.8518} dD \quad (38)$$

or in matrix form:

$$\begin{bmatrix} \text{Matrix Coefficients} \\ \text{are either} \\ \pm 1.8518\ KQ^{0.8518} \\ \text{or } 0 \\ \\ \end{bmatrix} \begin{bmatrix} A_1 dv_1 \\ A_2 dv_2 \\ A_3 dv_3 \\ \vdots \\ A_m dv_m \end{bmatrix} = \begin{bmatrix} \text{Matrix coefficients} \\ \text{are either} \\ \pm 1.1664 \frac{KQ^{1.8518}}{D} \\ \text{or } 0 \\ \\ \end{bmatrix} \begin{bmatrix} dD_1 \\ dD_2 \\ dD_3 \\ \vdots \\ dD_m \end{bmatrix} \quad (39)$$

By combining all independent equations derived from the basic hydraulic relations, (32) and (39), the following simultaneous equations are obtained.

$$\begin{matrix} \text{CONTINUITY EQUATIONS} \\ \\ \text{HEAD LOSS EQUATIONS} \end{matrix} \begin{bmatrix} \text{Matrix Coefficients} \\ \text{are } \pm 1 \text{ or } 0 \\ \hline \text{Matrix Coefficients} \\ \text{are} \\ \pm 1.8518\ KQ^{0.8518} \\ \text{or } 0 \end{bmatrix} \begin{bmatrix} A_1 dv_1 \\ A_2 dv_2 \\ A_3 dv_3 \\ \\ \\ A_m dv_m \end{bmatrix} = \begin{bmatrix} \text{Matrix} \\ \text{coefficients are} \\ -\frac{2Q}{D} \text{ or } 0 \\ \hline \text{Matrix} \\ \text{coefficients are} \\ \pm 1.1664 \frac{KQ^{1.8518}}{D} \\ \text{or } 0 \end{bmatrix} \begin{bmatrix} dD_1 \\ dD_2 \\ dD_3 \\ \\ \\ dD_m \end{bmatrix} \quad (40)$$

It is observed that, if m is the number of pipes in the network then the matrices on both sides of equations (40) are of order m x m.

The cost function, as given by eqn. (27) is expressed in terms of the diameter, D, flow Q, and Hazen-Williams' loss coefficients. Q is a function of D and v, K is a function of D. Thus the cost function can be expressed in terms of D and v and the first derivative of the cost function can be put in a form similar to (40).

The cost function previously defined can be written as

$$R = \Sigma a_i D_i^{1.29} + \Sigma b_i K_i Q_i^{2.8518}$$

where a_i and b_i are constants. Differentiation of R with respect to D_i's and v_i's gives

$$dR = \left(1.29\, \Sigma a_i D_i^{0.29} + \Sigma b_i \frac{\partial K_i}{\partial D_i} Q_i^{2.8518}\right) dD_i$$

$$+\ 2.8518\ \Sigma b_i K_i Q_i^{1.8518} \frac{\partial Q_i}{\partial D_i} dD_i$$

$$+\ 2.8518\ \Sigma b_i K_i Q_i^{1.8518} \frac{\partial Q_i}{\partial v_i} dv_i$$

Since $\frac{\partial K_i}{\partial D_i} = -4.87 \frac{K_i}{D_i}$ (from 35) and from (29) $\frac{\partial Q_i}{\partial D_i} = \frac{2Q_i}{D_i}$

and $\frac{\partial Q_i}{\partial v_i} = A_i$ the derivative becomes

$$dR = 1.29\, \Sigma a_i D_i^{0.29} dD_i + 0.8336\ \Sigma b_i \frac{K_i Q_i^{2.8518}}{D_i} dD_i$$

$$+\ 2.8518\ \Sigma b_i K_i Q_i^{-.8518} (A_i dv_i) \qquad (41)$$

This can be expressed in matrix form as follows:

$$\begin{bmatrix} 1 & e_1 & e_2 & e_3 \cdots e_m \end{bmatrix} \begin{bmatrix} dR \\ A_1 dv_1 \\ A_2 dv_2 \\ \vdots \\ A_m dv_m \end{bmatrix} = \begin{bmatrix} f_1 & f_2 & f_3 \cdots f_m \end{bmatrix} \begin{bmatrix} dD_1 \\ dD_2 \\ dD_3 \\ \vdots \\ dD_m \end{bmatrix} \qquad (42)$$

where $e_1, e_2, e_3, \ldots, e_m$ are coefficients of A_1dv_1, A_2dv_2, $A_3dv_3, \ldots, A_mdv_m$ and $f_1, f_2, f_3, \ldots, f_m$ are coefficients of $dD_1, dD_2, dD_3, \ldots, dD_m$.

By combining (40) and (42) the left hand side becomes a matrix of order (m+1)x (m+1) and the right hand one of (m+1)x m. It is also observed that "dR" only occurs once and that it is on the left hand side. Thus one of the columns on this side would have coefficients of

$$1,\ 0,\ 0,\ 0,\ \ldots 0.$$

Thus the following sets of equations are obtained

$$\begin{array}{c|c} \text{CONTINUITY EQUATIONS} & \\ \hline \text{LOOP EQUATIONS} & \end{array} \underbrace{\begin{bmatrix} 1 & e_1 \quad e_2 \cdots e_m \\ 0 & \text{Matrix coefficients are } \pm 1 \text{ or } 0 \\ 0 & \\ \hline & \text{Matrix coefficients are } \pm 1.8518KQ^{0.8518} \text{ or } 0 \\ 0 & \end{bmatrix}}_{(m+1)(m+1)} \begin{bmatrix} dR \\ A_1dv_1 \\ A_2dv_2 \\ A_3dv_3 \\ \vdots \\ A_mdv_m \end{bmatrix} = \underbrace{\begin{bmatrix} f_1 \quad f_2 \quad f_3 \cdots f_m \\ \text{Matrix coefficients are } -\frac{2Q}{D} \text{ or } 0 \\ \hline \text{Matrix coefficients are } \pm 1.1664\frac{KQ^{1.8518}}{D} \text{ or } 0 \end{bmatrix}}_{(m+1)(m)} \begin{bmatrix} dD_1 \\ dD_2 \\ dD_3 \\ \vdots \\ dD_m \end{bmatrix}$$

(43)

Before (43) can be used it is necessary to have values of Q_i and D_i. Values of D are assumed and values of Q are calculated. The pressure at the junctions are now computed and, if the pressure constraints are broken, new values of D are assumed and the process repeated until these constraints are satisfied.

In this initial stage, if new values of D are required it has been found convenient to add or subtract a constant increment to each of the diameters. It is usual to add the increments, but if the pressure at any point is greater than ABMAX the increment is subtracted.

After values of D and Q are obtained which are feasible solutions the partial derivatives with respect to diameters and velocities are computed and the matrix coefficients for (43) are not known. From these equations, by manipulating the matrices the following is obtained:

$$\begin{bmatrix} dR \\ A_1 dv_1 \\ A_2 dv_2 \\ A_3 dv_3 \\ \vdots \\ A_m dv_m \end{bmatrix} = \begin{bmatrix} \\ (m+1) x\ m \\ \text{matrix} \\ \\ \end{bmatrix} \begin{bmatrix} dD_1 \\ dD_2 \\ dD_3 \\ \\ \\ dD_m \end{bmatrix} \quad \text{-------------} \quad (44)$$

Since

$$dR = g_1 dD_1 + g_2 dD_2 + g_3 dD_3 + \ldots + g_m dD_m$$

(where $g_1, g_2, \ldots, g_m$ are coefficients of the first row of matrix in (44), then

$$\frac{dD_1}{g_1} = \frac{dD_2}{g_2} = \frac{dD_3}{g_3} = \ldots . = \frac{dD_m}{g_m} = \rho \qquad (45)$$

To obtain an optimal value of ρ, using the method of steepest ascent, it is necessary to solve the equation $\frac{dR}{d\rho} = 0$. This is a complex equation and calculating the value of ρ to satisfy it is very cumbersome and at best approximate. The value assigned to ρ is a critical factor in using the steepest ascent (descent) method. Too big a value of ρ oversteps the optimum, too small a value requires too many unnecessary calculations. This makes it difficult to find the optimum value of ρ and thus a form of direct search technique was developed to determine this value.

Of the coefficients, $g_1, g_2, g_3, \ldots, g_m$ the one with the biggest absolute value is chosen, say g_k, and then dD_k corresponding to g_k is given a value. The ratio dD_k/g_k is now known and from (45) all values of $dD_1, dD_2, \ldots, dD_m$ are computed. These increments, $dD_1, dD_2, dD_3, \ldots, dD_m$ may be either positive or negative. If g_k is negative these increments are first of all algebraically added and if g_k is positive they are subtracted.

If there is a reduction in cost and no constraints are broken, then the process of algebraically adding or subtracting can be continued, or new partial derivatives computed and the entire procedure so far repeated. If there is no reduction in cost within the specified constraints then the increments are halved and the process of algebraically adding or subtracting is continued. If again there is no reduction, the procedure of halving increments and algebraically adding or subtracting is continued until there is such reduction or the tolerance level is reached. At this stage new matrix coefficients are computed and the whole process is repeated. Thus, for an initial increment size of two inches and tolerance level of half inch, the possible increments likely to be tried after the first are one inch and half-inch.

The process eventually converges to a situation where changes in cost are negligible. At this stage most of the diameters do not satisfy constraint number (vi) that is D_i $(d_1, d_2, d_3, \ldots, d_m)$ and the final step in the procedure is to impose this condition.

If D_i is the diameter corresponding to g_i where g_i >

any other g and D_i lies between d_p and d_q -- where d_p and d_q are members of the class $(d_1, d_2, d_3, \ldots, d_m)$ and D_i is not -- then D_i is increased or decreased to d_p or d_q by making $dD_i = d_p - D_i$ or $dD_i = d_q - D_i$. The other dD's are computed and all the diameters are altered proportionately. The cost is computed for both cases when $dD_i = d_p - D_i$ and $dD_i = d_q - D_i$. The cost which is cheaper is noted and the corresponding diameters are chosen.

From now on the diameter for the i-th pipe is fixed and $dD_i = 0$. The derivative of cost function with respect to D_i will not be a term of equation (40), also any derivative with respect to D_i that is involved in the matrices of (40). The process is repeated until all the diameters are now in an acceptable set.

If, in making incremental changes in diameters, constraint (v) is broken, ($D \geq$ DIMIN) and if the diameter is not that of the pipe with the largest absolute partial derivative of cost function, the diameter of the pipe breaking this constraint will be assigned the value of DIMIN. The diameter increments for the other pipes will be unaffected. If the pipe breaking the diameter constraint is the one with the largest absolute partial derivative of cost function, then not only is this diameter assigned the value DIMIN, but the changes in the other diameters are proportionately adjusted. If a pressure constraint is broken the incremental changes are reduced proportionately until the pressure constraint is satisfied.

The method just outlined above is dependent on the shape of the cost function to obtain the optimum. If the function is convex then the global optimum is obtained. The pressure constraints, especially (iv) ($p_A \geq P_A$) may have the effect of rendering the function non-convex, i.e., a "hole" in the feasible region. In this case it is advisable to ignore the pressure constraint in the initial stages of the process.

Computer Program - Using the method just described a computer

FLOWCHART FOR OPTIMUM DESIGN

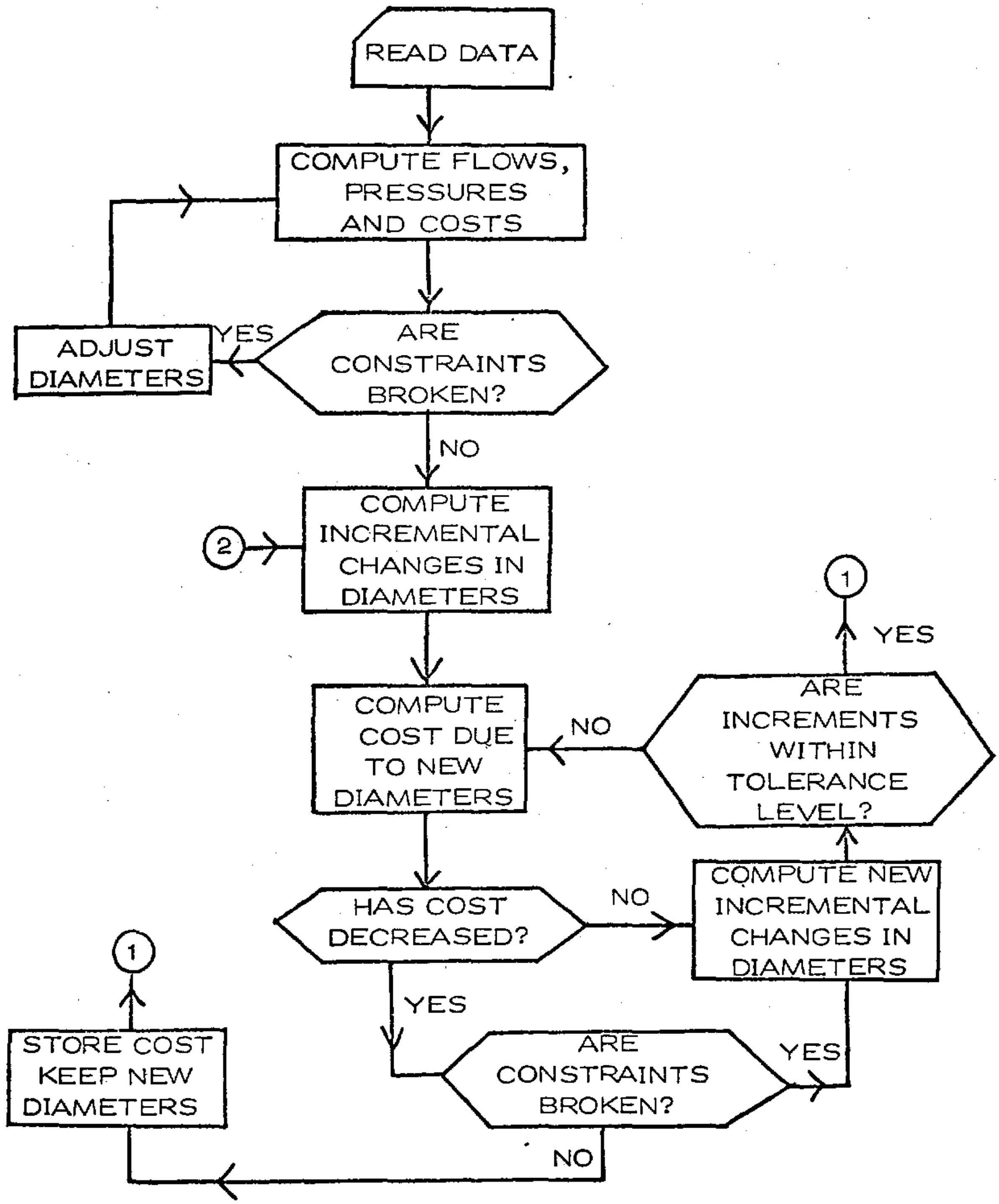


FIGURE 1A

FLOWCHART FOR OPTIMUM DESIGN
(CONT'D)

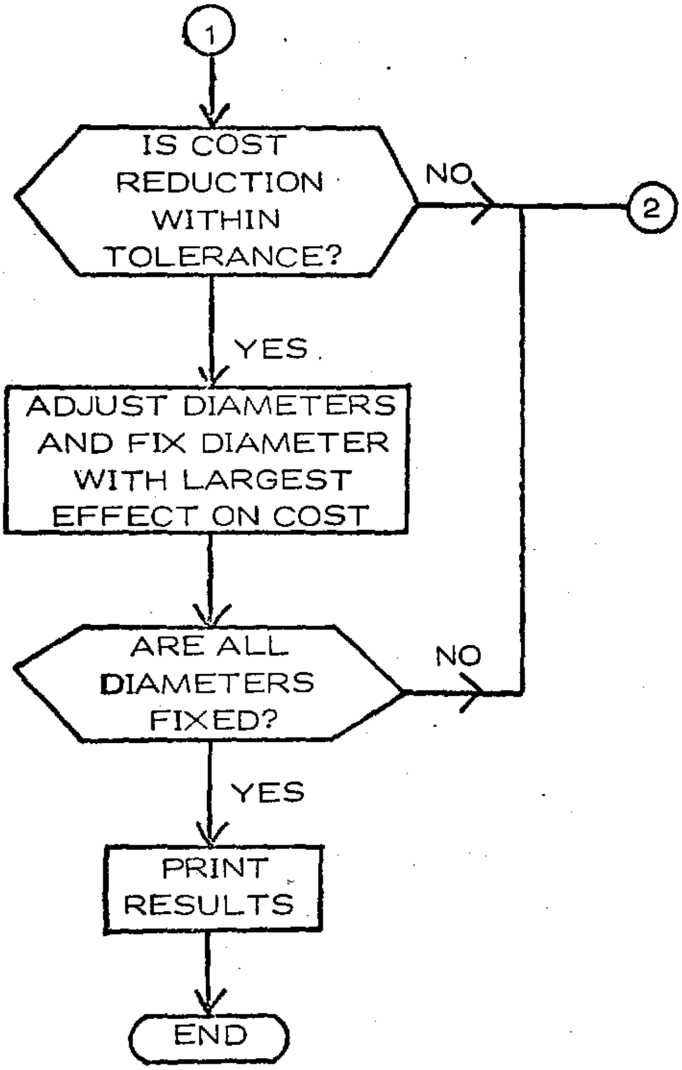
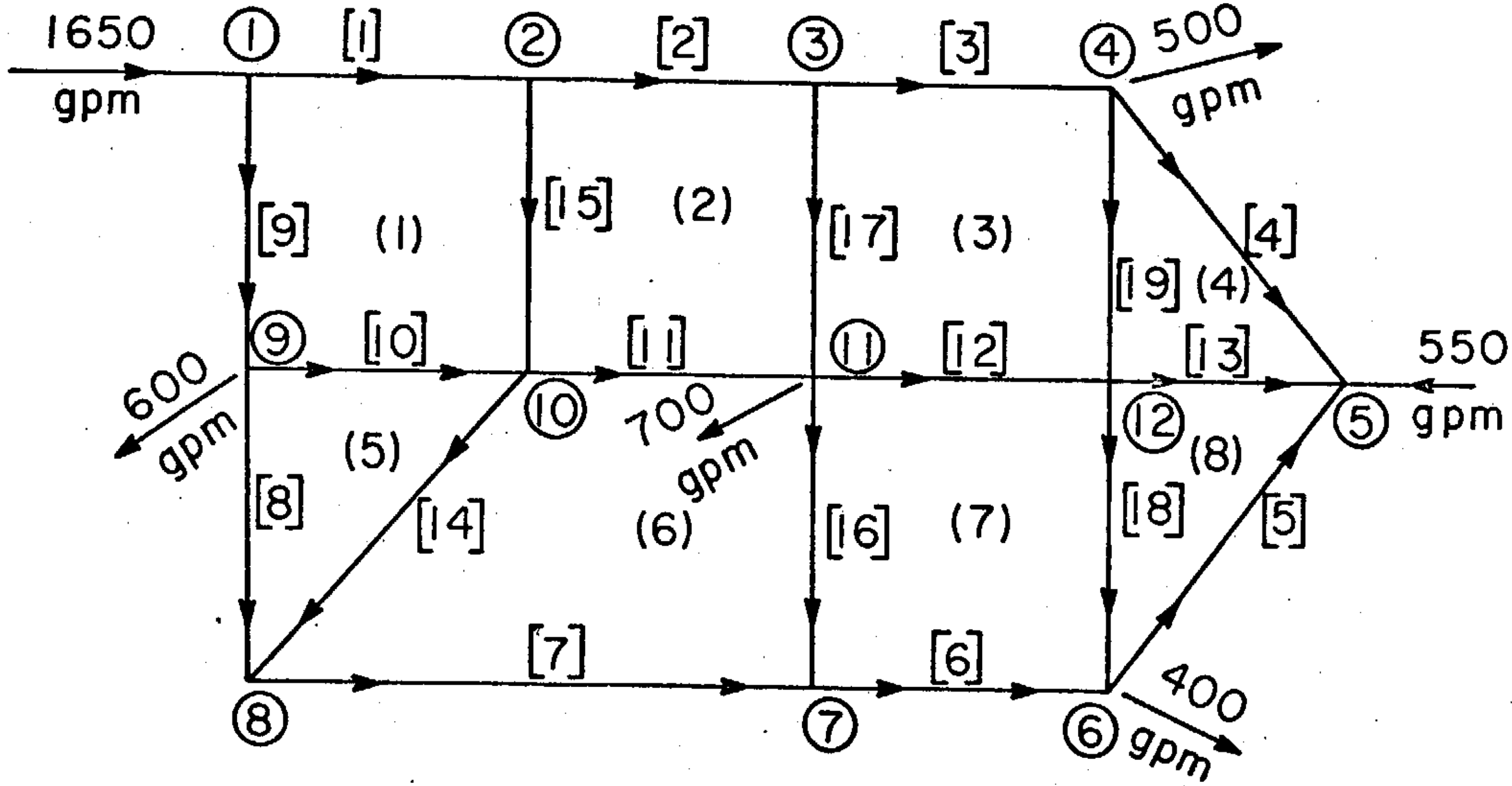


FIGURE 1B

program was written for the optimum design of closed pipe networks. A flowchart depicting the logic of this program is given in Figure 1. A listing of the program is presented in Appendix I. The program is fairly involved and the reader is referred to Reference 5 for the details.

Example - A pipe network of nineteen pipes was analyzed by this program to determine the minimum cost design. Figure 2 shows the geometry of the network. The computer output for this example is presented on the following pages to illustrate the type of information output by the computer. For the given cost information and constraints it would have been highly unlikely that this configuration could have been determined without such an aid as this program. At the present time additional efforts are being made to refine and improve this computer program. When this is completed the program will be made available to potential users.



Code: Pipe Number [ ]
Junction Number ◯
Loop Number ( )
Assumed Flow Direction ⟶

Figure 2 Hydraulic Network for Example (to convert flowrate to m^3/s multiply gpm by 6.309×10^{-5})

Pipe Number	Diameter (in.)	Length (ft.)	Roughness
1	12.0	1500	130
2	8.0	1000	130
3	8.0	1200	120
4	8.0	2000	120
5	8.0	2800	120
6	8.0	1100	120
7	8.0	1000	120
8	8.0	2500	120
9	8.0	800	100
10	6.0	1300	100
11	6.0	1000	100
12	10.0	1100	130
13	10.0	1000	130
14	6.0	1800	120
15	6.0	1100	120
16	6.0	1800	120
17	10.0	1200	130
18	6.0	1800	120
19	6.0	1300	120

TABLE I PIPELINE DATA FOR EXAMPLE

COMPUTER OUTPUT FOR EXAMPLE

MAXIMUM PRESSURE = 150.000 LBS PER SQ IN

MINIMUM PRESSURE= 30.000 LBS PER SQ IN

MINIMUM PRESSURE ALLOWED AT JUNCTION 9 IS 50.000 LBS PER SQ IN

SMALLEST DIAMETER ALLOWED= 6.000 INCHES

GREATEST INCREMENTAL CHANGE IN DIAMETER= 6.000 INCHES

WHEN THE LARGEST INCREMENTAL CHANGE IN DIAMETERS IS LESS THAN OR EQUAL TO 0.750 INCHES
SUCH CHANGES ARE IGNORED

TOLERANCE ON FLOW=15.0000 GPM

LIFE OF PROJECT= 50.000 YEARS

RATE OF INTEREST= 5.000 PER ANNUM

COST OF ELECTRICITY=$ 0.01 PER KILOWATT-HOUR

TOLERANCE ON MONEY=$ 10.00

BUILD-UP FACTOR= 1.000

ENGINEERING NEWS RECORD INDEX= 877.

RESULTS OF OPTIMAL TRIAL

PIPE NO	JUNCTION		LENGTH	ROUGHNESS	DIAMETER-INCHES		FLOW	PRESSURE AT JUNCTIONS	
					ORIGINAL	FINAL		BEGIN	END
1	1	2	1500.00	130.	10.00	6.00	311.71	120.000	123.172
2	2	3	1000.00	130.	14.00	6.00	390.92	123.172	117.600
3	3	4	1200.00	120.	16.00	6.00	286.12	117.600	106.317
4	4	5	2000.00	120.	18.00	18.00	-1106.09	106.317	105.871
5	5	6	2800.00	120.	20.00	20.00	-631.90	105.871	108.346
6	6	7	1100.00	120.	12.00	6.00	498.41	108.346	124.691
7	7	8	1000.00	120.	24.00	16.00	2431.60	124.691	126.298
8	8	9	2500.00	120.	20.00	20.00	2611.29	126.298	121.777
9	9	1	800.00	100.	18.00	24.00	4638.29	121.777	120.000
10	9	10	1300.00	100.	16.00	6.00	227.00	121.777	123.541
11	10	11	1000.00	100.	14.00	6.00	327.47	123.541	112.683
12	11	12	1100.00	130.	12.00	6.00	265.47	112.683	114.023
13	12	5	1000.00	130.	10.00	6.00	87.99	114.023	105.871
14	10	8	1800.00	120.	12.00	6.00	-179.69	123.541	126.298
15	2	10	1100.00	120.	14.00	6.00	-79.22	123.172	123.541
16	7	11	1800.00	120.	16.00	12.00	-1933.19	124.691	112.683

17	3	11	1200.00	130.	18.00	6.00	104.81	117.600	112.683
18	6	12	1800.00	120.	20.00	6.00	69.68	108.346	114.023
19	4	12	1300.00	120.	24.00	6.00	-107.79	106.317	114.023

CAPITAL COST=$ 12350.43

OPTIMAL COST=$ 13970.22

CORE USAGE OBJECT CODE= 31512 BYTES,ARRAY AREA= 119800 BYTES,TOTAL AREA AVAILABLE= 170080 BYTES

DIAGNOSTICS NUMBER OF ERRORS= 0, NUMBER OF WARNINGS= 0, NUMBER OF EXTENSIONS= 0

COMPILE TIME= 4.45 SEC,EXECUTION TIME= 716.00 SEC, WATFIV - VERSION 1 LEVEL 3 MARCH 1971 DATE= 72/344

CONCLUSIONS

The linear method developed as a result of this study for the hydraulic analyses of water distribution systems is proving to be a valuable aid to practicing engineers. This is supported by the large number of engineers who are presently using the program.

This method is also a useful tool in the minimum cost study of water distribution systems. The feasibility of developing a routine for minimum cost design also has been established through this study. A working program for closed loop systems has been developed.

A method for solving the hydraulics of water distribution system on a standard analog computer also has been developed. This provides the basic tool for an analog-digital model for optimum network design. However, while this appears to be a very promising technique its practicality is limited. This is because most practicing engineers do not have access to the necessary analog-digital systems.

PUBLICATIONS

1. Wood, D. J., "Analog Analysis of Water Distribution Networks," Transportation Engineering Journal, Proceedings, ASCE, Vol. 97, No. TE2, May 1971, 281-290.

2. Wood, D. J., and Charles, C. O., "Hydraulic Network Analyses Using Linear Theory," Proceedings, ASCE, Vol. 98, No. HY7, July 1972, pp. 1157-1170.

3. Charles, C. O., "Optimum Design of Hydraulic Networks Using Steepest Descent (Ascent) Method and Dynamic Programming," Ph. D. thesis, University of Kentucky, January 1973 (unpublished).

4. Wood, D. J., "Users Guide for Linear Method of Analysis of Water Distribution Systems," Department of Civil Engineering, University of Kentucky, 1973 (unpublished).

5. Wood, D. J., "Users Guide for Path Method of Analysis of Water Distribution Systems," Department of Civil Engineering, University of Kentucky, 1973 (unpublished).

APPENDIX I

LISTING OF FORTRAN PROGRAM FOR OPTIMUM DESIGN OF CLOSED LOOP SYSTEMS

```
  1          IMPLICIT REAL*8 (A-H,O-Z)
  2          INTEGER UNITS,TREE(50)
  3          REAL*8 K(50),L(50),DABS,LIFE,DELEV(50),STAN(2500),SCAN(50)
  4         0DIMENSION A(50,50),C(50,50),SAVE(2500),CHAIN(2500),
            1ROUGH(50),DPRE(50),PRESS(50),HT(50),Y(50),PECED(50),D(50),DIA(50),
            ABODE(50),DEMAND(50),SD(50),SDELD(50),SQ(50),SDPRE(50),SK(50),
            2DELD(50),Q(50),LOOP(50,50),ML(50),MN(50),JBIGIN(50),JEND(50),LZ(50
            3) ,NPIPE(50)
  5          READ (5,1000) UNITS,MXX,MMX
  6     1000 FORMAT (16I5)
  7     1700 FORMAT (//18X,'RATE OF INTEREST=',F6.3,1X,'PER ANNUM')
  8     1800 FORMAT (//18X,'WHEN THE LARGEST INCREMENTAL CHANGE IN DIAMETERS IS
            1 LESS THAN OR EQUAL TO ',1X,F6.3,1X,'INCHES'  /58X,'SUCH CHANGES
            2ARE IGNORED')
  9     1900 FORMAT ('1',18X,'MAXIMUM PRESSURE =', F9.3,1X,'LBS PER SQ IN')
 10     2010 FORMAT (//18X,'MINIMUM PRESSURE=',F9.3,1X,'LBS PER SQ IN')
 11     2020 FORMAT (//18X, 'MINIMUM PRESSURE ALLOWED AT JUNCTION',I3,1X,'IS',
            1F9.3,1X,'LBS PER SQ IN')
 12     2030 FORMAT (//18X, 'TOLERANCE ON FLOW=', F7.4,1X, 'CFS')
 13     2031 FORMAT (//18X, 'TOLERANCE ON FLOW=', F7.4,1X, 'GPM')
 14     2032 FORMAT (//18X, 'TOLERANCE ON FLOW=', F7.4,1X, 'MGD')
 15     2040 FORMAT (//18X,'TOLERANCE ON MONEY=$', F8.2,1X)
 16     2050 FORMAT (//18X,'GREATEST INCREMENTAL CHANGE IN DIAMETER=', F6.3,1X,
            1'INCHES')
 17     2060 FORMAT (//18X, 'SMALLEST DIAMETER ALLOWED=',F6.3,1X,'INCHES')
 18     2070 FORMAT (//18X, 'LIFE OF PROJECT=', F7.3,1X,'YEARS')
 19     2080 FORMAT (//18X, 'COST OF ELECTRICITY=$',F5.2,1X ,'PER KILOWATT-HOUR
            1')
 20     2090 FORMAT (//18X, 'ENGINEERING NEWS RECORD INDEX=', F7.0)
 21     1701 FORMAT (//18X, 'BUILD-UP FACTOR=', F6.3)
       C     IF UNITS=1, FLOW IS IN CFS, IF UNITS=2, FLOW IS IS GPM, IF UNITS=3 FLOW IS
       C     IN MGD.  MXX=1 INDICATES THAT UNITS OF PRESSURE AT INLET ARE LBS/IN**2,
       C     NOT 'FOOT-HEAD OF WATER'
 22          READ (5,1000) NJ,NP
 23          READ (5,6000) ABMAX,ABMIN,DIMIN,DD1,DD2,DD3,DD4
       C     NJ= NO OF JUNCTIONS,NP=NO OF PIPES
 24          READ (5,2000) (JBIGIN(I),JEND(I),D(I),ROUGH(I),L(I), I=1,NP)
 25          TOTDEM =0.0
 26          TOTAL=0.0
 27          NN=0
 28          KXXK=0
 29     1111 FORMAT (1X,1H,10X,'O U T P U T')
 30          WRITE (6,1111)
 31          WRITE (6,1900) ABMAX
 32          WRITE (6,2010) ABMIN
 33     3000 FORMAT   (5F10.4)
 34     2000 FORMAT (2I5,F5.2,2F10.5)
 35          DO 11 I=1,NJ
 36          READ (5,3000)  PECED(I),DEMAND(I),DELEV(I),DPRE(I),HT(I)
       C     PECED(I) =1.0 INDICATES NODE IS AN INLET.  DEMAND (WHETHER INLET OR OUTLET
       C     ) IS ALWAYS POSITIVE. DELEV IS ELEVATION IN FEET. DPRE  AND  HT ARE IN LBS
       C     /IN**2 UNITS, EXCEPT WHEN PECED=1.0 AND MXX .NE. 1 - IN WHICH CASE DPRE
       C     MAY BE IN ' FOOT-HEAD OF WATER' UNITS FOR THAT PARTICULAR INLET
 37          IF (PECED(I) .NE. 1.0) TOTDEM =TOTDEM+ DEMAND(I)
 38          IF (PECED(I) .EQ. 1.0) TOTAL =TOTAL + DEMAND(I)
 39          IF (PECED(I) .NE. 1.0) DEMAND(I)=-DEMAND(I)
 40          IF (PECED(I) .EQ. 1.0) NN=NN+1
 41          IF (HT(I) .NE. 0.0) WRITE (6,2020) I,HT(I)
 42       11 CONTINUE
 43          IF (TOTDEM .EQ. TOTAL) GO TO 100
```

```
      WRITE (6,5000) TOTAL,TOTDEM
 5000 FORMAT(//20X,'TOTAL=',1X,F10.2,2X,'TOTDEM=',1X,F10.2,2X, 'INFLOW
     1IS DIFFERENT FROM OUTFLOW')
      STOP
  100 DO 12  I=1,NP
      SD(I)=0.0
      BODE(I)=D(I)
      IF (D(I) .LT. DIMIN) D(I)=DIMIN
      SCAN(I)=0.0
      Q(I)=1.0
      NPIPE(I)=0
      IF (I .GE. NJ) DEMAND(I)=0.0
      ML(I)=0
      IF (ML(I) .EQ. 0) MM=1
      IF (ML(I) .EQ. 0) LL=16
  101 READ (5,1000) (LOOP(I,J),J=MM,LL)
C     LOOP(I,J) MAY BE POSITIVE OR NEGATIVE ACCORDING TO ASSUMED DIRECTION OF
C     FLOW. (EFFLUX FROM JUNCTION IS POSITIVE, INFLUX IS NEGATIVE).  REGARD LAST
C     JUNCTION AS REDUNDANT AND IGNORE IT.  FROM I .EQ. NJ THRU I .EQ. NP
C     LOOP(I,J) REFERS TO INDIVIDUAL COMPONENTS OF LOOP EQUATIONS (I.E. SUM OF
C     HEAD LOSSES =0.0) WHILE FROM I .EQ. 1 THRU I .EQ. (NJ-1) LOOP(I,J) REFERS
C     INDIVIDUAL COMPONENTS OF CONTINUITY EQUATIONS
      DO 13 J=MM,LL
      IF (LOOP (I,J) .EQ. 0) GO TO 12
   13 ML(I)=ML(I)+1
      MM=MM+16
      LL=LL+16
      GO TO 101
C     IF J=16 OR MULTIPLE OF 16   SUPPLY A BLANK CARD
   12 CONTINUE
      IF (UNITS .EQ. 1) B=1.
      IF (UNITS .EQ. 2) B=.002228
      IF (UNITS .EQ. 3) B=1.5473
 6000 FORMAT (7F10.4)
          READ(5,9000) RATE,LIFE,CENTS,X,BUILD,ENR
 9000 FORMAT (8F10.5)
      KL=NJ-1
      READ (5,1000) (TREE(I),I=1,KL)
      IF (ENR .EQ. 0.0) ENR=877.
      FAC= (RATE)*.01*(1.+(RATE)*.01)**LIFE/((1.+(RATE)*.01)**LIFE -1.)
      IF (X .EQ. 0.0) X=1.0
      IF (BUILD .EQ. 0.0) BUILD=1.0
      WRITE (6,2060) DIMIN
      WRITE (6,2050) DD1
      WRITE (6,1800) DD2
      IF (UNITS .EQ. 1) WRITE (6,2030) DD4
      IF (UNITS .EQ. 2) WRITE (6,2031) DD4
      IF (UNITS .EQ. 3) WRITE (6,2032) DD4
      WRITE (6,2070) LIFE
      WRITE (6,1700) RATE
      WRITE (6,2080) CENTS
      WRITE (6,2040) DD3
      WRITE (6,1701) BUILD
      WRITE (6,2090) ENR
      N=1
      INN=16
  141 READ(5,1100) (DIA(I),I=N,INN)
      N=N+16
      INN=INN+16
 1100 FORMAT (16F5.2)
C     IF 16 OR A MULTIPLE OF 16,THEN A BLANK CARD MUST BE INSERTED
```

```
      IF (DIA(INN-16) .NE. 0.0) GO TO 141
      N1=0
      DO 44 I=1,INN
      IF (DIA(I) .NE. 0.0) N1=N1+1
      IF (DIA(I) .EQ. 0.0) GO TO 223
   44 CONTINUE
  223 INN=N1
      N=0
      AID=DD1
      LPL=0
      CHANGE=0.0
      EXTRA=0.0
      CT2=0.0
      KX=0
      CT1=0.0
      KK=0
      M=1
      LX=16
      IF (MMX .EQ. 0) GO TO 118
  809 READ (5,1000) (MN(I),I=M,LX)
      LX=LX+16
      M=M+16
      IF ((LX-16) .LT. MMX) GO TO 809
      DO 674 I=1,MMX
      LM=MN(I)
  674 NPIPE(LM)=1
 118  DO 14 I=1,NP
      IF (D(I) .LT. DIMIN) SCAN(I)=D(I)
      IF (D(I) .LT. DIMIN) D(I)=DIMIN
      K(I)=4.77*12.**4.87*L(I)*(DABS(Q(I)))**.8518/(ROUGH(I)**1.8518*
     1D(I)**4.87)
      IF (UNITS .EQ. 2) K(I)=.000012*K(I)
      IF (UNITS .EQ. 3) K(I)=2.24416*K(I)
      DO 14 J=1,NP
   14 A(I,J)=0.0
  602 LL=NJ-1
      DO 16 I=1,LL
      LXK=ML(I)
      DO 16 J=1,LXK
      LN=LOOP(I,J)
      LM=IABS(LN)
      IF (LN .LT. 0) A(I,LM)=-1.0
      IF (LN .GT. 0) A(I,LM)=1.0
   16  CONTINUE
  102 DO 17 I=NJ,NP
      LXK=ML(I)
      DO 17 J=1,LXK
      LN=LOOP(I,J)
      LM=IABS(LN)
      A(I,LM)=K(LM)
      IF (LN .LT. 1) A(I,LM) =-A(I,LM)
   17 CONTINUE
      DO 18 I=1,NP
      DO 18 J=1,NP
      IG= J+(I-1)*NP
   18  SAVE(IG) =A(J,I)
      CALL MINV (SAVE,NP,DDD,LZ,MN,NP*NP)
      CALL GMPRD (SAVE,DEMAND,Y,NP,NP,1,NP*NP,NP*1,NP*1)
      DO 19 I=1,NP
      IF (KK .EQ. 0) Q(I)=Y(I)
      IF (KK .NE. 0) Q(I)=(Q(I)+Y(I))/2.
```

```
      K(I)=4.77*12.**4.87*L(I)*(DABS(Q(I)))**.8518/(ROUGH(I)**1.8518*
     1D(I)**4.87)
      IF (UNITS .EQ. 2) K(I)=.000012*K(I)
      IF (UNITS .EQ. 3) K(I)=2.24416*K(I)
   19 CONTINUE
      KK=KK+1
      IF (KK .EQ. 1) GO TO 102
      CT=0.0
      DO 20 I=1,NP
      DIFF=DABS(Q(I)-Y(I))
      IF (KK .EQ. 5) GO TO 807
      IF (DIFF .GT. DD4) GO TO 102
  807 CT=CT+ENR*.358*D(I)**1.29*L(I)/(877.*BUILD)*FAC
      CT=CT+ .746/550.*24.*365.*X*CENTS*K(I)*Q(I)**2*B*62.4
   20 CONTINUE
      CT3=CT
      CT4=0.0
  652 KK=0
  125 KL=NJ-1
      NL=0
      LK=0
      DO 21 I=1,KL
      DO 22 J=1,NJ
   22 A(I,J)=0.0
      LM=IABS(TREE(I))
      LN=JBIGIN(LM)
      LO=JEND(LM)
      IF (LN .EQ. LK) GO TO 113
      IF (LO .EQ. LK) GO TO 113
      IF (PECED(LN) .EQ. 1.0) NL=NL+1
      IF (PECED(LN) .EQ. 1.0) LK=LN
      IF (PECED(LO) .EQ. 1.0) NL=NL+1
      IF (PECED(LO) .EQ. 1.0) LK=LO
      IF (PECED(LN) .NE. 1.0 .AND. PECED(LO) .NE. 1.0) GO TO 113
      IF ((NL+KL) .GT. NJ) GO TO 801
      DO 23 J=1,NJ
   23 A((KL+NL),J)=0.0
  113 IF (PECED(LN) .EQ. 1.0) A((NL+KL),LN)=1.0
      IF (PECED(LO) .EQ. 1.0) A((NL+KL),LO)=1.0
      IF (PECED(LN) .EQ. 1.0 .AND. MXX .EQ. 1) PRESS(NL+KL)=DPRE(LN)*
     1144./62.4+DELEV(LN)
      IF (PECED(LO) .EQ. 1.0 .AND. MXX .EQ. 1) PRESS(NL+KL)=DPRE(LO)*
     1144./62.4+DELEV(LO)
      IF (PECED(LN) .EQ. 1.0 .AND. MXX .NE.1) PRESS(NL+KL)=DPRE(LN)+
     1DELEV(LN)
      IF (PECED(LO) .EQ. 1.0 .AND. MXX .NE.1) PRESS(NL+KL)=DPRE(LO)+
     1DELEV(LO)
      IF (PECED(LN) .NE. 1.0) A(I,LN)=1.0
      IF (PECED(LO) .NE. 1.0) A(I,LO)=-1.0
  801 IF ((NL+KL) .GT. NJ) A(I,LN)=1.0
      IF ((NL+KL) .GT. NJ) A(I,LO)=-1.0
      PRESS(I)=K(LM)*Q(LM)
      IF (TREE(I) .LT. 0) PRESS(I)=-PRESS(I)
      IF ((NL+KL) .GT. NJ) GO TO 21
      IF (PECED(LN) .EQ. 1.0) PRESS(I)=PRESS(I)-PRESS(NL+KL)
      IF (PECED(LO) .EQ. 1.0) PRESS(I)=PRESS(I)+PRESS(NL+KL)
   21 CONTINUE
      DO 24 I=1,NJ
      DO 24 J=1,NJ
      IG=J+(I-1)*NJ
   24 SAVE(IG)=A(J,I)
```

```
      CALL MINV (SAVE,NJ,DDD,LZ,MN,NJ*NJ)
      CALL GMPRD (SAVE,PRESS,Y,NJ,NJ,1,NJ*NJ,NJ*1,NJ*1)
  676 DO 25 I=1,NJ
      DPRE(I)=(Y(I)-DELEV(I))*62.4/144.
      IF (KX .NE. 0) GO TO 104
      IF (DPRE(I) .GT. ABMAX) GO TO 105
      IF (DPRE(I) .LT. ABMIN) GO TO 105
      IF (DPRE(I) .LT. HT(I) .AND. HT(I) .NE. 0.0) GO TO 105
      GO TO  25
  105 IF (DD1 .LE. DD2) GO TO 673
      DO 26 J=1,NP
      D(J)=D(J)+DD1
      IF (DPRE(I) .GT. ABMAX) D(J)=D(J)-2*(DD1)
   26 CONTINUE
      GO TO 118
  104 IF (DPRE(I) .GT. ABMAX) GO TO 106
      IF (DPRE(I) .LT. ABMIN) GO TO 106
      IF (DPRE(I) .LT. HT(I) .AND. HT(I) .NE. 0.0) GO TO 106
      GO TO 25
  106 DIFF=DABS(D(N)-SD(N))
      IF (DIFF .LE. .0001 .AND. CT4 .NE. 0.0) CT3=CT4
      IF (DIFF .LE. .0001)  GO TO 208
      DIFF=DABS(DELD(N)-EXTRA)
      IF (DIFF .LE. .0001) DELD(N)=EXTRA
      IF (DELD(N) .NE. EXTRA) GO TO 671
      DIFF=DABS(DELD(N)-0.0)
      IF (DIFF .LE. .0001) GO TO 671
  661 DO 207 J=1,NP
      D(J)=D(J)-DELD(J)+SDELD(J)
      Q(J)=SQ(J)
  207 CONTINUE
      NPIPE(N)=1
      CHANGE=0.0
      EXTRA=0.0
      KX=0
      GO TO 118
  208 DO 209 J=1,NP
      SD(J)=0.0
  209 D(J)=D(J)+DELD(J)-SDELD(J)
      NPIPE(N)=1
      CHANGE=0.0
      EXTRA=0.0
      KX=0
      GO TO 118
  671 CT4=0.0
      GO TO 205
   25 CONTINUE
      DIFF=DABS(EXTRA-0.0)
      IF (DIFF .LE. .0001) EXTRA=0.0
      IF (EXTRA .EQ. 0.0) GO TO 669
      DIFF= DABS(DELD(N)-EXTRA)
      IF (DIFF .LE. .0001) DELD(N)=EXTRA
      IF (DELD(N) .EQ. EXTRA) NPIPE(N)=1
  669 KX=KX+1
      IF (CT4 .EQ. 0.0) GO TO 654
  660 IF (CT1 .NE. 0.0 .AND.  CT1 .GT.  CT4) CT3=CT4
  654 IF (CT1 .NE. 0.0 .AND. CT3 .EQ. CT4) GO TO 139
      IF (CT1 .NE. 0.0 .AND. CT1 .LT. CT4)  CT3=CT1
  653 IF (CT1 .NE. 0.0 .AND. CT3 .EQ. CT1) GO TO 139
      IF (CT4 .EQ. 0.0) GO TO 646
      IF (CT3 .GT. CT4 .AND. DD1 .LE. DD2) CT3=CT4
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  646 IF (KX .EQ. 1) GO TO 139
      IF (CT3  .GT. CT4 .AND.  CT4 .NE. 0.0) CT3=CT4
      IF (CT3 .NE. CT4) GO TO 139
      IF (DD1 .NE. AID) DD1=DD1/2.
      DIFF=DABS(CT2-CT3)
      IF (DIFF .LE. DD3 .AND. DD1 .LE. DD2) GO TO 673
      IF (DD1 .LE. DD2) GO TO 139
  139 NM=NP+1
      KXKX=0
      KX=1
      DD1=AID
      EXTRA=0.0
      CHANGE=0.0
      CT1=0.0
      LPL=0
      CT2=CT3
      NNN=0
      N=1
      DO 34 I=1,NM
      IF (I .EQ. NM) GO TO 666
      SD(I)=0.0
      IF (NPIPE(I) .EQ. 1) NNN=NNN+1
      IF (NPIPE(N) .EQ. 1) N=N+1
      NPP=NP-1
      IF (NNN .GT. NPP/2) DD1=AID/2.
      IF (DD1 .LE. DD2) DD1=AID
      IF (NNN .EQ. NP) GO TO 137
  666 DO 35 J=1,NM
      A(I,J)=0.0
      C(I,J)=0.0
   35 CONTINUE
   34 CONTINUE
      A(1,1)=1.0
      DO 32 I=1,NM
      DO 33 J=1,NM
      IF (I .NE. 1) GO TO 121
      IF (J .EQ. 1) GO TO 142
      A(I,J)=.746/550.*24.*365.*X*CENTS*2.8518*K(J-1)*Q(J-1)*B*62.4
     1*(-1.)
      IF (J .EQ. NM) GO TO 33
  142 C(I,J)=ENR*.358*1.29* D(J)**.29*L(J)/(877.*BUILD)*FAC+.746*24.*365
     1.*X*CENTS*.8336*Q(J)*DABS(Q(J))*K(J)*62.4/550.*B/D(J)
      IF (NPIPE(J) .EQ. 1) C(I,J)=0.0
      GO TO 33
  121 IF (J .EQ. 1) GO TO 33
      LN=IABS(LOOP((I-1),(J-1)))
      IF (LOOP((I-1),(J-1)))120,32,122
  120 IF (I .LE. NJ) A(I,(LN+1))=-1.0
      IF (I .LE. NJ)  C(I,LN) =2.*Q(LN)/D(LN)
      IF (NPIPE(LN) .EQ. 1) C(I,LN)=0.0
      IF (I .GT. NJ) A(I,(LN+1))=-1.8518*K(LN)
      IF (Q(LN) .LT. 0.0) A(I,(LN+1))=-A(I,(LN+1))
      IF (I .GT. NJ)  C(I,LN)=-1.1664*K(LN)*Q(LN)/D(LN)
      IF (NPIPE(LN) .EQ. 1) C(I,LN)=0.0
      GO TO 33
  122 IF  (I .LE. NJ) A(I,(LN+1))=1.0
      IF  (I .LE. NJ)  C(I,LN)=-2.0*Q(LN)/D(LN)
      IF (NPIPE(LN) .EQ. 1) C(I,LN)=0.0
      IF (I .GT. NJ) A(I,(LN+1))=1.8518*K(LN)
      IF (Q(LN) .LT. 0.0) A(I,(LN+1))=-A(I,(LN+1))
      IF (I .GT. NJ) C(I,LN)=1.1664*K(LN)*Q(LN)/D(LN)
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      IF (NPIPE(LN) .EQ. 1) C(I,LN)=0.0
   33 CONTINUE
   32 CONTINUE
      DO 59 I=1,NM
      DO 60 J=1,NM
      IG=J+(I-1)*NM
      SAVE(IG)=A(J,I)
      IF (I .EQ. NM) GO TO 60
      IG=J+(I-1)*NM
      CHAIN(IG)=C(J,I)
      SCAN(I)=0.0
   60 CONTINUE
   59 CONTINUE
      CALL MINV (SAVE,NM,DDD,LZ,MN,NM*NM)
      CALL GMPRD (SAVE,CHAIN,STAN,NM,NM,NP,NM*NM,NM*NP,NM*NP)
      N=1
      DO 36 I=1,NP
      IF (NPIPE(N) .EQ. 1) N=N+1
      NT=1+(N-1)*NM
      NK=1+(I-1)*NM
      IF (DABS(STAN(NK)) .GT. DABS(STAN(NT)))N=I
      NT=1+(N-1)*NM
      C(1,I)=STAN(NK)
   36 CONTINUE
      SCAN(N)=0.0
      IF (C(1,N) .GT..0 .AND. KXKX .EQ. 0) LOOK=1
      IF (C(1,N) .LT..0 .AND. KXKX .EQ. 0) LOOK=0
      IF (DD1 .GT. DD2) GO TO 123
  673 DO 672 J=1,INN
      DIFF=DABS(D(N)-DIA(J))
      IF (DIFF .LE. .0001) D(N)=DIA(J)
      IF (DIA(J) .LT. D(N)) GO TO 672
      IF (KX .EQ. 0) DELD(N)=DIA(J+1)-D(N)
      IF (D(N) .EQ. DIA(J)) NPIPE(N)=1
      IF (D(N) .EQ. DIA(J)) GO TO 118
      LPL=0
      IF (D(N) .LE. DIMIN) DELD(N)=DIMIN-D(N)
      IF (D(N) .GT. DIMIN) DELD(N)=DIA(J-1)-D(N)
      EXTRA=DIA(J)-D(N)
  644 CHANGE=DELD(N)
      LOOK=3
      IF (KX .EQ. 0) KX=1
      GO TO 668
  672 CONTINUE
  123 IF (DD1 .GT. DD2) DELD(N)=DD1
      NPP=NP-1
  668 NNN=0
      DIFF=DABS(CHANGE-0.0)
      IF (DIFF .LE. .0001) CHANGE=0.0
      DIFF=DABS(C(1,N)-0.0)
      IF (DIFF .LE. .0001) C(1,N)=0.0
      DO 37 I=1,NP
      IF (NPIPE(I) .EQ. 1) NNN=NNN+1
      IF (C(1,N) .EQ. 0.0) DD1=DD2
      IF (C(1,N) .EQ. 0.0) GO TO 651
      DELD(I)=DELD(N)*C(1,I)/C(1,N)
      IF (LOOK .EQ. 1) DELD(I)=-DELD(I)
      IF (SCAN(I) .EQ. 0.0) D(I)=D(I)+DELD(I)
      M=0
      IF (SCAN(I) .NE. 0.0 .AND. D(I) .EQ. DIMIN) M=I
      IF (D(I) .EQ. DIMIN .AND. SCAN(I) .NE. 0.0) D(I)=SCAN(I)+DELD(I)
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      IF (M .EQ. I) GO TO 804
      IF (SCAN(I) .NE. 0.0 .AND. D(I) .NE. DIMIN) D(I)=D(I)+DELD(I)
  804 IF (D(I) .GE. DIMIN) GO TO 37
      LPL=I
      IF (LPL .NE. N) SCAN(I)=D(I)
      IF (LPL .NE. N) GO TO 37
      D(LPL)=D(LPL)-DELD(LPL)
      DELD(LPL)=DIMIN-D(LPL)
      DO 670 J=1,NP
      IF (J .LT. LPL) D(J)=D(J)-DELD(J)
      DELD(J)=DELD(LPL)*C(1,J)/C(1,LPL)
      D(J)=D(J)+DELD(J)
      D(LPL)=DIMIN
  670 CONTINUE
      IF (DELD(N) .EQ. 0.0) DD1=DD2
      IF (DELD(N) .EQ. 0.0) GO TO 651
      GO TO 127
   37 CONTINUE
      GO TO 127
  127 KKK=0
      DO 78 I=1,NP
      IF (D(I) .LT. DIMIN) SCAN(I)=D(I)
      IF (D(I) .LT. DIMIN) D(I)=DIMIN
      K(I)=4.77*12.**4.87*L(I)*(DABS(Q(I)))**.8518/(ROUGH(I)**1.8518*
     1D(I)**4.87)
      IF (UNITS .EQ. 2) K(I)=.000012*K(I)
      IF (UNITS .EQ. 3) K(I)=2.24416*K(I)
      IF (NNN .EQ. NPP .AND. I .NE. N) DELD(I)=0.0
      DO 78 J=1,NP
   78 A(I,J)=0.0
      DO 79 I=1,NP
      LXK=ML(I)
      DO 65 J=1,LXK
      LM=IABS(LOOP(I,J))
      A(I,LM)=1.0
      IF (LOOP(I,J) .LT. 0) A(I,LM)=-1.0
   65 CONTINUE
   79 CONTINUE
  144 DO 82 I=NJ,NP
      LP=ML(I)
      DO 83 J=1,LP
      LM=IABS(LOOP(I,J))
      A(I,LM)=K(LM)
      IF (LOOP(I,J) .LT. 0)A(I,LM)=-K(LM)
   83 CONTINUE
   82 CONTINUE
      DO 84 I=1,NP
      DO 84 J=1,NP
      IG= J+(I-1)*NP
   84 SAVE(IG)=A(J,I)
      CALL MINV (SAVE,NP,DDD,LZ,MN,NP*NP)
      CALL GMPRD (SAVE,DEMAND,Y,NP,NP,1,NP*NP,NP*1,NP*1)
      DO 85 I=1,NP
      IF (KKK .EQ. 0) Q(I)=Y(I)
      IF (KKK .NE. 0) Q(I)=(Q(I)+Y(I))/2.
      K(I)=4.77*12.**4.87*L(I)*(DABS(Q(I)))**.8518/(ROUGH(I)**1.8518*
     1D(I)**4.87)
      IF (UNITS .EQ. 2) K(I)=.000012*K(I)
      IF (UNITS .EQ. 3) K(I)=2.24416*K(I)
   85 CONTINUE
      KKK=KKK+1
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      IF (KKK .EQ. 1)  GO TO 144
      IF (DD1 .LE. DD2) GO TO 713
      IF (CT4 .EQ. 0.0) GO TO 713
      IF (KX  .NE. 1 .AND. CT3 .GT. CT4) CT3=CT4
  713 CT=0.0
      DO 86 I=1,NP
      DIFF =DABS(Q(I)-Y(I))
      IF (KKK .EQ. 5) GO TO 808
      IF (DIFF .GT. DD4) GO TO 144
  808 CT=CT+ENR*.358*D(I)**1.29*L(I)/(877.*BUILD)*FAC
      CT=CT+ .746/550.*24.*365.*X*CENTS*K(I)*Q(I)**2*8*62.4
   86 CONTINUE
      CT4=CT
      IF (LOOK .EQ. 3) GO TO 667
      IF (DD1 .GT. DD2) GO TO 212
  667 DIFF=DABS(CHANGE-0.0)
      IF (DIFF .LE. .0001) GO TO 212
      DIFF=DABS(CHANGE-DELD(N))
      IF (DIFF .LE. .0001)  CHANGE=DELD(N)
      IF (DELD(N) .NE. CHANGE) GO TO 212
  663 CT1=CT4
      DO 206 I=1,NP
      SDELD(I)=DELD(I)
      SD(I)=D(I)
      D(I)=D(I)-DELD(I)
      SQ(I)=Q(I)
      SK(I)=K(I)
  206 CONTINUE
      DELD(N)=EXTRA
      LPL=0
      GO TO 668
  212 DIFF=DABS(EXTRA-0.0)
      IF (DIFF .LE. .0001) EXTRA=0.0
      IF (EXTRA .NE. 0.0 .AND. CT4 .LT. CT1) GO TO 125
      IF (EXTRA .NE. 0.0) GO TO 655
      IF (KX .EQ. 1 .AND. DD1 .LE. DD2) GO TO 673
  662 DIFF=DABS(EXTRA-DELD(N))
      IF (DIFF .LE. .0001) EXTRA=DELD(N)
      IF (DELD(N) .EQ. EXTRA .AND. CT4 .LT. CT1)   GO TO 125
      IF (DELD(N) .NE. EXTRA) GO TO 718
      DIFF=DABS(DELD(N)-0.0)
      IF (DIFF .LE. .0001) GO TO 659
  651 IF (LOOK .NE. 3) LOOK=LOOK-1
      IF (LOOK .LT. 0) LOOK=1
      KXKX=KXKX+1
      IF (KXKX .EQ. 1 .AND. CT2 .LE. CT3)  DD1=1.5*DD2
      IF (KXKX .EQ. 1 .AND. CT2 .LE. CT3) GOTO 123
      IF (CT1 .EQ. 0.0 .AND. CHANGE .EQ. 0.0) GO TO 673
  655 DO 211 J=1,NP
      D(J)=D(J)-DELD(J)+SDELD(J)
      D(J)=SD(J)
      Q(J)=SQ(J)
      K(J)=SK(J)
  211 CONTINUE
      DIFF=DABS(EXTRA-DELD(N))
      IF (DIFF .LE. .0001) EXTRA=DELD(N)
      IF (DELD(N) .EQ. EXTRA) GO TO 125
  718 IF (CT3 .GT. CT4) GO TO 125
  205 DO 203 I=1,NP
      IF (SCAN(I) .EQ. 0.0) D(I)=D(I)-DELD(I)
      IF (SCAN(I) .NE. 0.0 .AND. D(I) .EQ. DIMIN) M=I
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      IF (D(I) .EQ. DIMIN .AND. SCAN(I) .NE. 0.0) D(I)=SCAN(I)-DELD(I)
      IF (M .EQ. I) GO TO 203
      IF (SCAN(I) .NE. 0.0 .AND. D(I) .NE. DIMIN) D(I)=D(I)-DELD(I)
  203 CONTINUE
      DD1=DD1/2.
      IF (KX .EQ. 1 .AND. DD1 .LE. DD2) GO TO 651
      DIFF=DABS(CT2-CT3)
      IF (DD1 .LE. DD2 .AND. DIFF .LE. DD3) GO TO 673
      IF (DD1 .LE. DD2) GO TO 118
  659 DIFF=DABS(EXTRA-0.0)
      IF (DIFF .LE. .0001) EXTRA=0.0
      IF (EXTRA .NE. 0.0) GO TO 667
      IF (KX .EQ. 2) GO TO  139
      GO TO 123
  137 CT=0.0
 1200 FORMAT (//40X,'RESULTS OF OPTIMAL TRIAL')
      WRITE (6,1200)
      WRITE (6,1500)
 15000FORMAT (//,1X,'PIPE NO',5X,'JUNCTION',5X,'LENGTH',5X,'ROUGHNESS',
     14X,'DIAMETER-INCHES',7X,'FLOW',7X,'PRESSURE AT JUNCTIONS',/50X,
     2'ORIGINAL',3X,'FINAL',18X,'BEGIN',8X,'END')
      CT=0.0
      CP=0.0
      DO 62 I=1,NP
      CT=CT+ENR*.358*D(I)**1.29*L(I)/(877.*BUILD)*FAC
      CP=CP+ENR*.358*D(I)**1.29*L(I)/(877.*BUILD)*FAC
      CT=CT+ .746/550.*24.*365.*X*CENTS*K(I)*Q(I)**2*B*62.4
      LN=JBIGIN(I)
      LG=JEND(I)
      WRITE (6,1300) I,LN,LG,L(I),ROUGH(I),BODE(I),D(I),Q(I),DPRE(LN),
     1DPRE(LG)
   62 CONTINUE
      WRITE (6,1600) CP
      WRITE (6,1400) CT3
 1600 FORMAT (//18X, 'CAPITAL COST=$',F15.2,1X)
 1400 FORMAT (//18X, 'OPTIMAL COST=$',F15.2,1X)
 1300 FORMAT (3X,I3,5X,I3,2X,I3,3X,F9.2,5X,F6.0,8X,F5.2,6X,F5.2,5X,F9.2,
     1 3X,F3.3,3X,F8.3)
      RETURN
      END
C                                                                       MINV
C        ...............................................................MINV
C                                                                       MINV
C        SUBROUTINE MINV                                                MINV
C                                                                       MINV
C        PURPOSE                                                        MINV
C           INVERT A MATRIX                                             MINV
C                                                                       MINV
C        USAGE                                                          MINV
C           CALL MINV(A,N,D,L,M)                                        MINV
C                                                                       MINV
C        DESCRIPTION OF PARAMETERS                                      MINV
C           A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY  MINV
C               RESULTANT INVERSE.                                      MINV
C           N - ORDER OF MATRIX A                                       MINV
C           D - RESULTANT DETERMINANT                                   MINV
C           L - WORK VECTOR OF LENGTH N                                 MINV
C           M - WORK VECTOR OF LENGTH N                                 MINV
C                                                                       MINV
C        REMARKS                                                        MINV
C           MATRIX A MUST BE A GENERAL MATRIX                           MINV
```

```
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
   30 A(JI) =HOLD
C
C        INTERCHANGE COLUMNS
C
   35 I=M(K)
      IF(I-K) 45,45,38
   38 JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI) =HOLD
C
C        DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
C        CONTAINED IN BIGA)
C
   45 IF(BIGA) 48,46,48
   46 D=0.0
      RETURN
   48 DO 55 I=1,N
      IF(I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
C
C        REDUCE MATRIX
C
      DO 66 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
   60 IF(J-K) 62,65,62
   62 KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
   65 CONTINUE
   66 CONTINUE
C
C        DIVIDE ROW BY PIVOT
C
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
C
C        PRODUCT OF PIVOTS
C
      D=D*BIGA
C
C        REPLACE PIVOT BY RECIPROCAL
C
      A(KK)=1.0/BIGA
```

```
   80 CONTINUE                                                          MINV1400
C                                                                       MINV1410
C        FINAL ROW AND COLUMN INTERCHANGE                               MINV1420
C                                                                       MINV1430
      K=N                                                               MINV1440
  100 K=(K-1)                                                           MINV1450
      IF(K) 150,150,105                                                 MINV1460
  105 I=L(K)                                                            MINV1470
      IF(I-K) 120,120,108                                               MINV1480
  108 JQ=N*(K-1)                                                        MINV1490
      JR=N*(I-1)                                                        MINV1500
      DO 110 J=1,N                                                      MINV1510
      JK=JQ+J                                                           MINV1520
      HOLD=A(JK)                                                        MINV1530
      JI=JR+J                                                           MINV1540
      A(JK)=-A(JI)                                                      MINV1550
  110 A(JI) =HOLD                                                       MINV1560
  120 J=M(K)                                                            MINV1570
      IF(J-K) 100,100,125                                               MINV1580
  125 KI=K-N                                                            MINV1590
      DO 130 I=1,N                                                      MINV1600
      KI=KI+N                                                           MINV1610
      HOLD=A(KI)                                                        MINV1620
      JI=KI-K+J                                                         MINV1630
      A(KI)=-A(JI)                                                      MINV1640
  130 A(JI) =HOLD                                                       MINV1650
      GO TO 100                                                         MINV1660
  150 RETURN                                                            MINV1670
      END                                                               MINV1680
C                                                                       GMPR  10
C     ..................................................................GMPR  20
C                                                                       GMPR  30
C        SUBROUTINE GMPRD                                               GMPR  40
C                                                                       GMPR  50
C        PURPOSE                                                        GMPR  60
C           MULTIPLY TWO GENERAL MATRICES TO FORM A RESULTANT GENERAL   GMPR  70
C           MATRIX                                                      GMPR  80
C                                                                       GMPR  90
C        USAGE                                                          GMPR 100
C           CALL GMPRD(A,B,R,N,M,L)                                     GMPR 110
C                                                                       GMPR 120
C        DESCRIPTION OF PARAMETERS                                      GMPR 130
C           A - NAME OF FIRST INPUT MATRIX                              GMPR 140
C           B - NAME OF SECOND INPUT MATRIX                             GMPR 150
C           R - NAME OF OUTPUT MATRIX                                   GMPR 160
C           N - NUMBER OF ROWS IN A                                     GMPR 170
C           M - NUMBER OF COLUMNS IN A AND ROWS IN B                    GMPR 180
C           L - NUMBER OF COLUMNS IN B                                  GMPR 190
C                                                                       GMPR 200
C        REMARKS                                                        GMPR 210
C           ALL MATRICES MUST BE STORED AS GENERAL MATRICES             GMPR 220
C           MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A         GMPR 230
C           MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX B         GMPR 240
C           NUMBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROWGMPR 250
C           OF MATRIX B                                                 GMPR 260
C                                                                       GMPR 270
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                   GMPR 280
C           NONE                                                        GMPR 290
C                                                                       GMPR 300
C        METHOD                                                         GMPR 310
C           THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A GMPR 320
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C           AND THE RESULT IS STORED IN THE N BY L MATRIX R.              GMPR 330
C                                                                         GMPR 340
C        ................................................................GMPR 350
C                                                                         GMPR 360
      SUBROUTINE GMPRD (A,B,R,N,M,L,NTM,MTL,NTL)
      DIMENSION A(NTM),B(MTL),R(NTL)
      DOUBLE PRECISION A,B,R
C
      IR=0                                                                GMPR 390
      IK=-M                                                               GMPR 400
      DO 10 K=1,L                                                         GMPR 410
      IK=IK+M                                                             GMPR 420
      DO 10 J=1,N                                                         GMPR 430
      IR=IR+1                                                             GMPR 440
      JI=J-N                                                              GMPR 450
      IB=IK                                                               GMPR 460
      R(IR)=0                                                             GMPR 470
      DO 10 I=1,M                                                         GMPR 480
      JI=JI+N                                                             GMPR 490
      IB=IB+1                                                             GMPR 500
   10 R(IR)=R(IR)+A(JI)*B(IB)                                             GMPR 510
      RETURN                                                              GMPR 520
      END                                                                 GMPR 530
                                                                          GMPR 540
```