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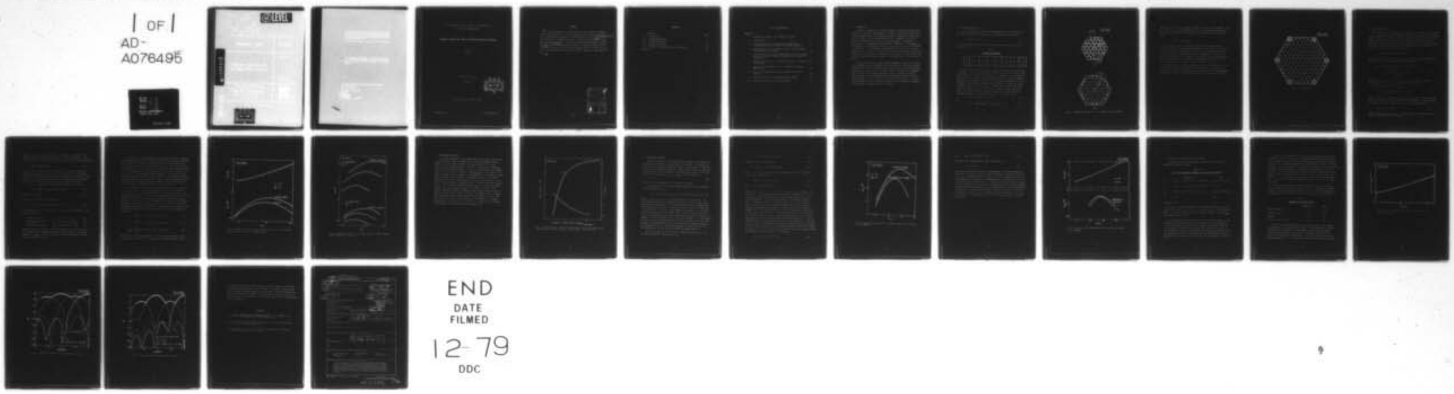

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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MINIMUM DIRECTIVE GAIN OF HOPPED-BEAM ANTENNAS

A. R. DION
Group 61

TECHNICAL NOTE 1979-33

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ABSTRACT

The optimum parameters of an antenna whose beam is hopped to uniformly spaced directions within a circular coverage are derived for a phased array and for a multifeed lens antenna. The minimum directive gain, (G_{MIN}) within the coverage is the parameter optimized. The analysis shows that, for small bandwidth-diameter products, the two antenna configurations exhibit about the same (G_{MIN}), but the optimum aperture diameter is about 30% smaller with the phased array. However, as the bandwidth-diameter product increases, the G_{MIN} of the lens antenna becomes progressively greater than that of the phased array.

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I. INTRODUCTION

The physical parameters of antennas designed to provide an optimum pencil beam which is uniformly hopped within a circular coverage are derived for two antenna configurations: (1) a phased array and (2) a lens with a feed array in its focal plane. The beam hopping geometry is a triangular lattice array such as is illustrated in Fig. 1a for a 37-beam antenna. Each beam is associated with an hexagonally shaped angular coverage cell. The parameter to be optimized is the minimum directive gain, G_{MIN}, which occurs at the corners of a coverage cell. This parameter is optimum for a specific value of aperture diameter. For the purpose of comparison, the number of elements in the phased array and in the feed array are taken equal to the number of hopped beams. The effect of a different number of elements in the phased array is also determined.

It will be shown that for small bandwidth-diameter products, both configurations exhibit about the same minimum directive gain, but the optimum phased array diameter is about 30% smaller than the optimum lens diameter and the gain variation, ΔG, within a cell of coverage is about 3.2 dB less for the phased array. However, as the bandwidth-diameter product increases the minimum directive gain of the lens antenna becomes progressively larger than that of the array antenna. The results of the analysis will be applied to a 37-beam, earth-coverage, synchronous satellite antenna, and the performance parameters of both antenna configurations will be presented.

II. OPTIMUM BEAM SPACING

Let N be the number of beams in the hexagonal beam array and let N_c be the number of beams on a diagonal; these two parameters are related by

$$N = (3N_c^2 + 1)/4 \tag{1}$$

and representative values of N and N_c for small to modest size arrays are presented in Table I.

TABLE I

HEXAGONAL BEAM ARRAYS

N_c → 3	5	7	9	11	13	15
N → 7	19	37	61	91	127	169

Minima of the directive gain occur in the corner directions of each cell of coverage. Since the desired overall coverage is a circle of radius θ_M (see Fig. 1a) while the coverage of the beam array is hexagonal, it is necessary to space the beams so that no direction within the circle of coverage is beyond an outer cell of coverage. This is achieved optimally by making the circle of coverage pass through the innermost corners (denoted 'A' in Fig. 1a) on the perimeter. These corners are located as shown in Figs. 1a and 1b for beam arrays with, respectively, an even and an odd number of beams forming their sides. The beam spacing, θ_s, required to satisfy the circular coverage requirement (assumed to be of small extent, i.e., $\theta_m < 10°$) is then

$$\theta_s = 4\sqrt{3}\ \theta_M/(3N_c - 1) \text{ for } N_s \text{ even} \tag{2}$$

and

$$\theta_s = 4\sqrt{3}\ \theta_M/(9N_c^2 - 6N_c + 13)^{1/2} \text{ for } N_s \text{ odd} \quad . \tag{3}$$

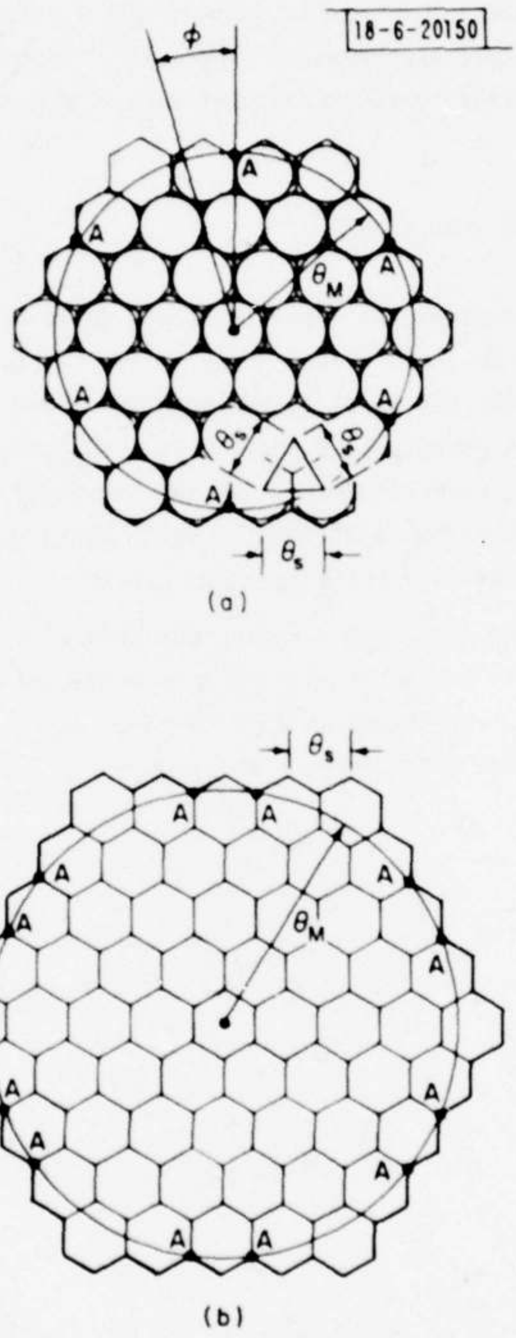

Fig. 1. Hexagonal beam arrays: (a) 37 beams, and (b) 61 beams.

where $N_s = (N_c + 1)/2$ is the number of beams on a side of the beam array. For $N_c \geq 5$, Eq. (3) is well approximated by Eq. (2). The angle between the beam axis and the directions to the corners of an associated cell of coverage is then

$$\theta_c = \theta_s/\sqrt{3} \simeq 4\theta_M/(3N_c-1) \qquad (4)$$

It should be noted that as the number of beams in the hexagonal array is increased, some of the beams become superfluous. This is illustrated in Fig. 2 for N=127, which shows the beams at the corners of the beam array completely outside the required coverage and therefore, a designated "127-beam" array would be implemented with 121 beams. As the number of beams in the hexagonal array is further increased, additional outside beams may be deleted, thus producing a beam array of overall coverage closer to circular.

The angle between the beam axes and the directions of minimum gain of an optimally spaced beam array has been derived above and is given by Eq. (4). In the next Section, the directive gain in these directions is *determined and* optimized for a phased array antenna and also for a lens-multifeed antenna.

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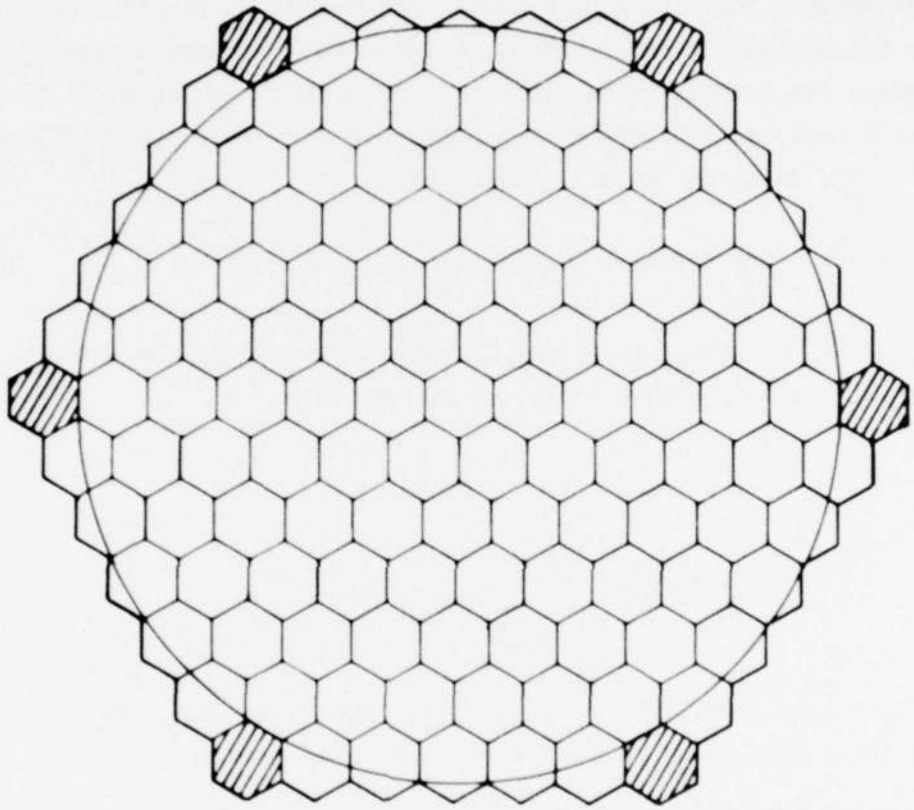

Fig. 2. 127-beam array.

III. OPTIMUM PHASED ARRAY

The configuration of the phased array considered is hexagonal, i.e., identical to that of the beam array and the number of elements is chosen equal to the number of hopped beams. The effect of a number of elements different from the number of beams is investigated further below. The N equally excited elements of the phased array are conical horns with aperture diameter, d, equal to their spacing. The elements are excited with a TE_{11} mode, and, therefore, their efficiency is 83.5%[1]. The directive gain of this array is*

$$G(\theta) = N[0.83(\pi d/\lambda)^2 G_e(\theta)] G_a(\theta) \tag{5}$$

where $G_a(\theta)$ is the array factor and the square bracket is the element directive gain with $G_e(\theta)$, the element radiation pattern given by [1]

$$G_e(\theta) = [\frac{2}{1 + \lambda/\lambda_g}]^2 [(1 + \frac{\lambda}{\lambda_g} \cos\theta) \frac{J_1(v)}{v} \cos^2\phi + (\frac{\lambda}{\lambda_g} + \cos\theta) \frac{J_1'(v)}{[1-(\frac{v}{1.84})^2]} \sin^2\phi]^2 \tag{6}$$

where $\lambda/\lambda_g = [1 - (\lambda/1.71d)^2]^{1/2}$, $v = 2\pi d \sin\theta/\lambda$, θ is the angle measured from the array axis and ϕ is the longitude defined in Fig. 1a.

The array factor in (5) is given by

$$G_a(\theta) = \frac{1}{N^2} [\sum_{i=1}^{N} \exp(-j2\pi(\vec{\rho} - \vec{\rho}_o) \cdot \vec{r}_i/\lambda)]^2 \tag{7}$$

where $\vec{\rho}$ and $\vec{\rho}_o$ are unit direction vectors with $\vec{\rho}_o$ the beam pointing direction and $\vec{r}_i$ is the vector position to the center of each element. For small angles off the array axis, the element pattern in (5) may be approximated by

$$G_e(\theta) \simeq [2J_1(v_1)/v_1]^2 \tag{8}$$

---

*Mutual coupling between elements is neglected since in most applications d/λ will be appreciably greater than unity.

where $v_1 = \pi d_1 \sin\theta/\lambda$ and $d_1 = 0.91d$ is the diameter of an element of unit aperture efficiency equivalent in gain to the TE_{11} excited aperture. Also, for small angles, ψ, from the beam axis the array factor is well approximated by

$$G_a(\psi) \simeq [2J_1(v_2)/v_2]^2 \quad (9)$$

where $v_2 = \pi\sqrt{N}\, d \sin\psi/\lambda$ ($\sqrt{N}\, d$ is the diameter of a uniformly illuminated aperture of area equal to the total area of the elements). An approximate expression for the minimum directive gain is obtained by substituting (8) and (9) in (5). The minimum directive gain is in the directions of the hexagon corners located on the coverage circle and is obtained by substituting $\theta = \theta_m$ and $\psi = \theta_c$ (from (4)) in the approximate expression, yielding

$$G(u) \simeq \frac{0.63N}{\theta_M^2} u^2[2J_1(\alpha u)/\alpha u]^2 \; [2J_1(u)/u]^2 \quad (10)$$

$$\text{where } u = \pi\sqrt{N}\, d \sin\theta_c/\lambda \quad (11)$$

and $\alpha = 0.23(3N_c - 1)/\sqrt{N}$. $G(u)$ is maximum for

$$G'(u) = J_1(\alpha u)[J_o(u) - J_2(u)] - 2\alpha J_1(u)J_2(\alpha u) = 0 \quad (12)$$

which is satisfied by $u \simeq 1.52$ for $N_c \geq 5$, yielding for the parameters of the optimum phased array:

Diameter of elements:	$d/\lambda \simeq 0.42(1-0.3\, N_c^{-1})/\theta_M$	(13)
Mean diameter of array:	$D/\lambda \simeq 0.39\, N_c(1-0.3\, N_c^{-1})/\theta_M$	(14)
Minimum directive gain:	$G_{MIN} \simeq 0.56\, N(1-0.6N^{-1/2})/\theta_M^2$	(15)

The gain variation, ΔG, within a cell of coverage is $\Delta G = (2J_1(1.52)/1.52)^2 = 2.6$ dB and the corresponding optimum ratio of beam spacing to half-power beamwidth, θ_s/θ_{HPBW}, is equal to 0.81, which in turn corresponds to a beam cross over level of -2.0 dB.

The validity of the approximations (8) and (9) was verified by using the exact expressions (6) and (7) to compute the gain in the directions of minima, as a function of d/λ, with the elements excited for circular polarization. Calculations were made for the particular case $\theta_M = 9°$, N=37, and the results are compared in Fig. 3 to the results obtained with the approximate expression (10). The agreement, which is within a few tenths of a dB, is indicative of the accuracy of the approximate optimum parameters. The minimum directive gain is observed to vary slowly with d/λ, allowing for substantial reductions of aperture size with little reduction of G_{MIN}. A further advantage of apertures smaller than optimum is the smaller amount of gain variation within a cell of coverage, as depicted by the ΔG vs d/λ curve of Fig. 3.

A disadvantage of phased arrays is the dependence of beam pointing direction on the frequency of operation. Because of this effect, the minimum directive gain of a given phased array is less than the value given by (15) as the operating frequency is varied from the design frequency. The effect of bandwidth on minimum directive gain is illustrated in Fig. 4 where G_{MIN} is plotted as a function of d/λ with the bandwidth as a parameter, for the same beam array characteristics as above, i.e., $\theta_M = 9°$, N=37 and also for N=127. As bandwidth increases, both G_{MIN} and the optimum aperture size decrease. The modified optimum parameters of a phased array of specified bandwidth may be shown to be:

$$d/\lambda \simeq \frac{0.42}{\theta_M} (1 - 0.3\, N_c^{-1})\ (1 - 0.15\, N_c\, \Delta f/f) \tag{16}$$

$$D/\lambda \simeq \frac{0.39 N_c}{\theta_M} (1 - 0.3\, N_c^{-1})\ (1 - 0.15\, N_c\, \Delta f/f) \tag{17}$$

$$G_{MIN} \simeq \frac{0.56 N}{\theta_M^2} (1 - 0.6\, N^{-1/2})\ (1 - 0.25\, N_c\, \Delta f/f) \tag{18}$$

where $\Delta f/f$ is the fractional bandwidth. Since the array diameter is proportional to N_c, the product $N_c\ \Delta f/f$ is called the bandwidth-diameter product.

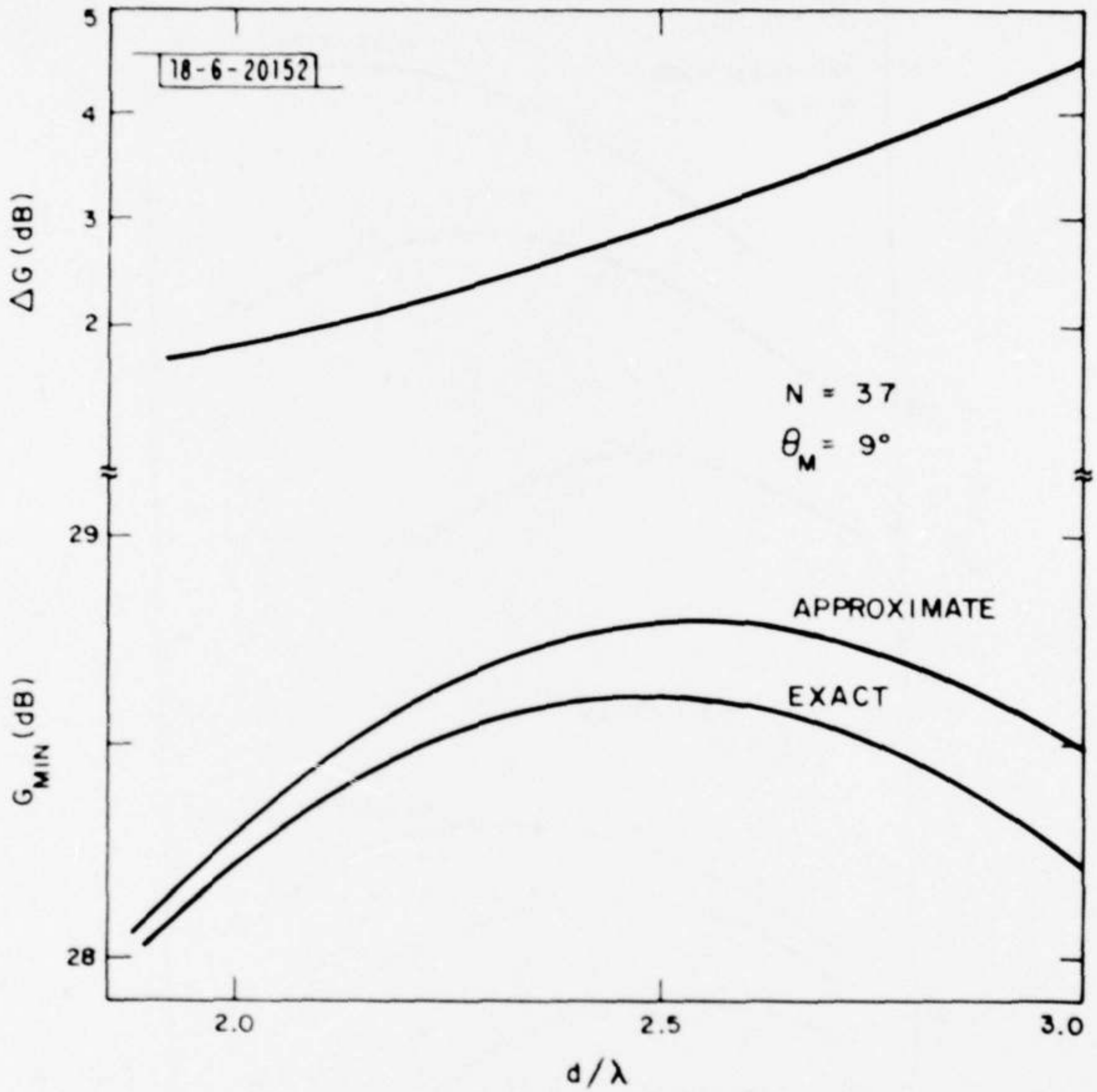

Fig.3. Minimum directive gain, G_{MIN}, and gain variation, ΔG, of 37-beam phased array as a function of element diameter.

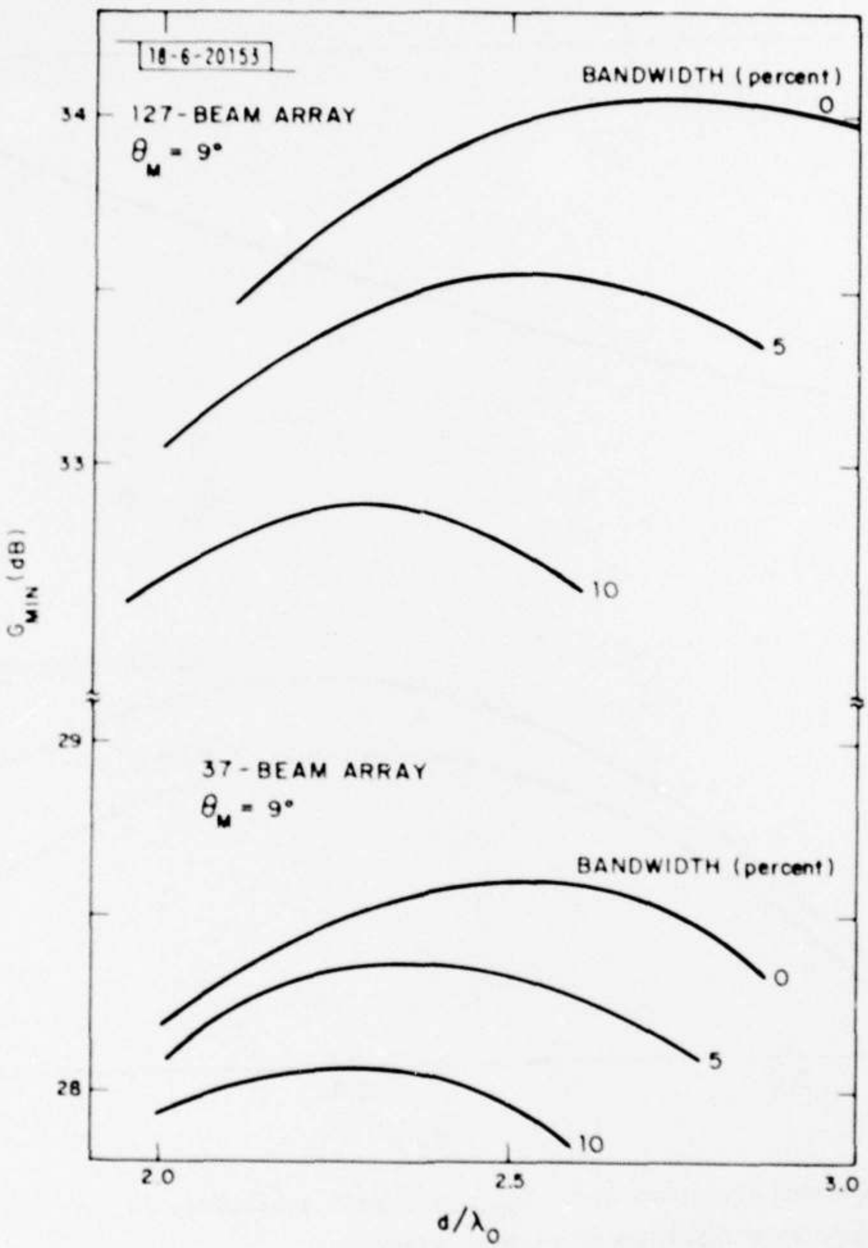

Fig. 4. Minimum directive gain of phased arrays vs element diameter with bandwidth as a parameter.

Performance when $N_e \neq N$

The optimum parameters of the hopped beam phased array have been derived for an array whose number of elements N_e is equal to the number of hopped beams N. For the present discussion, this specific array (i.e., $N=N_e$) will be called the reference array. The effect of N_e has been investigated using the exact formulations (Eqs. 5, 6, and 7), and the results are presented in Fig. 5 as a function of the ratio $r = N_{EL}/N_{BEAM}$ which is the ratio of the number of elements along a diagonal of the phased array to the number of beams along a diagonal of the beam array. The minimum directive gain is shown normalized to the gain of the reference array (r=1) as given by (18). It is observed that increasing r beyond unity does not yield an appreciable increase of G_{MIN}. For instance, the minimum directive gain of a 37-beam, 61-element array is only 0.5 dB greater than that of a 37-beam, 37-element array. Similarly, making the number of elements somewhat smaller than the number of beams only causes a small reduction of G_{MIN}. For example, with the 37-beam array, the minimum directive gain achieved with 19 elements is only 1.2 dB less than with 37 elements. The diameter of the elements of the optimum phased array is also shown in Fig. 5, where it is normalized to the diameter of the elements of the reference array as given by (16).

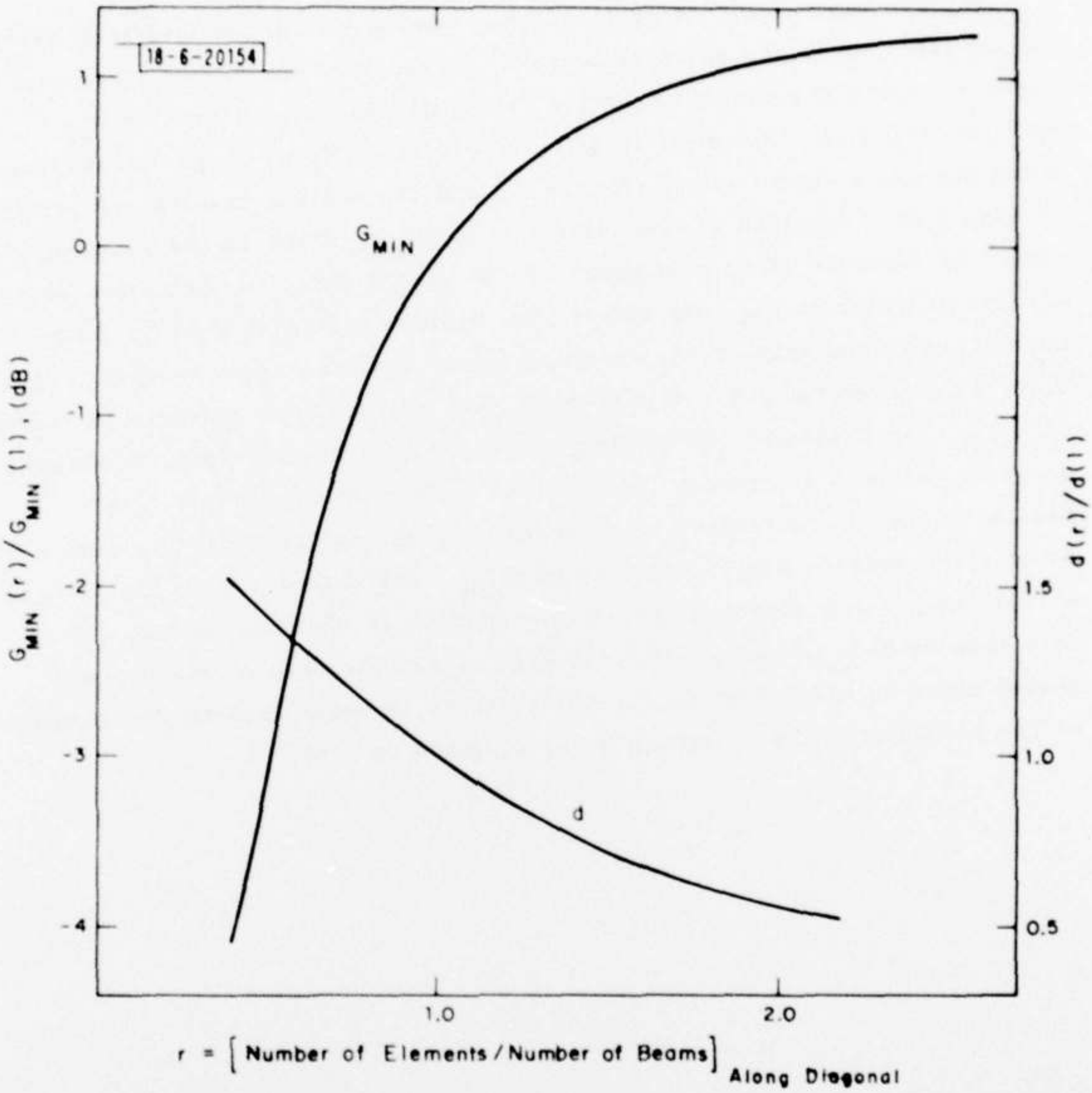

Fig. 5. Minimum directive gain and optimum element spacing of a phased array with a number of elements different from the number of hopped beams.

IV. OPTIMUM LENS ANTENNA

In this configuration, a feed array which is an image of the beam array is centered at the focus of a lens assumed reflectionless. The diameter, d, of the feed array elements (feedhorns) is equal to the spacing. The apertures of the array are located on a spherical surface centered on the lens axis and of radius equal to the focal length. Since the angular spacing of the feeds is equal to the beam spacing, θ_s, the diameter of the feedhorns is

$$d = F\theta_s \quad (19)$$

where F is the focal length and θ_s is expressed in radians.

The gain function of the lens-multifeed configuration can be expressed by the approximate formula

$$G(\psi) \simeq (\pi D/\lambda)^2 \; [2(1-J_o(u_f))/u_f]^2 \; [2J_1(\pi D \sin\psi/\lambda)/(\pi D \sin\psi/\lambda)]^2 \quad (20)$$

where $u_f = 0.52\ \pi D\theta_s/\lambda$ and ψ is the angle relative to the beam axis. The first square bracket in (20) expresses the loss due to spillover and to non-uniform aperture illumination. This expression was obtained considering the directive gain of each feed to be equal to the aperture gain of the unit cell of the feed array (the unit cell is an hexagon of area equal to $1.10\pi(d/2)^2$). For feeds of diameter not much larger than λ, such as is the case for $F/D \simeq 1$, unit cell performance may be achieved through end fire gain as obtained, for example, by placing a dielectric rod in each horn aperture[2]. The last bracket in (20) is the antenna radiation pattern which is approximated by that of the uniformly illuminated aperture. The error caused by this approximation will be considered later on. It should be noted that (20) does not account for the adverse effect of feed offset; all beams are assumed identical.

The directive gain in the directions of minima is obtained with $\psi = \theta_c$ in (20) yielding, after substitution of (4)

$$G(u) \simeq 0.30(3N_c-1)^2 \; F^2(u)/\theta_M^2 \qquad (21)$$

where $F(u) = (1-J_o(\alpha u)) \; (2J_1(u)/u)$ (22)

$$u = \pi D \sin\theta_c/\lambda \simeq (\pi D/\lambda) \; (4\theta_M/(3N_c-1)) \qquad (23)$$

and $\alpha = u_f/u \simeq 0.91$. Letting F_M be the maximum value of F(u), obtained with $u = u_M$, there results

$$G_{MIN} \simeq 0.30 \; (3N_c-1)^2 \; F_M^{\;2}/\theta_M^{\;2} \qquad (24)$$

and $D/\lambda \simeq (3N_c-1) \; u_M/(4\pi\theta_M)$ (25)

Making $F'(u) = \alpha J_1(\alpha u) - (1 - J_o(\alpha u)) \; J_2(u) = 0$ yields $u_M \simeq 2.2$ and $F_M \simeq 0.39$. The accuracy of these optimum values was verified by comparing G(u) vs. D/λ from (21) to results obtained using an accurate computer model of the lens-multifeed antenna.[3] The comparison was made for a 37-beam lens antenna (F/D = 1) designed to provide a coverage with $\theta_M = 9°$. With the computer model, the directive gain was computed for minima located on the circle of coverage; because gain drops slightly with beam offset these are absolute minima. The results, presented in Fig. 6, show the agreement to be good for D/λ less than optimum value (i.e., D/λ=22) but becoming increasingly poorer as D/λ increases beyond this value. The increasing difference is the result of neglecting the effects of amplitude taper on the radiation pattern. Since the feedhorn aperture increases proportionally to the lens diameter (Eq. (19) with F/D=1), the taper also increases thus causing a progressively larger difference with D/λ. The deficiencies of the approximate mathematical model may be removed by using for F_M and u_M in (24) and (25), values derived from the computer model. These values are $u_M = 2.4$ and $F_M = 0.43$, yielding

$$D/\lambda \simeq 0.57 \; N_c(1.0-3 \; N_c^{\;-1})/\theta_M \qquad (26)$$

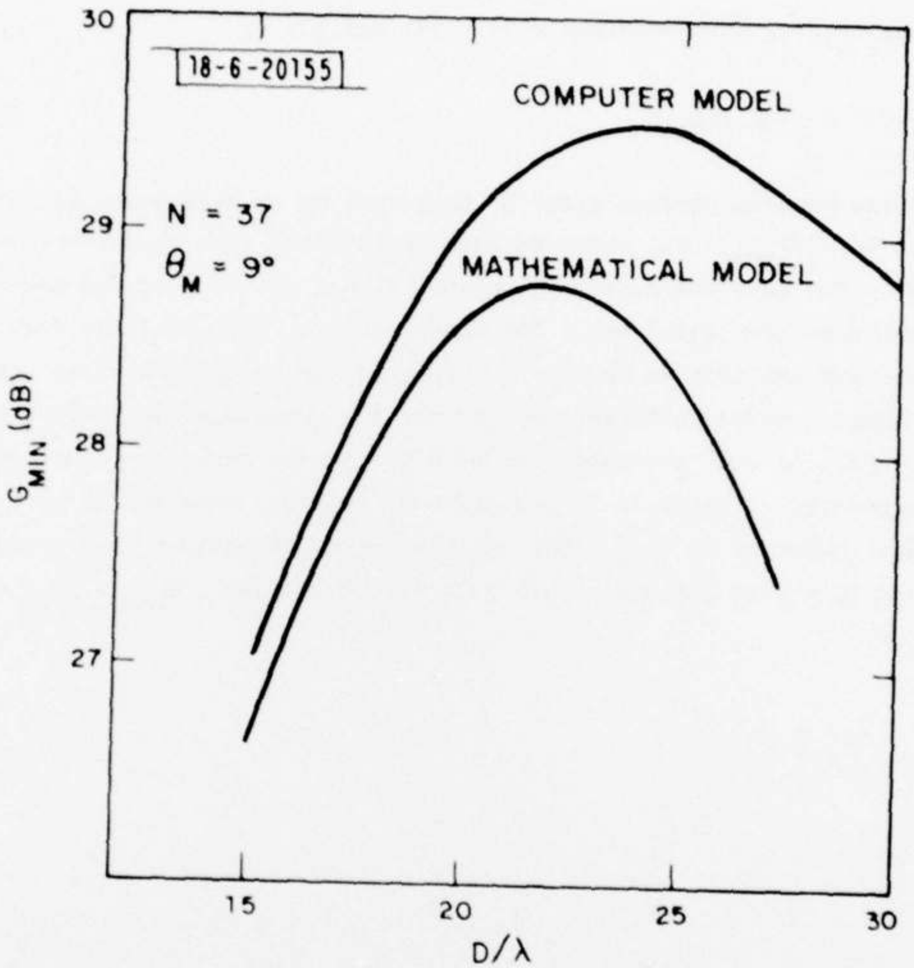

Fig. 6. Minimum directive gain of 37-beam lens antenna as a function of lens diameter.

and $$G_{MIN} \simeq 0.66\ N[1-0.6N^{-1/2}]/\theta_M^{\ 2} \tag{27}$$

The element spacing obtained from (19), (25) and (4) is

$$d/\lambda \simeq 1.32\ F/D \tag{28}$$

and the corresponding optimum ratio of beam spacing to half-power beamwidth is deduced to be $\theta_s/\theta_{HPBW} = 1.1$ which in turn corresponds to a beam cross over level of -4.5 dB. The gain variation, ΔG, within a cell of coverage and the value of the minimum directive gain over a 10% bandwidth were also obtained from the computer model and are plotted in Fig. 7. Contrary to the phased array, the G_{MIN} of the lens antenna is not affected much by varying frequency over modest bandwidths centered on the design frequency. As with the phased array, a less-than-optimum design allows for an appreciable reduction of aperture size and of ΔG without a significant reduction of G_{MIN}. For example, with D/λ optimum ($D/\lambda = 24$), $G_{MIN} =$ 29.5 dB and $\Delta G = 6$ dB while with $D/\lambda = 21$ (12.5% smaller), $G_{MIN} = 29.2$ dB and $\Delta G = 4.7$ dB.

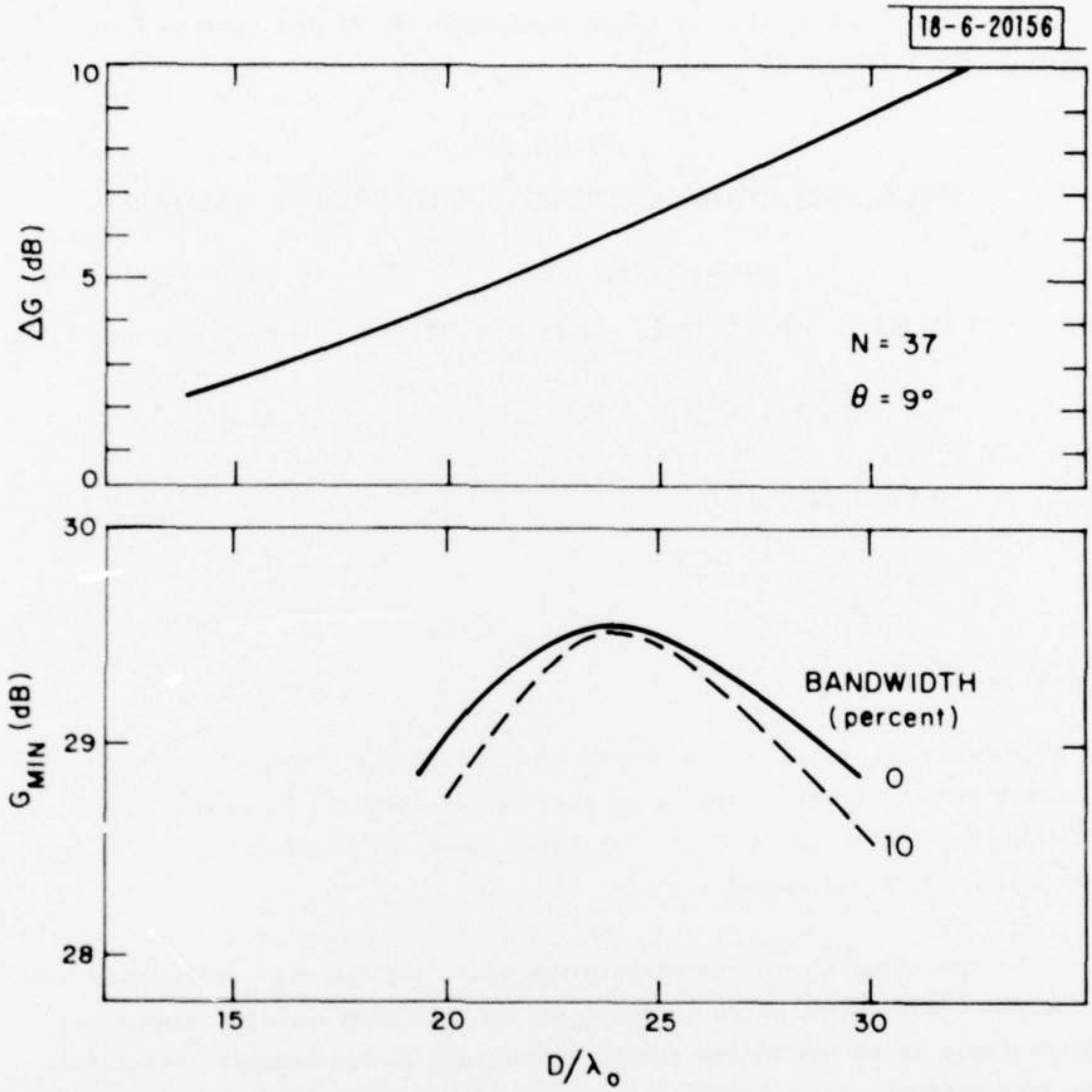

Fig. 7. G_{MIN} over a 10% bandwidth and ΔG of 37-beam lens antenna vs lens diameter.

V. COMPARISON OF PHASED ARRAY AND LENS ANTENNAS

The parameters of the optimum phased array and of the optimum lens antennas are compared in Table II.

TABLE II

DESIGN AND PERFORMANCE PARAMETERS OF BEAM HOPPING ANTENNAS

	Phased Array	Lens
D/λ	$0.39N_c(1 - 0.3\,N_c^{-1})(1 - 0.15\,N_c\,\Delta f/f)/\theta_m$	$0.57N_c(1-0.3\,N_c^{-1})/\theta_M$
d/λ	$0.42\,(1 - 0.3\,N_c^{-1})(1 - 0.15\,N_c\,\Delta f/f)/\theta_m$	$1.32\ F/D$
G_{MIN}	$0.56N(1-0.6\,N^{-1/2})\,(1-0.25\,N_c\Delta f/f)/\theta_M^{\,2}$	$0.66N(1-0.6N^{-1/2})/\theta_M^{\,2}$
ΔG	2.8 dB	6.0 dB
θ_s/θ_{HPBW}	0.81	1.10

In this table, D is the mean diameter of the hexagonal phased array or the diameter of the circular lens, d is the element spacing, N_c is the number of beams on the center row and N is the total number of beams; ΔG and θ_s/θ_{HPBW} are design-frequency values.

The most significant characteristics are: (1) the mean diameter of the hexagonal phased array which is about 30% smaller than the lens diameter, (2) the gain variation within a cell of coverage is appreciably larger for the lens antenna, indicating a larger gain slope variation with angle of observation which translates into more precise pointing requirement, and (3) the minimum directive gain of the phased array decreases with increasing bandwidth-diameter product while that of the lens is independent of this product.

The ratio of the minimum directive gain of the two antenna configurations is plotted in Fig. 8 as a function of the bandwidth-diameter product (N_c $\Delta f/f$). Over the practical range of values considered, the G_{MIN} differential increases to about 2 dB. For vanishing bandwidth, the minimum directive gain of the lens is about 0.7 dB larger than that of the phased array, which is accounted for by the different efficiency assumed for the elements of the phased array and of the feed array. The feed array elements being a little over one wavelength in diameter will have an efficiency close to unity[2], such as was assumed in the analysis. On the other hand, the phased array elements are appreciably larger (d/λ = 2.5 for θ_M = 9°), and their assumed efficiency of $\simeq$ 0.83 is considered appropriate.

A practical application of hopped beam antennas is for earth coverage communications from a synchronous satellite ($\theta_M \simeq$ 9°). Considering a 37-beam antenna for this application and a bandwidth of 5%, the optimum parameters of the phased array and of the lens (F/D=1) configurations are given in Table III.

TABLE III

PARAMETERS OF 37-BEAM ANTENNA

	Phased Array	Lens
Aperture diameter D/λ	15.8	24.4
Element spacing d/λ	2.43	1.32
G_{MIN}(dB)	28.4	29.5
ΔG(dB) at center frequency	2.8	6.0

The center-frequency radiation patterns corresponding to these optimum configurations are presented in Figs. 9 and 10. These patterns were obtained using the exact formulations for the phased array and using the computer model for the lens antenna. The patterns are shown in the plane ϕ = 60° which is a plane passing through minima of the directive gain including the absolute minimum at θ = 9°. The heavy line shows the directive gain as would

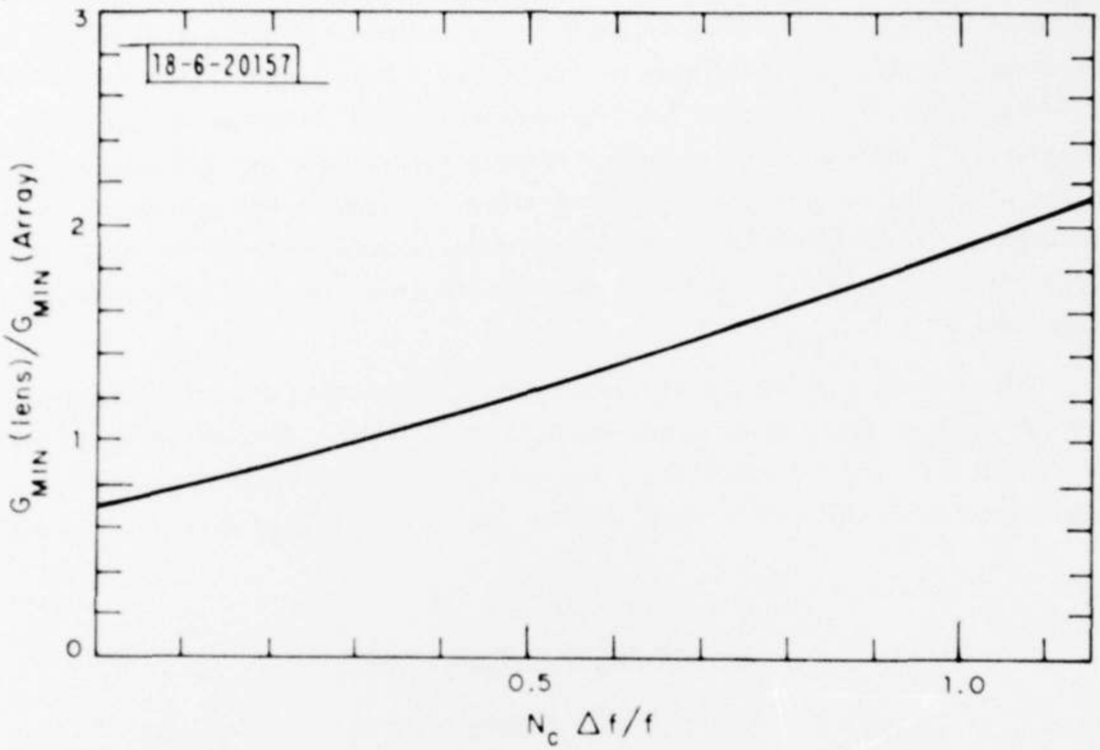

Fig. 8. Ratio of minimum directive gain of phased array and lens antennas.

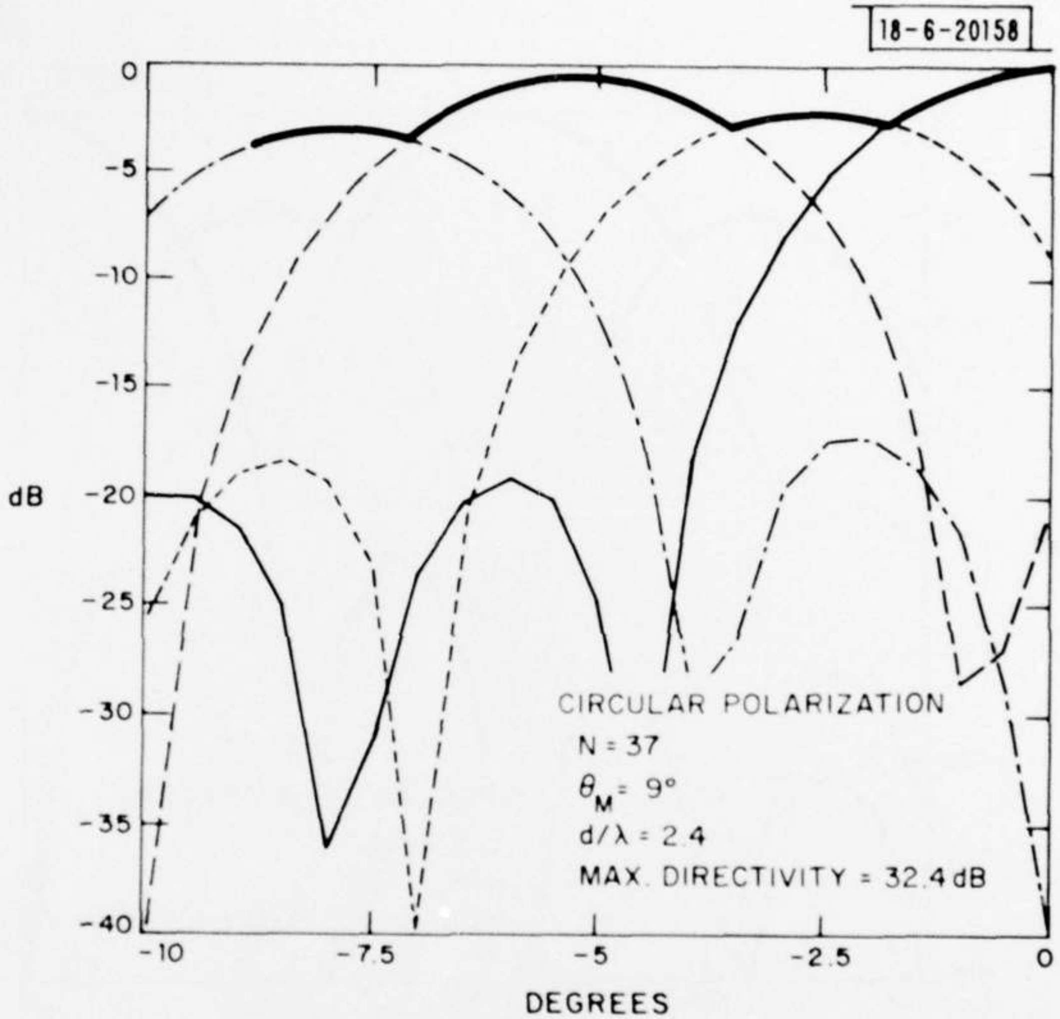

Fig. 9. Radiation patterns of 37-beam phased array antenna.

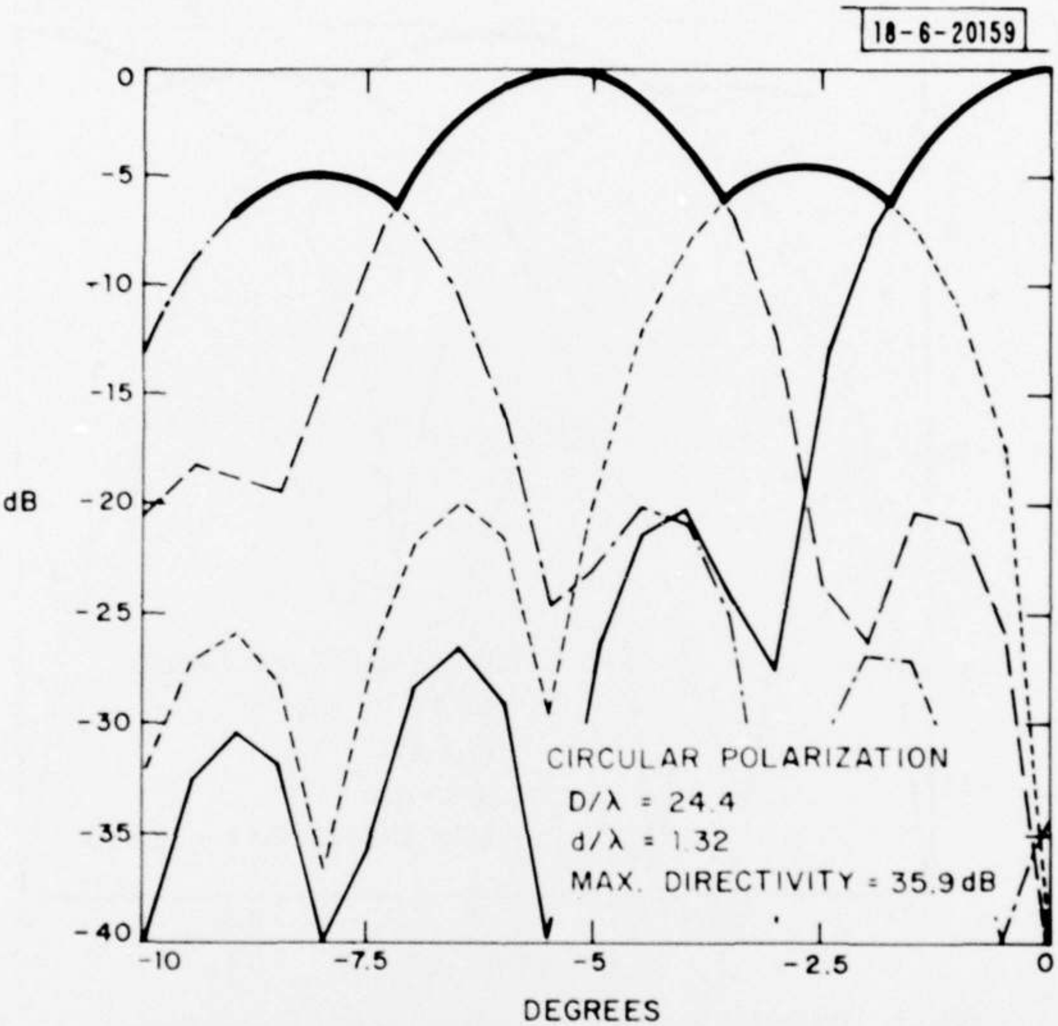

Fig. 10. Radiation patterns of 37-beam lens antenna.

be observed by users in this plane when accessing the communication satellite via the beam pointing closest to their location. It should be observed again that since G_{MIN} is nearly optimum over a broad range of aperture diameter, the latter may be reduced appreciably from the optimum value with little change in G_{MIN}, and with the added benefit of a substantial reduction of the gain variation over a coverage cell.

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The optimum parameters of an antenna whose beam is hopped to uniformly spaced directions within a circular coverage are derived for a phased array and for a multifeed lens antenna. The minimum directive gain, G_{MIN}, within the coverage is the parameter optimized. The analysis shows that, for small bandwidth-diameter products, the two antenna configurations exhibit about the same G_{MIN}, but the optimum aperture diameter is about 30% smaller with the phased array. However, as the bandwidth-diameter product increases, the G_{MIN} of the lens antenna becomes progressively greater than that of the phased array.

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