MINIMUM WEIGHT DESIGN OF FUSELAGE TYPE STIFFENED CIRCULAR CYLINDRICAL SHELLS SUBJECT TO UNIFORM AXIAL COMPRESSION

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SUMMARY

A procedure is outlined by which one may design a fuselage-type stiffened circular cylindrical shell under a given uniform axial compression with minimum weight. The precise statement of the problem is as follows:

Given an internally stiffened circular cylindrical shell of specified material, radius, and length, find the size, shape and spacings of the stiffeners and thickness of the skin such that it can safely carry a given uniform axial compression with minimum weight.

The objective function is the cylinder weight. The behavioral equality constraint is the general instability load. The behavioral inequality constraints are the panel buckling, skin wrinkling, local instability of stringers, limitation on the stress level in the skin, stringers, and rings, and simultaneous occurrence of failure modes. By a proper grouping of the parameters involved, the solution is accomplished by separation into two phases: "Phase 1" and "Phase 2." "Phase 1" leads to design charts and tables, which are then used in "Phase 2" to arrive at a minimum weight configuration satisfying all constraints. The solution in "Phase 1" is accomplished by using the irregular simplex search method of Nelder and Mead in combination with the golden section method.

The cylinder geometries, taken up in the design examples, correspond to the moderately and heavily loaded shell and a geometry similar to C-141 fuselage immediately after the wing box. The design results have shown that the minimum weight design configuration is not unique. The design approach allows the designer to deviate from the minimum weight solution with minimum weight penalty, in order to avoid simultaneous occurrence of failure modes and/or unrealistic design variables. For one particular design, the moderately loaded shell, the effect of the shapes of the stiffening members is assessed by considering a number of stiffener shapes. For this case with the geometric constraint that no design dimensions are less than .02 in., it has been found out that the circular cylindrical shell stiffened by tee stringers and rectangular rings is most efficient. The design for this case, without minimum gauge restriction, has also been done, using rectangular rings and stringers, for comparison purposes. The resulting design has shown a weight improvement of 45.3 per cent over the best previously obtained result which has been reported in the open literature. For all cases, the curves of weight vs. skin thickness are relatively flat. Thus, large variations in the skin thickness yield design configurations with small differences in weight.

NOTATIONS

A_x, A_y	Stringer and ring cross-sectional area, in^2
C_x, C_y	Stringer and ring shape parameter
D	Flexural stiffness of the skin, in-lb
D_{xx}, D_{yy}, D_{xy}	Orthotropic flexural and twisting stiffnesses, in-lb
D_{xxst}, D_{yyr}	Flexural stiffnesses of stringer and ring, in-lb
E, E_x, E_y	Young's moduli of elasticity of skin, stringer, and ring, psi
E_{xx}, E_{yy}	Orthotropic extensional stiffnesses, lb/in
E_{xxp}, E_{yyp}	Extensional stiffnesses of skin, lb/in
E_{xxst}, E_{yyr}	Extensional stiffnesses of stringer and ring, lb/in
$(GJ)_{x \text{ or } y}$	Stiffener contributions to torsional stiffness, in^2-lb
G_{xy}	Inplane skin shear stiffness, lb/in
I_{xc}, I_{yc}	Stringer and ring moment of inertia about their centroidal axes, in^4
$\bar{K}_{xx}$, $\bar{K}_{yy}$, $\bar{K}_s$	Buckling load coefficient of axial compression, pressure, and torsion
$\bar{K}_{xxp}$	Panel buckling load coefficient
L	Total length of the shell, in
M_{xx}, M_{yy}, M_{xy}	Moment resultants, in-lb/in
$\bar{N}$	Applied axial compressive load, lb/in
N_{xx}, N_{yy}, N_{xy}	Stress resultants, lb/in
$\bar{N}_{xx_{cr}}$	Critical axial compressive load, lb/in
$\bar{N}^*$	Nondimensional load parameter

R	Radius of the shell, in
T	Applied torque, in-lb
W	Weight of the shell, lb
$\overline{W}$	Nondimensional weight parameter
W^*	Composite weight function
$\overline{W}^*$	Nondimensional composite weight function
Z	Curvature parameter, $\frac{L^2(1-\nu^2)^{1/2}}{Rh}$
b_{fx}, b_{fy}	Flange widths of stringer and ring, in
c_{fx}, c_{fy}	Flange to web thickness ratios of stringer and ring
d_{wx}, d_{wy}	Stringer and ring depths, in
e_x, e_y	Stringer and ring eccentricities, in
$\overline{e}_x$, $\overline{e}_y$	Nondimensional stringer and ring eccentricities
h	Skin thickness, in
k_s, k_r	Width to depth ratios of stringer and ring
ℓ_x, ℓ_y	Stringer and ring spacings, in
m, n	Number of axial and circumferential waves for general instability
m_p, n_p	Number of axial and circumferential waves for panel instability
q	Applied pressure (positive outward), psi
t_{wx}, t_{wy}	Thickness of web of stringer and ring, in
t_{fx}, t_{fy}	Thickness of flange of stringer and ring, in
u, v, w	Displacement components of reference surface points, in
x, y, z	Coordinate system
$\overline{\alpha}_x$, $\overline{\alpha}_y$	Nondimensional radii of gyration of stringer and ring

γ	Shear strain at any point
γ_{xy}	Shear strain of point on reference surface
ε_x, ε_y	Normal strains at any point
ε_{xx}, ε_{yy}	Normal strains of point on reference surface
κ_{xx},κ_{yy},κ_{xy}	Changes of curvatures
λ	Lagrange multiplier
λ^*	Nondimensional Lagrange multiplier
$\bar{\lambda}_{xx}$, $\bar{\lambda}_{yy}$	Nondimensional extensional stiffnesses of stringer and ring
ν	Poisson's ratio
ρ_x, ρ_y	Weight density of stringer and ring, lb/in^3
ρ_{sk}	Weight density of skin, lb/in^3
$\bar{\rho}_{xx}$, $\bar{\rho}_{yy}$	Nondimensional flexural stiffnesses of stringer and ring
σ_o	Yield stress
σ_{xxsk},σ_{yysk}	Prebuckling stresses of the skin, psi
σ_{xxst}, σ_{yyr}	Prebuckling stresses of stringer and ring, psi
$\sigma_{xxsf_{cr}}$,$\sigma_{xxsw_{cr}}$	Critical stresses of stringer flange and web, psi
$\sigma_{xxsk_{cr}}$	Critical local skin buckling stress, psi

Superscript "o" indicates membrane state

Superscript "1" indicates an additional quantity necessary to bring the membrane state to the classical buckling state

GLOSSARY OF ABBREVIATIONS

AR	Angle ring
AS	Angle stringer
ASRR	Angle stringer and rectangular ring
CR	Channel ring
CS	Channel stringer
CSCR	Channel stringer and ring
CSRR	Channel stringer and rectangular ring
CSTR	Channel stringer and tee ring
GB	Gross buckling, $\overline{N}/\overline{N}_{xx_{cr}}$
IR	I ring
IS	I stringer
IAR	Inverted angle ring
IAS	Inverted angle stringer
ISIR	I stringer and ring
MG	Minimum gauge
PB	Panel buckling, $\overline{N}/\overline{N}_{xxp_{cr}}$
RR	Rectangular ring
RS	Rectangular stringer
RYT	Ring yielding in tension, σ_{yyr}/σ_o
RSRR	Rectangular stringer and ring
SB	Skin buckling, $\sigma_{xxsk}/\sigma_{xxsk_{cr}}$
SY	Skin yielding, σ_{xxsk}/σ_o

STB	Stringer buckling, $\sigma_{xxst}/\sigma_{xxst_{cr}}$
STFB	Stringer flange buckling, $\sigma_{xxst}/\sigma_{xxsf_{cr}}$
STWB	Stringer web buckling, $\sigma_{xxst}/\sigma_{xxsw_{cr}}$
STYC	Stringer yielding in compression, σ_{xxst}/σ_{o}
TR	Tee ring
TS	Tee stringer
TSCR	Tee stringer and channel ring
TSRR	Tee stringer and rectangular ring
TSTR	Tee stringer and ring
WMG	Without minimum gauge
ZR	Zee ring
ZS	Zee stringer
ZSZR	Zee stringer and ring

CHAPTER I

INTRODUCTION

Statement of the Problem

As the size of modern aerospace vehicles increases, the demand for light weight structures increases. This has made the structural engineer, engaging in this area, more and more conscious of minimum weight design. A structural configuration that is used widely in aerospace vehicles is the stiffened thin cylindrical shell. Since stiffened thin cylindrical shells have been used extensively in the past thirty years, a tremendous effort has been exerted in designing such a configuration for minimum weight. Gerard [1] has presented a comprehensive bibliography on the subject of optimal structural design. His work has been extended by Niordson and Pedersen [2]. Better understanding, during the past decade, of the failure modes of the stiffened thin cylindrical shells, for aerospace use, has produced some important results in the attempt to achieve minimum weight design [3-17]. A detailed discussion of these efforts is presented in the next section.

The precise statement of the problem considered in this research effort is as follows: Given an internally stiffened circular cylindrical shell of specified material, radius, and length, find the size, shape, and spacings of the stiffeners, and the thickness of the skin, such that the resulting design configuration can safely carry a given uniform axial compressive load with minimum weight.

The design objective is minimum weight. The general instability load is taken to be as an equality constraint, because it represents the principal catastrophic mode of failure for present day aircrafts. Panel instability, another catastrophic mode of failure, is considered as an inequality constraint. This means that the material of the design configuration is distributed in such a way that this mode of failure is avoided. Other behavioral inequality constraints are the wrinkling of the skin, local instability of the stringers and limitations on the stress level of the skin, stringers, and rings and simultaneous occurrence of failure modes. In addition, geometric inequality constraints are used, which represent the realistic dimensions for the design variables (thickness and spacings of stiffeners, etc.).

Depending on the size of the fuselage, the level of the applied loads, and the section of the fuselage to be designed, different primary criterion must be used. For example, for some section of the fuselage, the primary consideration in the design process is strength, for others it is stiffness. Finally, for a large section, usually in the middle part, it is general instability. Therefore the role of the primary consideration and constraints are interchanged for different sections. The present thesis is concerned with the minimum weight design of that part which the primary consideration is general instability.

For this case, the dependence of the general instability load on the geometric parameters is obtained from linear, smeared theory for eccentrically stiffened thin circular cylindrical shells. Since linear theory is used, there is no assessment of the effect of geometric imperfections. In addition, the effects of prebuckling deformations and edge restraints have been ignored. Because of these, the proposed solution provides an interim solution within the current state-of-the-art and all these effects may be lumped into a desired "knockdown factor." The load case chosen, uniform axial compression, can represent an upbending design case for fuselages when the maximum bending stresses are equal to the stresses due to uniform axial compression. Justification is given in [18].

The solution to this problem is accomplished in two stages. First, by a proper grouping of the design variables, the number of parameters that optimizes the weight is reduced to a minimum. On the basis of this, a mathematical search technique is employed and design charts and tables are prepared. This first stage is called "Phase 1." Next, these charts and tables are employed to arrive at a minimum weight configuration satisfying all constraints. This stage is called "Phase 2."

This procedure, effectively, leads to a minimum weight configuration against general instability and satisfies all other possible constraints (behavioral and geometric) as well.

The proposed procedure has many advantages over the past attempts. Firstly, the design charts and tables will provide the necessary insight and information to the designer in order to deviate from the optimum solution when other considerations, such as availability and cost of construction, become important. Secondly, the designer can avoid the simultaneous occurrence of various failure modes and thus minimize the possibility of arriving at a configuration which is unnecessarily more imperfection sensitive (see discussion in the next section). Finally, this procedure allows the consideration of many different shapes of stiffening members.

Review of Previous Work

In the past, there have been two types of attempt at the minimum weight design of the thin circular cylindrical shell subject to a uniform axial compression. One approach is to make a parametric study with regard to the general instability mode of failure and investigate the effects of various parameters on the cylinder weight, [3-5], [7-10], by keeping several parameters fixed. These investigations are also based on the premise that minimum weight is accomplished if all possible modes of failure occur simultaneously. This conjecture has been disproved by another group of investigators, [11-16], who have not imposed this limitation on their formulations. In addition, recently, Thompson and Lewis [22] have quantitatively verified the suspicion of van der Neut [19], Koiter and Kuiken [20], and Graves-Smith [21], that a structural element which is designed for simultaneous occurrence of all possible modes of failure is extremely sensitive to geometric imperfections. Because of these two reasons, the resulting designs based on this approach are somewhat unreliable in terms of load carrying capacity.

The second approach is based on convenient mathematical search techniques applied to the objective function, which contains all of the constraints as penalty functions. The objective function is expressed in terms of the design variables. This approach used in [11-16], is in accord with the philosophy of the present time, that is to achieve a fully automated design, but the author has serious reservations concerning the desirability and the useful applicability of such techniques. First of all, the number of the design variables for rectangular cross-sectional stiffeners is seven. Admittedly, all of the investigators who have used mathematical search techniques in the 7-dimensional space have reported great difficulties and computational failures. Moreover, if one were to deal with T-shaped stiffeners, the number of design variables will be 11 and hence, more computational difficulties. Even if these difficulties can be overcome, there is still another question about the applicability of such techniques because Pappas and Amba-Rao [15] have reported that there exist several, if not many, nearly equal weight, and yet significantly different design configurations. This means that the minimum weight design may not be unique (it is shown in the present research that minimum weight design is not unique indeed). This suspicion has been supported by the design results of case 7-I of Jones and Hague [16], where they have reported a multitude of designs for nearly equal weight and yet significantly different design variables. These different designs have been obtained by either using different search techniques, or using the same technique with different starting point.

Another research paper along this line which does not fall into the above two approaches is by Rehfield [17]. His approach is indirect with the assumption of simultaneous occurrence of failure modes. The design procedure is an iterative one and the minimum weight is located by trial and error.

The above discussions imply that there are many combinations of the design variables which satisfy all behavioral constraints and lead to the same minimum weight. Finally, due to various behavioral constraints built into their objective function, their designs cannot purposely avoid the simultaneous occurrence of the various instability failure modes. Thus, the resulting design configuration may be unnecessarily more imperfection sensitive.

CHAPTER II

MATHEMATICAL FORMULATION OF THE PROBLEM

Introduction

The statement of the problem is as follows: given an internally stiffened circular cylindrical shell of specified material, radius and length, find the size, shape, and spacings of the stiffeners and the thickness of the skin such that the resulting design configuration can safely carry a given uniform axial compression with minimum weight.

There are three major failure modes for the problem posed above. These are, general instability, panel instability and yielding of the material of the stiffened cylindrical shell. In the present problem one is concerning with large thin circular cylindrical shells for fuselage application only. In such an application the loading will not cause the yielding of the material to become critical. Thus, the remaining two principal catastrophic modes of failure are general and panel instabilities. Since the stress level in the rings, in this problem, is very low, one can always adjust the ring spacing such that the panel instability load is higher than the general instability load for the same weight. Hence, the objective function chosen for "Phase 1" is the weight of the shell with the equality constraint of general instability built into it. The other constraints to be satisfied in "Phase 2" are the behavioral inequality constraints of panel instability, wrinkling of the skin, local instability of the stringers, limitation of the stress level in the skin, stringers and rings, and simultaneous occurrence of failure modes. In addition, the geometric inequality constraints which represent the realistic design dimensions of the design variables are to be satisfied as well.

In the next sections, the analysis of thin stiffened circular cylindrical shells, the mathematical formulation of "Phase 1" and "Phase 2," and the mathematical search technique are presented.

Analysis of Stiffened Circular Cylindrical Shell

Assumptions

In this section all the equations needed to analyze the stiffened circular cylindrical shell are presented. These include the development of the buckling equations (for general instability, panel instability, and local instabilities) and the stress analysis of the skin and stiffeners. The assumptions in this development are:

1. x, y, z are reference surface coordinates which are orthogonal and along the directions of principal curvatures.

2. The shell is thin.

3. The deflections are small.

4. The rotations about the inplane axes are much larger than that about the normal axis.

5. The normals to the reference surface before deformation remain normal to the reference surface after deformation and they are inextensional. That is $\gamma_{xz} = \gamma_{yz} = \epsilon_{zz} = 0$.

6. Stiffeners are along the principal curvatures and their effects on flexural and extensional stiffness are distributed mathematically over the whole surface of the shell (smeared technique).

7. The connection is monolithic.

8. The stiffeners do not transmit shear force. The shear membrane force is carried entirely by the skin.

9. Stiffeners are in the uniaxial stress state.

10. Stiffeners are torsionally weak (open-section stiffeners).

Stress-Strain Relations

The skin of the stiffened circular cylinder is assumed to be in a biaxial state of stress. The x-axis is in the longitudinal direction and the y-axis is in the circumferential direction (see Figure 1). With these assumptions the stress-strain relations in the skin are

$$\sigma_{xxsk} = \frac{E}{1-\nu^2} (\epsilon_x + \nu\epsilon_y) \tag{1}$$

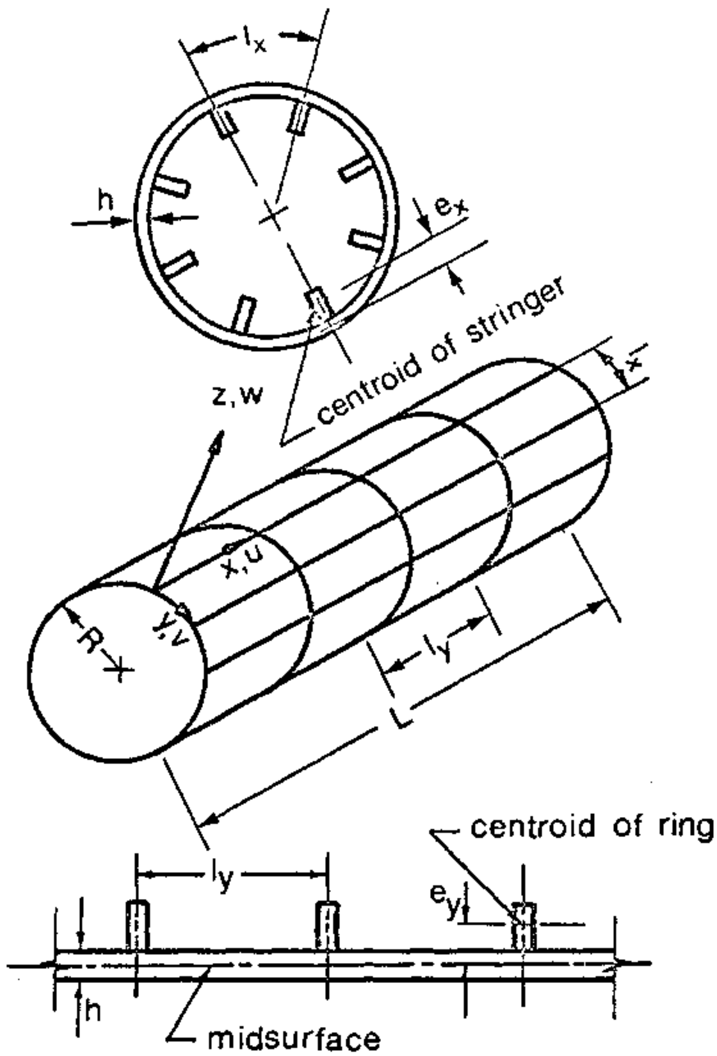


Fig. 1 Shell Geometry

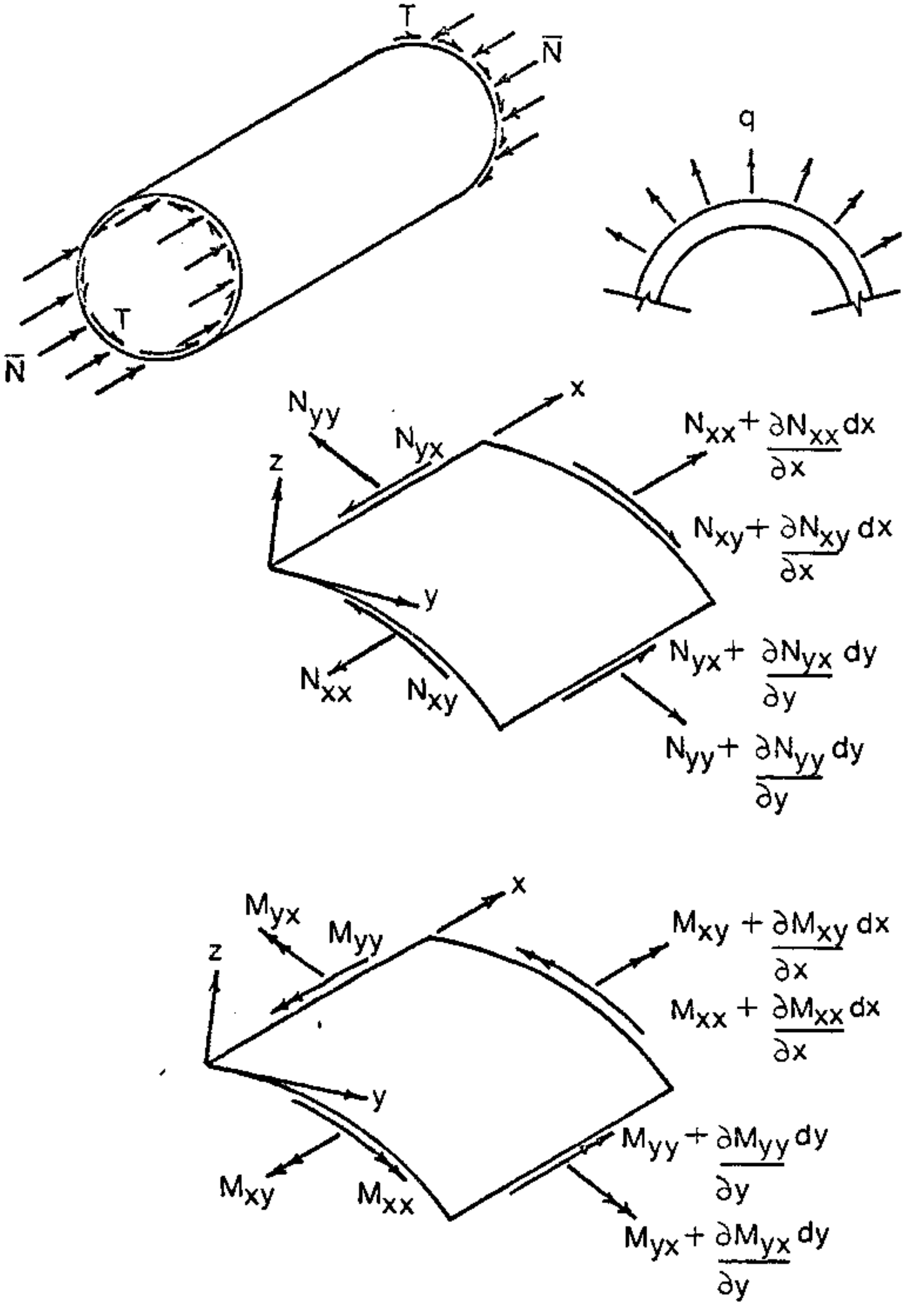


Fig. 2 Sign Convention

$$\sigma_{yysk} = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\sigma_{xysk} = \frac{E}{2(1+\nu)}\gamma$$

The stiffeners are assumed to be in a uniaxial state of stress so that the stress-strain relations are

$$\sigma_{xxst} = E_x \varepsilon_x \qquad (2)$$

$$\sigma_{yyr} = E_y \varepsilon_y$$

for the longitudinal and circumferential stiffeners respectively.

Strain-Displacement Relations

The reference surface of the shell is taken as the midsurface of the skin. The coordinate system is as shown in Figure 1 and u, v, and w being the deformations of material points on the reference surface. The strain-displacement relations are

$$\varepsilon_x = \varepsilon_{xx} + z\kappa_{yy}$$

$$\qquad (3)$$

$$\varepsilon_y = \varepsilon_{yy} + z\kappa_{yy}$$

$$\gamma = \gamma_{xy} + 2z\kappa_{xy}$$

$$\kappa_{xx} = -w,_{xx}$$

$$\kappa_{yy} = -w,_{yy}$$

$$\kappa_{xy} = -w,_{xy} \tag{3}$$

$$\varepsilon_{xx} = u,_{x}$$

$$\varepsilon_{yy} = v,_{y} + \frac{w}{R}$$

$$\gamma_{xy} = u,_{y} + v,_{x}$$

Stress and Moment Resultants

The stress and moment resultants (per unit length) are obtained by the appropriate integrations of the stresses over the thickness of the shell and then adding to these the corresponding stiffener contributions. According to assumption 6, the effects of the flexural and extensional stiffness of the stiffeners are assumed to be smeared over the surface of the skin. The stress and moment resultants are

$$N_{xx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xxsk}\, dz + \frac{1}{\ell_x} \int_{A_x} \sigma_{xxst}\, dA_x \tag{4}$$

$$N_{yy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{yysk}\, dz + \frac{1}{\ell_y} \int_{A_y} \sigma_{yyr}\, dA_y$$

$$N_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xysk}\, dz$$

$$M_{xx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_{xxsk}\, dz + \frac{1}{\ell_x} \int_{A_x} z\, \sigma_{xxst}\, dA_x$$

$$M_{yy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_{yysk}\, dz + \frac{1}{\ell_y} \int_{A_y} z\, \sigma_{yyr}\, dA_y$$

$$M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\, \sigma_{xysk}\, dz + \frac{(GJ)_x}{\ell_x}\, \kappa_{xy}$$

$$M_{yx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\, \sigma_{xysk}\, dz + \frac{(GJ)_y}{\ell_y}\, \kappa_{yx}$$

Substitution of the stress-strain and kinematic relations, from equations (1) and (3), into equations (4), and performing the indicated integrations yields

$$N_{xx} = \frac{Eh}{1-\nu^2}(\varepsilon_{xx}+\nu\varepsilon_{yy}) + \frac{E_xA_x}{\ell_x}\varepsilon_{xx} + \frac{E_xA_x}{\ell_x}e_x\kappa_{xx} \quad (5)$$

$$N_{yy} = \frac{Eh}{1-\nu^2}(\varepsilon_{yy}+\nu\varepsilon_{xx}) + \frac{E_yA_y}{\ell_y}\varepsilon_{yy} + \frac{E_yA_y}{\ell_y}e_y\kappa_{yy}$$

$$N_{xy} = \frac{Eh}{2(1+\nu)}\gamma_{xy}$$

$$M_{xx} = \frac{Eh^3}{12(1-\nu^3)}(\kappa_{xx}+\nu\kappa_{yy}) + \frac{E_xA_x}{\ell_x}e_x\varepsilon_{xx} + \frac{E_x}{\ell_x}(I_{xc}+e_x^2A_x)\kappa_{xx}$$

$$M_{yy} = \frac{Eh^3}{12(1-\nu^2)}(\kappa_{yy}+\nu\kappa_{xx}) + \frac{E_yA_y}{\ell_y}e_y\varepsilon_{yy} + \frac{E_y}{\ell_y}(I_{yc}+e_y^2A_y)\kappa_{yy}$$

$$M_{xy} = \frac{Eh^3}{12(1+\nu)}\kappa_{xy} + \frac{(GJ)_x}{\ell_x}$$

$$M_{yx} = \frac{Eh^3}{12(1+\nu)}\kappa_{yx} + \frac{(GJ)_y}{\ell_y}$$

Because of assumption 10

$$M_{xy} = M_{yx} = \frac{Eh^3}{12(1+\nu)} \kappa_{xy}$$

A number of new parameters is defined by:

$$E_{xxp} = E_{yyp} = \frac{Eh}{1-\nu^2}$$

$$E_{xxst} = \frac{E_x A_x}{\ell_x}$$

$$E_{yyr} = \frac{E_y A_y}{\ell_y}$$

$$G_{xy} = \frac{Eh}{2(1+\nu)}$$

$$E_{xx} = E_{xxp} + E_{xxst}$$

$$E_{yy} = E_{yyp} + E_{yyr}$$

$$D_{xxp} = D_{yyp} = D = \frac{Eh^3}{12(1-\nu^2)}$$

$$D_{xxst} = \frac{E_x I_{xc}}{\ell_x}$$

$$D_{yyr} = \frac{E_y I_{yc}}{\ell_y}$$

$$D_{xy} = (1-\nu)D_{xxp}$$

$$D_{xx} = D_{xxp} + D_{xxst}$$

$$D_{yy} = D_{yyp} + D_{yyr}$$

With these new parameters equations (5) become

$$N_{xx} = E_{xx}\varepsilon_{xx} + \nu E_{xxp}\varepsilon_{yy} + e_x E_{xxst}\kappa_{xx} \qquad (6)$$

$$N_{yy} = \nu E_{yyp}\varepsilon_{xx} + E_{yy}\varepsilon_{yy} + e_y E_{yyr}$$

$$N_{xy} = G_{xy}\gamma_{xy}$$

$$M_{xx} = (D_{xx}+e_x^2 E_{xxst})\kappa_{xx} + \nu D_{xxp}\kappa_{yy} + e_x E_{xxst}\varepsilon_{xx}$$

$$M_{yy} = \nu D_{xxp}\kappa_{xx} + (D_{yy}+e_y^2 E_{yyr})\kappa_{yy} + e_y E_{yyr}\varepsilon_{yy}$$

$$M_{xy} = D_{xy}\kappa_{xy}$$

Prebuckling Stresses

It is assumed that when the cylinder is loaded there is a uniform change in length and radius, that is, a membrane state exists. Let the superscript "o" denote the membrane state parameters. Under this membrane state u is a linear function of x only, and v and w are independent of x and y. Therefore

$$\varepsilon_x^o = \varepsilon_{xx}^o = \frac{\partial u}{\partial x}$$

$$\varepsilon_y^o = \varepsilon_{yy}^o = \frac{w}{R}$$

$$\gamma^o = o$$

The membrane state stress resultants become

$$N_{xx}^o = E_{xx}\varepsilon_{xx}^o + \nu E_{xxp}\varepsilon_{yy}^o \qquad (7)$$

$$N_{yy}^o = \nu E_{yyp}\varepsilon_{xx}^o + E_{yy}\varepsilon_{yy}^o$$

$$N_{xy}^o = o$$

For a circular cylindrical shell under uniform axial compression

$$N_{xx}^o = -\overline{N}$$

$$N_{yy}^o = o$$

Hence, equations (7) yield the prebuckling strains

$$\varepsilon_{xx}^{o} = \frac{-\bar{N}E_{yy}}{E_{xx}E_{yy} - \nu^2 E_{xxp}E_{yyp}} \tag{8}$$

$$\varepsilon_{yy}^{o} = \frac{\nu\bar{N}E_{yyp}}{E_{xx}E_{yy} - \nu^2 E_{xxp}E_{yyp}}$$

Substitution of equations (8) into equations (1) and (2) yields for the skin, stringer, and rings

$$\sigma_{xxsk} = -\frac{\bar{N}}{\bar{h}} E_{xxp}\left(\frac{E_{yy} - \nu^2 E_{xxp}}{E_{xx}E_{yy} - \nu^2 E_{xxp}E_{yyp}}\right)$$

$$\sigma_{yysk} = -\frac{\bar{N}}{\bar{h}} \nu E_{xxp}\left(\frac{E_{yy} - E_{yyp}}{E_{xx}E_{yy} - \nu^2 E_{xxp}E_{yyp}}\right)$$

$$\sigma_{xxst} = \frac{-\bar{N}E_{x}E_{yy}}{E_{xx}E_{yy} - \nu^2 E_{xxp}E_{yyp}}$$

$$\sigma_{yyr} = \frac{\bar{N}\nu E_{y}E_{yyp}}{E_{xx}E_{yy} - \nu^2 E_{xxp}E_{yyp}}$$

In terms of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ which are defined in the next section, the prebuckling stresses are

$$\sigma_{xxsk} = \frac{-\bar{N}(1+\bar{\lambda}_{yy}-\nu^2)}{h[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2]}$$

$$\sigma_{yysk} = \frac{-\nu\bar{\lambda}_{yy}\bar{N}}{h[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2]}$$

$$\tag{9}$$

$$\sigma_{xxst} = \frac{-E_x(1-\nu^2)(1+\bar{\lambda}_{yy})\bar{N}}{Eh[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2]}$$

$$\sigma_{yyr} = \frac{E_y\nu(1-\nu^2)\bar{N}}{Eh[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2]}$$

Buckling Equations

The well-known equilibrium equations of the linear thin shell theory are

$$N_{xx,x} + N_{xy,y} + q^x = o \tag{10a}$$

$$N_{xy,x} + N_{yy,y} + q^y = o$$

$$M_{xx,xx} + M_{yy,yy} + 2M_{xy,xy} + (N_{xx}w_{,x})_{,x} + (N_{yy}w_{,y})_{,y} +$$

$$(N_{xy}w_{,x})_{,y} + \frac{N_{yy}}{R} + (N_{xy}w_{,y})_{,x} - q^z = o$$

where q^x, q^y, and q^z are the loads in the x, y, and z directions, respectively.

Investigation of instability of eccentrically stiffened cylinders under the action of single load application have been reported by a number of authors [23]-[27]. Most of these authors have used orthotropic thin shell theory and have reduced the problem to an eigenvalue problem, with three differential equations. Using the geometry and sign convention shown in Figures 1 and 2, and letting the superscript "1" refer to the additional quantities necessary to bring the membrane state to the adjacent buckled state, these three governing equations are

$$[E_{xx} \frac{\partial^2}{\partial x^2} + G_{xy} \frac{\partial^2}{\partial y^2}]u^1 + [(G_{xy}+\nu E_{yyp}) \frac{\partial^2}{\partial x \partial y}]v^1 = \qquad (10b)$$

$$[(q-\frac{\nu}{R} E_{yyp}) \frac{\partial}{\partial x} + e_x E_{xxst} \frac{\partial^3}{\partial x^3}]w^1$$

$$[(G_{xy}+\nu E_{xxp}) \frac{\partial^2}{\partial x \partial y}]u^1 + [E_{yy} \frac{\partial^2}{\partial y^2} + G_{xy} \frac{\partial^2}{\partial x^2}]v^1 =$$

$$[(q-\frac{E_{yy}}{R}) \frac{\partial}{\partial y} + e_y E_{yyr} \frac{\partial^3}{\partial y^3}]w^1$$

$$[(D_{xx}+e_x^2E_{xxst})\frac{\partial^4}{\partial x^4} + 2(D_{xy}+\frac{\nu}{2}D_{xxp}+\frac{\nu}{2}D_{yyp})\frac{\partial^4}{\partial x^2\partial y^2}$$

$$+ (D_{yy}+e_y^2E_{yyr})\frac{\partial^4}{\partial y^4} + \frac{E_{yy}}{R^2} - 2\frac{e_y}{R}E_{yyr}\frac{\partial^2}{\partial y^2}]w^1$$

$$+ [\frac{\nu}{R}E_{xxp}\frac{\partial}{\partial x} - e_xE_{xxst}\frac{\partial^3}{\partial x^3}]u^1 + [\frac{E_{yy}}{R}\frac{\partial}{\partial y} - e_yE_{yyr}\frac{\partial^3}{\partial y^3}]v^1$$

$$= N_{xx}^o w^1_{,xx} + N_{yy}^o w^1_{,yy} + 2N_{xy}^o w^1_{,xy}$$

Note that equations (10) are the buckling equations of the stiffened cylinder subjected to the uniform axial compression, torsion, and hydrostatic pressure and that the pressure load q remains normal to the deflected midsurface during the buckling process. The eigenvalues for the problem are

$$N_{xx}^o = \frac{qR}{2} - \bar{N} \tag{11}$$

$$N_{yy}^o = qR$$

$$N_{xy}^o = \frac{T}{2\pi R^2}$$

By a judicious choice of groups of parameters to be used in "Phase 1" and "Phase 2", the following nondimensional groups of parameters are defined.

$$\bar{\lambda}_{xx} = \frac{E_{xxst}}{E_{xxp}} = \frac{E_x A_x (1-\nu^2)}{Eh\ \ell_x}$$

$$\bar{\lambda}_{yy} = \frac{E_{yyr}}{E_{yyp}} = \frac{E_y A_y (1-\nu^2)}{Eh\ \ell_y}$$

$$\bar{\rho}_{xx} = \frac{D_{xxst}}{D} = \frac{E_x I_{xc}}{D\ \ell_x}$$

$$\bar{\rho}_{yy} = \frac{D_{yyr}}{D} = \frac{E_y I_{yc}}{D\ \ell_y}$$

$$\bar{e}_x = \frac{\pi^2 R e_x}{L^2}$$

$$\bar{e}_y = \frac{\pi^2 R e_y}{L^2}$$

$$Z = \frac{L^2 (1-\nu^2)^{1/2}}{Rh}$$

$$\bar{K}_{xx} = \frac{\bar{N}\ L^2}{\pi^2 D}$$

$$\bar{K}_{yy} = \frac{qR\ L^2}{\pi^2 D}$$

$$\bar{K}_s = \frac{N^o_{xy} L^2}{\pi^2 D}$$

Since the operators in equations (10) are commutative, it is possible to derive a single higher order Donnell-Batdorf type of an equation by eliminating u^1 and v^1. This has been done in [25] and in terms of the new group of parameters the single buckling equation is

$$(1+\bar{\rho}_{yy})\nabla_D w^1 + \nabla_E^{-1}[\frac{12Z^2}{1-\nu^2}(1+\bar{\lambda}_{xx})\nabla_c w^1 - (\frac{L}{\pi R})^2 \bar{K}_{yy}\nabla_p w^1]$$

$$= (\frac{L}{\pi})^2[(\frac{1}{2}\ \bar{K}_{yy}-\bar{K}_{xx})w^1{}_{,xx} + \bar{K}_{yy}w^1{}_{,yy} + 2\bar{K}_s w^1{}_{,xy}] \qquad (12)$$

where

$$\nabla_E = (\frac{L}{\pi})^4[\frac{\partial^4}{\partial x^4} + \frac{2}{(1-\nu)(1+\bar{\lambda}_{xx})}\{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu\}\ \frac{\partial^4}{\partial x^2 \partial y^2}$$

$$+ \frac{1+\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}}\ \frac{\partial^4}{\partial y^4}]$$

$$\nabla_D = \left(\frac{L}{\pi}\right)^4 \left[\frac{1+\bar{\rho}_{xx}}{1+\bar{\rho}_{yy}} \frac{\partial^4}{\partial x^4} + \frac{2}{1+\bar{\rho}_{yy}} \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\right]$$

$$\nabla_p = \left(\frac{L}{\pi}\right)^6 \frac{1}{1+\bar{\lambda}_{xx}} \left[\bar{e}_x \bar{\lambda}_{xx} \frac{\partial^6}{\partial x^6} + \frac{(2\bar{\lambda}_{yy}+1-\nu)}{1-\nu} \bar{e}_x \bar{\lambda}_{xx} \frac{\partial^6}{\partial x^4 \partial y^2}\right.$$

$$+ \frac{(2\bar{\lambda}_{xx}+1-\nu)}{1-\nu} \bar{e}_y \bar{\lambda}_{yy} \frac{\partial^6}{\partial x^2 \partial y^4} + \bar{e}_y \bar{\lambda}_{yy} \frac{\partial^6}{\partial y^6} -$$

$$\nu \left(\frac{\pi}{L}\right)^2 \frac{\partial^4}{\partial x^4} + \left(\frac{\pi}{L}\right)^2 \frac{1}{1-\nu} \{\nu(1+\nu) - (1-\bar{\lambda}_{yy})(2\bar{\lambda}_{xx}+1+\nu) \quad \frac{\partial^4}{\partial x^2 \partial y^2}$$

$$\left. - \left(\frac{\pi}{L}\right)^2 (1+\bar{\lambda}_{yy}) \frac{\partial^4}{\partial y^4}\right]$$

$$\nabla_c = \frac{(L/\pi)^8}{(1+\bar{\lambda}_{xx})^2} \left[\bar{e}_x^2 \bar{\lambda}_{xx} \frac{\partial^8}{\partial x^8} + \frac{2\bar{e}_x^2 \bar{\lambda}_{xx} (\bar{\lambda}_{yy}+1-\nu)}{1-\nu} \frac{\partial^8}{\partial x^6 \partial y^2}\right.$$

$$+ \{\bar{e}_x^2 \bar{\lambda}_{xx} (1+\bar{\lambda}_{yy}) + 2\bar{e}_x \bar{e}_y \bar{\lambda}_{xx} \bar{\lambda}_{yy} \frac{(1+\nu)}{(1-\nu)} + \bar{e}_y^2 \bar{\lambda}_{yy} (1+\bar{\lambda}_{xx})\} \frac{\partial^8}{\partial x^4 \partial y^4}$$

$$+ \frac{2\bar{e}_y^2 \bar{\lambda}_{yy} (1+\bar{\lambda}_{xx}-\nu)}{1-\nu} \frac{\partial^8}{\partial x^2 \partial y^6} + \bar{e}_y^2 \bar{\lambda}_{yy} \frac{\partial^8}{\partial y^8} + 2\nu \left(\frac{\pi}{L}\right)^2 \bar{e}_x \bar{\lambda}_{xx} \frac{\partial^6}{\partial x^6}$$

$$- 2\left(\frac{\pi}{L}\right)^2 \{\bar{e}_x \bar{\lambda}_{xx} (1+\bar{\lambda}_{yy}) + \bar{e}_y \bar{\lambda}_{yy} (1+\bar{\lambda}_{xx})\} \frac{\partial^6}{\partial x^4 \partial y^2} + 2\nu \left(\frac{\pi}{L}\right)^2$$

$$\left. \bar{e}_y \bar{\lambda}_{yy} \frac{\partial^6}{\partial x^2 \partial y^4} + \left(\frac{\pi}{L}\right)^4 \{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2\} \frac{\partial^4}{\partial x^4}\right]$$

where ∇^{-1} is an inverse differential operator such that $\nabla^{-1}\nabla = \nabla\nabla^{-1} = 1$.

Instabilities Under Uniform Axial Compression

General Instability. For uniform axial compression the buckling equation (12) becomes

$$(1+\overline{\rho}_{yy})\nabla_D w^1 + \frac{12Z^2}{1-\nu^2}(1+\overline{\lambda}_{xx})\nabla_E^{-1}\nabla_C w^1 + (\frac{L}{\pi})^2\overline{K}_{xx}w^1_{,xx} = o \qquad (13)$$

The classical simply supported boundary conditions are

$$\begin{aligned} w^1(0,y) &= 0 \;, & w^1(L,y) &= 0 \\ v^1(0,y) &= 0 \;, & v^1(L,y) &= 0 \\ M_{xx}(0,y) &= 0 \;, & M_{xx}(L,y) &= 0 \\ N^1_{xx}(0,y) &= 0 \;, & N^1_{xx}(L,y) &= 0 \end{aligned} \qquad (14)$$

The displacement function which satisfies all boundary conditions is

$$w^1 = W_{mn} \sin\frac{m\pi x}{L} \sin\frac{ny}{R}$$

The expression for the buckling load is obtained by substituting into the buckling equation the assumed displacement function. The resulting expression for the buckling coefficient contains two integer parameters, m and n, representing the mode shape. The critical load coefficient is then obtained by searching for the mode shape which yields the lowest buckling load.

Let $\beta = \frac{nL}{\pi R}$, then the buckling coefficient is

$$\bar{K}_{xx} = \frac{1}{m^2}[(1+\bar{\rho}_{xx})m^4+2m^2\beta^2+(1+\bar{\rho}_{yy})\beta^4]+\frac{12Z^2}{m^2\pi^4(1-\nu^2)}[\bar{e}_x^2\bar{\lambda}_{xx}m^8 \quad (15)$$

$$+ \frac{2}{1-\nu}\bar{e}_x^2\bar{\lambda}_{xx}(1-\nu+\bar{\lambda}_{yy})m^6\beta^2+\{e_x^2\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) +$$

$$\frac{2(1+\nu)}{1-\nu}\bar{e}_x\bar{e}_y\bar{\lambda}_{xx}\bar{\lambda}_{yy} + \bar{e}_y^2\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\}m^4\beta^4 +$$

$$\frac{2}{1-\nu}\bar{e}_y^2\bar{\lambda}_{yy}(1-\nu+\bar{\lambda}_{xx})m^2\beta^6 + \bar{e}_y^2\bar{\lambda}_{yy}\beta^8 - 2\nu\bar{e}_x\bar{\lambda}_{xx}m^6 +$$

$$2\{\bar{e}_x\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + \bar{e}_y\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\}m^4\beta^2 - 2\nu\bar{e}_y\bar{\lambda}_{yy}m^2\beta^4 +$$

$$\{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2\}m^4]/[(1+\bar{\lambda}_{xx})m^4 + \frac{2}{1-\nu}\{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu\}m^2\beta^2$$

$$+ (1+\bar{\lambda}_{yy})\beta^4]$$

For any given stiffened shell geometry the critical load coefficient, $\overline{K}_{xx_{cr}}$, is obtained through minimization of equation (15) with respect to all integer values of m and n, except m = 0.

Let $\overline{\beta} = \frac{nL}{m\pi R}$, and also note that for an internally stiffened shell $\overline{e}_x$ and $\overline{e}_y$ are negative numbers; therefore, after changing the signs of $\overline{e}_x$ and $\overline{e}_y$, equation (15) can be rearranged as

$$\overline{K}_{xx} = Pm^2 + \frac{Q}{m^2} + S \tag{16}$$

where

$$P = 1+\overline{\rho}_{xx}+2\overline{\beta}^2+(1+\overline{\rho}_{yy})\overline{\beta}^4 + \frac{12Z^2}{\pi^4(1-\nu^2)}[\overline{e}_x^2\overline{\lambda}_{xx}+\frac{2}{1-\nu}\overline{e}_x^2\overline{\lambda}_{xx}(1-\nu+\overline{\lambda}_{yy})\overline{\beta}^2$$

$$+ \{\overline{e}_x^2\overline{\lambda}_{xx}(1+\overline{\lambda}_{yy}) + \frac{2(1+\nu)}{1-\nu}\overline{\lambda}_{xx}\overline{\lambda}_{yy}\overline{e}_x\overline{e}_y+\overline{e}_y^2\overline{\lambda}_{yy}(1+\overline{\lambda}_{xx})\}\overline{\beta}^4$$

$$+ \frac{2}{1-\nu}\overline{e}_y^2\overline{\lambda}_{yy}(1-\nu+\overline{\lambda}_{xx})\overline{\beta}^6+\overline{e}_y^2\overline{\lambda}_{yy}\overline{\beta}^8]/B$$

$$B = 1+\overline{\lambda}_{xx} + \frac{2}{1-\nu}\{(1+\overline{\lambda}_{xx})(1+\overline{\lambda}_{yy})-\nu\}\overline{\beta}^2+(1+\overline{\lambda}_{yy})\overline{\beta}^4$$

$$Q = \frac{12Z^2}{\pi^4(1-\nu^2)}[(1+\overline{\lambda}_{xx})(1+\overline{\lambda}_{yy})-\nu^2]/B$$

$$S = \frac{24Z^2}{\pi^4(1-\nu^2)}[\nu\overline{e}_x\overline{\lambda}_{xx}-\{\overline{e}_x\overline{\lambda}_{xx}(1+\overline{\lambda}_{yy})+\overline{e}_y\overline{\lambda}_{yy}(1+\overline{\lambda}_{xx})\}\overline{\beta}^2+\nu\overline{e}_y\overline{\lambda}_{yy}\overline{\beta}^4]/B$$

For the purpose of the first stage of computer program analysis of the buckling mode, m^2 is first treated as a continuous variable. Minimization of equation (16) with respect to m^2 yields

$$\overline{K}_{xx} = 2\sqrt{PQ} + S \qquad (17)$$

$$m^2 = \sqrt{\frac{Q}{P}}$$

Panel Instability. The panel instability is the instability when all stringers and skin between two adjacent rings participate. This is the special case of the general instability. Thus, the expression for panel instability can be obtained from equation (15) by setting all ring parameters to zero. That is

$$\overline{e}_y = o \quad , \quad \overline{\lambda}_{yy} = o$$

$$\overline{\rho}_{yy} = o \quad , \quad L = \ell_y$$

The resulting expression for panel instability with the sign of $\bar{e}_x$ changed for inside stiffeners is

$$\bar{K}_{xxp} = (1+\bar{\rho}_{xx})m^2+2\beta^2+\frac{\beta^4}{m^2}+\frac{12Z^2}{\pi^4(1-\nu^2)}[\bar{e}_x^2\bar{\lambda}_{xx}(m^2+\beta^2)^2 - \tag{18}$$

$$2\bar{e}_x\bar{\lambda}_{xx}(\beta^2-\nu m^2)+1-\nu^2+\bar{\lambda}_{xx}]/[(1+\bar{\lambda}_{xx})m^2$$

$$+ \frac{2}{1-\nu}(1-\nu+\bar{\lambda}_{xx})\beta^2+\frac{\beta^4}{m^2}]$$

For any given stiffened geometry the critical load coefficient for panel buckling is obtained through minimization of equation (18) with respect to all integer values of m and n.

Local Stringer and Skin Buckling

For closely spaced stiffeners the local skin buckling and the stringer buckling are governed by the equation of a flat plate. The critical stress of a flat plate with various edge conditions is given in Bleich [28] as

$$\sigma_{cr} = K\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{a}{b}\right)^2 \tag{19}$$

where a = skin thickness, thickness of stiffener web, or thickness of stiffener flange.

b = stringer spacing, height of stiffener web, or width of stiffener flange.

K = 4, for four sides simply-supported

$K = (\frac{d}{\ell_y})^2 + 0.425$, for three sides simply-supported and one unloading side free.

In the design analysis of the local buckling, it is assumed that all edges of stiffeners and skin connecting to any part of the cylinder are simply supported. With both rings and stringers inside, the possible buckling failure modes are the following.

Skin Wrinkling. The skin wrinkling is considered as the buckling of a flat plate of size ℓ_x by ℓ_y. The critical stress is

$$\sigma_{xxsk} = \frac{\pi^2 E}{3(1-\nu^2)} \left(\frac{h}{\ell_x}\right)^2 \tag{20}$$

Local Stringer Buckling. When rings are deepest the portion of a stringer between any adjacent rings is treated as a flat plate of length ℓ_y. The stringer web is considered as four sides simply supported while the flange portion, a flat plate with three sides simply supported and the unloaded side free.

In the case when stringers are deepest, the material of the stringer web below the ring material is assumed to buckle as a flat plate of length ℓ_y with four sides simply supported while the outstanding portion of the stringer web is considered as a flat plate of length L with four sides simply supported. The stringer flange, which is above the ring material, is also treated as a flat plate of length L with three sides simply supported and the unloaded side free. For rectangular stringers there is no stringer flange, therefore the stringer material above the ring material is treated as a flat plate of length L with three sides simply supported and the unloaded side free.

During the design process, however, it has been discovered that when stringers are deepest, and in the region where $\bar{\alpha}_x > \bar{\alpha}_y$, either the resulting design configuration will always have the ring thickness and stringer thickness which are too thin to be fabricated or the stringers will buckle. Thus, this subcase of the local stringer failure can be disregarded in the designing process by concentrating only in the region where $\bar{\alpha}_y > \bar{\alpha}_x$ in favor of practical limitation on fabrication. It is worthy to mention at this time that since both rings and stringers are inside and the rings are in tension therefore there is no possible buckling failure of the rings.

The critical stresses of stringers for several types of stiffening members for the configuration when rings are deepest, are tabulated in Table 1.

Table 1. Critical Stresses of Stringers

Stringer Type	Stringer Web, $\sigma_{xxsw_{cr}}$	Stringer Flange, $\sigma_{xxsf_{cr}}$
RS	$\frac{\pi^2 E_x}{12(1-\nu^2)}(\frac{t_{wx}}{d_{wx}})^2[(\frac{d_{wx}}{\ell_y})^2 + .425]$	—
TS	$\frac{\pi^2 E_x}{3(1-\nu^2)}(\frac{t_{wx}}{d_{wx}})^2$ *	$\frac{\pi^2 E_x}{12(1-\nu^2)}(\frac{2t_{fx}}{b_{fx}-t_{wx}})^2[(\frac{b_{fx}-t_{wx}}{2\ell_y})^2+.425]$
IAS	$\frac{\pi^2 E_x}{3(1-\nu^2)}(\frac{t_{wx}}{d_{wx}})^2$ *	$\frac{\pi^2 E_x}{12(1-\nu^2)}(\frac{t_{fx}}{b_{fx}-t_{wx}})^2[(\frac{b_{fx}-t_{wx}}{\ell_y})^2+.425]$
CS,ZS	$\frac{\pi^2 E_x}{3(1-\nu^2)}(\frac{t_{wx}}{d_{wx}-2t_{fx}})^2$ *	$\frac{\pi^2 E_x}{12(1-\nu^2)}(\frac{t_{fx}}{b_{fx}})^2[(\frac{b_{fx}}{\ell_y})^2 + .425]$
IS	$\frac{\pi^2 E_x}{3(1-\nu^2)}(\frac{t_{wx}}{d_{wx}-2t_{fx}})^2$ *	$\frac{\pi^2 E_x}{12(1-\nu^2)}(\frac{2t_{fx}}{b_{fx}})^2[(\frac{b_{fx}}{2\ell_y})^2 + .425]$
AS	$\frac{\pi^2 E_x}{12(1-\nu^2)}(\frac{t_{wx}}{d_{wx}-t_{fx}})^2[(\frac{d_{wx}-t_{fx}}{\ell_y})^2+.425]$	—

*In the case of the design without geometric constraint one may have short plate, $\ell_y/d_{wx}<1$, then $\sigma_{xxsw_{cr}}$ has the form $\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{12(1-\nu^2)}(\frac{a}{b})^2(\frac{b}{\ell_y} + \frac{\ell_y}{b})^2$. Example for IS, $\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{12(1-\nu^2)}(\frac{t_{wx}}{d_{wx}-2t_{fx}})^2(\frac{d_{wx}-2t_{fx}}{\ell_y} + \frac{\ell_y}{d_{wx}-2t_{fx}})^2$.

Mathematical Formulation

Phase 1

Assuming that the eccentricities of the stiffening members are small in comparison to the radius of the stiffened circular cylindrical shell, such that the common stiffener material at the intersection of stringers and rings is negligible, then the weight of the stiffened shell is given by

$$W = 2\pi RLh\rho_{sk} + \rho_x \int_0^L \int_0^{2\pi R} \frac{A_x}{\ell_x}\, dydx + \rho_y \int_0^L \int_0^{2\pi R} \frac{A_y}{\ell_y}\, dydx \qquad (21)$$

In terms of the nondimensional parameters defined in the previous section, the weight of the stiffened circular cylindrical shell is

$$W = 2\pi RLh\rho_{sk}[1+\frac{1}{1-\nu^2}(\frac{E\rho_x}{E_x\rho_{sk}}\,\bar{\lambda}_{xx} + \frac{E\rho_y}{E_y\rho_{sk}}\,\bar{\lambda}_{yy})] \qquad (22)$$

The classical general instability buckling parameter of the thin stiffened circular cylindrical shell subject to a uniform axial compression with simply supported boundary conditions is given by equation (16). The requirement for minimum weight against general instability leads to the objective function (composite weight function)

$$W^* = W + \lambda |\bar{N}_{xx_{cr}} - \bar{N}| \tag{23}$$

where W is the weight of the stiffened shell, $\bar{N}$ the applied compressive load, $\bar{N}_{xx_{cr}}$ the general instability load obtained from minimization of equation (16) with respect to m^2 and $\bar{\beta}^2$, and λ a Lagrange multiplier. To incorporate the effect of imperfection sensitivity, a "knockdown" factor must be included in the design load $\bar{N}$.

Equation (23) can be put into nondimensional form as

$$\bar{W}^* = \frac{\bar{W}}{Z} + \lambda^* |\bar{K}^*_{xx_{cr}} - \bar{N}^*| \tag{24}$$

where

$$\bar{W}^* = \frac{W^*}{2\pi L^3 \rho_{sk} (1-\nu^2)^{1/2}} \quad , \quad \bar{K}^*_{xx_{cr}} = \frac{\bar{K}_{xx_{cr}}}{Z^3} \tag{25}$$

$$\bar{N}^* = \frac{12 R^3 \bar{N}}{\pi^2 E L^4 (1-\nu^2)^{1/2}} \quad , \quad \lambda^* = \frac{\pi E L \lambda}{24 \rho_{sk} R^3}$$

$$\bar{W} = 1 + \frac{1}{1-\nu^2} \left(\frac{E\rho_x}{E_x \rho_{sk}} \bar{\lambda}_{xx} + \frac{E\rho_y}{E_y \rho_{sk}} \bar{\lambda}_{yy} \right)$$

Thus, $\bar{W}^*$ is a function of the following parameters,

$$\overline{W}^* = F(Z, \overline{\lambda}_{xx}, \overline{\lambda}_{yy}, \overline{\rho}_{xx}, \overline{\rho}_{yy}, \overline{e}_x, \overline{e}_y, m^2, \overline{\beta}^2) \qquad (26)$$

It is seen that $\overline{W}^*$ behaves like 1/Z, therefore, it can be concluded that there is no minimum $\overline{W}^*$ with respect to a finite Z. In other words, there is no minimum weight against general instability with respect to a finite Z.

It can be seen from equation (15) that the buckling load or $\overline{K}^*_{xx_{cr}}$ increases with the increase of $\overline{\rho}_{xx}$, $\overline{\rho}_{yy}$, $\overline{e}_x$, and $\overline{e}_y$, therefore there is no minimum $\overline{W}^*$ with respect to reasonably finite values of these parameters. This implies that if a given stiffener material is distributed in such a manner that, although its contribution to the extensional stiffness is the same, its contribution to the flexural stiffness, $\overline{\rho}$, is continuously increasing (within bounds), then the critical load for general instability will be continuously increasing. Of course, during this process of distributing the material the local instability failures will dominate the problem. Thus, there are some limiting values (upper bounds) on both $\overline{e}$ and $\overline{\rho}$. In addition for fixed values of Z, $\overline{e}$, and $\overline{\rho}$ there is a minimizing set of values for $\overline{\lambda}_{xx}$ and $\overline{\lambda}_{yy}$. Because of this, charts may be generated, in which for a specified set of Z, $\overline{e}$, and $\overline{\rho}$ one can have minimum $\overline{W}$ with the corresponding minimizing values of $\overline{\lambda}_{xx}$ and $\overline{\lambda}_{yy}$.

At this point it is convenient to introduce four new parameters, $\bar{\alpha}_x$, $\bar{\alpha}_y$, C_x, and C_y. The new parameters, $\bar{\alpha}$, denote the ratio of the radius of gyration of the stiffeners to that of the skin of unit width. Their expressions for various types of stiffening members are given in Table A1 of Appendix A. The new parameters, C_x and C_y, called shape parameters, are just numbers characterizing the shapes of the stiffeners. For example, C is equal to one for rectangular stiffeners, greater than one for tee and inverted angle stiffeners, and less than one for channel, zee, I, and angle stiffeners. Using these new parameters one can eliminate the parameters $\bar{e}_x$, $\bar{e}_y$, $\bar{\rho}_{xx}$, and $\bar{\rho}_{yy}$ in equation (26) through the relations of equations (A2). Hence

$$\bar{W}^* = F[\bar{\lambda}_{xx}, \bar{\lambda}_{yy}, m^2, \bar{\beta}^2, (Z, \bar{\alpha}_x, \bar{\alpha}_y, C_x, C_y)] \tag{27}$$

The change of parameters from $\bar{\rho}_{xx}$, $\bar{\rho}_{yy}$, $\bar{e}_x$, and $\bar{e}_y$ to $\bar{\alpha}_x$, $\bar{\alpha}_y$, C_x, and C_y are convenient because the ranges of these new parameters are known. For example, using rectangular rings $\bar{\alpha}_y = \frac{d_{wy}}{h}$. But for the assumption of thin ring theory $\frac{R}{d_{wy}} > 20$, therefore

$$\bar{\alpha}_y < \frac{R}{20h}$$

Therefore, it is proposed to generate the design charts and tables in the $\bar{\alpha}_x$-$\bar{\alpha}_y$ space for each type of stiffening members. The precise statement of the mathematical formulation in "Phase 1" is as follows.

In the $\overline{\alpha}_x$-$\overline{\alpha}_y$ space, for each type of stiffeners and for each Z and a given load parameter, $\overline{N}$, minimize the weight parameter of the stiffened circular cylindrical shell, $\overline{W}$, with respect to $\overline{\lambda}_{xx}$ and $\overline{\lambda}_{yy}$ subject to the equality constraint of general instability. That is

$$\underset{"\overline{\lambda}_{xx},\overline{\lambda}_{yy}"}{\text{Minimize}}\ \overline{W} \text{ subject to } \overline{K}^{*}_{xx_{cr}} = \overline{N}^{*} \qquad (28)$$

It has been shown in [29] that, provided λ^{*} is sufficiently large, the solution of the unconstrained minimization of equation (24) will approach the solution of the constrained minimization of equation (28). The exact solution will be obtained when λ^{*} approaches infinity.

This implies that, if one uses the optimum weight parameter W, one will find in the $\overline{\alpha}_x$-$\overline{\alpha}_y$ space families of curves of constant optimum $\overline{W}$ and the corresponding optimizing values of $\overline{\lambda}_{xx}$ and $\overline{\lambda}_{yy}$ which will be employed in "Phase 2" to arrive at a minimum weight geometry, satisfying all constraints.

Phase 2

Assuming that the stresses in the stringers and rings are in uniaxial state and the stresses in the skin are in biaxial state, then these stress components before buckling are given by equation (9). The possible local buckling failures of the skin and stringers have already been discussed. The expressions for the critical stresses of stringers of several types of cross-section are given in Table 1.

Considering only the absolute values of these stresses during the design process, the stresses of the local buckling of the skin and stringers given in Table 1 must be greater than the applied stresses given by equations (9) accordingly. Furthermore, the applied stresses must be less than a certain appropriate stress level, for example, the yield stress of the material. Of all ring spacings ℓ_y, obtained from the constraint of stringer buckling, one must select the one (there are many) which does not yield panel buckling. The details of the steps in the minimum weight design procedure of the stiffened circular cylindrical shell for stiffeners of rectangular, tee, inverted angle, channel, zee, I, and angle cross-sections are outlined in Chapter III. The typical design examples are demonstrated in Appendix C.

Mathematical Search Technique

Selection Criteria

Because of the complexity of the objective function in the present problem, the derivative-free unconstrained minimization method is preferable. Depending on the type of the function, some or all criteria to be considered in the selection of the method should be the reliability or the success in obtaining an optimal solution to within a certain precision, the computer time required, and the number of functional evaluations.

The first criterion, the reliability, must be the primary concern in every algorithm. The number of functional evaluations might not be a good measure of the effectiveness of an algorithm because one can design an algorithm which reduces the number of functional evaluations by incorporating in the algorithm all sorts of time-consuming tests, matrix operations, and so forth. On the other hand, if the time-consuming subroutine must be called for each functional evaluation, this criterion might be fruitful. Thus, the ultimate decision in selecting an algorithm should be the reliability and the total computer time required to obtain an optimal solution within the desired degree of precision (including all concerned subprograms). In reality there is no clear-cut evidence that indicates which algorithm is the best. For the present two-dimensional minimization problem, the author has selected the irregular simple or flexible polyhedron method of Nelder and Mead [30] because the simplex has been designed to adapt itself to the topography of the objective function, hence, high reliability.

Search Technique of Nelder and Mead

The search technique of Nelder and Mead consists of four basic operations: The reflection, expansion, contraction, and reduction of the simplex. The method minimizes a function of n independent variables using (n+1) vertices of a simplex in the n-dimensional euclidean space. In the present two-dimensional problem a simplex is a triangle. The vertex which yields the highest value of the objective function is projected through the center of gravity or centroid of the remaining vertices. Improved values of the objective function are found by successively replacing the point with the highest value of the objective function by better points until the minimum is found. For further details of the method the reader is referred to reference [30].

CHAPTER III

SOLUTION PROCEDURE

The solution to the present problem is accomplished in two stages: Phase 1 and Phase 2. In "Phase 1" the search technique of Nelder and Mead is employed and design charts and tables are prepared. These charts and tables are then used in "Phase 2" to arrive at a minimum weight configuration satisfying all constraints.

Phase 1: Development of Design Charts and Tables

In moving a simplex towards the minimum $\overline{W}$ in $\overline{\lambda}_{xx}$ - $\overline{\lambda}_{yy}$ space for each Z, stiffener shape, and a pair of $(\overline{\alpha}_x, \overline{\alpha}_y)$ one needs to evaluate $\overline{K}_{xx_{cr}}$ at every vertex of the simplex. To accomplish this, the well-known and probably the most efficient one dimensional search technique, the gold section, is employed [31]. To find $\overline{K}_{xx_{cr}}$ for each vertex or point in the $\overline{\lambda}_{xx}$ - $\overline{\lambda}_{yy}$ space when m is an integer, the golden section has to be applied twice. The process is as follows:

At a point in the $\overline{\lambda}_{xx}$ - $\overline{\lambda}_{yy}$ space, during the optimum seeking procedure, all quantities, except m and $\overline{\beta}$, in equation (16) are known. First, one treats m as a continuous variable and equation (17) is used in the golden section to find $\overline{\beta}$ for $\overline{K}_{xx_{cr}}$. From this, one can compute m according to equation (17). This m, in general, will not be an integer.

Next, one considers m as two consecutive integers, except 0, bounding the non-integer m found previously. For these two m's, one uses equation (16) to find $\overline{\beta}$'s and thus $\overline{K}_{xx_{cr}}$'s. The integer m and the corresponding $\overline{\beta}$, giving the smaller $\overline{K}_{xx_{cr}}$, will be taken as the solution for $\overline{K}_{xx_{cr}}$ at this point. The instructions and computer listings used in "Phase 1" are given in Appendix E. There is no convergence problem in finding the minimum $\overline{W}$ with this method.

Figures 3 through 7 are some results of the design charts for $\overline{N}^* = 1.233 \times 10^{-8}$, (corresponding to case 7-I in [16]) using RSRR (rectangular stringers and rings). For the case of $\overline{N}^* = 4.10306 \times 10^{-8}$ (corresponding to case 6-I in [16]), the surface of optimum $\overline{W}$ becomes wavy, thus smooth curves as in Figures 3 through 6 cannot be drawn. In this case an example of one chart with the value of optimum $\overline{W}$ at each pair of $(\overline{\alpha}_x,\overline{\alpha}_y)$ is shown in Figure 7. The solid and dashed lines in Figure 7 are the schematic paths showing the possible movement towards minimum weight design, without geometric and with geometric constraints, respectively. The design procedure at each $(\overline{\alpha}_x,\overline{\alpha}_y)$ will be described in detail in the next section.

It should be pointed out that in addition to the design charts (Figures 3 through 7), one needs to have at hand the tables showing the values of optimum $\overline{W}$ and their corresponding $\overline{\lambda}_{xx}$ and $\overline{\lambda}_{yy}$ for many pairs of $(\overline{\alpha}_x,\overline{\alpha}_y)$. Thus, these tables must be considered part of the design charts.

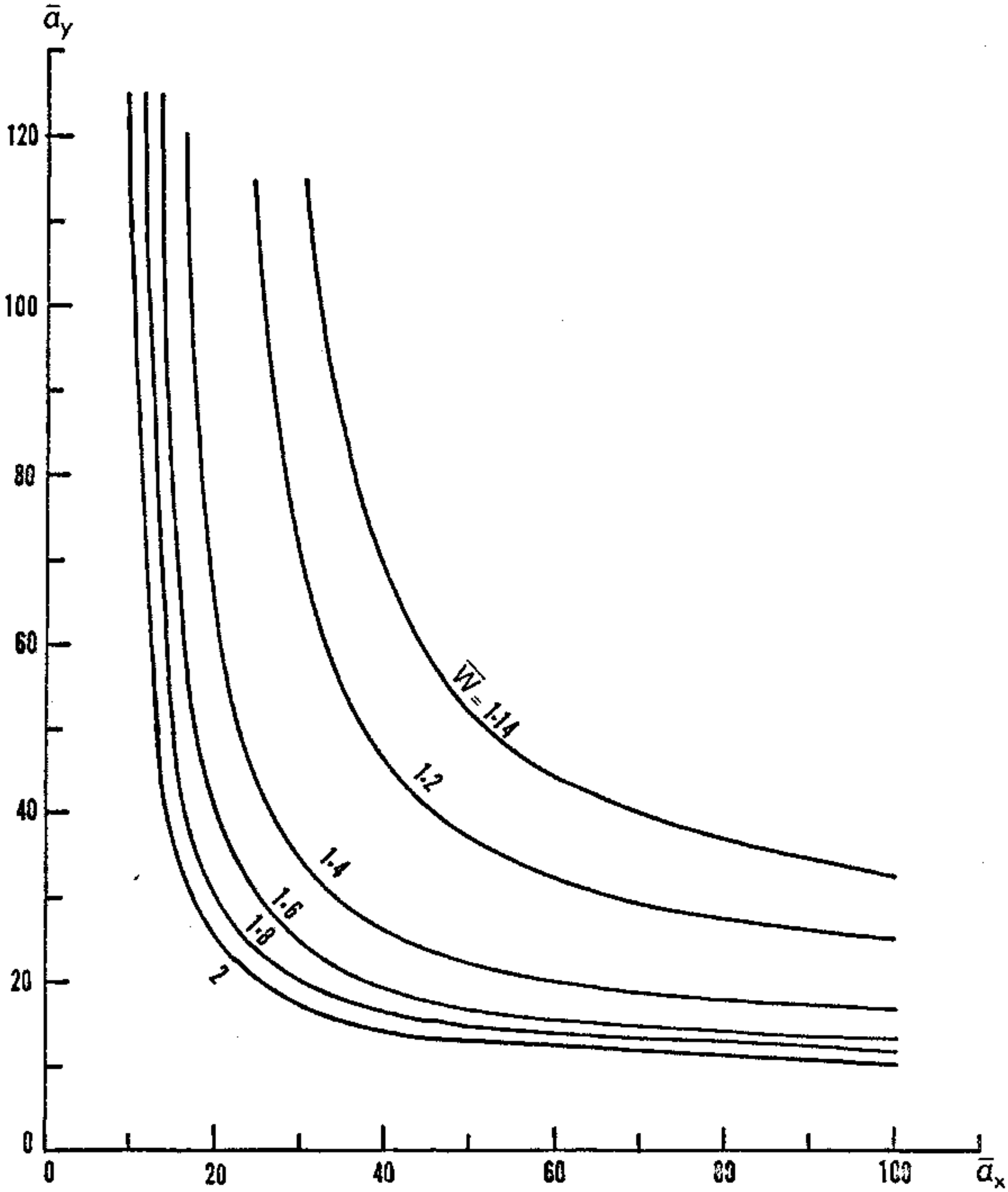


Fig. 3 Design Chart for Optimum $\overline{W}$. RSRR. Z = 30000.

$\bar{N}^* = 1{\cdot}233 \times 10^{-8}$

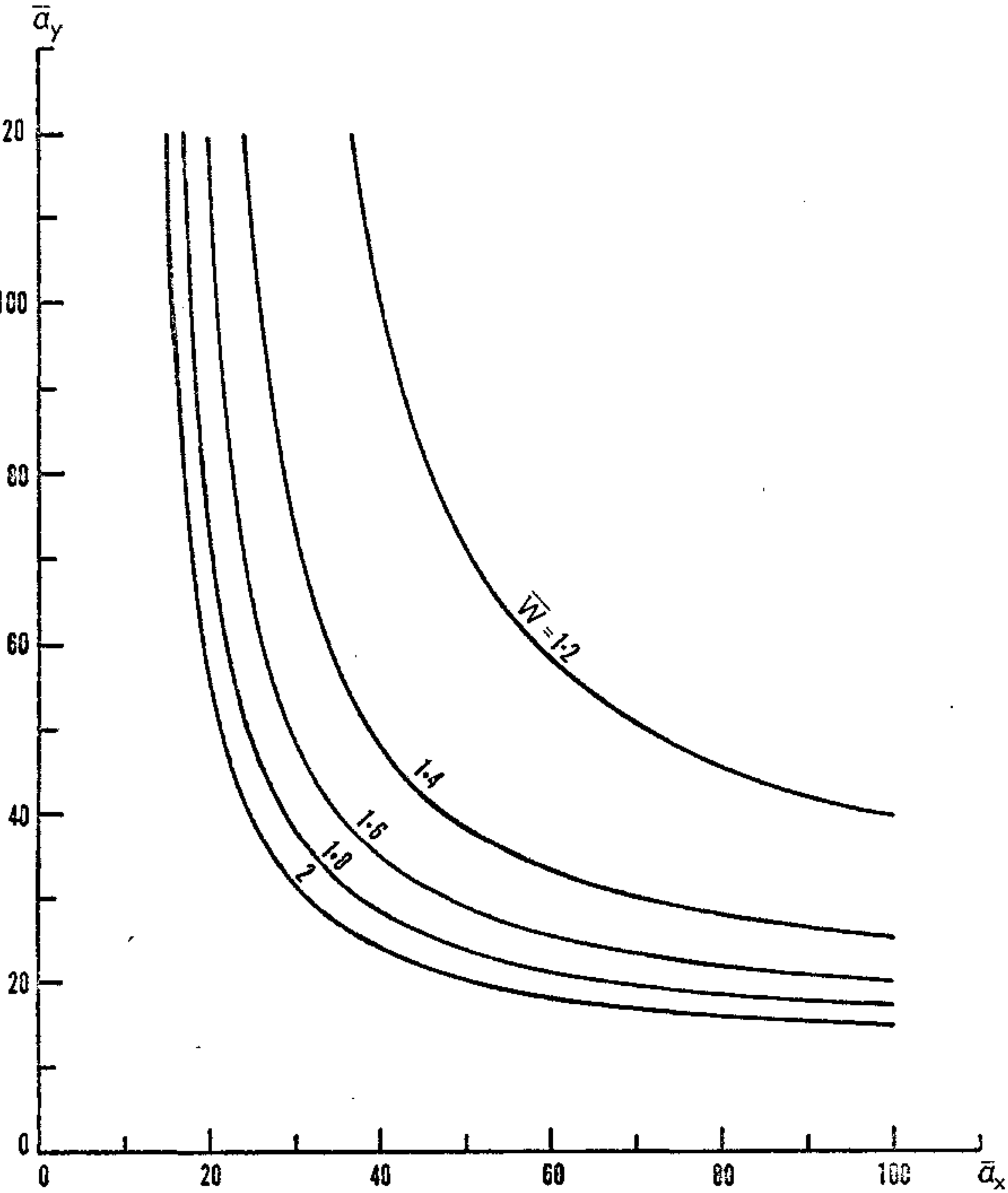


Fig. 4 Design Chart for Optimum $\overline{W}$. RSRR. Z = 35000,

$\bar{N}^* = 1{\cdot}233 \times 10^{-8}$

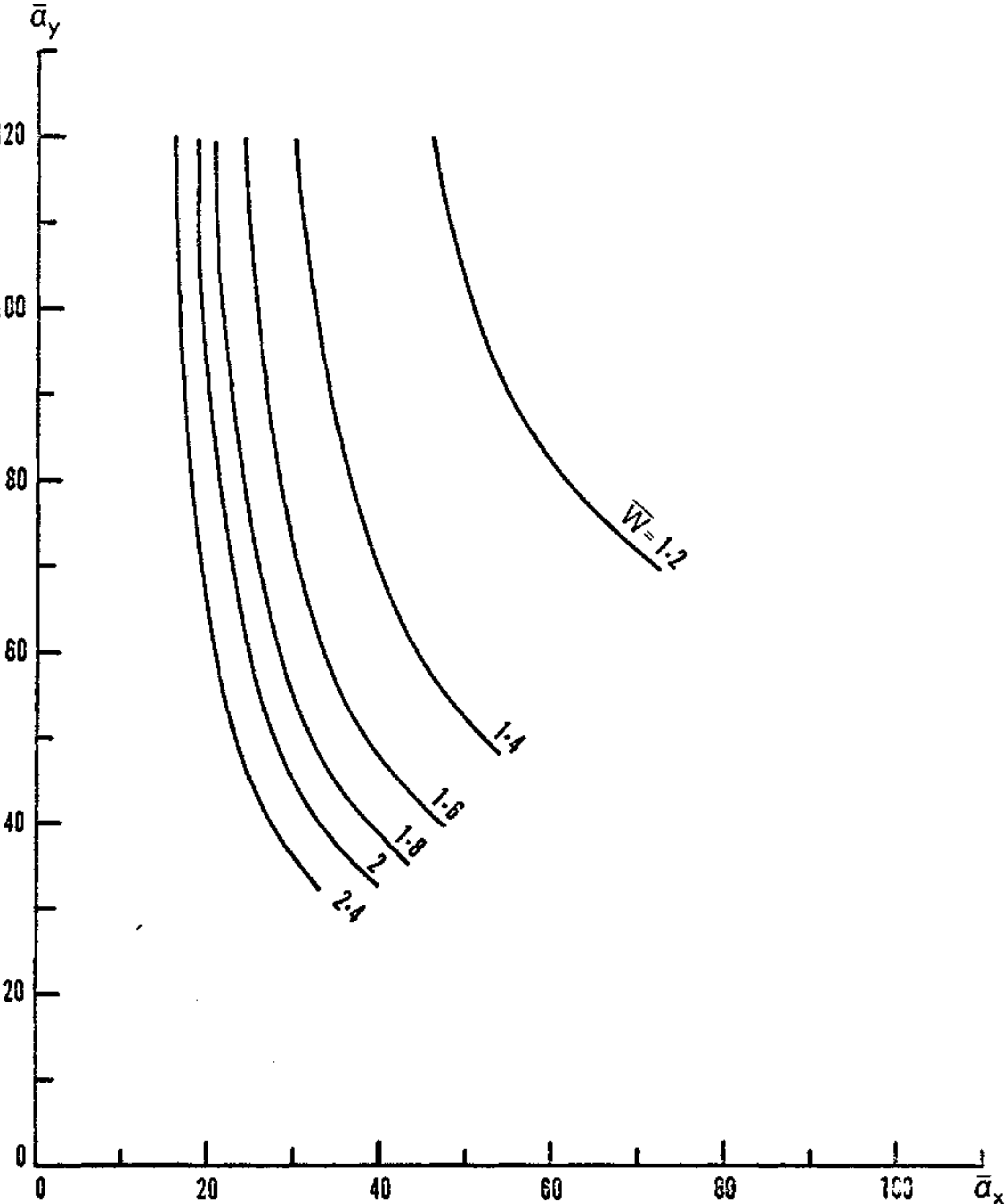


Fig. 5 Design Chart for Optimum $\overline{W}$. RSRR. Z = 38000.

$\overline{N}^* = 1{\cdot}233 \times 10^{-8}$

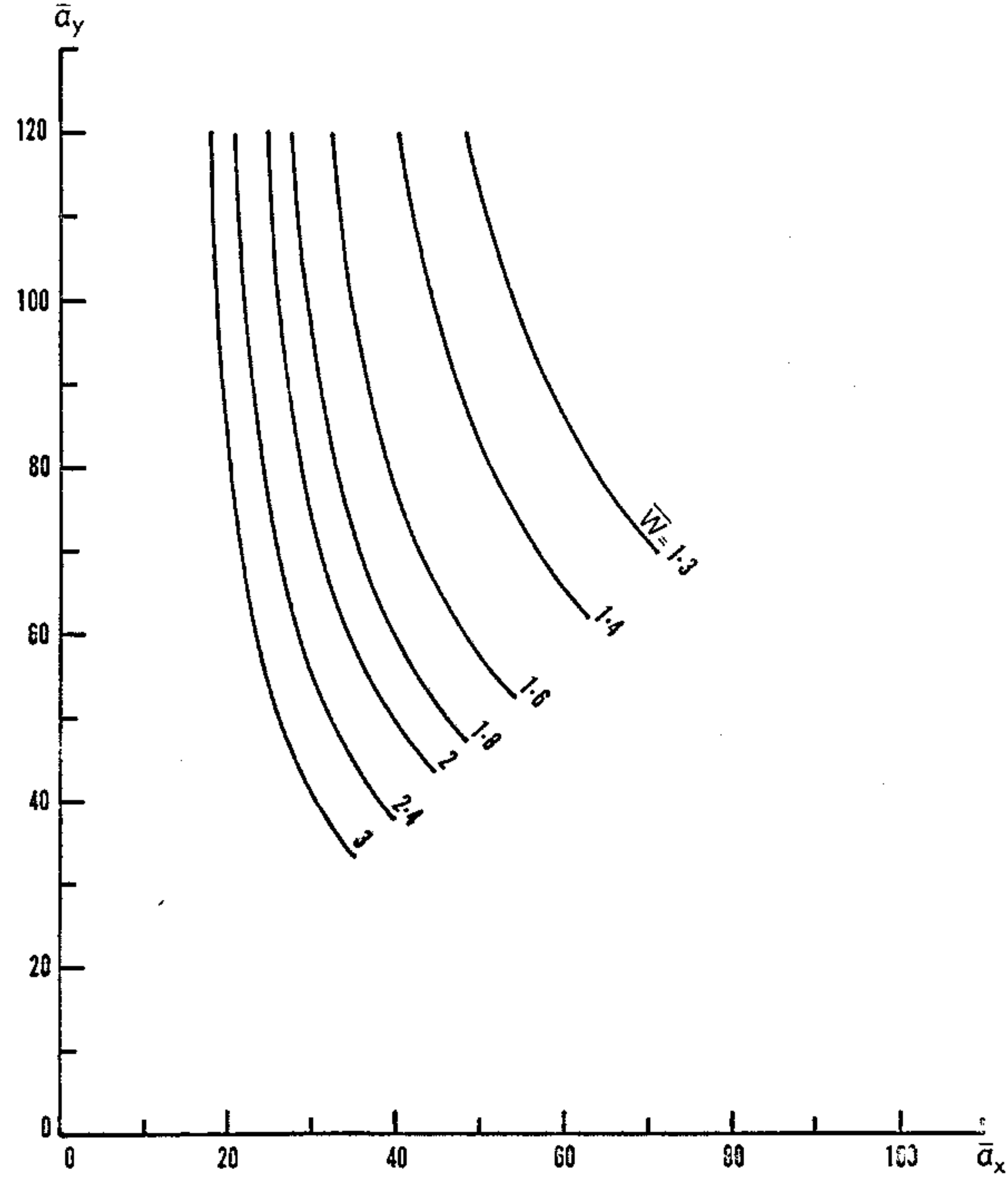
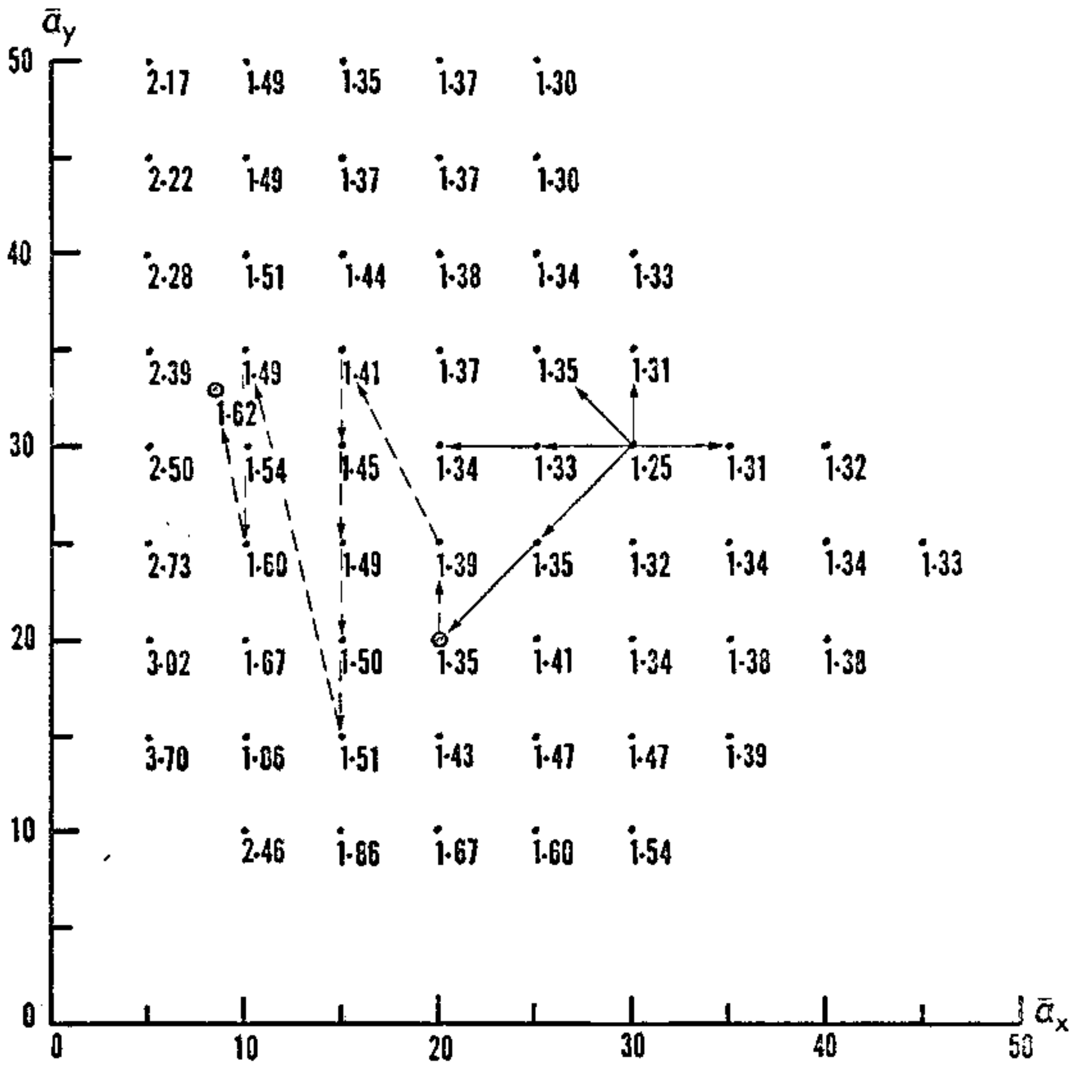


Fig. 6 Design Chart for Optimum $\overline{W}$. RSRR. Z = 42000,

$\overline{N}^* = 1{\cdot}233 \times 10^{-8}$



Fig·7 Design Chart for Optimum $\overline{W}$. RSRR. Z = 12000.

$\vec{N}^{*} = 4{\cdot}10306 \times 10^{-8}$

Examples of such tables are shown in Appendix B for TSRR (tee stringer and rectangular ring) and CSTR (channel stringer and tee ring). For more data of this type, with different shapes of stiffening members (different C_x and C_y), one should refer to the supplementary notes to this dissertation [32].

Phase 2: Design Procedure

In the design of the stiffened circular cylindrical shell the following quantities are known.

1. The applied uniform axial compressive load.
2. The radius and length of the stiffened shell.
3. The skin and stiffener materials and their associated properties.
4. The position of the stiffeners (inside).

The design variables to be determined are the skin thickness, the ring and stringer shapes, sizes and spacings. In this section the steps in designing the stiffened shells for minimum weight using different types of stiffening members are outlined. Expressions of stringer buckling for various types of stiffener section are given in Table 1 of Chapter II.

Design for RSRR and ASRR

1. For each Z, locate the minimum weight parameter $\overline{W}$ in the $\overline{\alpha}_x$-$\overline{\alpha}_y$ space (charts or tables) and the corresponding $\overline{\lambda}$'s. Since the expression for the stress in the rings is based on thin ring theory, $\frac{R}{d_{wy}}$ must be greater than 20.

This implies that $\bar{\alpha}_y \leq \frac{R}{20h}$. One then follows steps 2 through 7 such that no constraints are violated. If any constraint is violated one must increase the weight and repeat the procedure. Note that in many cases minimum $\bar{W}$ is a line rather than a point.

2. Calculate the stresses in the skin, stringers and rings by employing equations (9).

If all stresses are less than or equal to the yield stress or certain limiting stress level the next step is executed. Otherwise, one must move to the next higher W and repeat step 2. Note that since the skin is in a biaxial state of stress one should use an appropriate yield criterion.

3. The stringer and ring heights are computed from the definitions of $\bar{\alpha}_x$ and $\bar{\alpha}_y$. For the definitions of all new parameters, such as d_{wx}, c_{fx}, t_{wx}, b_{fx}, etc., see Appendix A.

$$d_{wx} = \frac{(1+c_{fx}k_s)h\bar{\alpha}_x}{(1+4c_{fx}k_s)^{1/2}} \quad , \quad d_{wy} = h\bar{\alpha}_y .$$

Note that the knowledge of Z implies the knowledge of h. For RS (rectangular stringers), $k_s = c_{fx} = 0$.

4. The ratios of the stiffener thickness to the stiffener spacing are determined from the definitions of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$.

$$\frac{t_{wx}}{\ell_x} = \frac{E\bar{\lambda}_{xx} h}{E_x d_{wx}(1-\nu^2)} \quad , \quad \frac{t_{wy}}{\ell_y} = \frac{E\bar{\lambda}_{yy} h}{E_y d_{wy}(1-\nu^2)}$$

5. The stringer spacing is determined by requiring that the stress in the skin be less than the skin buckling stress, $|\sigma_{xxsk_{cr}}| > |\sigma_{xxsk}|$ or

$$\ell_x < h\sqrt{\frac{\pi^2 E}{3(1-\nu^2)|\sigma_{xxsk}|}}$$

6. From the selected ℓ_x, calculate the stringer web thickness, t_{wx}, from step 4. Then the stringer flange thickness and width are determined from

$$t_{fx} = c_{fx} t_{wx} \quad , \quad b_{fx} = k_s d_{wx}$$

7. The ring spacing is determined by requiring that the stringer stress be less than the stringer buckling stress, $|\sigma_{xxst_{cr}}| > |\sigma_{xxst}|$ or

$$\ell_y < \frac{d_{wx} - t_{fx}}{\sqrt{\frac{12(1-\nu^2)}{\pi^2 E_x}\left(\frac{d_{wx} - t_{fx}}{t_{wx}}\right)^2 |\sigma_{xxst}| - .425}}$$

If the quantity under the radical sign is negative, then any ℓ_y will satisfy this constraint. In this step, ℓ_y must be checked to insure that no panel instability occurs. Furthermore, the number of rings must be greater than three for the smeared technique employed herein to apply [33].

8. Calculate the ring thickness, t_{wy}, from step 4. Observe that the simultaneous occurrence of general instability, panel instability, and local instabilities of skin and stringer can be avoided by proper choice of ℓ_x and ℓ_y. Note that steps 4 through 8 yield several combinations of t_{wx}, t_{fx}, t_{wy}, ℓ_x, and ℓ_y for the same cylinder weight (examples of this are presented in Appendix C).

9. The weight of the stiffened shell is

$$W = 2\pi RLh \, \rho_{sk} \, \overline{W}$$

10. Repeat the above steps for a number of Z values (h) and plot W vs. h. At least three values of h are needed. From the plot, one can then locate the absolute minimum weight with the corresponding value of h, and hence Z.

11. With the value of Z for minimum weight in step 10, one then generates the required data (design charts and tables) and repeats step 1 through 9 to finalize the dimensions. This last step is performed only when the exact minimum weight configurations is desired.

Design of TSTR and TSRR

Note that for inverted angle stringers (IAS) only the design step 7 has to be modified. For rectangular rings (RR) one puts $c_{fy} = k_r = 0$.

Step 1 and 2 are the same as those of RSRR except

$$\bar{\alpha}_y \leq \frac{R}{20h} \frac{(1+4c_{fy}k_r)^{1/2}}{1+c_{fy}k_r}$$

3. The stringer and ring heights are computed from the definitions of $\bar{\alpha}_x$ and $\bar{\alpha}_y$.

$$d_{wx} = \frac{(1+c_{fx}k_s)h\bar{\alpha}_x}{(1+4c_{fx}k_s)^{1/2}} , \quad d_{wy} = \frac{(1+c_{fy}k_r)h\bar{\alpha}_y}{(1+4c_{fy}k_r)^{1/2}}$$

4. The ratios of the stiffener thickness to the stiffener spacing are determined from the definitions of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$.

$$\frac{t_{wx}}{\ell_x} = \frac{E\bar{\lambda}_{xx}h}{E_x(1-\nu^2)(1+c_{fx}k_s)d_{wx}}$$

$$\frac{t_{wx}}{\ell_y} = \frac{E\bar{\lambda}_{yy}h}{E_y(1-\nu^2)(1+c_{fy}k_r)d_{wy}}$$

5. From the constraint of skin wrinkling

$$|\sigma_{xxsk_{cr}}| > |\sigma_{xxsk}|$$

one has,

$$\ell_x < h\sqrt{\frac{\pi^2 E}{3(1-\nu^2)\,|\sigma_{xxsk}|}}$$

6. From the selected ℓ_x, calculate the stringer web thickness, t_{wx}, from step 4. Then the stringer flange thickness and width are determined from

$$t_{fx} = c_{fx} t_{wx} \quad , \quad b_{fx} = k_s d_{wx}$$

7. From the constraints of stringer flange buckling

$$|\sigma_{xxsf_{cr}}| > |\sigma_{xxst}|$$

one has,

$$\ell_y < \frac{d_{fx}}{\sqrt{\frac{12(1-\nu^2)}{\pi^2 E_x} \left(\frac{d_{fx}}{t_{fx}}\right)^2 |\sigma_{xxst}| - .425}}$$

where

$$d_{fx} = \frac{1}{2}(b_{fx} - t_{wx}) \text{ for TS}$$

$$d_{fx} = b_{fx} - t_{wx} \text{ for IAS}$$

If the quantity under the radical sign is negative, then any ℓ_y will satisfy this constraint. The selected ℓ_y must be checked to insure that panel instability must not occur.

For small k_s (i.e. d_{fx} is small), the stringers are equivalent to the bulb-head stringers; therefore there will be no stringer flange buckling. Thus, one will not have the above expression for ℓ_y, but ℓ_y is determined on the basis of panel instability alone, with the number of rings being greater than three.

8. From the selected ℓ_y, calculate t_{wy} from step 4. Next the ring flange thickness and width are determined from

$$t_{fy} = c_{fy} t_{wy} \quad , \quad b_{fy} = k_r d_{wy}$$

The simultaneous occurrence of general instability, panel instability and local instabilities can be avoided by proper choice of ℓ_x and ℓ_y.

9. Check the local stringer web buckling.

$$\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{3(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx}}\right)^2 \quad \text{for } \frac{\ell_y}{d_{wx}} \geq 1$$

$$\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx}}\right)^2 \left(\frac{d_{wx}}{\ell_y} + \frac{\ell_y}{d_{wx}}\right)^2 \quad \text{for } \frac{\ell_y}{d_{wx}} < 1$$

If $|\sigma_{xxsw_{cr}}| > |\sigma_{xxst}|$, one goes to the next step. Otherwise, the weight must be increased and step 2 through 8 are repeated.

10. Calculate the weight of the stiffened shell.

$$W = 2\pi RLh \ \rho_{sk} \ \overline{W}$$

The last two steps are the same as those in the design of RSRR.

Design for TS and Other Types of Ring Shape

To design a stiffened shell using tee-shaped stringer (TS) with other types of ring shape only the step 1 through 4 of the design TSTR are needed to be modified as follows.

CR or ZR or IR. For channel (CR), or zee (ZR), or I rings the thin ring theory in step 1 implies that

$$\bar{\alpha}_y \leq \frac{R}{20h} \left(\frac{1+6c_{fy}k_r}{1+2c_{fy}k_r}\right)^{1/2}$$

The changes in step 3 and 4 are

$$d_{wy} = \left(\frac{1+2\ c_{fy}k_r}{1+6\ c_{fy}k_r}\right)^{1/2} h\bar{\alpha}_y$$

$$\frac{t_{wy}}{\ell_y} = \frac{E\bar{\lambda}_{yy}h}{E_y(1-\nu^2)(1+2c_{fy}k_r)d_{wy}}$$

Angle-Shaped Ring (AR). Using TS, the corresponding modification in TSTR design to angle-shaped rings is in step 3 only, namely

$$d_{wy} = \frac{(1+c_{fy}k_r)h\bar{\alpha}_y}{(1+4c_{fy}k_r)^{1/2}}$$

Design for Channel (C), Zee (Z), or I-Shaped Stringers and Rings

The design steps for channel and zee stringers and rings are identical but for I-section, only the following design step 7 has to be modified. For rectangular ring (RR) one puts $c_{fy} = k_r = 0$.

Step 1 and 2 are the same as those of RSRR except

$$\bar{\alpha}_y \leq \frac{R}{20h}\left(\frac{1+6c_{fy}k_r}{1+2c_{fy}k_r}\right)^{1/2}$$

3. The stringer and ring heights are computed from the definitions of $\bar{\alpha}_x$ and $\bar{\alpha}_y$.

$$d_{wx} = \left(\frac{1+2c_{fx}k_s}{1+6c_{fx}k_s}\right)^{1/2} h\bar{\alpha}_x$$

$$d_{wy} = \left(\frac{1+2c_{fy}k_r}{1+6c_{fy}k_r}\right)^{1/2} h\bar{\alpha}_y$$

4. From the definitions of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$, one has

$$\frac{t_{wx}}{\ell_x} = \frac{E\bar{\lambda}_{xx}h}{E_x(1-\nu^2)(1+2c_{fx}k_s)d_{wx}}$$

$$\frac{t_{wy}}{\ell_y} = \frac{E\bar{\lambda}_{yy}h}{E_y(1-\nu^2)(1+2c_{fy}k_r)d_{wy}}$$

5. From the constraint of skin wrinkling, $|\sigma_{xxsk_{cr}}| > |\sigma_{xxsk}|$ one has,

$$\ell_x < h\sqrt{\frac{\pi^2 E}{3(1-\nu^2)|\sigma_{xxsk}|}}$$

6. From the selected ℓ_x, calculate t_{wx} from step 4. Then

$$t_{fx} = c_{fx}t_{wx} \;, \quad b_{fx} = k_s d_{wx}$$

7. From the constraint of stringer flange buckling

$$|\sigma_{xxsf_{cr}}| > |\sigma_{xxst}|$$

one has,

$$\ell_y < \frac{b_{fx}}{\sqrt{\frac{12(1-\nu^2)}{\pi^2 E_x} (\frac{b_{fx}}{t_{fx}})^2 |\sigma_{xxst}| - .425}} \text{ for CS or ZS}$$

$$\ell_y < \frac{b_{fx}/2}{\sqrt{\frac{12(1-\nu^2)}{\pi^2 E_x} (\frac{b_{fx}}{2t_{fx}})^2 |\sigma_{xxst}| - .425}} \text{ for IS}$$

If the quantity under the radical sign is negative, then any ℓ_y will satisfy this constraint. Check the selected ℓ_y for panel instability with the number of rings being greater than three.

8. From the selected ℓ_y, calculate t_{wy} from step 4. Then

$$t_{fy} = c_{fy} t_{wy} , \quad b_{fy} = k_r d_{wy}$$

The simultaneous occurrence of general instability, panel instability, and local instabilities can be avoided by proper choice of ℓ_x and ℓ_y.

9. Check the local stringer web buckling.

$$\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{3(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx}-2t_{fx}}\right)^2 \quad \text{for } \frac{\ell_y}{d_{wx}-2t_{fx}} \geq 1$$

$$\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx}-2t_{fx}}\right)^2 \left(\frac{d_{wx}-2t_{fx}}{\ell_y}+\frac{\ell_y}{d_{wx}-2t_{fx}}\right)^2$$

for $\frac{\ell_y}{d_{wx}-2t_{fx}} < 1$.

If $|\sigma_{xxsw_{cr}}| > |\sigma_{xxst}|$, one goes to the next step. Otherwise, the weight must be increased and step 2 through 8 are repeated.

Steps 10 through 12 are the same as the design of TSTR.

<u>Design for CS, ZS, or IS and Other Types of Ring Shape</u>

<u>Tee and Angle-Shaped Ring (TR, IAR, AR)</u>. In this case, only the design step 1, 3, and 4 in the last design (CSCR, ZSZR, CSZR, etc.) are modified as

$$\bar{\alpha}_y \leq \frac{R}{20h} \frac{(1+4c_{fy}k_r)^{1/2}}{1+c_{fy}k_r}$$

$$d_{wy} = \frac{(1+c_{fy}k_r)h\bar{\alpha}_y}{(1+4c_{fy}k_r)^{1/2}}$$

$$\frac{t_{wy}}{\ell_y} = \frac{E\bar{\lambda}_{yy}h}{E_y(1-\nu^2)(1+c_{fy}k_r)d_{wy}}$$

CHAPTER IV

DESIGN RESULTS AND DISCUSSIONS OF THE RESULTS

The cylinder geometries and load taken as design examples are the following.

Case 1: $R = 95.5$ in. , $L = 291$ in.

$\bar{N} = 800$ lb/in., $\bar{N}^* = 1.233 \times 10^{-8}$

$\nu = .33$, $\sigma_o = 50{,}000$ psi

$E = E_x = E_y = 10.5 \times 10^6$ psi

$\rho_{sk} = \rho_x = \rho_y = .101$ lb/in^3

Case 2: $R = 9.55$ in. , $L = 38$ in.

$\bar{N} = 800$ lb/in., $\bar{N}^* = 4.10306 \times 10^{-8}$

$\nu = .33$, $\sigma_o = 50{,}000$ psi

$E = E_x = E_y = 10.5 \times 10^6$ psi

$\rho_{sk} = \rho_x = \rho_y = .101$ lb/in.^{3}

Case 3: $R = 85$ in. , $L = 100$ in.

$\bar{N} = 2700$ lb/in, $\bar{N}^* = 2.036 \times 10^{-6}$

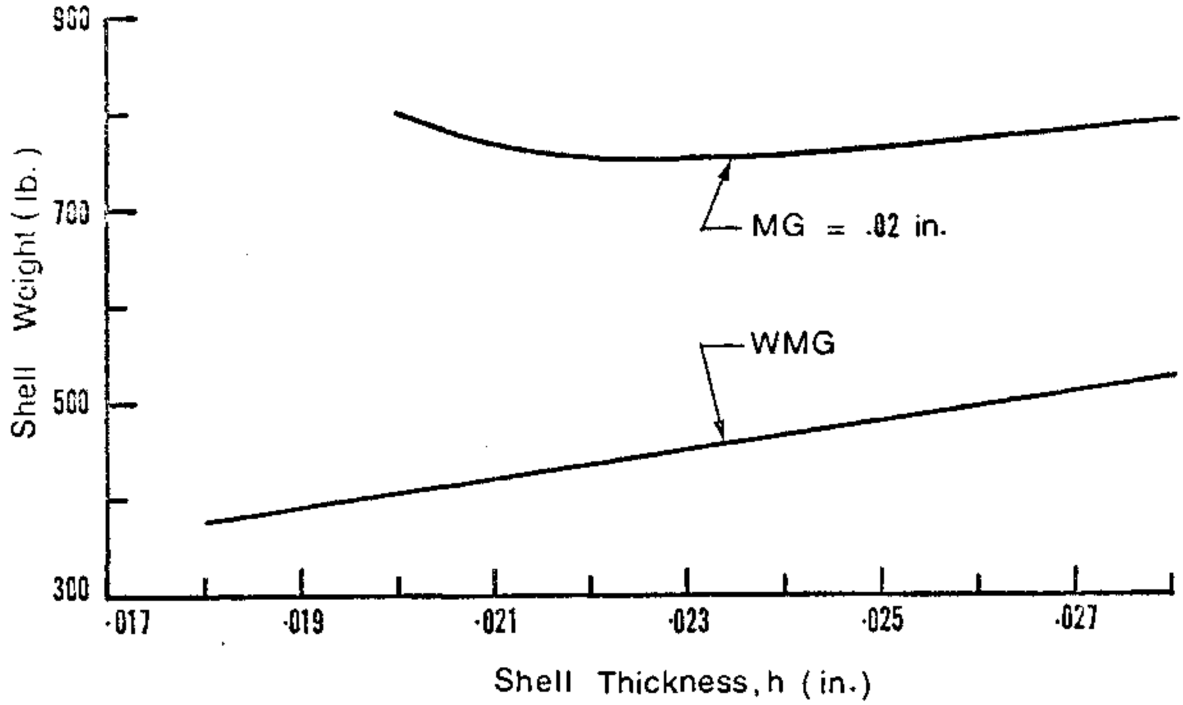
$\nu = .33$, $\sigma_o = 45{,}000$ psi

$E = E_x = E_y = 10.5 \times 10^6$ psi

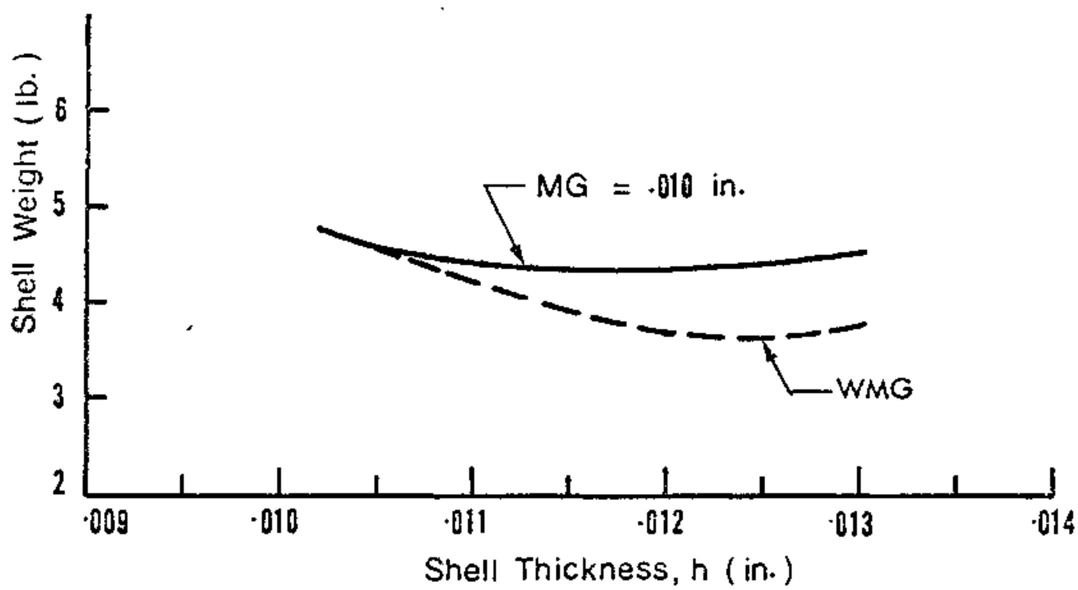
$\rho_{sk} = \rho_x = \rho_y = .101$ lb/in^3

Case 1 and 2 correspond to case 7-I and 6-I in reference [16] respectively. Case 1 represents a moderately loaded shell while Case 2, a heavily loaded shell. To compare the design results with those of Jones and Hague the design WMG (without minimum gauge) has been done for RSRR (rectangular stringer and ring). The results of the design analysis are shown in Figures 8 and 9 and the comparisons with their results in Table 2. For moderately loaded shell where yielding is not a strong factor the plot of W vs. h is a straight line as in Figure 8. For case 2, the heavily loaded shell, where yielding is critical the curve of W vs. h concaves downward. These designs (WMG) give unrealistic design dimensions beyond practice but they have been illustrated here to show the applicability of the method and also for comparison purpose. In such cases it is suggested to interchange the role of general instability and skin yielding in the formulation of the problem. This means that skin yielding is used as an equality constraint to generate design charts and general instability is considered as an inequality constraint in "Phase 2."

Case 1 shows a weight improvement of 45.3% over that of Jones and Hague but there is no improvement for Case 2. Note that, from Figure 9, the more exact location of minimum weight for the design WMG is at h = .0124 in. but the design has not been done for this skin thickness because the weight savings is only slightly different. Also in Figures 8 and 9, and Table 2 show the results of the design MG (with minimum gauge), which correspond to realistic design geometries. The corresponding results of Jones and Hague



Fig·8 Case 1, RSRR. Calculations to Determine Minimum Weight Design of Cylindrical Shell.



Fig·9 Case 2, RSRR. Calculation to Determine Minimum Weight Design of Cylindrical Shell.

Table 2. Some Design Results and Comparisons

	Case 1. RSRR			Case 2. RSRR		
	WMG(present)	WMG(Ref.16)	MG=.02 in.	WMG(present)	WMG(Ref.16)	MG=.01 in.
W	373	682.54	755	3.707	3.700	4.360
h	.018000	.03044	.022105	.011895	.00998	.010980
t_{wx}	.000527	.02760	.032620	.004424	.01244	.014921
t_{wy}	.000004	.000022	.022720	.000235	.00027	.014937
d_{wx}	2.07000	.3879	.44210	.23789	.11348	.09882
d_{wy}	2.07000	20.0000	2.10000	.23789	1.00850	.32939
ℓ_x	.51970	1.3162	.91985	.32072	.23791	.29114
ℓ_y	.00800	3.2290	9.38710	.05994	1.65190	1.18750
GB	1.0000	1.0028	1.0000	1.0000	1.0042	1.0000
PB	.0003	.2173	.9017	.0006	.9943	.7339
SB	.8511	1.0051	.9542	.9029	.7486	.9198
STB	.9427	1.0071	.9292	.8879	1.0007	.5159
SY	.7964	.4145	.4269	.9687	1.0030	1.0130
STYC	.7925	.4146	.4186	.9620	1.0039	.9893
RYT	.2487	.1375	.1146	.2966	.3295	.2430
m	8	27	18	7	13	16
n	10	6	9	8	7	7
m_p	1	1	1	1	1	1
n_p	over 600	62	36	272	21	25

are not available. Observe also that the present methodology avoids the simultaneous occurrence of failure modes while procedures based on mathematical search techniques with an objective function containing all constraints as penalty functions have no control over this point.

Some design results of Case 1 with MG = .02 in. using the combination of rectangular, tee, and channel stiffening members are shown in Figures 10 through 13. The design results indicate that the location of the minimum weight configurations for various shapes of stiffening members (different values of C_x and C_y) correspond to approximately the same value for h (.022 in.). Furthermore, the curves are very flat therefore a relatively large variation of the skin thickness will result in designs which differ only by a small percentage. This implies that in order to design the same case for other shapes of stiffening members one needs to generate data at the value of Z corresponding to the skin thickness of .022 in. only.

The effects of stringer and ring shapes of all cross-sections considered herein are investigated in order to obtain the absolute minimum weight configuration of Case 1. Consider the minimum weight design of various shapes of stiffening members as a three dimensional figure in the space of W, C_x, C_y, and if the plane $C_y = 1$ (rectangular ring) is cut through this figure, one has a two-dimensional case shown on Figure 14. That is, using rectangular rings,

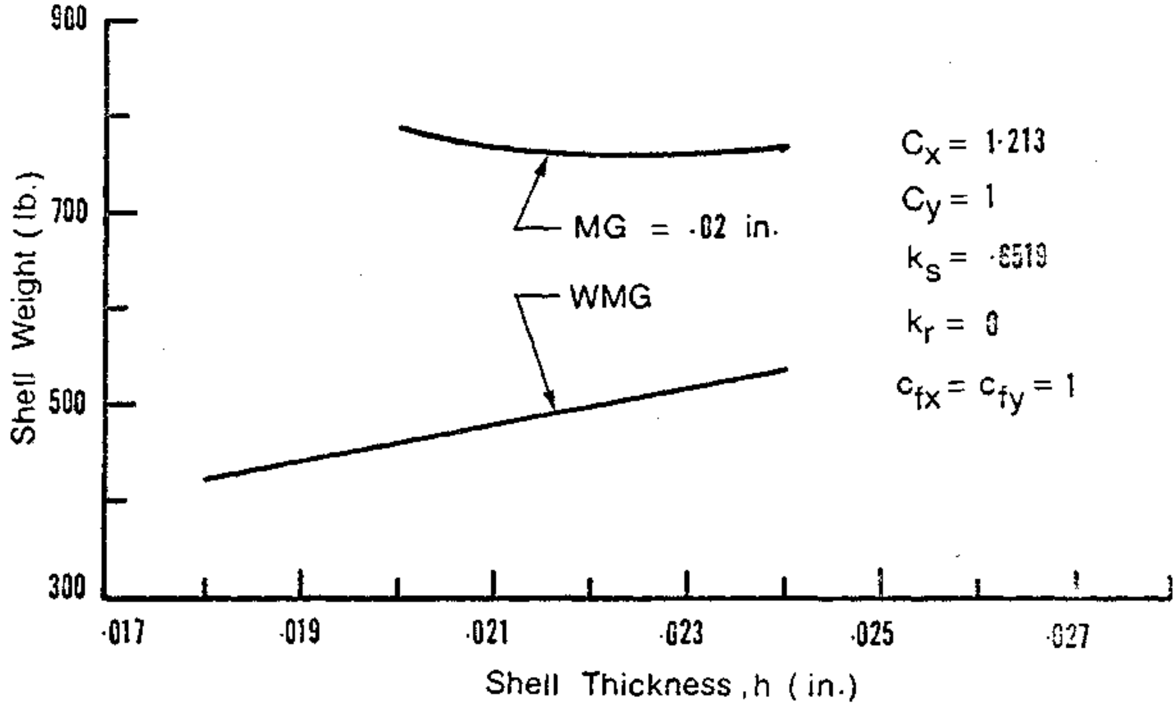


Fig.10 Case 1, TSRR. Calculations to Determine Minimum Weight Design of Cylindrical Shell.

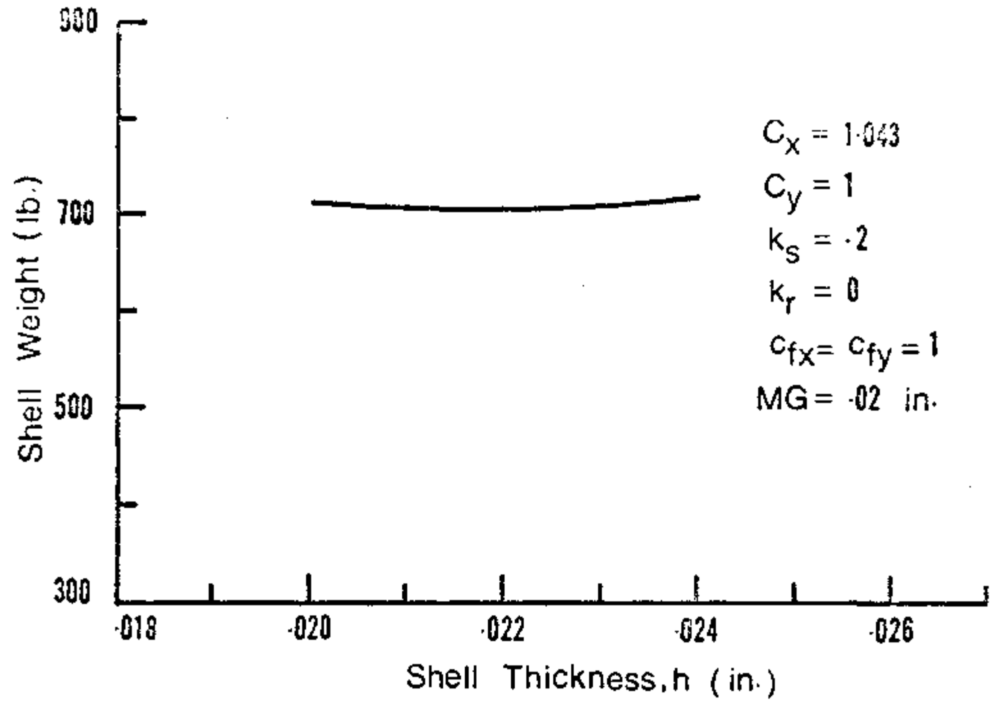


Fig.11 Case 1, TSRR. Calculations to Determine Minimum Weight Design of Cylindrical Shell.

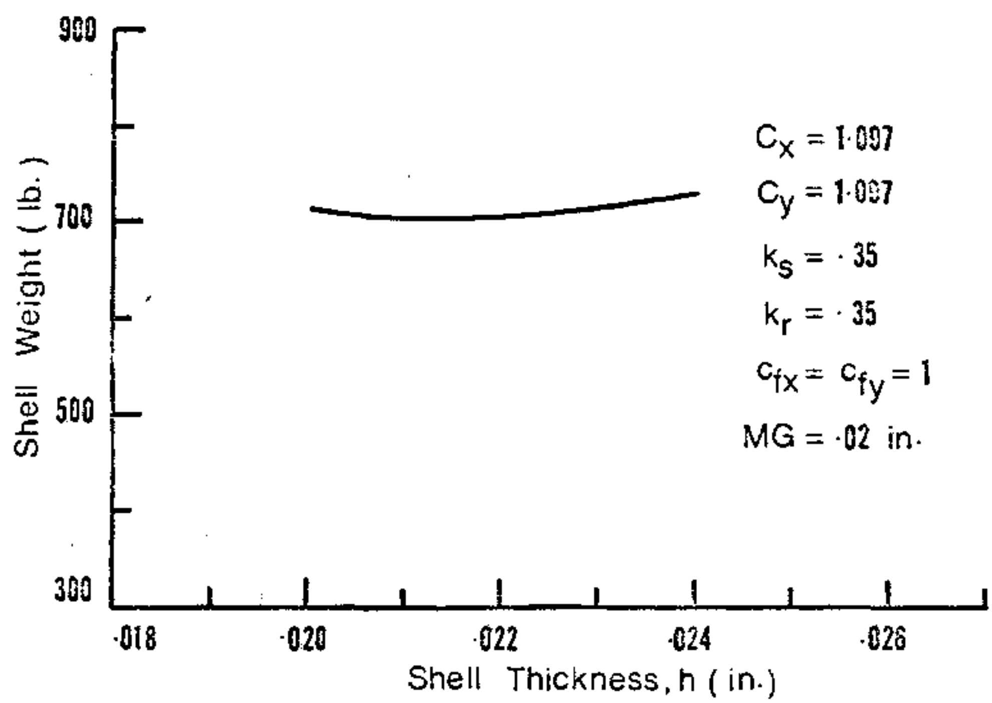


Fig.12 Case 1, TSTR. Calculations to Determine Minimum Weight Design of Cylindrical Shell.

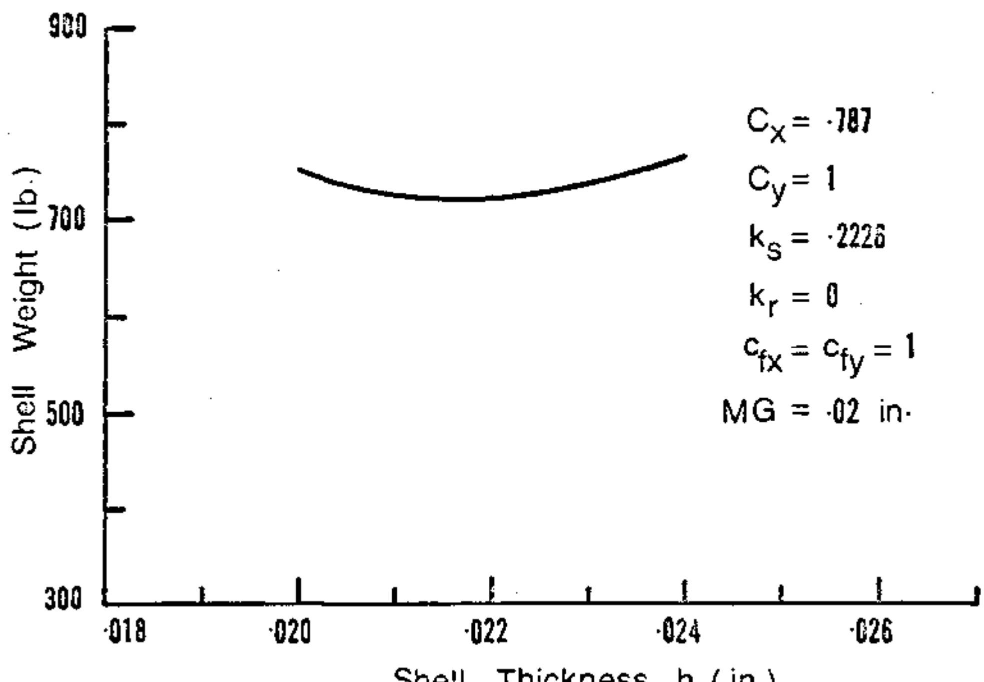


Fig.13 Case 1, CSRR· Calculations to Determine Minimum Weight Design of Cylindrical Shell·

the tee or inverted angle stringer (TS or IAS) with $C_x \simeq 1.09$ gives the least weight while the best weight of channel, zee, or I stringer (CS, ZS, IS) is at about $C_x \simeq .86$. The angle stringer (AS) shows the best weight at its degeneration into a rectangular stringer. Table 3 shows the minimum weight design geometry considering the effect of stringer shapes using rectangular rings.

The effect of ring shapes on the cylinder weight is investigated by passing the plane with different values for C_y through the minimum weight figure in W, C_x, C_y space. The results shown on Figures 15 and 16 are for $C_x = 1.097$ and .866 only because these two C_x's give the best weight for each type of stringer (TS and CS) (see Figure 14). The results show that the rectangular ring is the most efficient in designing a circular cylindrical shell under a uniform axial compression. This suggests that the extensional stiffness of the ring plays an important role for this load case (uniform axial compression) but not its flexural stiffness. The resulting design configurations are shown in Tables 4 and 5.

Case 3 is a geometry similar to the C-141 fuselage immediately after the wing box. Figures 17, 18, and Table 6 present the necessary data and results for minimum weight design using TSRR with MG = .05 in. The curve of W vs. C_x is very flat. The result indicates that the absolute minimum weight using TSRR is at $C_x = 1.08$.

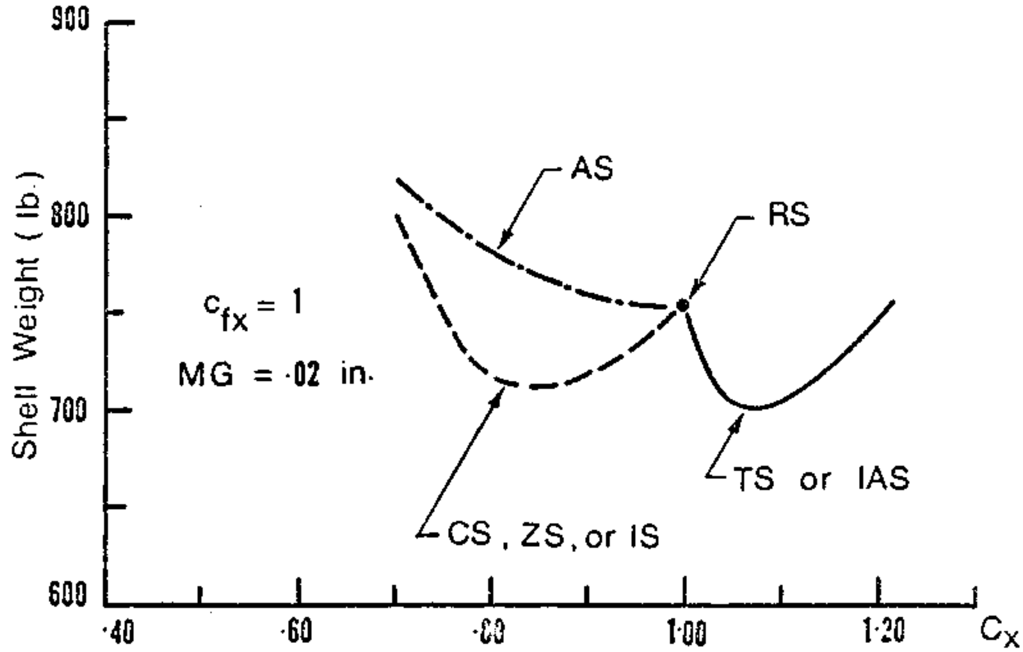


Fig.14 Case 1. Effect of Stringer Shapes on Cylinder Weight using Rectangular Ring ($C_y = 1$).

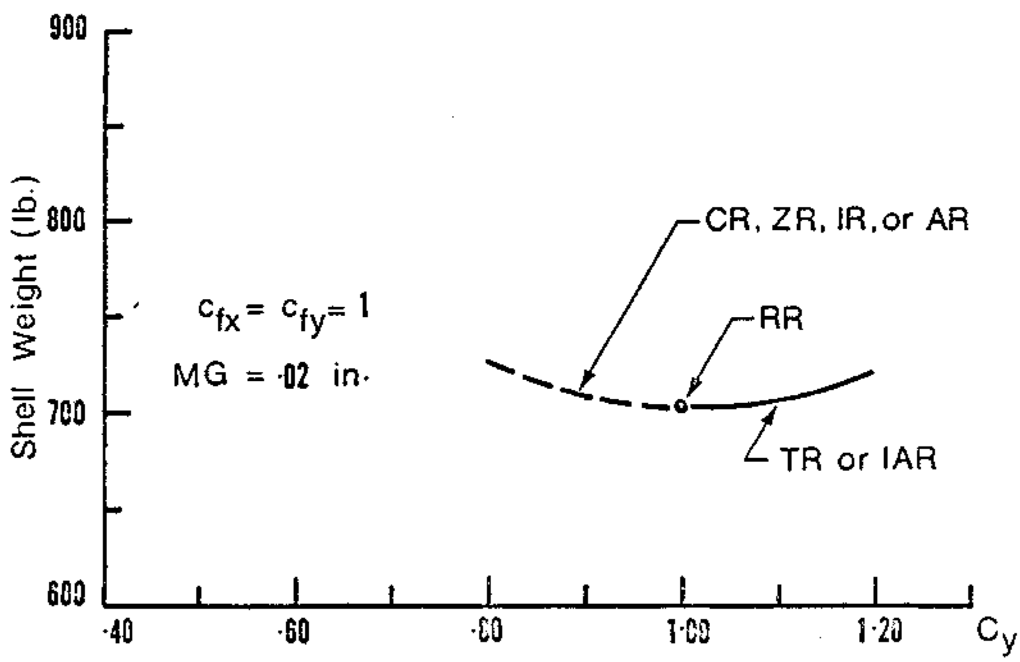
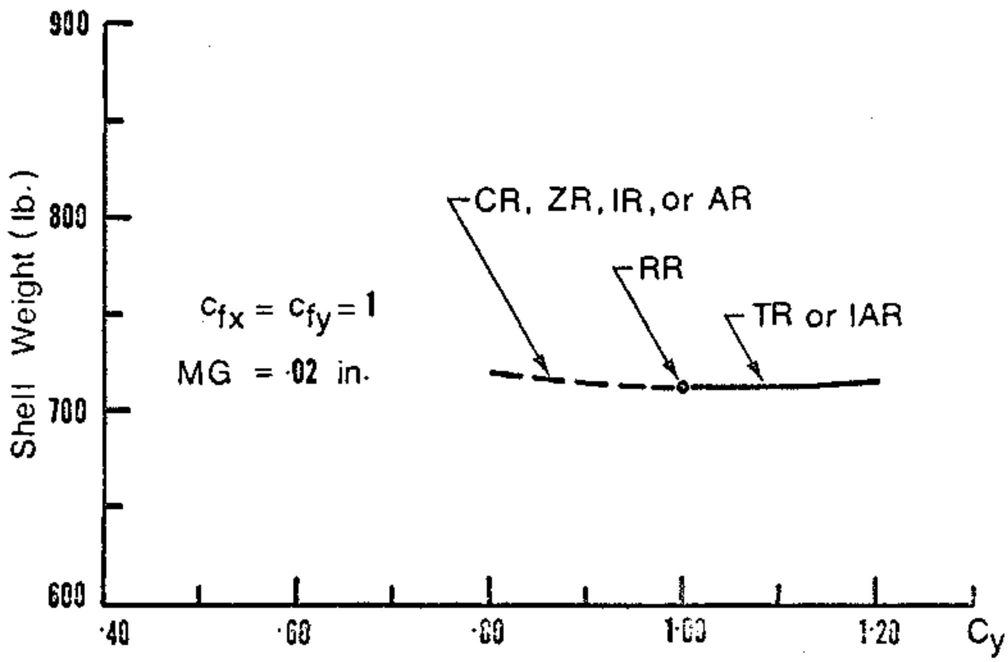
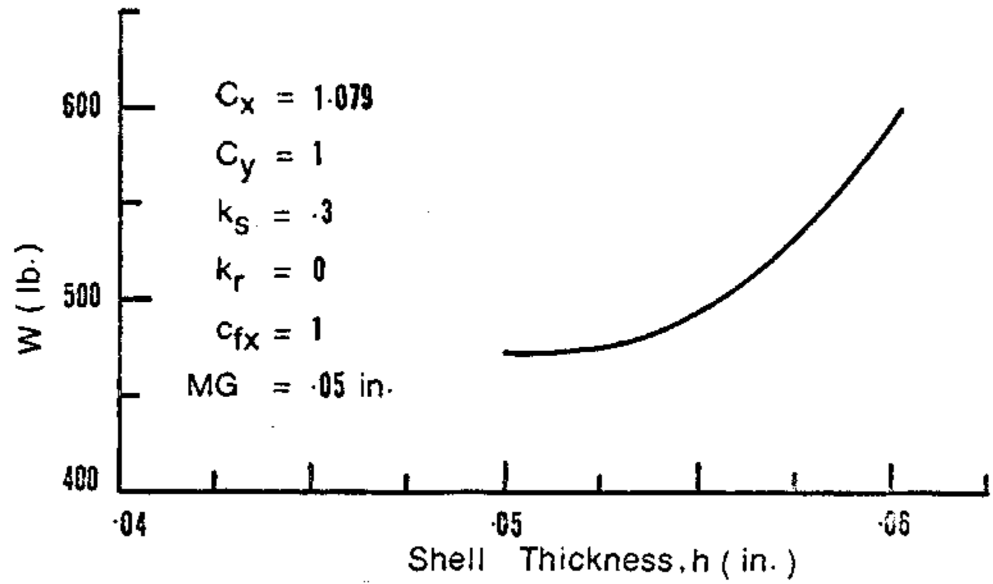


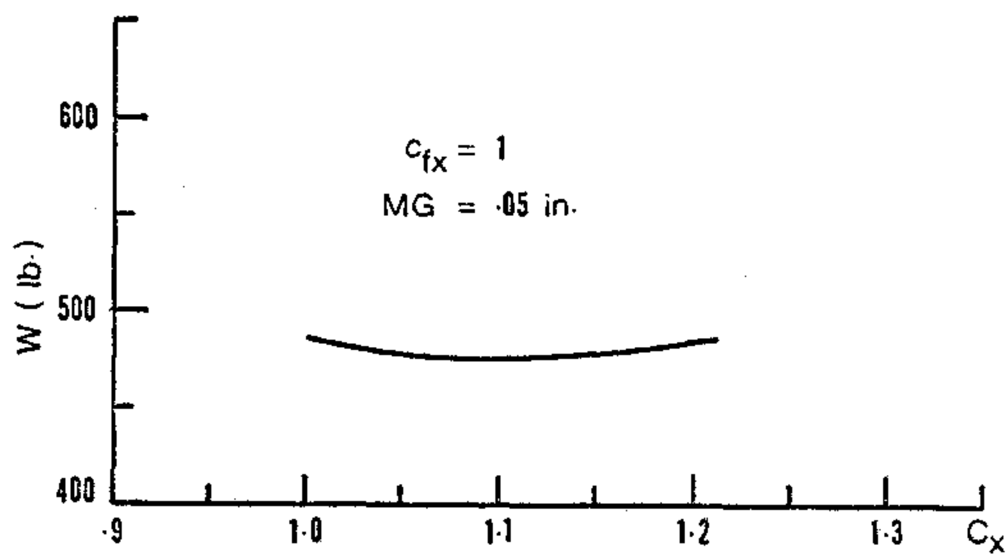
Fig.15 Case 1. Effect of Ring Shapes on Cylinder Weight using Most Efficient Stringer (TS or IAS, $C_x = 1{\cdot}067$)



Fig·16 Case 1· Effect of Ring Shapes on Cylinder Weight using Most Efficient Channel Stringer (or ZS or IS, C_x = ·866)·



Fig·17 Case 3, TSRR· Calculations to Determine Minimum Weight Design of Cylindrical Shell·



Fig·18 Case 3· Calculations to Determine the Minimum Weight among TSRR·

Table 3. Case 1. Effect of Stringer Shapes Using RR(C_y=1)

MG = .02 in. *STFB for IAS

Stringer Type	TS or IAS			RS
k_s	.6519	.3500	.2000	0
C_x	1.213	1.097	1.043	1
w	755	703	706	755
h	.02210	.02203	.02203	.02210
t_{wx}, t_{fx}	.02006	.02015	.02100	.03262
t_{wy}	.02722	.02768	.01991	.02272
d_{wx}	.32683	.44147	.42357	.44210
b_{fx}	.21306	.15451	.08471	--
d_{wy}	2.54210	1.65197	2.53302	2.10000
ℓ_x	.85433	.88068	.84115	.91985
ℓ_y	8.55882	10.77778	9.38710	9.38710
GB	1.0000	1.0000	1.0000	1.0000
PB	.9217	.8969	.9569	.9017
SB	.8963	.9318	.8742	.9542
STWB	.1548	.2506	.2389	.9292
STFB	.5073*	.2398*	.0508*	--
STFB	.1916	.0600	0	--
SY	.4648	.4515	.4644	.4629
STYC	.4516	.4440	.4548	.4186
RYT	.1124	.1250	.1233	.1146
m	20	15	18	18
n	8	10	9	9
m_p	1	1	1	1
n_p	38	33	37	36

Table 3. (continued)

MG = .02 in. *STFB for IS

Stringer Type	CS, ZS, or IS			AS	
k_s	.5071	.2226	.1000	.2518	.1536
C_x	.706	.787	.866	.706	.787
W	800	721	714	817	787
h	.02203	.02203	.02203	.02203	.02203
t_{wx}, t_{fx}	.02072	.02008	.02070	.02958	.03042
t_{wy}	.02390	.02180	.02436	.02729	.02452
d_{wx}	.36537	.41583	.55317	.42816	.42996
b_{fx}	.18528	.09256	.05532	.10781	.06604
d_{wy}	2.64315	2.20263	1.54184	2.31276	2.31276
ℓ_x	.91703	.87810	.90051	.92126	.91985
ℓ_y	9.38710	9.38710	11.64000	8.81820	9.09375
GB	1.0000	1.0000	1.0000	1.0000	1.0000
PB	.9291	.9302	.8230	.9060	.9411
SB	.9301	.9247	.9188	.9315	.9466
STWB	.1287	.1996	.3360	.8821	.8570
STFB	.0991*	0*	0*		
STFB	.3961	.1141	.0369	--	--
SY	.4184	.4507	.4314	.4125	.4205
STYC	.4080	.4418	.4258	.4018	.4107
RYT	.1058	.1208	.1243	.1028	.1082
m	19	18	13	19	19
n	9	9	11	8	9
m_p	1	1	1	1	1
n_p	37	37	32	39	38

Table 4. Case 1. Effect of Ring Shapes Using Most Efficient Stringer

(TS or IAS, C_x = 1.097) MG = .02 in. *STFB for IAS

Ring Type	TR or IAR		CR,ZR,IR	AR
k_r	.3500	.6000	.2000	.1429
C_y	1.097	1.193	.798	.798
W	705	721	727	727
h	.02203	.02203	.02203	.02203
t_{wx}, t_{fx}	.02037	.02081	.02127	.02127
t_{wy}, t_{fy}	.02105	.02664	.02050	.02197
d_{wx}	.47026	.46066	.36469	.36469
b_{fx}	.16459	.16123	.12764	.12764
d_{wy}	1.34359	1.14676	2.10851	2.40972
b_{fy}	.47026	.68806	.42170	.34435
ℓ_x	.88197	.88197	.84829	.84829
ℓ_y	11.64000	11.64000	8.81820	8.81820
GB	1.0000	1.0000	1.0000	1.0000
PB	.8782	.9132	.9335	.9335
SB	.9073	.9083	.8994	.8994
STWB	.2975	.2730	.1737	.1737
STFB	.2632*	.2387*	.1390*	.1390*
STFB	.0658	.0597	.0348	.0348
SY	.4348	.4388	.4697	.4697
STYC	.4326	.4316	.4579	.4579
RYT	.1260	.1217	.1346	.1346
m	13	13	19	19
n	11	10	9	9
m_p	1	1	1	1
n_p	31	31	38	38

Table 5. Case 1. Effect of Ring Shapes Using Most Efficient CS, ZS, or IS (C_x = .866)

MG = .02 in. *STFB for IS

Ring Type	TR or IAR		CR,ZR,IR	AR
k_r	.3500	.6000	.2000	.1429
C_y	1.097	1.193	.798	.798
W	711	716	719	719
h	.02203	.02203	.02203	.02203
t_{wx}, t_{fx}	.02027	.02012	.02005	.02005
t_{wy}, t_{fy}	.02437	.02411	.02526	.02708
d_{wx}	.55317	.53409	.55317	.55317
b_{fx}	.05532	.05341	.05532	.05532
d_{wy}	1.24762	1.14676	1.31782	1.50607
b_{fy}	.43667	.68806	.26356	.21522
ℓ_x	.90051	.90051	.90051	.90051
ℓ_y	12.12500	10.39286	11.64000	11.64000
GB	1.0000	1.0000	1.0000	1.0000
PB	.9070	.7380	.8439	.8439
SB	.9389	.9561	.9440	.9440
STWB	.3541	.3389	.3640	.3640
STFB	.0097*	.0093*	.0100*	.0100*
STFB	.0388	.0373	.0398	.0398
SY	.4351	.4431	.4375	.4375
STYC	.4292	.4357	.4306	.4306
RYT	.1246	.1227	.1223	.1223
m	13	14	13	13
n	11	11	11	11
m_p	1	1	1	1
n_p	32	33	32	32

Table 6. Case 3. Minimum Weight Design Using RR

MG = .05 in.

Stringer Type	TS			RS
k_s	.650	.450	.300	0
C_x	1.212	1.135	1.079	1
W	484	478	473	486
h	.05000	.05000	.05000	.05000
t_{wx}, t_{fx}	.05018	.05052	.05258	.06050
t_{wy}	.05519	.06235	.05419	.05078
d_{wx}	.60874	.51992	.70117	.60000
b_{fx}	.39568	.23395	.21035	--
d_{wy}	1.75000	2.25000	1.75000	3.00000
ℓ_x	1.62249	1.53833	1.58397	1.54725
ℓ_y	11.11111	10.00000	12.50000	9.09091
GB	1.0000	1.0000	1.0000	1.0000
PB	.7270	.8662	.7270	.8821
SB	.9111	.8898	.8765	.9168
STWB	.1036	.0972	.1531	.8521
STFB	.0949	.0285	.0182	--
SY	.7448	.8091	.7517	.8240
STYC	.7326	.7896	.7408	.8013
RYT	.2094	.2085	.2154	.2073
m	4	5	4	6
n	9	8	9	7
m_p	1	1	1	1
n_p	26	30	25	31

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The important conclusions of the present research are

1. The solution of the minimum weight design problem is not unique. This means that there are several combinations of the design variables for the same minimum weight.

2. The present approach allows the designer to deviate from the minimum weight solution with minimum penalty in weight, in order to avoid interaction among failure modes and/or unrealistic design variables.

3. Among all combinations of the rectangular, tee, zee, channel, I, angle, and inverted angle stiffening members, the circular cylindrical shell stiffened by tee stringers and rectangular rings is most efficient (least weight). The minimum weight configuration of Case 1 has tee stringers corresponding to $C_x = 1.09$, that of Case 3, $C_x = 1.08$.

4. The generated data can be used to design other circular cylindrical shells and loading whose nondimensional load parameter, $\overline{N}^*$, is about the same. If the data are stored, eventually all the possible cases of Z and $\overline{N}^*$ will be covered, thus, there will be no need to generate additional

data but simply use the stored ones in "Phase 2."

5. The curves of minimum W vs. h have wide flat portion. This implies that large variations in skin thickness (up to about 10%) yield design configurations with small difference in weight. Consequently, no exact Z is required for the minimum weight design.

Recommendations

In the aerospace application such as an airplane fuselage, the critical load case is a combined torsion with bending. Up to the present time there has been no reported work on the minimum weight design of stiffened cylindrical shells under torsion. Furthermore, several fuselage configurations are not complete circular cylindrical shells but a combination of cylindrical panels. Thus, the approach and search technique in the present work can be extended to the following possible investigations in the future.

1. Minimum weight design of stiffened cylindrical shells under torsion.

2. Minimum weight design of stiffened cylindrical shells under combined torsion and axial compression.

3. Minimum weight design of stiffened cylindrical panels under combined torsion and axial compression.

In addition, the following comments and recommendations are pertinent for the minimum weight design of fuselages. The methodology developed herein is applicable to that part of the fuselage which is subject to general instability failure. As a consequence, the resulting design has an overall bending stiffness, $(EI)_{eff}/L$, and torsional stiffness, $(GJ)_{eff}/L$. These stiffnesses must be acceptable for the dynamic respond of the vehicle. To insure this one must perform an aeroelastic investigation and arrive at the acceptable stiffness requirements which can be incorporated in the design procedure (Phase 2) as additional geometric constraints.

Finally, it is seen from the actual examples considered, especially cases 1 and 3, that the weight contribution of the different elements is as follows: skin weight 60%, stringer weight 30%, and ring weight 10%. Note that this holds true for the load case under consideration, a uniform axial compression. This distribution of weight suggests that if further improvement is to be accomplished by radically new fuselage configurations, most of the attention is warranted in the design of the skin (layered composite skin) and stringers (layered composite straps attached on the flange of the T-stringers in the stringer direction, x). This suggestion does not exclude the possibility that the ultimate solution might lie in an all composite configuration or even in a sandwich construction configuration.

APPENDICES

APPENDIX A

PROPERTIES OF STIFFENER CROSS-SECTIONS

Rectangular Stiffener

The radius of gyration of a rectangular cross-section is

$$\alpha = \frac{d}{\sqrt{12}}$$

Through nondimensionalization with respect to the radius gyration of the skin per unit width one obtains

$$\overline{\alpha} = \frac{d}{h}$$

The nondimensionalized stiffener flexural stiffness and eccentricity parameters are

$$\overline{\rho} = \frac{E_{st} I_{st_c}}{\ell D}$$

$$\overline{e} = \frac{\pi^2 Re}{L^2}$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad \text{and} \quad I_{st_c} = \frac{td^3}{12} = \frac{Ad^2}{12}$$

These two quantities can be expressed as

$$\bar{\rho} = \bar{\alpha}^2\bar{\lambda} \tag{A1}$$

$$\bar{e} = \frac{\pi^2(1-\nu^2)^{1/2}}{2Z}\,(1+\bar{\alpha})$$

<u>Other Types of Stiffeners</u>

With the assumption that t_w, $t_f \ll d_w$, $\bar{\rho}$ and $\bar{e}$ of the tee, angle, channel, zee, I, and inverted angle cross-sections can be expressed as

$$\bar{\rho}_{xx} = \bar{\alpha}_x^{\ 2}\ \bar{\lambda}_{xx} \tag{A2}$$

$$\bar{\rho}_{yy} = \bar{\alpha}_y^{\ 2}\ \bar{\lambda}_{yy}$$

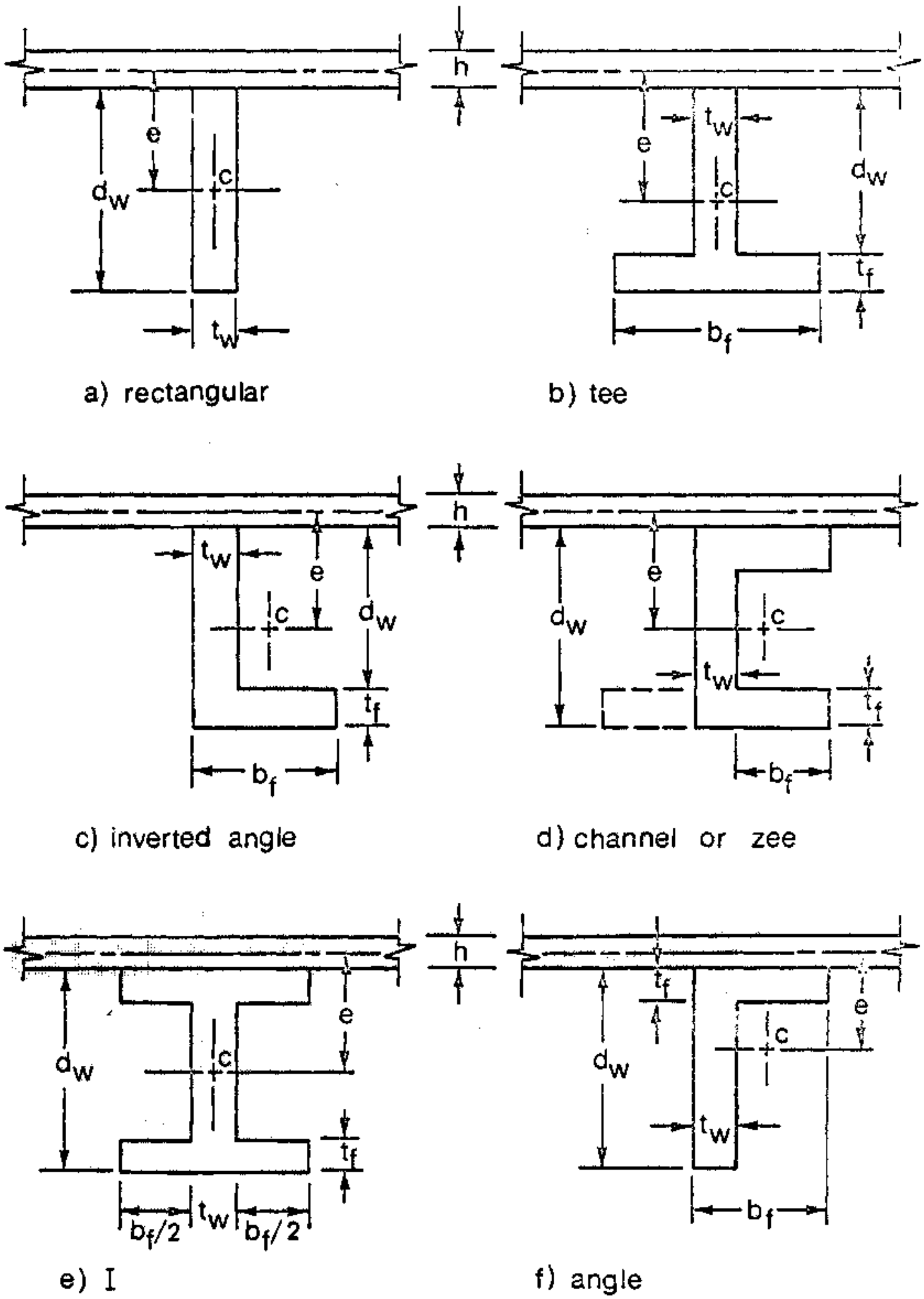
$$\bar{e}_x = \frac{\pi^2(1-\nu^2)^{1/2}}{2Z}\,(1+C_x\bar{\alpha}_x)$$

$$\bar{e}_y = \frac{\pi^2(1-\nu^2)^{1/2}}{2Z}\,(1+C_y\bar{\alpha}_y)$$

The expressions for $\bar{\alpha}$ and C, for each type of stiffener cross-section, are given in Table A1.

Table A1. Properties of Stiffener Cross-Sections

Section	Area, A	$\overline{\alpha}$	C
Rectangular	td	$\frac{d}{h}$	1.0
Tee or Inverted Angle	$d_w t_w (1+c_f k)$	$(\frac{d_w}{h}) \frac{(1+4c_f k)^{1/2}}{1+c_f k}$	$\frac{1+2c_f k}{(1+4c_f k)^{1/2}}$
Channel, I, or Z	$d_w t_w (1+2c_f k)$	$(\frac{d_w}{h}) (\frac{1+6c_f k}{1+2c_f k})^{1/2}$	$(\frac{1+2c_f k}{1+6c_f k})^{1/2}$
Angle	$d_w t_w (1+c_f k)$	$(\frac{d_w}{h}) \frac{(1+4c_f k)^{1/2}}{1+c_f k}$	$\frac{1}{(1+4c_f k)^{1/2}}$



$$b_f = k d_w \quad ; \quad t_f = c_f t_w$$

Fig. A1 Geometry of Stiffener Cross-Section

APPENDIX B

EXAMPLES OF DESIGN TABLES

Table B1. Design Table for TSRR. $c_{fx} = 1$

ν	C_x	C_y	k_s	k_r	$\bar{N}^*$	Z
.33	1.097	1	.35	0	1.233×10^{-8}	38000

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	β
24.0	60.0	1.94190	.61152	.22780	13	10.160
25.0	60.0	1.94633	.66046	.18282	12	10.321
26.0	60.0	1.89040	.61891	.17453	12	10.410
27.0	60.0	1.88881	.63906	.15295	11	10.399
28.0	60.0	1.85360	.61550	.14514	11	10.509
24.0	65.0	1.90379	.62564	.17973	13	10.226
25.0	65.0	1.90625	.65737	.15019	12	10.363
26.0	65.0	1.74052	.45464	.20524	14	10.179
27.0	65.0	1.81598	.59264	.13448	12	10.580
28.0	65.0	1.87208	.66365	.11346	11	10.618
23.0	70.0	1.87705	.59838	.18316	14	10.073
24.0	70.0	1.78124	.44090	.25526	16	9.764
25.0	70.0	1.82466	.59716	.13770	13	10.442
26.0	70.0	1.85996	.65222	.11408	12	10.567
27.0	70.0	1.85719	.66020	.10364	11	10.498
23.0	75.0	1.81165	.55163	.17163	15	10.048
24.0	75.0	1.83524	.61741	.12687	13	10.300
25.0	75.0	1.84977	.65017	.10706	12	10.410
26.0	75.0	1.85194	.66368	.09548	12	10.649
27.0	75.0	1.81369	.63437	.09070	11	10.509
23.0	80.0	1.76194	.41452	.26445	17	9.259
23.5	80.0	1.74834	.38506	.28179	17	9.142
24.0	80.0	1.83963	.64451	.10369	13	10.398
25.0	80.0	1.85234	.66974	.08978	12	10.512

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	β
26.0	80.0	1.80983	.63650	.08513	12	10.608
27.0	80.0	1.78625	.62135	.07927	11	10.509
22.0	85.0	1.77913	.44067	.25361	17	9.075
23.0	85.0	1.72114	.40362	.23899	17	9.142
24.0	85.0	1.67116	.37017	.22790	17	9.227
25.0	85.0	1.74356	.57287	.08972	13	10.433
19.0	90.0	1.97453	.57544	.29296	18	8.712
21.0	90.0	1.84246	.44224	.30848	18	8.621
22.0	90.0	1.75762	.41513	.25998	18	8.864
23.0	90.0	1.70805	.37498	.25597	18	8.892
20.0	95.0	1.89785	.46853	.33155	19	8.402
21.0	95.0	1.82680	.42210	.31466	19	8.447
22.0	95.0	1.73928	.39508	.26369	18	8.621
23.0	95.0	1.67527	.36768	.23406	18	8.774
24.0	95.0	1.65972	.32173	.26615	18	8.606
20.0	100.0	1.85909	.46078	.30476	19	8.292
21.0	100.0	1.77330	.42375	.26534	19	8.477
22.0	100.0	1.69228	.39966	.21723	18	8.683
23.0	100.0	1.64793	.35965	.21772	18	8.692
24.0	100.0	1.60325	.33005	.20751	18	8.732
19.0	105.0	1.86124	.54373	.22372	19	8.542
20.0	105.0	1.83630	.44740	.29783	19	8.129
21.0	105.0	1.74771	.41467	.25162	19	8.351
22.0	105.0	1.70008	.37173	.25211	19	8.337
23.0	105.0	1.64194	.34250	.22953	19	8.447
18.0	110.0	1.95648	.56285	.28947	20	8.053
19.0	110.0	1.83767	.60903	.13742	18	9.081
19.5	110.0	1.87682	.45181	.32952	20	7.881
20.0	110.0	1.81658	.43595	.29171	20	8.021
21.0	110.0	1.72367	.40661	.23826	19	8.226
22.0	110.0	1.66617	.37016	.22346	19	8.306
19.5	113.0	1.85600	.44919	.31359	20	7.835
18.0	115.0	1.92815	.55422	.27286	20	7.952

$\overline{\alpha}_x$	$\overline{\alpha}_y$	$\overline{W}$	$\overline{\lambda}_{xx}$	$\overline{\lambda}_{yy}$	m	β
19.0	115.0	1.86159	.48149	.28627	20	7.881
20.0	115.0	1.81622	.42099	.30635	20	7.801
21.0	115.0	1.67899	.41605	.18901	19	8.381
22.0	115.0	1.63063	.37317	.18879	19	8.367
17.0	120.0	1.98484	.64104	.23654	20	7.984
19.0	120.0	1.84158	.47248	.27746	20	7.744
19.5	120.0	1.88946	.42043	.37217	21	7.471
20.0	120.0	1.80784	.41018	.30969	20	7.620
21.0	120.0	1.74734	.36990	.29606	20	7.643
18.0	125.0	1.90706	.52103	.28726	20	7.552
19.0	125.0	1.88537	.44366	.34529	21	7.389
20.0	125.0	1.76413	.41178	.26914	20	7.620
21.0	125.0	1.68650	.37768	.23406	20	7.770
17.0	130.0	1.95655	.60088	.25151	20	7.573
18.0	130.0	1.85125	.53581	.22273	20	7.723
19.0	130.0	1.83809	.44550	.30133	21	7.389
20.0	130.0	1.79082	.39388	.31082	21	7.350

Table B2. Design Table for CSTR. $c_{fx} = c_{fy} = 1$

ν	C_x	C_y	k_s	k_r	$\vec{N}^*$	Z
.33	.866	1.193	.10	.60	1.233×10^{-8}	38000

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	β
30.0	50.0	1.90513	.59771	.20885	12	10.281
31.0	50.0	1.90422	.62647	.17928	12	10.538
32.0	50.0	1.87300	.60972	.16821	11	10.423
33.0	50.0	1.85005	.59874	.15874	11	10.508
34.0	50.0	1.83458	.59573	.14797	11	10.613
35.0	50.0	1.86596	.64016	.13150	10	10.556
29.0	55.0	1.88922	.61726	.17532	13	10.409
30.0	55.0	1.87309	.62199	.15602	12	10.396
31.0	55.0	1.77837	.51909	.17455	13	10.409
32.0	55.0	1.78987	.55854	.14532	12	10.538
33.0	55.0	1.87465	.66472	.11469	11	10.743
34.0	55.0	1.88157	.67878	.10679	10	10.556
27.0	60.0	1.91563	.64857	.16735	14	10.291
28.0	60.0	1.84324	.57923	.17222	14	10.240
29.0	60.0	1.77795	.51061	.18263	14	10.157
30.0	60.0	1.85539	.64045	.12178	12	10.511
31.0	60.0	1.85070	.64820	.10986	11	10.463
32.0	60.0	1.79153	.59576	.10957	11	10.508
33.0	60.0	1.83014	.64359	.09615	11	10.743
26.0	65.0	1.91325	.45849	.35531	17	9.129
27.0	65.0	1.82380	.44671	.28738	16	9.336
28.0	65.0	1.82820	.38591	.35210	17	9.129
29.0	65.0	1.71858	.40334	.26399	16	9.602
30.0	65.0	1.85173	.66187	.09711	12	10.626
31.0	65.0	1.82340	.64219	.09154	11	10.508

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	β
32.0	65.0	1.81806	.64340	.08557	11	10.613
25.0	70.0	1.90404	.48264	.32295	17	8.966
26.0	70.0	1.83872	.45060	.29678	17	9.067
27.0	70.0	1.75341	.46137	.21000	16	9.507
28.0	70.0	1.74030	.38820	.27148	17	9.191
29.0	70.0	1.68192	.37298	.23468	16	9.336
30.0	70.0	1.63870	.35766	.21149	16	9.507
24.0	75.0	1.89106	.52820	.26582	17	8.966
25.0	75.0	1.86068	.46116	.30579	18	8.829
26.0	75.0	1.78873	.43516	.26768	17	8.942
27.0	75.0	1.74661	.39878	.26652	17	8.942
28.0	75.0	1.69585	.37584	.24423	17	9.067
23.0	80.0	1.94542	.53186	.31060	18	8.591
24.0	80.0	1.87721	.48690	.29479	18	8.632
25.0	80.0	1.83427	.43893	.30450	18	8.591
26.0	80.0	1.78095	.40516	.29074	18	8.657
28.0	80.0	1.68723	.35013	.26225	18	8.763
30.0	80.0	1.56664	.33828	.16665	17	9.392
21.0	85.0	2.08090	.61731	.34589	19	8.297
22.0	85.0	1.93564	.62527	.20848	18	8.911
23.0	85.0	1.89143	.52400	.27035	18	8.525
24.0	85.0	1.86954	.45779	.31706	19	8.367
25.0	85.0	1.79285	.42739	.27913	18	8.459
26.0	85.0	1.72305	.40498	.23933	18	8.657
20.0	90.0	2.10084	.69690	.28406	19	8.297
21.0	90.0	2.01960	.61343	.29513	19	8.254
22.0	90.0	1.97223	.53609	.33027	19	8.115
23.0	90.0	1.83696	.60767	.13814	17	9.268
24.0	90.0	1.83396	.44723	.29591	19	8.228
25.0	90.0	1.77124	.41326	.27400	19	8.297
26.0	90.0	1.71500	.38350	.25364	19	8.410
20.0	95.0	2.08740	.65612	.31286	19	7.976

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	β
21.0	95.0	1.96673	.61255	.24891	19	8.254
22.0	95.0	1.91690	.53313	.28392	19	8.072
23.0	95.0	1.89809	.46553	.33475	20	7.923
24.0	95.0	1.80115	.43787	.27603	19	8.115
25.0	95.0	1.76723	.39579	.28789	19	8.046
20.0	100.0	2.07561	.63070	.32777	20	7.776
21.0	100.0	1.93052	.60346	.22574	19	8.185
22.0	100.0	1.83895	.60429	.14330	18	8.829
23.0	100.0	1.88094	.45390	.33111	20	7.731
24.0	100.0	1.77398	.42828	.26141	19	7.976
20.0	105.0	2.04716	.61722	.31590	20	7.658
21.0	105.0	1.88911	.61529	.17699	19	8.340
22.0	105.0	1.84611	.51777	.23620	20	7.968
23.0	105.0	1.87979	.43983	.34416	20	7.540
24.0	105.0	1.75702	.41826	.25632	20	7.849
20.0	110.0	2.00802	.61114	.28711	20	7.585
21.0	110.0	1.95505	.53658	.31446	20	7.466
22.0	110.0	1.89435	.48179	.31516	20	7.466
23.0	110.0	1.84196	.43558	.31469	20	7.466
24.0	110.0	1.76629	.40308	.27976	20	7.585
26.0	110.0	1.66689	.34285	.25141	20	7.658
28.0	110.0	1.50869	.33266	.12064	18	8.632
18.0	115.0	2.21857	.74910	.33677	21	7.280
19.0	115.0	2.10990	.65945	.32958	21	7.298
20.0	115.0	2.00616	.58988	.30671	21	7.357
21.0	115.0	1.91688	.53258	.28445	20	7.421
22.0	115.0	1.81540	.49426	.23234	20	7.613
23.0	115.0	1.81174	.43073	.29261	20	7.348
24.0	115.0	1.70742	.40832	.22206	20	7.658
18.0	120.0	2.22365	.72462	.36577	21	7.050
19.0	120.0	2.05833	.66094	.28258	21	7.298

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	β
20.0	120.0	1.97197	.58592	.28020	21	7.298
21.0	120.0	1.89371	.52562	.27113	21	7.328
22.0	120.0	1.84745	.46871	.28646	21	7.251
23.0	120.0	1.78326	.42691	.27106	21	7.298

APPENDIX C

DESIGN EXAMPLES

The following two design examples illustrate the design of Case 1 using different stringers and rings. The given quantities are:

$$R = 95.5 \text{ in.}, \quad L = 291 \text{ in.}, \quad \overline{N} = 800 \text{ lbs/in.}$$

$$E = E_x = E_y = 10.5 \times 10^6 \text{ psi}$$

$$\rho_{sk} = \rho_x = \rho_y = .101 \text{ lbs/in}^3$$

$$\nu = .33, \quad \sigma_o = 50{,}000 \text{ psi}$$

$$\overline{N}^* = \frac{12R^3\overline{N}}{\pi^2 E L^4 (1-\nu^2)^{1/2}} = 1.233 \times 10^{-8}$$

Design for TSRR

$$c_{fy} = k_r = o$$

$$C_x = 1.097$$

MG (minimum gauge) = .02 in.

$$c_{fx} = 1, \quad k_s = .35$$

All design steps are listed in Chapter III.

$$Z = 38000$$

$$h = \frac{L^2(1-\nu^2)^{1/2}}{RZ} = .02203$$

$$\overline{\alpha}_x = 23, \quad \overline{\alpha}_y = 75$$

From Table B1, one has

$\bar{\lambda}_{xx} = .55163$, $\bar{\lambda}_{yy} = .17163$; $\bar{W} = 1.81165$

$m = 15$, $\beta = 10.048$

Calculate the stresses in the skin, stringers, and rings using equation (9)

$\sigma_{xxsk} = -22576$ psi $\qquad \sigma_{yysk} = -1203$ psi

$\sigma_{xxst} = -22199$ psi $\qquad \sigma_{yyr} = 6252$ psi

From Steps 3 and 4,

$$d_{wx} = \frac{(1+c_{fx}k_s)h\bar{\alpha}_x}{(1+4c_{fx}k_s)^{1/2}} = .44147 \text{ in.}$$

$$d_{wy} = h\bar{\alpha}_y = 1.65197 \text{ in.}$$

$$\frac{t_{wx}}{\ell_x} = \frac{E\bar{\lambda}_{xx}h}{E_x(1-\nu^2)(1+c_{fx}k_s)d_{wx}} = .02288$$

$$\frac{t_{wy}}{\ell_y} = \frac{E\bar{\lambda}_{yy}h}{E_y(1-\nu^2)d_{wy}} = .00257$$

Then

$$\ell_x < h\sqrt{\frac{\pi^2 E}{3(1-\nu^2)|\sigma_{xxsk}|}}$$

or $$\ell_x < .91206$$

Select the stringer spacing such that one has a whole number of stringers and yet stays away from skin buckling. Choose

$$\ell_x = .88068 \text{ in.}$$

Therefore

$$t_{wx} = .02015 \text{ in.}$$

$$t_{fx} = .02015 \text{ in.}$$

$$b_{fx} = .15451 \text{ in.}$$

$$t_{fy} = b_{fy} = 0$$

From Step 7, one finds that any ring spacing, ℓ_y, will satisfy the constraint

$$|\sigma_{xxsf_{cr}}| > |\sigma_{xxst}|.$$

Thus, the determination of ℓ_y must be based on panel instability only. Using the computer program in Appendix E one has

$$\ell_y = 10.77778 \text{ in.}$$

$$N_{xxp_{cr}} = 892 \text{ lbs/in.}$$

$$m_p = 1,\ n_p = 33$$

Thus,

$$t_{wy} = .02768 \text{ in.}$$

Next, calculate the local buckling stresses using appropriate equations in Table 1.

$$\sigma_{xxsk_{cr}} = \frac{\pi^2 E}{3(1-\nu^2)} \left(\frac{h}{\ell_x}\right)^2 = 24228 \text{ psi.}$$

$$\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{3(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx}}\right)^2 = 88603 \text{ psi.}$$

$$\sigma_{xxsf_{cr}} = \frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{2t_{fx}}{b_{fx}-t_{wx}}\right)^2 \left[\left(\frac{b_{fx}-t_{wx}}{2\ell_y}\right)^2 + .425\right]$$

$$= 370{,}261 \text{ psi.}$$

Finally, compute the ratios of actual load to failure load, which clearly demonstrate the desired separation of failure modes.

$$PB = \overline{N}/N_{xxp_{cr}} = .8969$$

$$SB = \sigma_{xxsk}/\sigma_{xxsk_{cr}} = .9318$$

$$STWB = \sigma_{xxst}/\sigma_{xxsw_{cr}} = .2505$$

$$STFB = \sigma_{xxst}/\sigma_{xxsf_{cr}} = .0600$$

$$SY = \sigma_{xxsk}/\sigma_{o} = .4515$$

$$STYC = \sigma_{xxst}/\sigma_{o} = .4440$$

$$RYT = \sigma_{yyr}/\sigma_{o} = .1250$$

$$W = 2\pi RLh\rho_{sk}\overline{W} = 703.4 \text{ lb.}$$

Other designs, with the same weight, which satisfy all constraints (including geometric constraints) are

(1) $\ell_y = 10.03448$ in.

$t_{wy} = .02579$ in.

$N_{xxp_{cr}} = 1028$ lbs/in.

$m_p = 1,\ n_p = 34$

$PB = .7782$

(2) $\ell_y = 9.70$ in.

$t_{wy} = .02493$

$N_{xxp_{cr}} = 1100$ lbs/in.

$m_p = 1,\ n_p = 34$

$PB = .7273$

<u>Design for CSTR</u>

$C_x = .866$

$C_y = 1.193$

$MG = .02$ in.

$c_{fx} = 1,\ k_s = .10$

$c_{fy} = 1,\ k_r = .60$

All design steps are referred to those in Chapter III.

$Z = 38000$

$$h = \frac{L^2(1-\nu^2)^{1/2}}{RZ} = .02203 \text{ in.}$$

$$\bar{\alpha}_x = 28, \ \bar{\alpha}_y = 60$$

From Table B2, one has

$$\bar{\lambda}_{xx} = .57923$$

$$\bar{\lambda}_{yy} = .17222$$

$$\bar{W} = 1.84328$$

$$m = 14$$

$$\beta = 10.240$$

Calculate the stresses in the skin, stringers, and rings using equation (9).

$$\sigma_{xxsk} = -22157 \text{ psi.}$$

$$\sigma_{yysk} = -1184 \text{ psi.}$$

$$\sigma_{xxst} = -21786 \text{ psi.}$$

$$\sigma_{yyr} = 6133 \text{ psi.}$$

Steps 3 and 4 give

$$d_{wx} = \left(\frac{1+2c_{fx}k_s}{1+6c_{fx}k_s}\right)^{1/2} h\bar{\alpha}_x = .53409 \text{ in.}$$

$$d_{wy} = \frac{(1+c_{fy}k_r)h\bar{\alpha}_y}{(1+4c_{fy}k_r)^{1/2}} = 1.14676 \text{ in.}$$

$$\frac{t_{wx}}{\ell_x} = \frac{E\bar{\lambda}_{xx}h}{E_x(1-\nu^2)(1+2c_{fx}k_s)d_{wx}} = .02234$$

$$\frac{t_{wy}}{\ell_y} = \frac{E\bar{\lambda}_{yy}h}{E_y(1-\nu^2)(1+c_{fy}k_r)d_{wy}} = .00232$$

Then

$$\ell_x < h\sqrt{\frac{\pi^2 E}{3(1-\nu^2)|\sigma_{xxsk}|}}$$

or

$$\ell_x < .92373$$

Select the stringer spacing such that one has a whole number of stringers and yet stays away from skin buckling. Choose

$$\ell_x = .90051 \text{ in.}$$

therefore

$$t_{wx} = .02012 \text{ in.}$$

$$t_{fx} = .02012 \text{ in.}$$

$$b_{fx} = .05341 \text{ in.}$$

From Step 7, one finds that any ring spacing will satisfy the constraint

$$|\sigma_{xxsf_{cr}}| > |\sigma_{xxst}|.$$

Therefore ℓ_y must be selected on the basis of panel instability only. Using the computer program in Appendix E one has

$$\ell_y = 10.39286 \text{ in.}, \; N_{xxp_{cr}} = 1084 \text{ lbs/in.}$$

$$m_p = 1, \quad n_p = 33$$

Thus,

$$t_{wy} = .02411 \text{ in.}$$

$$t_{fy} = .02411 \text{ in.}$$

$$b_{fy} = .68806 \text{ in.}$$

Next, calculate the local buckling stresses using appropriate equations in Table 1.

$$\sigma_{xxsk_{cr}} = \frac{\pi^2 E}{3(1-\nu^2)} \left(\frac{h}{\ell_x}\right)^2 = 23173 \text{ psi}$$

$$\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{3(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx} - 2t_{fx}}\right)^2 = 64287 \text{ psi}$$

$$\sigma_{xxsf_{cr}} = \frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{t_{fx}}{b_{fx}}\right)^2 \left[\left(\frac{b_{fx}}{\ell_y}\right)^2 + .425\right]$$

$$= 584{,}032 \text{ psi}$$

Finally, the ratios of actual to failure load are:

$$PB = \overline{N}/N_{xxp_{cr}} = .7380$$

$$SB = \sigma_{xxsk}/\sigma_{xxsk_{cr}} = .9561$$

$$STWB = \sigma_{xxst}/\sigma_{xxsw_{cr}} = .3389$$

$$STFB = \sigma_{xxst}/\sigma_{xxsf_{cr}} = .0373$$

$$SY = \sigma_{xxsk}/\sigma_{o} = .4431$$

$$STYC = \sigma_{xxst}/\sigma_{o} = .4357$$

$$RYT = \sigma_{yyr}/\sigma_{o} = .1227$$

$$W = 2\pi RLh\rho_{sk}\,\overline{W} = 715.7 \text{ lb.}$$

APPENDIX D

GUIDELINE FOR DATA GENERATION

In several design cases the approximate value of the skin thickness can be estimated, therefore the interval of Z,

$$Z = \frac{L^2(1-\nu^2)^{1/2}}{Rh}$$

for which the data must be generated, is greatly reduced. But without priori knowledge of the skin thickness the following procedure to establish the range of Z values is recommended.

It is well-known that the skin thickness of an unstiffened circular cylindrical shell subject to a given axial compressive load is given by

$$h_u = \sqrt{\frac{\bar{N}R}{.61E}}$$

Since the weight of the unstiffened geometry is greater than that of a stringer- and ring-stiffened geometry, h_u will provide a lower bound for the value of Z. It may also be anticipated that the optimum stiffened geometry has a skin thickness not less than 15 per cent of h_u. This may be considered as a lower bound for h or an upper bound for the value of Z. Thus, if one defines Z_u by

$$Z_u = \frac{L^2(1-\nu^2)^{1/2}}{Rh_u}$$

then the range of Z values, in which the optimum configuration will lie, is

$$Z_u \leq Z \leq 6Z_u$$

In the case of uniform axial compression, from designing experience, one generally expects the optimum configuration to have both rings and stringers with rings being deeper than stringers to strengthen the local stringer buckling. Furthermore, when stringers are deeper than rings and in the region $\bar{\alpha}_x > \bar{\alpha}_y$, the design dimensions (stringer and ring thickness, ring spacing, etc.) become too small to accept. Also from thin ring theory one must have approximately

$$\bar{\alpha}_y \leq \frac{R}{20h} .$$

Hence, the region for which the data must be generated, for each Z, is where

$$\bar{\alpha}_x \leq \bar{\alpha}_y \text{ and } \bar{\alpha}_y < \frac{R}{20h}.$$

Now the question is: What value of Z in (Z_u, $6Z_u$) interval should be tried first? The following procedure is recommended.

1. Divide Z into 6 intervals: Z_u, $2Z_u$, ..., $6Z_u$.

2. Obtain data at $Z = 4Z_u$ and design the stiffened shell according to design procedure outlined in Chapter III, such that the resulting configuration has the lowest weight with all constraints being satisfied. Call this weight W_4.

3. Repeat Step 2 with $Z = 5Z_u$ and obtain the cylinder weight W_5.

4. If $W_4 < W_5$, one repeats Step 2 with $Z = 3Z_u$. If $W_4 > W_5$, one repeats Step 2 with $Z = 6Z_u$. If $W_4 \simeq W_5$ then the minimum weight configuration is between $4Z_u$ and $5Z_u$.

5. Plot W vs. h. If necessary, Step 2 is repeated with $Z = 2Z_u$.

In this systematic way one can eventually locate the thickness of the skin for minimum weight by generating data of not more than four values of Z.

APPENDIX E

COMPUTER PROGRAMS

Program for the Development of Design Charts and Tables

The structure of this program consists of a main program and five subprograms. The purpose of each program is as follows.

Main Program is the search method of Nelder and Mead.

SUBROUTINE START sets up an initial simplex from a given starting point.

SUBROUTINE SUMR contains nondimensional composite weight function, $\overline{W}^*$.

SUBROUTINE KXX is the search method of Golden Section.

FUNCTION F(Z) is the $\overline{K}_{xx}$ expression with m as a continuous variable.

FUNCTION G(Z) is the $\overline{K}_{xx}$ expression with m as an integer.

Descriptions of Inputs and Outputs

The symbols of the computer listings, with their corresponding representations, necessary to operate the Optimization Program are:

$$ALP = \overline{N}^*$$

$$ALX = \overline{\alpha}_x$$

ALY = $\bar{\alpha}_y$

BET = β

CX = C_x

CY = C_y

CFX = c_{fx}

CFY = c_{fy}

DIFER = Standard deviation of the $\bar{W}^*$ of the simplex to determine convergence.

FCX = k_s

FCY = k_r

GZ = $\bar{K}_{xx_{cr}}$

II = Number of iterations.

M = m

$$PO = \nu$$

$$SUM(IN) = \overline{W}*$$

$$SUML = \overline{W}* \text{ for minimum weight}$$

$$WP = \overline{W}$$

$$XI(KOUNT,1) = \overline{\lambda}_{xx}$$

$$XI(KOUNT,2) = \overline{\lambda}_{yy}$$

$$ZZZ = Z$$

To use the program, Lines 34 through 42 in the main program must be modified according to the type of stiffening member, load parameter, and curvature parameter. The data cards, to be read in, are $\overline{\alpha}_x$ and $\overline{\alpha}_y$. Each pair of $\overline{\alpha}_x$ and $\overline{\alpha}_y$ is punched on the same card with the Format (2F10.5) of Line 51. There can be any number of data cards. The complete program listings are shown on the next page.

Panel Buckling Program

The computer program for panel buckling analysis consists of a main program and two subprograms.

Main Program is the search method of Golden Section.

```
C     MINIMIZATION OF THE WEIGHT OF THE STIFFENED SHELL BY FLEXIBLE
C     POLYHEDRON METHOD OF NELDER AND MEAD.
C     ALLOWANCE HAS BEEN MADE FOR A 10-DIMENSIONAL PROBLEM.
C     NX IS THE NUMBER OF INDEPENDENT VARIABLES.
C     STEP IS THE INITIAL STEP SIZE.
C     X(I) IS THE ARRAY OF INITIAL GUESSES.
C     X(1) = LAMBDA XX BAR.
C     X(2) = LAMBDA YY BAR.
C     10**X(3) = LAGRANGE MULTIPLIER.
C     ZZZ = CURVATURE PARAMETER.
C     Z = BETA BAR, ARGUMENT IN THE KXX EXPRESSION.
C     M OR AM = NUMBER OF AXIAL WAVES.
C     ALP = APPLIED LOAD PARAMETER.
C     SUM(IN) = COMPOSITE WEIGHT FUNCTION.
C     GZ = KXXCR.
C     WP = WEIGHT PARAMETER.
C     PO = POISSON RATIO
C     CFX = STRINGER THICKNESS RATIO.
C     CFY = RING THICKNESS RATIO.
C     FCX = KS = STRINGER FLANGE WIDTH RATIO.
C     FCY = KR = RING FLANGE WIDTH RATIO.
C     FOR PROPER PRINT  OUT FORMAT STATEMENT 2002 AND 101 MUST BE
C     REVISED ACCORDINGLY.
C
      DIMENSION X1(10,10),X(10),SUM(10)
      COMMON/S/X1,NX,STEP,K1,SUM,IN
      COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZZ
      COMMON/EE/Z,AM,GZ
      COMMON/SR/ALP
      WRITE(6,2005)
 2005 FORMAT(//10X,'GENERAL INSTABILITY OPTIMIZATION-CSCR'//)
      NX = 2
      STEP = .1
      PO = 0.33
```

```
      ZZZ = 38000.
      ALP = 1.233E-8
      CFX = 1.0
      CFY = 1.0
      FCX = .1
      FCY = .2
      CX = SQRT((1.0+2.0*CFX*FCX)/(1.0+6.0*CFX*FCX))
      CY = SQRT((1.0+2.0*CFY*FCY)/(1.0+6.0*CFY*FCY))
      WRITE (6,111)
  111 FORMAT(/8X,'NU',5X,'CX',5X,'CY',7X,'Z',6X,'CFX',4X,'CFY',4X,'KS',5
     1X,'KR')
      WRITE (6,113) PO,CX,CY,ZZZ,CFX,CFY,FCX,FCY,ALP
  113 FORMAT (6X,F5.3,F6.3,F7.3,3X,F8.2,4F7.4,E15.6//)
      WRITE (6,2002)
 2002 FORMAT (6X,'ALX',4X,'ALY',3X,'WP',10X,'KXXCR',5X,'X(1)',6X,'X(2)',
     15X,'M',4X,'BETA',8X,'WPSTAR',4X,'DIFFER',5X,'II'/)
  100 READ (5,110,END=999) ALX,ALY
  110 FORMAT (2F10.5)
C     GUESS STARTING VALUES OF X(1) AND X(2).
      X(1) =.60
      X(2) = .25
      X(3) = 10.
C
      ALFA = 1.0
      BETA = 0.5
      GAMA = 2.0
      DIFER = 0.
      XNX = NX
      IN = 1
      CALL SUMR
      K1 = NX+1
      K2 = NX+2
      K3 = NX+3
      K4 = NX+4
```

```
      CALL START
      DO 3 I = 1,K1
      DO 4 J = 1,NX
    4 X(J) = X1(I,J)
      IN = I
      CALL SUMR
    3 CONTINUE
C
   63 II = 0
   28 II=II+1
      IF (II.LT.61) GO TO 60
      GO TO 888
C
C     SELECT LARGEST VALUE OF SUM(I) IN SIMPLEX
   60 SUMH = SUM(1)
      INDEX = 1
      DO 7 I = 2,K1
      IF(SUM(I).LE.SUMH) GO TO 7
      SUMH = SUM(I)
      INDEX = I
    7 CONTINUE
C     SELECT MINIMUM VALUE OF SUM(I) IN SIMPLEX
      SUML = SUM(1)
      KOUNT = 1
      DO 8 I = 2,K1
      IF(SUML.LE.SUM(I)) GO TO 8
      SUML = SUM(I)
      KOUNT = I
    8 CONTINUE
C     FIND CENTROID OF POINTS WITH I DIFFERENT THAN INDEX
      DO 9 J = 1,NX
      SUM2 = 0.
      DO 10 I = 1,K1
   10 SUM2 = SUM2+X1(I,J)
      X1(K2,J) = 1./XNX*(SUM2-X1(INDEX,J))
```

```
C     FIND REFLECTION OF HIGH POINT THROUGH CENTROID
      X1(K3,J) = (1.+ALFA)*X1(K2,J)-ALFA*X1(INDEX,J)
      IF(X1(K3,J).LT.0.)X1(K3,J) = 0.
    9 X(J) = X1(K3,J)
      IN = K3
      CALL SUMR
      IF(SUM(K3).LT.SUML) GO TO 11
C     SELECT SECOND LARGEST VALUE IN SIMPLEX
      IF(INDEX.EQ.1) GO TO 38
      SUMS = SUM(1)
      GO TO 39
   38 SUMS = SUM(2)
   39 DO 12 I = 1,K1
      IF((INDEX-I).EQ.0) GO TO 12
      IF(SUM(I).LE.SUMS) GO TO 12
      SUMS = SUM(I)
   12 CONTINUE
      IF(SUM(K3).GT.SUMS) GO TO 13
      GO TO 14
C     FORM EXPANSION OF NEW MINIMUM IF REFLECTION HAS PRODUCED ONE MINI.
   11 DO 15 J = 1,NX
      X1(K4,J) = (1-GAMA)*X1(K2,J)+GAMA*X1(K3,J)
      IF(X1(K4,J).LT.0.)X1(K4,J) = 0.
   15 X(J) = X1(K4,J)
      IN = K4
      CALL SUMR
      IF(SUM(K4).LT.SUML) GO TO 16
      GO TO 14
   13 IF(SUM(K3).GT.SUMH) GO TO 17
      DO 18 J = 1,NX
   18 X1(INDEX,J) = X1(K3,J)
   17 DO 19 J = 1,NX
      X1(K4,J) = BETA*X1(INDEX,J)+(1.-BETA)*X1(K2,J)
      IF(X1(K4,J).LT.0.)X1(K4,J) = 0.
   19 X(J) = X1(K4,J)
```

```
          IN = K4
          CALL SUMR
          IF(SUMH.GT.SUM(K4)) GO TO 16
C         REDUCE SIMPLEX BY HALF IF REFLECTION HAPPENS TO PRODUCE A LARGER
C         VALUE THAN THE MAXIMUM
          DO 20 J = 1,NX
          DO 20 I = 1,K1
       20 X1(I,J) = 0.5*(X1(I,J)+X1(KOUNT,J))
          DO 29 I = 1,K1
          DO 30 J = 1,NX
       30 X(J) = X1(I,J)
          IN = I
          CALL SUMR
       29 CONTINUE
          GO TO 26
       16 DO 21 J = 1,NX
          X1(INDEX,J) = X1(K4,J)
       21 X(J) = X1(INDEX,J)
          IN = INDEX
          CALL SUMR
          GO TO 26
       14 DO 22 J = 1,NX
          X1(INDEX,J) = X1(K3,J)
       22 X(J) = X1(INDEX,J)
          IN = INDEX
          CALL SUMR
       26 DO 23 J = 1,NX
       23 X(J) = X1(K2,J)
          IN = K2
          CALL SUMR
C         TO TERMINATE THE SEARCH, DIFER MUST BE LESS THAN EPSILON.
          DIFER = 0.
          DO 24 I = 1,K1
       24 DIFER = DIFER+(SUM(I)/SUM(K2)-1.)**2
          DIFER = SQRT(1./(XNX+1.0)*DIFER)
```

```
      IF(DIFER.GE.0.00001) GO TO 28
  888 BET = Z*AM
      M = AM
      WP  = 1.+(X1(KOUNT,1)+X1(KOUNT,2))/(1.-PO*PO)
      WRITE (6,101) ALX,ALY,WP,GZ,(X1(KOUNT,J),J=1,NX),M,BET,SUML,DIFER,
     1II
  101 FORMAT(1X,F8.1,F7.1,F10.5,F10.0,2F10.5,I5,F8.3,4X,F10.5,E12.5,I5)
      GO TO 100
  999 CONTINUE
      END
```

```
C     SET UP THE INITIAL SIMPLEX FROM ONE STARTING POINT.
      SUBROUTINE START
      DIMENSION X1(10,10),X(10),SUM(10),A(10,10)
      COMMON/S/X1,NX,STEP,K1,SUM,IN
      COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZZ
      VN = NX
      STEP1 = STEP/(VN*SQRT(2.))*(SQRT(VN+1.)+VN-1.)
      STEP2 = STEP/(VN*SQRT(2.))*(SQRT(VN+1.)-1.)
      DO 1 J = 1,NX
    1 A(1,J) = 0.
      DO 2 I = 2,K1
      DO 2 J = 1,NX
      A(I,J) = STEP2
      L = I-1
      A(I,L) = STEP1
    2 CONTINUE
      DO 3 I = 1,K1
      DO 3 J = 1,NX
    3 X1(I,J) = X(J)+A(I,J)
      RETURN
      END
```

```
      SUBROUTINE SUMR
C     SUMR IS THE WEIGHT EXPRESSION.
      DIMENSION X1(10,10),X(10),SUM(10)
      COMMON/S/X1,NX,STEP,K1,SUM,IN
      COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZZ
      COMMON/EE/Z,AM,GZ
      COMMON/SR/ALP
      DO 10 J=1,NX
   10 IF(X(J).LT.0.) X(J)=0.
      CALL KXX
      SUM(IN) = 1.0+(X(1)+X(2))/(1.-PO*PO)+10.**X(3)*ABS(GZ/(ZZZ*ZZZ)-
     1ALP*ZZZ)
      RETURN
      END
```

```
      SUBROUTINE KXX
C     CALCULATE BETA BAR AND M FOR KXXCR FOR EACH MOVEMENT OF X(I)
C     UNIDIMENSIONAL SEARCH BY GOLDEN SECTION METHOD USING FIBONACCI
C     FRACTIONS.
C     FIBONACCI FRACTION = F1 = 0.382
C
      DIMENSION X1(100),X2(100),X3(100),Y1(100),Y2(100),DEL(100),X(10)
     1,M(5),GG(5),Z1(5)
      COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZZ
      COMMON/CC/P,Q,R
      COMMON/DD/M,JJ
      COMMON/EE/Z,AM,GZ
      DATAX1(1),X2(1),X3(1),F1,EPS/.00,4.00,5.00,0.381966011,0.01/
      K = 1
      L = 0
   11 IF(F(X2(K))-F(X3(K))) 10,10,20
   20 X3(K) = X3(K)+0.2*X3(K)
      IF(X3(K).LT.15.) GO TO 11
```

```
      L = L+1
      IF(L.LT.10) GO TO 11
      X1(1) = 0.00001
      X2(1) = 0.8
      X3(1) = 1.0
      IF(L.LT.11) GO TO 11
C     BETA BAR CURVE IS TOO FLAT. SET A TRIAL M = 1
      AM = 1.0
      GO TO 8
   10 DEL(K) = X3(K)-X1(K)
   12 Y1(K) = X1(K)+F1*DEL(K)
      Y2(K) = X3(K)-F1*DEL(K)
      IF(F(Y1(K))-F(Y2(K))) 30,31,32
   30 DEL(K+1) = Y2(K)-X1(K)
      X1(K+1) = X1(K)
      X3(K+1) = Y2(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   31 DEL(K+1) = Y2(K)-X1(K)
      X1(K+1) = Y1(K)
      X3(K+1) = X3(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   32 DEL(K+1) = X3(K)-Y1(K)
      X1(K+1) = Y1(K)
      X3(K+1) = X3(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   40 Z   = (X1(K)+X3(K))/2.
      FX = F(Z)
      AM = (Q/P)**0.25
      BE = Z*AM
    8 JJ = 1
      IF(AM-1.0) 41,41,42
   41 M(JJ) = 1
      GO TO 49
   42 JJ = JJ+1
      M(JJ) = AM
      GO TO 49
   43 JJ = JJ+1
      M(JJ) = M(JJ-1)+1
      GO TO 49
   49 X1(1) = 0.01
      X2(1) = 4.5
      X3(1) = 5.
      K = 1
      L = 0
   71 IF(G(X2(K))-G(X3(K))) 72,72,73
```

```
   73 X3(K) = X3(K)+0.2*X3(K)
      IF(X3(K).LT.15.) GO TO 71
      L = L+1
      IF(L.LT.20) GO TO 71
      WRITE(6,101)
  101 FORMAT (/5X,'BETA BAR HAS BEEN LOST IN GZ')
      STOP
   72 DEL(K) = X3(K)-X1(K)
   74 Y1(K) = X1(K)+F1*DEL(K)
      Y2(K) = X3(K)-F1*DEL(K)
      IF(G(Y1(K))-G(Y2(K))) 75,76,77
   75 DEL(K+1) = Y2(K)-X1(K)
      X1(K+1) = X1(K)
      X3(K+1) = Y2(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 78
      GO TO 74
   76 DEL(K+1) = Y2(K)-X1(K)
      X1(K+1) = Y1(K)
      X3(K+1) = X3(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 78
      GO TO 74
   77 DEL(K+1) = X3(K)-Y1(K)
      X1(K+1) = Y1(K)
      X3(K+1) = X3(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 78
      GO TO 74
   78 Z1(JJ)= (X1(K)+X3(K))/2.
      GG(JJ) = G(Z1(JJ))
      IF(JJ.EQ.1) GO TO 51
      IF(JJ.EQ.3) GO TO 44
      GO TO 43
   44 IF ((GG(JJ)-GG(JJ-1)))51,51,52
   51 GZ = GG(JJ)
      Z = Z1(JJ)
      AM = M(JJ)
      GO TO 47
   52 GZ = GG(JJ-1)
      Z =Z1(JJ-1)
      AM = M(JJ-1)
   47 CONTINUE
      RETURN
      END
```

```
C     F IS THE KXX EXPRESSION TREATED M AS CONTINUOUS VARIABLE.
C
      FUNCTION F(Z)
      DIMENSION X(10)
      COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZZ
      COMMON/CC/P,Q,R
      RHOX = ALX*ALX*X(1)
      RHOY = ALY*ALY*X(2)
      EX = 3.14*3.14*SQRT(1.-PO*PO)*(1.0+CX*ALX)/(2.0*ZZZ)
      EY = 3.14*3.14*SQRT(1.-PO*PO)*(1.0+CY*ALY)/(2.0*ZZZ)
      A = 1.+RHOX+2.*Z*Z+(1.+RHOY)*Z**4
      B = 12.*ZZZ*ZZZ/(3.14**4*(1.-PO*PO))
      C = B*(EX*EX*X(1)+2.*EX*EX*X(1)*(1.-PO+X(2))*Z*Z/(1.-PO)+(EX*EX
     1*X(1)*(1.+X(2))+2.0*(1.0+PO)*X(1)*X(2)*EX*EY/(1.-PO)+EY*EY*X(2)
     2*(1.0+X(1)))*Z**4+2.*EY*EY*X(2)*(1.-PO+X(1))/(1.-PO)*Z**6+EY*EY*
     3X(2)*Z**8)
      D = 2.0*B*(PO*EX*X(1)-(EX*X(1)*(1.+X(2))+EY*X(2)*(1.+X(1)))*Z*Z+
     1PO*EY*X(2)*Z**4)
      E = B*((1.0+X(1))*(1.0+X(2))-PO*PO)
      FF = 1.0+X(1)+2.0/(1.-PO)*((1.+X(1))*(1.+X(2))-PO)*Z*Z+(1.+X(2))
     1*Z**4
      P = A+C/FF
      Q = E/FF
      R = D/FF
      F = 2.0*SQRT(P*Q)+R
      RETURN
      END
```

```
      FUNCTION G(Z)
C     G IS THE KXX EXPRESSION TREATED M AS INTEGER.
      DIMENSION X(10),M(5)
      COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZZ
      COMMON/DD/M,JJ
      RHOX = ALX*ALX*X(1)
      RHOY = ALY*ALY*X(2)
      EX = 3.14*3.14*SQRT(1.-PO*PO)*(1.0+CX*ALX)/(2.0*ZZZ)
      EY = 3.14*3.14*SQRT(1.-PO*PO)*(1.0+CY*ALY)/(2.0*ZZZ)
      A = 1.+RHOX+2.*Z*Z+(1.+RHOY)*Z**4
      B = 12.*ZZZ*ZZZ/(3.14**4*(1.-PO*PO))
      C = B*(EX*EX*X(1)+2.*EX*EX*X(1)*(1.-PO+X(2))*Z*Z/(1.-PO)+(EX*EX
     1*X(1)*(1.+X(2))+2.0*(1.0+PO)*X(1)*X(2)*EX*EY/(1.-PO)+EY*EY*X(2)
     2*(1.0+X(1)))*Z**4+2.*EY*EY*X(2)*(1.-PO+X(1))/(1.-PO)*Z**6+EY*EY*
     3X(2)*Z**8)
      D = 2.0*B*(PO*EX*X(1)-(EX*X(1)*(1.+X(2))+EY*X(2)*(1.+X(1)))*Z*Z+
     1PO*EY*X(2)*Z**4)
      E = B*((1.0+X(1))*(1.0+X(2))-PO*PO)
      FF = 1.0+X(1)+2.0/(1.-PO)*((1.+X(1))*(1.+X(2))-PO)*Z*Z+(1.+X(2))
     1*Z**4
      P = A+C/FF
      Q = E/FF
      R = D/FF
      G = P*M(JJ)*M(JJ)+Q/(M(JJ)*M(JJ))+R
      RETURN
      END
```

FUNCTION F(Z) is the $\overline{K}_{xxp}$ expression with m as a continuous variable.

FUNCTION G(Z) is the $\overline{K}_{xxp}$ expression with m as an integer.

Descriptions of Inputs and Outputs

The symbols of the computer listings, with their corresponding representations, necessary to operate the program are:

ALX = $\overline{\alpha}_x$

BET = β

CX = C_x

CMW = n

E = E

GZ = $\overline{K}_{xxp_{cr}}$

MM = m

PO = ν

$$PCR = \overline{N}_{xxp_{cr}}$$

$$W1 = \overline{\lambda}_{xx}$$

$$W2 = \ell_y$$

$$W3 = R$$

$$W4 = h$$

$$ZZZ = Z_p$$

To use the program the value of ν in line 22 of the main program listings must be changed according to the material used in the design. The data card contains seven quantities, E, C_x, R, $\overline{\alpha}_x$, $\overline{\lambda}_{xx}$, h, ℓ_y, punched on one card according to the Format of line 24. There can be any number of data cards. The computer listings are as follows.

```
C     PROGRAM FOR CHECKING PANEL INSTABILITY.
C     UNIDIMENSIONAL SEARCH BY GOLDEN SECTION.
C     F1 = FIBONACCI FRACTION.
C     ZZZ = CURVATURE PARAMETER.
C     CMW = NO. OF CIRCUMFERENTIAL WAVES.
C     Z = BETA BAR, ARGUMENT IN THE FUNCTION.
C     PO = POISSON RATIO.
C     M = NO. OF AXIAL WAVWS.
C     ALX = ALPHA X BAR.
C     PCR = CRITICAL LOAD.
C     GZ = PANEL BUCKLING COEFFICIENT.
C     W1 = LAMBDA X BAR.
C     W2 = LY.
C     W3 = RADIUS.
C     W4 = SKIN THICKNESS.
C
      DIMENSION X1(100),X2(100),X3(100),Y1(100),Y2(100),DEL(100)
     1,M(5),GG(5),Z1(5)
      COMMON/KXXP/ALX,CX,PO,ZZZ,W1,W2,W3,W4
      COMMON/FFF/P,Q
      COMMON/GGG/M,JJ
      PO = .33
    4 READ(5,2,END=999) E,CX,W3,ALX,W1,W4,W2
    2 FORMAT (F10.0,6F10.5)
      DATAX1(1),X2(1),X3(1),F1,EPS/.01,4.00,5.00,0.381966011,0.01/
      WRITE(6,105)
  105 FORMAT (//9X,'E',4X,'CX',8X,'RADIUS',4X,'ALX',7X,'X(1)',7X,'H',8X,
     1'LY')
      WRITE(6,7)E,CX,W3,ALX,W1,W4,W2
    7 FORMAT(F10.0,6F10.5)
      WRITE (6,3)
    3 FORMAT (11X,'KXXPCR',8X,'ZP',6X,'M',4X,'BETA',4X,'N',5X,'NXXCR')
      ZZZ = W2*W2*SQRT(1.-PO*PO)/(W3*W4)
      K = 1
      L = 0
```

```
   11 IF(F(X2(K))-F(X3(K))) 10,10,20
   20 X3(K) = X3(K)+0.2*X3(K)
      IF(X3(K).LT.15.) GO TO 11
      L = L+1
      IF(L.LT.10) GO TO 11
      X1(1) = 0.01
      X2(1) = 0.8
      X3(1) = 1.0
      IF(L.LT.11) GO TO 11
C     BETA BAR CURVE IS TOO FLAT. SET M = 1.
      AM = 1.0
      GO TO 8
   10 DEL(K) = X3(K)-X1(K)
   12 Y1(K) = X1(K)+F1*DEL(K)
      Y2(K) = X3(K)-F1*DEL(K)
      IF(F(Y1(K))-F(Y2(K))) 30,31,32
   30 DEL(K+1) = Y2(K)-X1(K)
      X1(K+1) = X1(K)
      X3(K+1) = Y2(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   31 DEL(K+1) = Y2(K)-X1(K)
      X1(K+1) = Y1(K)
      X3(K+1) = X3(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   32 DEL(K+1) = X3(K)-Y1(K)
      X1(K+1) = Y1(K)
      X3(K+1) = X3(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   40 Z    = (X1(K)+X3(K))/2.
      FX = F(Z)
      AM = (Q/P)**0.25
      BE = Z*AM
    8 JJ = 1
      IF(AM-1.0) 41,41,42
   41 M(JJ) = 1
      GO TO 49
   42 JJ = JJ+1
      M(JJ) = AM
      GO TO 49
   43 JJ = JJ+1
      M(JJ) = M(JJ-1)+1
      GO TO 49
   49 X1(1) = 0.01
      X2(1) = 4.5
      X3(1) = 5.
```

```
      K = 1
      L = 0
   71 IF(G(X2(K))-G(X3(K))) 72,72,73
   73 X3(K) = X3(K)+0.2*X3(K)
      IF(X3(K).LT.15.) GO TO 71
      L = L+1
      IF(L.LT.20) GO TO 71
      WRITE(6,101)
  101 FORMAT (/5X,'BETA BAR HAS BEEN LOST IN GZ')
      GO TO 4
   72 DEL(K) = X3(K)-X1(K)
   74 Y1(K) = X1(K)+F1*DEL(K)
      Y2(K) = X3(K)-F1*DEL(K)
      IF(G(Y1(K))-G(Y2(K))) 75,76,77
   75 DEL(K+1) = Y2(K)-X1(K)
      X1(K+1) = X1(K)
      X3(K+1) = Y2(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 78
      GO TO 74
   76 DEL(K+1) = Y2(K)-X1(K)
      X1(K+1) = Y1(K)
      X3(K+1) = X3(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 78
      GO TO 74
   77 DEL(K+1) = X3(K)-Y1(K)
      X1(K+1) = Y1(K)
      X3(K+1) = X3(K)
      K = K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 78
      GO TO 74
   78 Z1(JJ)= (X1(K)+X3(K))/2.
      GG(JJ) = G(Z1(JJ))
      IF(JJ.EQ.1) GO TO 51
      IF(JJ.EQ.3) GO TO 44
      GO TO 43
   44 IF ((GG(JJ)-GG(JJ-1)))51,51,52
   51 GZ = GG(JJ)
      Z = Z1(JJ)
      AM = M(JJ)
      GO TO 47
   52 GZ = GG(JJ-1)
      Z =Z1(JJ-1)
      AM = M(JJ-1)
   47 CONTINUE
      BET = Z*AM
      MM = AM
      CMW = 3.14*BET*W3/W2
```

```
      PCR = 3.14*3.14*E*W4**3*GZ/(W2*W2*12.*(1.-PO*PO))
      WRITE (6,102) GZ,ZZZ,MM,BET,CMW,PCR
  102 FORMAT (5X,2F12.3,I5,F8.3,F7.1,E14.7)
      GO TO 4
  999 CONTINUE
      END
```

```
      FUNCTION F(Z)
C     F IS THE KXXP EXPRESSION TREATED M AS CONTINUOUS VARIABLE.
      COMMON/KXXP/ALX,CX,PO,ZZZ,W1,W2,W3,W4
      COMMON/FFF/P,Q
      RHOX = ALX*ALX*W1
      EX = 3.14*3.14*SQRT(1.-PO*PO)*(1.0+CX*ALX)/(2.0*ZZZ)
      A = 1.+RHOX+2.*Z*Z+Z**4
      B = 12.*ZZZ*ZZZ/(3.14**4*(1.-PO*PO))
      C = 1.+W1+2./(1.-PO)*(1.-PO+W1)*Z*Z+Z**4
      P = A+B*EX*EX*W1*(1.+Z*Z)*(1.+Z*Z)/C
      Q = B*(1.-PO*PO+W1)/C
      R = 2.*B*EX*W1*(PO-Z*Z)/C
      F = 2.*SQRT(P*Q)+R
      RETURN
      END
```

```
      FUNCTION G(Z)
C     G IS THE KXXP EXPRESSION TREATED M AS DISCRETE VARIABLE.
      COMMON/KXXP/ALX,CX,PO,ZZZ,W1,W2,W3,W4
      COMMON/GGG/M,JJ
      DIMENSION M(5)
      RHOX = ALX*ALX*W1
      EX = 3.14*3.14*SQRT(1.-PO*PO)*(1.0+CX*ALX)/(2.0*ZZZ)
      A = 1.+RHOX+2.*Z*Z+Z**4
      B = 12.*ZZZ*ZZZ/(3.14**4*(1.-PO*PO))
      C = 1.+W1+2./(1.-PO)*(1.-PO+W1)*Z*Z+Z**4
      P = A+B*EX*EX*W1*(1.+Z*Z)*(1.+Z*Z)/C
      Q = B*(1.-PO*PO+W1)/C
      R = 2.*B*EX*W1*(PO-Z*Z)/C
      G = P*M(JJ)*M(JJ)+Q/(M(JJ)*M(JJ))+R
      RETURN
      END
```

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