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Minimum distance based precoder for MIMO-OFDM systems using a 16-QAM modulation

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***Abstract*—A precoder based on the exact optimization of the minimum Euclidean distance $d_{\min}$ between signal points at the receiver side is proposed for MIMO-OFDM systems using a 16-QAM modulation. Assuming that channel state information (CSI) can be made available at the transmitter, the channel is diagonalized and a precoder can be derived. A numerical approach shows that the precoder design depends on the channel characteristics, leading to 8 different precoder expressions. Comparisons with maximum signal-to-noise ratio (SNR) strategy and other precoders based on criteria, such as water-filling (WF), minimum mean square error (MMSE), and maximization of the minimum singular value of the global channel matrix, are performed to illustrate the significant bit-error-rate (BER) improvement of the proposed precoder. In order to make its implementation easier, it is shown that it can be expressed by only two ways without significant performance degradation.**

I. Introduction

The future generation of mobile broadband wireless systems will probably be based on the association of Multiple Input Multiple Output (MIMO) and Orthogonal Frequency Division Multiplex (OFDM) techniques because of the large gains in spectral efficiency, capacity and quality they can achieve compared with single antenna or single carrier links [1], [2]. In order to fully exploit the presence of multiple antennas and add resiliency against ill-conditioning, the linear precoding proposes to adapt the transmitted signal to the channel.

Assuming that the channel state information (CSI) is available at the transmitter, linear precoders can be designed in order to optimize a pertinent criterion such as beamforming, waterfilling (WF), minimum mean square error (MMSE) [3], quality of service (QoS), or maximization of the minimum eigenvalue ($\max -\lambda_{\min}$). Those solutions lead to power allocation with diagonal solutions based on the singular value decomposition.

On the other hand, the precoder in [4] maximizes the minimum Euclidean distance of the received constellation ($\max -d_{\min}$). The exact solution for two transmit symbols and 4-QAM modulation is given and is not a diagonal solution. Performances in terms of BER are significantly enhanced. However, it is difficult to give a general form of this precoder because of the complexity of the optimization. Indeed, the problem depending on the number of symbols, the modulation, and the channel matrix, is still open [5]. A heuristic suboptimal precoder based on the exact solution $\max -d_{\min}$ was derived in [6] and allows to increase the number of transmit symbols with a complexity trade-off.

In this paper, we propose the exact solution of the maximization of $d_{\min}$ with two 16-QAM symbols. In Section II, the system model is described with the matrix notation and the eigenmode representation, before the optimization method is exposed. Section III presents the $\max -d_{\min}$ solution for two 16-QAM symbols. Performances in terms of minimum distance and BER compared to diagonal precoders are presented in Section IV. In order to reduce the complexity, a simplification of the precoder is also proposed in Section V. Our conclusions are drawn in Section VI.

II. Optimization of the minimum distance

A. Channel model

When CSI is available at the transmitter, it was shown that every MIMO system can be simplified and virtually diagonalized. Let us consider a MIMO system with n_R receive and n_T transmit antennas over which we want to achieve b independent data streams. The received signal can then be expressed as

$$\mathbf{y} = \mathbf{G}_D \mathbf{H}_v \mathbf{F}_D \mathbf{s} + \mathbf{G}_D \nu_v \tag{1}$$

where $\mathbf{s}$ is the $b \times 1$ vector of transmitted symbols, $\mathbf{H}_v = \mathbf{G}_v \mathbf{H} \mathbf{F}_v$ is the virtual channel matrix of size $b \times b$, $\nu_v = \mathbf{G}_v \nu$ is an additive white Gaussian noise vector of size $b \times 1$ and $\mathbf{F}_D$ and $\mathbf{G}_D$ are $b \times b$ matrices, representing respectively the precoder and decoder that maximize $d_{\min}$.

The virtual channel matrix $\mathbf{H}_v$ is

$$\mathbf{H}_v = \text{diag}(\sigma_1, ..., \sigma_b) \tag{2}$$

where σ_i stands for every subchannel gain (sorted by decreasing order).

The power constraint can be expressed as

$$\text{trace}\{\mathbf{F}_D \mathbf{F}_D^*\} = p_0 \tag{3}$$

where p_0 is the mean available transmit power. As only an ML detection is considered in the rest of the paper, the decoder matrix $\mathbf{G}_D$ has no impact on the performance and is consequently assumed to be $\mathbf{G}_D = \mathbf{I}_b$, with $\mathbf{I}_b$ the identity matrix of size $b \times b$.

B. Minimum Euclidean distance

Let us denote $\mathcal{S}$ the set of all possible transmitted vectors $\mathbf{s}$. The precoder matrix maximizing the minimum Euclidean distance $d_{\min}$ of the received constellation under the power constraint has to be determined. $d_{\min}$ can be expressed as

$$d^2_{\min} = \min_{\mathbf{s}_k, \mathbf{s}_l \in S, \mathbf{s}_k \neq \mathbf{s}_l} \|\mathbf{H}_v \mathbf{F}_D (\mathbf{s}_k - \mathbf{s}_l)\|^2 \quad (4)$$

Let us define a difference vector $\breve{\mathbf{x}} = \mathbf{s}_k - \mathbf{s}_l$ with $\mathbf{s}_k \neq \mathbf{s}_l$. The reduced set $\breve{X}$ contains difference vectors $\breve{\mathbf{x}}$ without any redundancy. The minimum distance can then be expressed as

$$d^2_{\min} = \min_{\breve{\mathbf{x}} \in \breve{X}} \|\mathbf{H}_v \mathbf{F}_D \breve{\mathbf{x}}\|^2 \quad (5)$$

Determining the precoder matrix $\mathbf{F}_D$ which maximizes $d_{\min}$ is difficult for two reasons: the solution depends on the symbol alphabet and the space of solutions is large. To simplify, the proposed technique is only derived for $b = 2$ virtual channels using cosine function. Let us remark that $b \leq \text{rank}(\mathbf{H}) \leq \min(n_T, n_R)$.

C. Reduction of optimization space

Considering this bi-dimensional virtual system, the virtual channel matrix can be expressed as

$$\mathbf{H}_v = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} = \rho \begin{pmatrix} \cos\gamma & 0 \\ 0 & \sin\gamma \end{pmatrix} \quad (6)$$

where $\rho = \sqrt{\sigma_1^2 + \sigma_2^2}$ and $\gamma = \arctan \frac{\sigma_2}{\sigma_1}$ stand respectively for the channel gain and the channel angle. Since $\sigma_1 \geq \sigma_2 > 0$, we have $0 < \gamma \leq \pi/4$.

The precoder matrix can be simplified as in [4]:

$$\mathbf{F}_D = \sqrt{p_0} \begin{pmatrix} \cos\psi & 0 \\ 0 & \sin\psi \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \quad (7)$$

Considering all symmetries in usual constellations, the influence of the angles on the Euclidean distance has to be studied only for $0 \leq \varphi \leq \pi/2$ and $0 \leq \theta \leq \pi/4$, $0 \leq \psi \leq \pi/2$. The angles θ and φ correspond to scaling and rotation of the received constellation, respectively. When both are equal to 0, the matrix $\mathbf{F}_D$ is diagonal and leads to the power allocation. The angles corresponding to the optimal precoder with the $d_{\min}$ criterion can now be determined.

D. Optimal $\max -d_{\min}$ precoder for a QPSK modulation

Firstly, if a QPSK modulation with $b = 2$ data streams is considered, the symbols belong to the set

$$S_{QPSK} = \frac{1}{\sqrt{2}} \{(1+i), (1-i), (-1+i), (-1-i)\} \quad (8)$$

It was shown in [4] that this QPSK solution is rather simple, with only two precoder expressions

i) $0 \leq \gamma \leq \gamma_0$

$$\mathbf{F}_D = \mathbf{F}_{r_1} = \sqrt{p_0} \begin{pmatrix} \sqrt{\frac{3+\sqrt{3}}{6}} & \sqrt{\frac{3-\sqrt{3}}{6}} e^{i\frac{\pi}{12}} \\ 0 & 0 \end{pmatrix} \quad (9)$$

ii) $\gamma_0 \leq \gamma \leq \pi/4$

$$\mathbf{F}_D = \mathbf{F}_{octa} = \sqrt{\frac{p_0}{2}} \begin{pmatrix} \cos\psi & 0 \\ 0 & \sin\psi \end{pmatrix} \begin{pmatrix} 1 & e^{i\frac{\pi}{4}} \\ -1 & e^{i\frac{\pi}{4}} \end{pmatrix} \quad (10)$$

$$\text{where} \begin{cases} \psi = \arctan \frac{\sqrt{2}-1}{\tan\gamma} \\ \gamma_0 = \arctan \sqrt{\frac{3\sqrt{3}-2\sqrt{6}+2\sqrt{2}-3}{3\sqrt{3}-2\sqrt{6}+1}} \approx 17.28^\circ \end{cases}$$

The parameter ψ is linked to the power allocation, and the constant threshold γ_0 allows the precoder to use one or two sub-channels. γ_0 is obtained when considering that the two precoders give the same minimum Euclidean distance $d_{\min}$. This one depends on ρ and γ and is expressed as

$$d_{\min} = \begin{cases} \sqrt{p_0}\rho\sqrt{1 - \frac{1}{\sqrt{3}}}\cos\gamma & \text{if } 0 < \gamma \leq \gamma_0 \\ \sqrt{p_0}\rho\sqrt{\frac{(4-2\sqrt{2})\cos^2\gamma\sin^2\gamma}{1+(2-2\sqrt{2})\cos^2\gamma}} & \text{if } \gamma_0 < \gamma \leq \pi/4 \end{cases} \quad (11)$$

III. Extension of $\max -d_{\min}$ precoder for a 16-QAM modulation

In the case of a 16-QAM modulation, the symbols belong to the following set

$$S = \frac{1}{\sqrt{10}} \{(\pm 1 \pm i), (\pm 1 \pm 3i), (\pm 3 \pm i), (\pm 3 \pm 3i)\} \quad (12)$$

A numerical search on ψ, θ and φ to maximize the Euclidean distance for every angle γ leads to eight different expressions. If γ stays under γ_0, then only the best sub-channel is used as in the max-SNR strategy and the precoder will be denoted $\mathbf{F}_{r1}$. On the other hand, if $\gamma_i < \gamma < \gamma_{i+1}$, the precoder leads to a 256-points constellation on both receivers, and it will be denoted as $\mathbf{F}_{T_i}$, respectively.

A. Expression of $\mathbf{F}_{r1}$ precoder

For every $\gamma \leq \gamma_0$, the numerical maximization of $d_{\min}$ gives an angle $\psi = 0$, meaning that only the best virtual sub-channel is used (i.e. the first one, since $\sigma_1 \geq \sigma_2$). A received constellation on this subchannel is represented on Fig.1 (for $\psi = 0$ and arbitrary θ and φ). The points denoted from 1 to 256 correspond to the 256 received symbols.

$d_{\min}$ is optimized such that nearest neighbors have the same distance. On Fig.1, the optimized solution can be obtained with $d_{12,16} = d_{16,29} = d_{29,12}$ corresponding to the 3 difference vectors

$$\breve{x}_1 = \frac{1}{\sqrt{10}} \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \breve{x}_2 = \frac{1}{\sqrt{10}} \begin{pmatrix} 2 \\ -6 \end{pmatrix}, \breve{x}_3 = \frac{1}{\sqrt{10}} \begin{pmatrix} 2 \\ -6+2i \end{pmatrix} \quad (13)$$

The corresponding distances lead to the system

$$\begin{cases} d^2_{\breve{x}_1} = \frac{\cos^2\gamma}{10} \times (4 - 4\cos^2\theta) \\ d^2_{\breve{x}_2} = \frac{\cos^2\gamma}{10} \times (-32\cos^2\theta - 24\cos\theta.\sin\theta.\cos\varphi + 36) \\ d^2_{\breve{x}_3} = \frac{\cos^2\gamma}{10} \times (-8\cos\theta.\sin\theta.\sin\varphi - 36\cos^2\theta \\ \quad +40 - 24\cos\theta.\sin\theta.\cos\varphi) \end{cases} \quad (14)$$

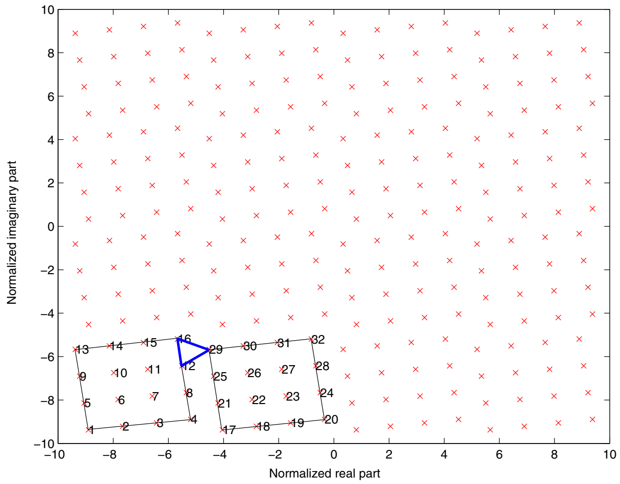


Fig. 1. First virtual subchannel constellation for $\psi = 0$

whose resolution gives

$$\begin{cases} \varphi_0 = \arctan \frac{1}{6+\sqrt{3}} \approx 7.3693^\circ \\ \theta_0 = \arctan(2\sin\varphi_0) \approx 14.3877^\circ \end{cases} \tag{15}$$

The received constellation looks like a 256-QAM constellation rotated by 7.369 °. This solution is close to the max-SNR strategy, but leads to a little higher $d_{\min}$. The optimization of $d_{\min}$ is always obtained by the difference vector $\breve{x}_1 = \frac{1}{\sqrt{10}}\begin{pmatrix}0\\2\end{pmatrix}$ and the distance obtained by $\mathbf{F}_{r1}$ is then

$$d_{r1}^2 = p_0\rho^2 \frac{2\sin^2\theta_0}{5}.\cos^2\gamma \tag{16}$$

B. Expression of $\mathbf{F}_{T_i}$ precoders

For every $\gamma_0 < \gamma \leq \gamma_1$, the angles θ et φ are fixed. Moreover, ψ depends on γ, allowing a power allocation over the two virtual subchannels. The minimum Euclidean distance of $\mathbf{F}_{T_1}$ is obtained when $\theta = 45^\circ$ and $\varphi = 45^\circ$. The precoder $\mathbf{F}_D$ is now expressed as a function of ψ

$$\mathbf{F}_{T_1} = \sqrt{p_0}\begin{pmatrix}\cos\psi & 0\\ 0 & \sin\psi\end{pmatrix}\frac{1}{2}\begin{pmatrix}\sqrt{2} & 1+i\\ -\sqrt{2} & 1+i\end{pmatrix} \tag{17}$$

When γ is explored from 0° to 45°, the value of ψ maximizing $d_{\min}$ is obtained with the 2 difference vectors $\breve{a}_1 = 1/\sqrt{10}\begin{pmatrix}2\\-2+2i\end{pmatrix}$ and $\breve{b}_1 = 1/\sqrt{10}\begin{pmatrix}4+4i\\-6\end{pmatrix}$.

If we denote respectively $d_{\breve{a}_1}$ and $d_{\breve{b}_1}$ the minimum Euclidean distance linked respectively to the difference vectors $\breve{a}_1$ and $\breve{b}_1$, the optimum precoder is obtained when $d_{\breve{a}_1} = d_{\breve{b}_1}$

$$\begin{cases} d_{\breve{a}_1}^2 = \frac{1}{10}(6 + 4\sqrt{2} + 12\cos^2\gamma.\cos^2\psi - 6\cos^2\psi \\ \quad -6\cos^2\gamma - 4\sqrt{2}\cos^2\psi - 4\sqrt{2}\cos^2\gamma) \\ d_{\breve{b}_1}^2 = 1/10 \times (34 + 24\sqrt{2} - 68\cos^2\gamma.\cos^2\psi - 34\cos^2\psi \\ \quad -34\cos^2\gamma - 24\sqrt{2}\cos^2\psi - 24\sqrt{2}\cos^2\gamma) \end{cases} \tag{18}$$

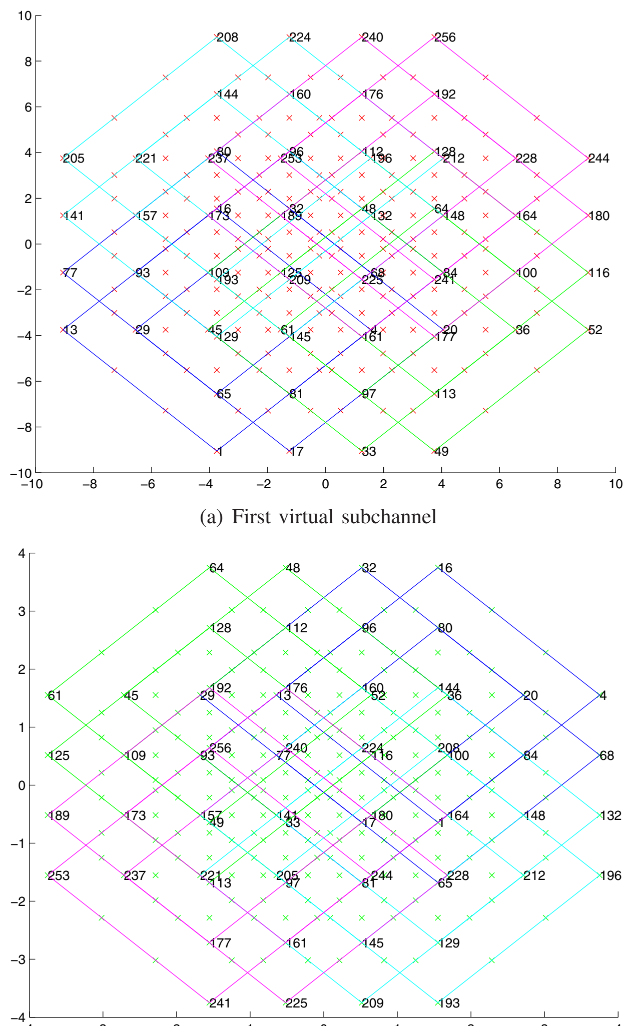
Considering $d_{\breve{a}_1} = d_{\breve{b}_1}$, we get ψ as a function of γ

$$\tan(\psi_1) = \frac{5\sqrt{2}-7}{\tan\gamma} \tag{19}$$

The precoder $\mathbf{F}_{T_1}$ is then obtained by substituting ψ_1 in equation (17), and the corresponding distance is finally given by

$$d_{T_1}^2 = p_0\rho^2\frac{20-14\sqrt{2}}{5}\frac{\sin^2\gamma}{\tan^2\gamma + (5\sqrt{2}-7)^2} \tag{20}$$

Using the same process and exploring all possible values of ψ, θ and φ, it is possible to obtain the six other precoders $\mathbf{F}_{T_i}$, $i = 2\ldots7$. The corresponding angles ψ_i, θ_i and φ_i are summarized in Tab. I ($\alpha = 1 + \frac{6}{\sqrt{34}}$). To obtain a precoder design depending only on the channel angle γ, these values or expressions have to be injected in (7). Every $\mathbf{F}_{T_i}$ precoder allows the use of both subchannels and consequently the power allocation at the transmitter. As an example, the received constellation using $\mathbf{F}_{T_7}$ is given by Fig.2.



(a) First virtual subchannel

(b) Second virtual subchannel

Fig. 2. Received constellation for precoder $\mathbf{F}_{T_7}$

C. *Evolution of the minimum distance*

Previously obtained optimal distances are only dependent on the parameter γ representing the channel. In order to choose between all the precoders and obtain the corresponding thresholds, we have to search for γ_0 such that $d_{r1} = d_{T_1}$ and γ_i such that $d_{T_i} = d_{T_{i+1}}$ with $i \geq 1$ (Fig.3).

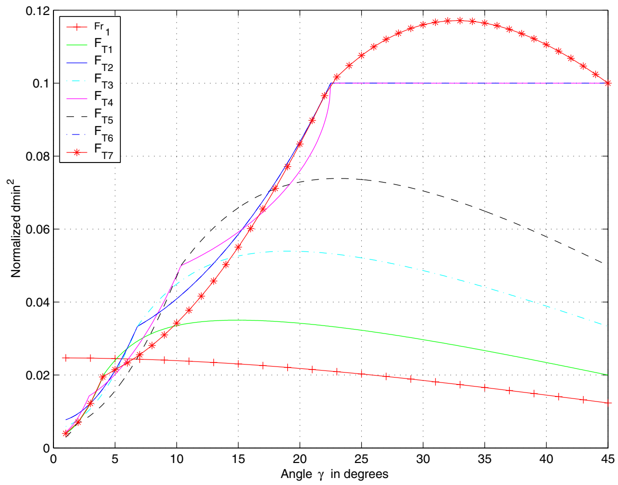


Fig. 3. Evolution of $d_{\min}$ as a function of γ for a 16-QAM modulation

For example, let us solve the equation $d_{r1} = d_{T_1}$ to obtain γ_0. These distances are given by

$$\begin{cases} d_{r1}^2 = p_0\rho^2 \frac{2\sin^2\theta_0}{5}.\cos^2\gamma \\ d_{T_1}^2 = p_0\rho^2 \frac{20-14\sqrt{2}}{5}\frac{\sin^2\gamma}{\tan^2\gamma+(5\sqrt{2}-7)^2} \end{cases} \tag{21}$$

Considering $d_{r1} = d_{T_1}$, we get

$$\gamma_0 = \arctan\sqrt{\frac{M_0}{1-M_0}(5\sqrt{2}-7)^2} \approx 5.128° \tag{22}$$

where $M_0 = \frac{\sin^2\theta_0}{10-7\sqrt{2}}$. The other thresholds γ_i are obtained using the same process and are expressed in Tab. I.

IV. Comparison to other precoders

Minimum Euclidean distances for every precoder are shown on Fig. 4 in the case of a 16-QAM modulation. The dotted black curve represents the upper bound of $\mathbf{F}_{d_{\min}}$, meaning that its expression is selected among F_{r1} and F_{T_i}, $\quad i = 1..7$ depending on the value of γ.

When $\gamma \leq \gamma_0$, performances of $\max -d_{\min}$ and *Beamforming* are very close with the same difference. The light advantage of $\mathbf{F}_{d_{\min}}$ is due to the rotation of $7.36°$. These two precoders are the only ones whose $d_{\min}$ remains different from 0 when γ reaches small values.

When γ increases, the $\max(\lambda_{\min})$ solution is better than MMSE, WF, QoS 3dB, and WF solutions in terms of $d_{\min}$, but it is really outperformed by the $\max -d_{\min}$ precoder.

Due to this considerable improvement of $d_{\min}$, a significant increase of BER performance of the $\mathbf{F}_{d_{\min}}$ is expected compared to the diagonal precoders.

This is confirmed by Fig. 5 representing BER with respect to the SNR for a MIMO-OFDM system using a 16-QAM

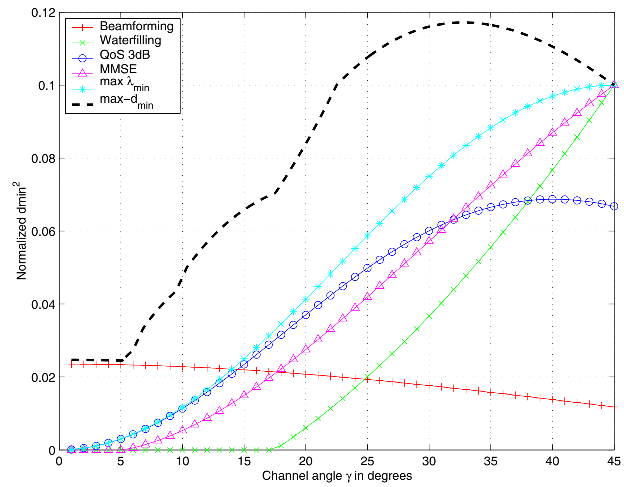


Fig. 4. Minimum Euclidean distance of precoders for a 16-QAM modulation

modulation, $n_T = 4$ transmitters, $n_R = 2$ receivers, 128 subcarriers and a Rayleigh channel. The $\max -d_{\min}$ precoder proposes a gain of 3 dB compared to diagonal precoders for a BER $= 10^{-5}$, which clearly demonstrates its interest when an ML receiver is used (this gain will even be higher if the number of antennas increases).

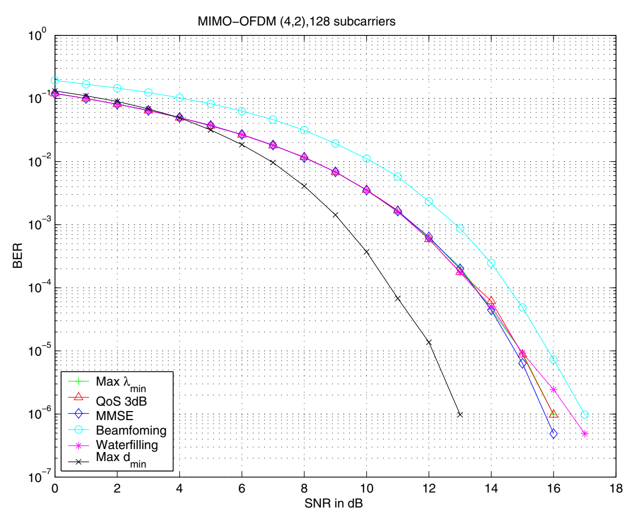


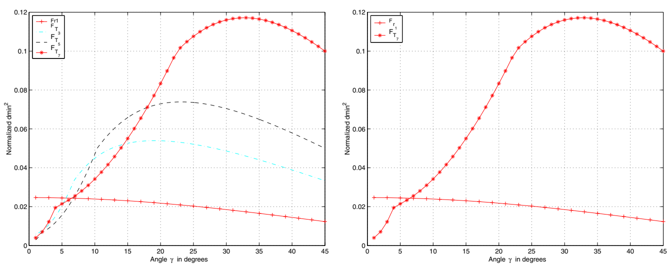
Fig. 5. Comparison of precoders in terms of BER for a MIMO-OFDM system using a 16-QAM modulation, $n_T = 4$ transmitters, $n_R = 2$ receivers, 128 subcarriers and a Rayleigh channel

V. Simplification of the precoder

The $\mathbf{F}_{d_{\min}}$ precoder can be simplified in order to implement it in a easier way. Indeed, if we look again at Fig. 3, we observe that some precoders are useful for a very small range of γ (e.g., $\gamma_1 - \gamma_0 = 0.84°$), and may be canceled. Two sets of precoders are represented in terms of minimum distance on Fig. 6 for only two ($\mathbf{F}_{r1}$ and $\mathbf{F}_{T_7}$) and four ($\mathbf{F}_{r1}$, $\mathbf{F}_{T_3}$, $\mathbf{F}_{T_5}$ and $\mathbf{F}_{T_7}$) different expressions.

Precoder	ψ (in degrees)	φ (in degrees)	θ (in degrees)	γ (in degrees)
F_{r1}	0	7.37	14.39	$\gamma_0 \approx 5.128$
F_{T_1}	$\psi_1 = \arctan \frac{5\sqrt{2}-7}{\tan\gamma}$	45	45	$\gamma_1 \approx 5.96$
F_{T_2}	45	$\varphi_2 = \arcsin \frac{1}{2\cos 2\gamma}$	45	$\gamma_2 \approx 6.81$
F_{T_3}	$\psi_3 = \arccos \sqrt{\frac{\alpha-\alpha.\cos^2\gamma}{\alpha-2\cos^2\gamma}}$	$\varphi_3 = \arctan \frac{3}{5}$	45	$\gamma_3 \approx 9.45$
F_{T_4}	45	$\varphi_4 = \arctan \frac{\cos^2 2\gamma+1-\sqrt{6\cos^2 2\gamma-3}}{2-\cos^2 2\gamma}$	$\theta_4 = \frac{1}{2}\arctan \frac{1}{\cos\varphi_4-\sin\varphi_4}$	$\gamma_4 \approx 10.35$
F_{T_5}	$\psi_5 = \arctan \frac{\sqrt{10/\sqrt{14}-1}}{\sqrt{10/\sqrt{14}+1}\times\tan\gamma}$	$\varphi_5 = \arctan \frac{1}{3}$	$\theta_5 = \frac{1}{2}\arctan \frac{\sqrt{10}}{2}$	$\gamma_5 \approx 17.53$
F_{T_6}	45	$\varphi_6 = \arcsin \frac{1}{2\cos 2\gamma}$	45	$\gamma_6 \approx 22.50$
F_{T_7}	$\psi_7 = \arctan \frac{\sqrt{2}-1}{\tan\gamma}$	45	45	

TABLE I
ANGLE VALUES FOR $d_{\min}$ PRECODER



(a) 4 different expressions (b) 2 different expressions

Fig. 6. Simplification of $\mathbf{F}_{d_{\min}}$ precoder

The corresponding BERs for the same communication system as in Section IV are represented on Fig. 7. The simplest version of the precoder, $\max - d_{\min} - v2$, offers very good performances and remains very close to the other sets $\max - d_{\min} - v4$ and $\max - d_{\min} - v8$.

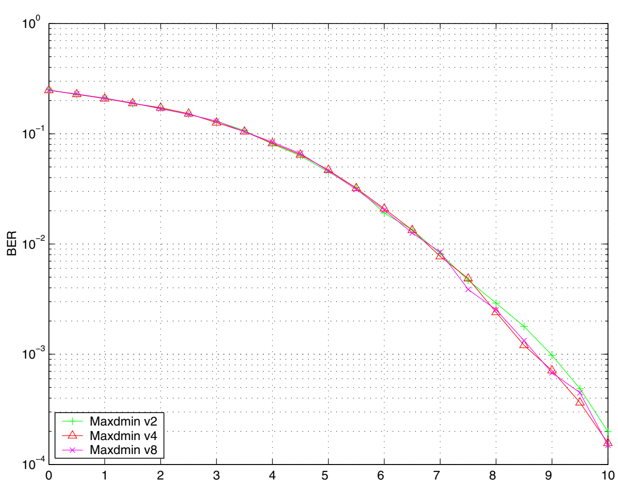


Fig. 7. BER performance of the different sets of precoders for a MIMO-OFDM system using a 16-QAM modulation, $n_T = 4$ transmitters, $n_R = 2$ receivers, 128 subcarriers and a Rayleigh channel

VI. CONCLUSION

A new exact solution of the maximization of the minimum Euclidean distance between received symbols has been proposed for two 16-QAM modulated symbols. This precoder shows an important enhancement of the $d_{\min}$ compared to diagonal precoders which leads to a significant BER improvement. For a MIMO-OFDM system using $n_T = 4$ transmitters, $n_R = 2$ receivers, 128 subcarriers and transmitting $b = 2$ data streams over a Rayleigh channel, it outperforms traditional precoding strategies such as waterfilling, beamforming, or minimizing the mean square error by over 3 dB at $BER = 10^{-5}$.

This new strategy selects the best precoding matrix among eight different expressions, depending on the value of the channel angle γ. In order to decrease the complexity, other sets of precoders have been proposed and the performances of the simplest one, composed only of $\mathbf{F}_{r_1}$ and $\mathbf{F}_{T_7}$ remain very close to the optimal in terms of BER.

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