Minimum weight shape design for the natural vibration problem of plate and shell structures

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Abstract

In this paper we present a solution to a shape optimization problem involving plate and shell structures subject to natural vibration. The volume is chosen as the response to be minimized under a specified eigenvalue constraint with mode tracking. The designed boundaries are assumed to be movable in the in-plane direction so as to maintain the initial curvatures. The surfaces are discretized by plane elements based on the Mindlin-Reissner plate theory. A non-parametric or a distributed shape optimization problem is formulated and the shape gradient function is theoretically derived using the material derivative method and the Lagrange multiplier method. The traction method, a shape optimization method developed by the authors, is applied to obtain the optimal shape in this problem. The validity of the numerical solution for minimizing the weight of the plate and shell structures is verified through application to fundamental design problems and an actual automotive suspension component.
Keywords: shell, shape optimization, traction method, structural optimization, optimal shape, non-parametric optimisation, natural vibration, eigenvalue.

1 Introduction

Thin structures like shells and folded plates are used as the basic structural components of a wide range of industrial products such as automotive, ship, airplane, architecture and containers, etc. They cover broad areas, support large applied loads, have excellent formability and also contribute to cost and weight reductions. However, their light weight often causes noise and vibration problems. It is important and necessary to improve and optimise the design of plate and shell structures to avoid such problems.

This paper proposes a numerical solution to shape optimization problems with regard to the natural vibration of plate and shell structures, which is one crucial aspect of their structural design. Specifically, we treat a volume minimization problem subject to a specified eigenvalue (i.e., squared natural frequency) constraint with mode tracking. As the design variables that determine shell shapes, one can consider in-plane variables that move in the tangential direction to the surface, and/or out-of-plane variables that move in the normal direction to the surface. In this paper, we will consider only the shape variation in the in-plane direction in order to maintain the curvatures of the initial shape.

Most of the reports in the literature regarding the shape optimization of shell structures have been based on parametric methods in which basis vectors, polynominals, or splines have been introduced to reduce the design variables [1][2]. In contrast, we have developed the traction method, a non-parametric shape optimization method, and applied it to various design problems [3]-[7]. The traction method eliminates the need to use specific parametric functions in advance and also allows smooth boundary shapes to be found without any limitation on the number of design variables. In this paper, we will apply the traction method to the aforementioned vibration design problem of plate and shell structures. The problem will be formulated as a distributed shape optimization problem. The shape sensitivity function (i.e., shape gradient function [8]) will be theoretically derived using the material derivative method and the Lagrange multiplier method. It will be shown that optimal plate and shell shapes can be obtained with the traction method by applying it to fundamental problems and an actual suspension component.

2 Variational equation for natural vibration of plate and shell

Consider that a shell is a set of piecewise plane elements occupying a bounded domain $\Omega \subset \mathbb{R}^3$ as shown in fig. 1 and eqn. (1) below.

$$\Omega = \left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 \mid (x_1, x_2) \in A \subset \mathbb{R}^2, x_3 \in (-h/2, h/2) \right\} \tag{1}$$

where S denotes the boundary of the design domain A and h is the plate thickness. Additionally, it is assumed that the mapping of the local coordinate system $(x_1, x_2, 0)$, which gives the position of the midsurface of the plate, to the global coordinate system (X_1, X_2, X_3), i.e., $\phi : (x_1, x_2, 0) \in \mathbb{R}^2 \mapsto (X_1, X_2, X_3) \in \mathbb{R}^3$, is piecewise smooth. The displacement expressed by the local coordinates, $\boldsymbol{u} = \{u_i\}_{i=1,2,3}$, is considered by dividing it into the displacement in the in-plane direction, $\{u_\alpha\}_{\alpha=1,2}$, and the displacement in the out-of-plane direction, u_3. The subscripts of the Greek letters are expressed as α = 1, 2, and the tensor subscript notation with respect to α = 1, 2 uses a summation convention and a partial differential notation for the spatial coordinates.

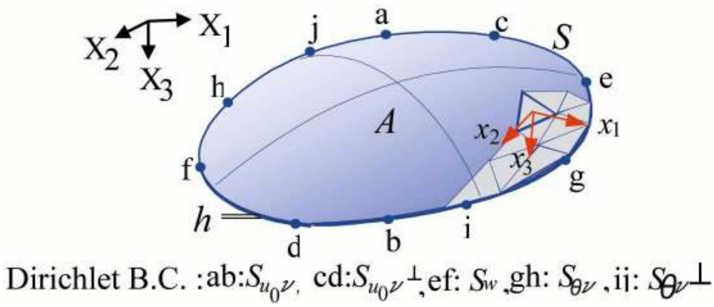

Figure 1: Geometry and boundary conditions for shell consisting of plane elements.

For simplicity, plane elements consisting of a combination of membrane and bending are used in the finite element analysis. The Mindlin-Reissner plate theory is applied as the theory concerning plate bending, and coupling of the membrane stiffness and bending stiffness is ignored. Under these assumptions, the displacements are expressed as follows:

$$u_\alpha(x_1,x_2,x_3) \equiv u_{0\alpha}(x_1,x_2) - x_3\theta_\alpha(x_1,x_2) \quad (2)$$

$$u_3(x_1,x_2,x_3) \equiv w(x_1,x_2) \quad (3)$$

where $\{u_{0\alpha}\}_{\alpha=1,2}$, w and $\{\theta_\alpha\}_{\alpha=1,2}$ express the in-plane displacement, out-of-plane displacement and rotational angle of the midsurface of the plate, respectively.

The boundary conditions shown in fig. 1 for the shell are defined as follows. At the boundary S, the in-plane outward unit normal vector with respect to the boundary is expressed as $\boldsymbol{\nu} = \{\nu_\alpha\}_{\alpha=1,2}$ in relation to the local coordinate system (x_1, x_2) and the unit tangent vector as $\boldsymbol{\nu}^\perp = \{\nu_\alpha^\perp\}_{\alpha=1,2}$. Then, the following assumptions are made: the normal component of the in-plane displacement $\boldsymbol{u}_0$ at the subboundary $S_{u_0\nu} \subset S$ is constrained ($\boldsymbol{u}_0 \cdot \boldsymbol{\nu} = 0$); $\boldsymbol{u}_0 \cdot \boldsymbol{\nu}^\perp = 0$ is given at the subboundary $S_{u_0\nu^\perp} \subset S$; $w = 0$ at the subboundary $S_w \subset S$; $\boldsymbol{\theta}\cdot\boldsymbol{\nu} = 0$ at the subboundary $S_{\theta\nu} \subset S$; and $\boldsymbol{\theta}\cdot\boldsymbol{\nu}^\perp = 0$ at the subboundary $S_{\theta\nu^\perp} \subset S$.

Then, the weak form of the state equation for natural vibration with respect to $(\boldsymbol{u}_0, w, \boldsymbol{\theta}) \in U$ can be expressed as follows:

$$a((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) = \lambda_{(r)} b((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})),\ \forall(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}}) \in U \quad (4)$$

where $(\bar{\bullet})$, $(\bullet)_{(r)}$ and λ express a variation, rth mode, and an eigenvalue, respectively. In addition, the bilinear expressions $a(\bullet,\bullet)$ and $b(\bullet,\bullet)$ are respectively defined as shown below.

$$a((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = \int_A \{c^B_{\alpha\beta\gamma\delta}\theta_{(r)(\gamma,\delta)}\bar{\theta}_{(\alpha,\beta)}$$
$$+kc^S_{\alpha\beta}(w_{(r)},_\beta - \theta_{(r)\beta})(\bar{w},_\alpha - \bar{\theta}_\alpha) + c^M_{\alpha\beta\gamma\delta}u_{0(r)\gamma,\delta}\bar{u}_{0\alpha,\beta}\}dA \quad (5)$$

$$b((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = \rho\int_A \{h(w_{(r)}\bar{w} + u_{0(r)\alpha}\bar{u}_{0\alpha}) + I\theta_{(r)\alpha}\bar{\theta}_\alpha\}dA \quad (6)$$

where $\{c^B_{\alpha\beta\gamma\delta}\}_{\alpha,\beta,\gamma,\delta=1,2}$, $\{c^S_{\alpha\beta}\}_{\alpha,\beta=1,2}$ and $\{c^M_{\alpha\beta\gamma\delta}\}_{\alpha,\beta,\gamma,\delta=1,2}$ express an elastic tensor with respect to bending, transverse shear and membrane stress, respectively. The notations k, ρ and $I(=h^3/12)$ express a shear correction factor, mass density, and a second moment of area, respectively. In addition, $\{\kappa_{\alpha\beta}\}_{\alpha,\beta=1,2}$ expresses the curvature and is defined by the following expression.

$$\kappa_{\alpha\beta} = \frac{1}{2}(\theta_{\alpha,\beta} + \theta_{\beta,\alpha}) \equiv \theta_{(\alpha,\beta)} \quad (7)$$

U in eqn. (4) is given by the following equation.

$$U = \{(u_{01}, u_{02}, w, \theta_1, \theta_2) \in (H^1(A))^5 | u_0 \cdot \nu = 0 \text{ on } S_{u_0\nu},\ u_0 \cdot \nu^\perp = 0 \text{ on } S_{u_0\nu},$$
$$\boldsymbol{\theta}\cdot\boldsymbol{\nu} = 0 \text{ on } S_{\theta\nu},\ \boldsymbol{\theta}\cdot\boldsymbol{\nu}^\perp = 0 \text{ on } S_{\theta\nu^\perp},\ w = 0 \text{ on } S_w\} \quad (8)$$

3 Shape optimization problem for plate and shell structures

As shown in fig. 2, consider that a linear shell having an initial domain A and boundary S undergoes in-plane domain variation (i.e. the design velocity field) $\boldsymbol{V}$ such that its domain and boundary become A_s and S_s. The domain variation at this time can be expressed by a one-parameter family $T_S : \mathbb{R}^2 \mapsto \mathbb{R}^2$, $0 \le s \le \varepsilon$ (ε is a small integer) of the mapping from $(x_1, x_2) \in A$ to $(x_{s1}, x_{s2}) \in A_s$ [4]. The notation s indicates the iteration history of domain variation. In order to maintain the curvatures of the initial shape, shape variation in the in-plane direction is selected as the design variable for determining the shell shapes. Moreover, it is assumed that the plate thickness does not vary.

A distributed shape optimization problem is expressed as shown below, subject to the constraint eigenvalue $\hat{\lambda}_{(r)}$ of the specified rth order natural mode and the state equation (eqn. (4)) and with volume M as the objective functional.

$$\text{find} \quad A \text{ (or } \boldsymbol{V}) \quad (9)$$

$$\text{that minimize} \quad M(=\int_A h dA) \quad (10)$$

$$\text{subject to} \quad Eq.(4), \quad (11)$$

$$\lambda_{(r)} = \hat{\lambda}_{(r)} \tag{12}$$

For mode tracking of the specified rth natural mode of the initial shape, we use the following parameter t_s. The mode with a maximum value of t_s in all natural modes is regarded as the corresponding mode and is traced.

$$t_s = \phi_{0(r)}^T m_s \phi_s \tag{13}$$

where $\phi_{0(r)}^T$, ϕ_s and m_s indicate the transposed vector of the constrained rth mode, the normalized ith eigenvector consisting of $(\boldsymbol{u}_{0(i)}, w_{(i)}, \boldsymbol{\theta}_{(i)})$ and the mass matrix of the sth iteration, respectively.

Letting $(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})$ and Λ denote the Lagrange multipliers for the state equation and the eigenvalue constraint, respectively, the Lagrangian functional L associated with this problem can be expressed as

$$L((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}}), \Lambda) = M$$

$$+\lambda_{(r)} b((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) - a((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) + \Lambda(\lambda_{(r)} - \hat{\lambda}_{(r)}) \tag{14}$$

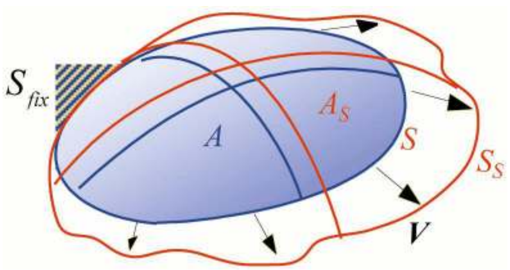

Figure 2: In-plane domain variation $\boldsymbol{V}$.

Using the design velocity field $\boldsymbol{V}(x)(= \partial T_s(x)/\partial s,\ x \in A_s)$ to represent the amount of domain variation, the derivative $\dot{L}$ of the domain variation of the Lagrangian functional L can be expressed as

$$\begin{aligned}\dot{L} = &-a((\boldsymbol{u}'_{0(r)}, w'_{(r)}, \boldsymbol{\theta}'_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) + \lambda_{(r)} b((\boldsymbol{u}'_{0(r)}, w'_{(r)}, \boldsymbol{\theta}'_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) \\ &- a((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}'_0, \overline{w}', \overline{\boldsymbol{\theta}}')) + \lambda_{(r)} b((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}'_0, \overline{w}', \overline{\boldsymbol{\theta}}')) \\ &+ \dot{\lambda}_{(r)} \{b((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) - \Lambda\} + \dot{\Lambda}(\lambda_{(r)} - \hat{\lambda}_{(r)}) + \int_S G v_i V_i ds,\ \boldsymbol{V} \in C_\Theta\end{aligned} \tag{15}$$

$$\begin{aligned}\boldsymbol{G} = [1 - \Lambda\{(-c^B_{\alpha\beta\gamma\delta} \theta_{(r)(\gamma,\delta)} \overline{\theta}_{(\alpha,\beta)} - kc^S_{\alpha\beta}(w_{(r),\beta} - \theta_{(r)\beta})(\overline{w}_{,\alpha} - \overline{\theta}_\alpha) - c^M_{\alpha\beta\gamma\delta} u_{0(r)\gamma,\delta} \overline{u}_{0\alpha,\beta} \\ + \lambda_{(r)} \rho \{h(w_{(r)} \overline{w} + u_{0(r)\alpha} \overline{u}_{0\alpha}) + I \theta_{(r)\alpha} \overline{\theta}_\alpha\}\}]\boldsymbol{\nu} \quad \text{on} \quad S_{design} \equiv S \setminus S_{fix}\end{aligned} \tag{16}$$

where $(\dot{\bullet})$ and $(\bullet)'$ indicate the material derivative and shape derivative, respectively [8]. $\boldsymbol{\nu}$ is an outward unit normal vector, and C_Θ is the suitably smooth function space that satisfies the constraints of domain variation.

The optimality conditions of the Lagrangian functional L with respect to $(\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)})$, $(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})$ and Λ are expressed as shown below.

$$a((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}'_0, \overline{w}', \overline{\boldsymbol{\theta}}')) = \lambda_{(r)} b((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}'_0, \overline{w}', \overline{\boldsymbol{\theta}}')),\ \forall (\overline{\boldsymbol{u}}'_0, \overline{w}', \overline{\boldsymbol{\theta}}') \in U \tag{17}$$

$$a((\boldsymbol{u}'_{0(r)}, w'_{(r)}, \boldsymbol{\theta}'_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) = \lambda_{(r)} b((\boldsymbol{u}'_{0(r)}, w'_{(r)}, \boldsymbol{\theta}'_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})), \forall (\boldsymbol{u}'_{0(r)}, w'_{(r)}, \boldsymbol{\theta}'_{(r)}) \in U \tag{18}$$

$$b((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) = \Lambda, \qquad \dot{\Lambda}(\lambda_{(r)} - \hat{\lambda}_{(r)}) = 0 \qquad (19)\ (20)$$

The following quasi self-adjoint relationship

$$(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}}) = \frac{\Lambda(\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)})}{b((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}))} = \Lambda(\tilde{\boldsymbol{u}}_{0(r)}, \tilde{w}_{(r)}, \tilde{\boldsymbol{\theta}}_{(r)}) \tag{21}$$

holds true between the state equation (17) and the adjoint equation (18). Eqns. (19) and (20) are the governing equations of Λ with respect to the constraint. $(\tilde{\boldsymbol{u}}_{0(r)}, \tilde{w}_{(r)}, \tilde{\boldsymbol{\theta}}_{(r)})$ expresses the natural mode normalized by equation (21).

By substituting $(\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)})$, $(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})$ and Λ determined by these optimality conditions for eqn. (15), the material derivative $\dot{L}$ can be expressed as the dot product of the shape gradient function $\boldsymbol{G}$ and the design velocity field $\boldsymbol{V}$ as shown in the following equation.

$$\dot{L} = l_G(\boldsymbol{V}) \equiv \int_{S_s} G_i V_i \, d\Gamma \tag{22}$$

$$\boldsymbol{G} = [1 - \Lambda\{(-c^B_{\alpha\beta\gamma\delta}\theta_{(\mathrm{r})_{(\gamma,\delta)}}\overline{\theta}_{(\alpha,\beta)} - kc^S_{\alpha\beta}(w_{(r),\beta} - \theta_{(r)\beta})(\overline{w}_{,\alpha} - \overline{\theta}_\alpha) - c^M_{\alpha\beta\gamma\delta}u_{0(r)\gamma,\delta}\overline{u}_{0\alpha,\beta}$$
$$+ \lambda_{(r)}\rho\{h(w_{(r)}\overline{w} + u_{0(r)\alpha}\overline{u}_{0\alpha}) + I\theta_{(r)\alpha}\overline{\theta}_\alpha\}\}]\boldsymbol{n} \quad \text{on} \quad S_{design} \equiv S \setminus S_{fix} \tag{23}$$

Since the shape gradient function $\boldsymbol{G}$ has been derived, the traction method [4][5] can be applied.

4 Numerical solution to optimal domain variation using the traction method

The traction method is based on the gradient method in Hilbert space [4][5]. With the traction method, a negative shape gradient function $-\boldsymbol{G}$ (traction force in the normal direction), based on eqn. (23), acts as an external force on a design boundary that allows shape variation under the shape constraint condition. The resultant displacement field represents the amount of domain variation (i.e., design velocity field $\boldsymbol{V}$). The governing equation (referred to as a velocity analysis) for applying the traction method to the plate and shell structures considered here is given as the expression noted below. In order to maintain the curvatures of the initial shape, we constrain the variation in the normal direction to the shell surface and find the amount of variation in the in-plane direction. Equation (24) can be solved by a finite element method.

$$a(\boldsymbol{V}, \boldsymbol{y}_0) = -l_G(\boldsymbol{y}_0) \quad \forall \boldsymbol{y}_0 \in C_\Theta \tag{24}$$

With this approach, the eigenvalue analysis (eqn. (17) or eqn. (18)) that yields the objective functional, constraint conditions and shape gradient function and the velocity analysis (eqn. (24)) that yields the amount of shape variation are alternately repeated to update the domain. As a result, the objective functional is reduced and a smooth converged shape is obtained in the end [4][5].

5 Results of numerical analysis

The proposed method was applied to two fundamental design problems and an automotive suspension component in order to confirm its validity and practical utility for application to plate and shell structures.

5.1 Quadratic shallow shell

The initial shape of a quadratic shallow shell cut from a spherical surface is shown in fig. 3-(a) along with the boundary conditions of the eigenvalue analysis. The constraint conditions for the velocity analysis are shown in fig. 3-(b). In the eigenvalue analysis, the four corners were clamped. In the velocity analysis, the four corners were clamped and all the remaining boundaries were treated as the design boundaries. The 4th natural mode of the initial shape was set as the constraint and traced, having the same value as that of the initial shape. Figure 4 shows the initial shape and the final shape obtained. The domains having low sensitivity boundaries were largely contracted in the in-plane direction. The optimal shape with the smooth boundaries was obtained while maintaining the curvatures of the initial shape. Iteration convergence histories of the volume and eigenvalue are shown in fig. 5. The volume decreased by about 10%, satisfying the constraint before finally converging.

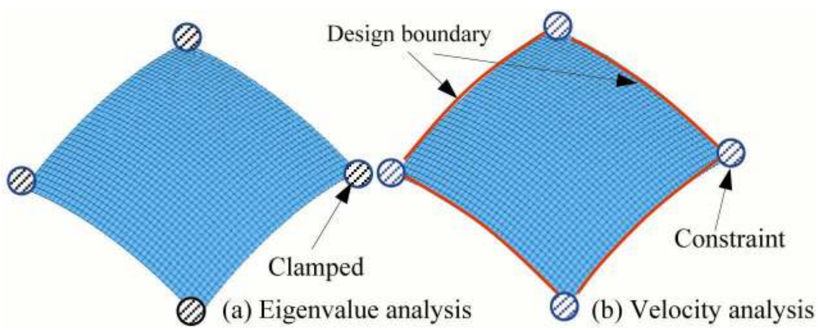

Figure 3: Boundary conditions for optimization of quadratic shallow shell.

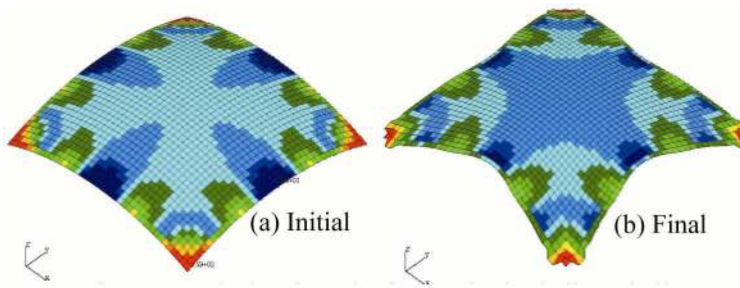

Figure 4: Calculated results for quadratic shallow shell.

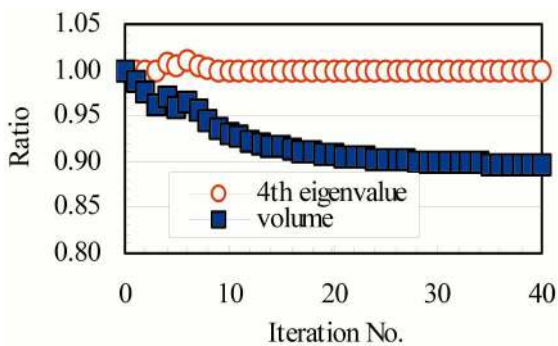

Figure 5: Iteration histories of quadratic shallow shell.

5.2 Curly plate

The shape of a curly plate, which is used as a plate spring, clamped at one side and with non-structural lumped masses at the other side was optimized. The initial 1st eigenvalue and natural mode were given as a constraint value and the specified mode. The initial shape and the final shape that was obtained are shown in fig. 6. Iteration histories of the constraint value and volume are shown in fig. 7. The results confirm that the final shape was approximately 17% lighter in weight while still satisfying the constraint condition and maintaining the smoothness and the curvatures of the initial shape.

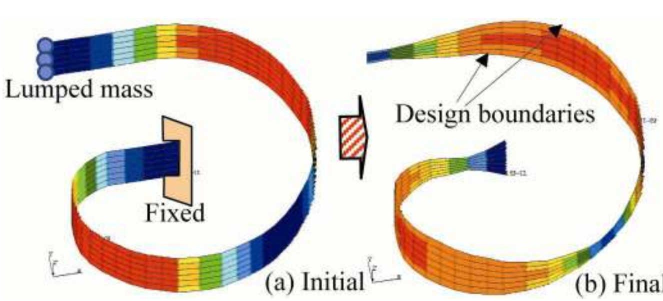

Figure 6: Optimization of curly plate.

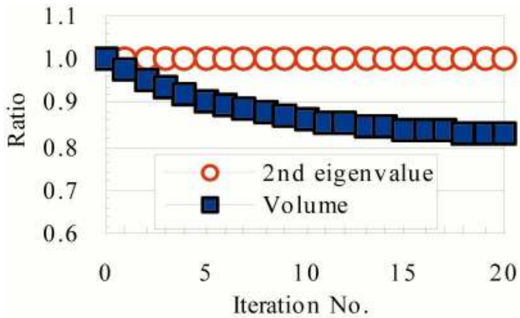

Figure 7: Iteration histories of curly plate.

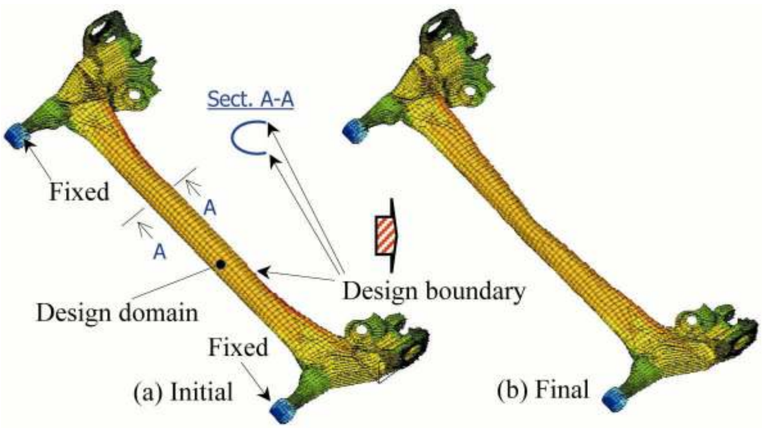

Figure 8: Optimization of automotive rear suspension component.

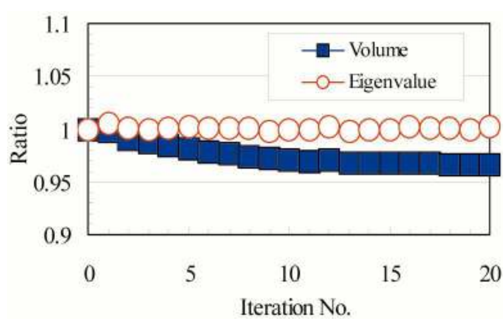

Figure 9: Iteration histories of rear suspension component.

5.3 Application to a suspension component

Figure 8-(a) shows the initial shape and design domain of a rear suspension component. The torsion beam with an open section connecting the right and left sides of the suspension was treated as the design domain. The design boundary was set at the edges of the open section. The constraint value was set at 1.0 times the 1st eigenvalue of the initial shape, and the 1st natural mode of the

initial shape was traced. The final shape that was obtained is shown in fig. 8-(b). The center portion having low sensitivity was obtained as a shape contracted in the in-plane direction. Iteration histories of the volume and 1st eigenvalue are shown in fig. 9. The results confirm that the final shape was approximately 4% lighter in weight while still satisfying the given constraint condition and maintaining the smoothness and the curvatures of the initial shape.

6 Conclusions

This paper has proposed a numerical solution based on the traction method to the issue of natural vibration in shape optimization problems for plate and shell structures, under the assumption that the curvatures of the initial shape are maintained. A volume minimization problem subject to an eigenvalue constraint was formulated, and the shape gradient function was theoretically derived. To validate the proposed method, it was applied to two typical design problems and an automotive component. The smooth optimal shapes and optimization histories were obtained maintaining the initial curvatures as intended. The results verified the validity and practical utility of the proposed method.

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