Mining graph evolution rules

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Abstract. In this paper we introduce *graph-evolution rules*, a novel type of frequency-based pattern that describe the evolution of large networks over time, at a local level. Given a sequence of snapshots of an evolving graph, we aim at discovering rules describing the local changes occurring in it. Adopting a definition of support based on *minimum image* we study the problem of extracting patterns whose frequency is larger than a minimum support threshold. Then, similar to the classical association rules framework, we derive graph-evolution rules from frequent patterns that satisfy a given minimum confidence constraint. We discuss merits and limits of alternative definitions of support and confidence, justifying the chosen framework. To evaluate our approach we devise *GERM* (Graph Evolution Rule Miner), an algorithm to mine all graph-evolution rules whose support and confidence are greater than given thresholds. The algorithm is applied to analyze four large real-world networks (i.e., two social networks, and two co-authorship networks from bibliographic data), using different time granularities. Our extensive experimentation confirms the feasibility and utility of the presented approach. It further shows that different kinds of networks exhibit different evolution rules, suggesting the usage of these local patterns to globally discriminate different kind of networks.

1 Introduction

With the increasing availability of large social-network data, the study of the temporal evolution of graphs is receiving a growing attention. While most research so far has been devoted to analyze the change of global properties of evolving networks, such as the diameter or the clustering coefficient, not much work has been done to study graph evolution at a microscopic level. In this paper, we consider the problem of searching for patterns that indicate local, structural changes in dynamic graphs. Mining for such local patterns is a computationally challenging task that can provide further insight into the increasing amount of evolving-network data.

Following a frequent pattern-mining approach, we introduce the problem of extracting *Graph Evolution Rules* (*GER*), which are rules that satisfy given constraints of minimum support and confidence in evolving graphs. An example

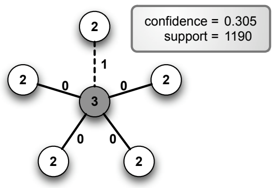


Fig. 1. A Graph Evolution Rule extracted from the DBLP co-authorship network.

of a real *GER* extracted form the DBLP co-authorship network is given in Fig. 1: nodes are authors, with an edge between two nodes if they co-authored a paper.

In this specific example, the node labels represent a class of degree of the node: the higher the label the higher the degree of the node. It is important to note that the label refers to the degree of the node in the input graph, not in the rule. In particular the label 3 indicates a node with degree > 50. In general, node labels may represent any property of a node. The labels on the edges instead are more important as they represent the (relative) year in which the first collaboration between two authors was established. Intuitively (later we provide all the needed definitions) the rule might be read as a sort of local evidence of *preferential attachment*, as it shows a researcher with a large degree (label 3) that at time t is connected to four medium degree researchers (labels 2), and that at time $t+1$ will be connected to another medium degree researcher. The definition, extraction and subsequent empirical analysis of such *Graph Evolution Rules* (*GER*) constitute the main body of our work.

The remainder of the paper is organized as follows: Section 2 describes the problem under investigation and defines the novel kind of pattern we are interested in. In Section 3 we describe the details of our algorithm. We report on our experimental results in Section 4 and present related work in Section 5. Finally, in Section 6 we discuss possible future research directions and in Section 7 we provide our conclusions.

2 Patterns of graph evolution

2.1 Time-evolving graphs

We start by describing how we *conceptually* represent an evolving graph, and subsequently discuss how to *actually* represent the graph in a more compact format. As usual the terminology $G = (V, E, \lambda)$ is used to denote a graph G over a set of nodes V and edges $E \subseteq V \times V$, with a labeling function $\lambda : V \cup E \mapsto \Sigma$, assigning to nodes and edges labels from an alphabet Σ. These labels represent properties, and for simplicity we assume that they do not change with time. As an example, in a social network where nodes model its members, node properties may be *gender*, *country*, *college*, etc., while an edge property can be the kind of connection between two users. The evolution of the graph over time

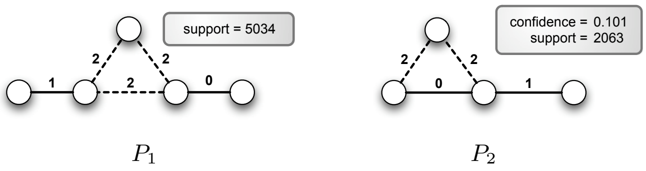


Fig. 2. Relative time patterns extracted from two different samples of the DBLP co-authorship network: respectively 1992-2002 for (P_1), and 2005-2007 (P_2). Dataset details are given in Sec. 4.1.

is conceptually represented by a series of undirected graphs $G_1, \dots, G_T$, so that $G_t = (V_t, E_t)$ represents the graph at time t. Since $G_1, \dots, G_T$ represent different snapshots of the same graph, we have $V_t \subseteq V$ and $E_t \subseteq E$. For simplicity of presentation, we assume that as the graph evolves, nodes and edges are only added and never deleted: i.e., $V_1 \subseteq V_2 \subseteq \dots V_T$ and $E_1 \subseteq E_2 \subseteq \dots E_T$.

It is worth noting that the number of edge deletions in social networks is so small to be negligible when analyzing the temporal evolution of networks. However, in our framework we can handle also deletions by slightly changing the matching operator as described in Section 6.

Our mining algorithm represents the dataset by simply collapsing all the snapshots $G_1, \dots, G_T$ in one undirected graph G, in which edges are *time-stamped* with their first appearance. Thus, we have $G = (V, E)$ with $V = \bigcup_{t=1}^{T} V_t = V_T$ and $E = \bigcup_{t=1}^{T} E_t = E_T$. To each edge $e = (u, v)$ a time-stamp $t(e) = \arg\min_j \{E_j \mid e \in E_j\}$ is assigned. Note that time-stamps on the nodes may be ignored as a node always comes with its first edge and hence this information is implicitly kept in edge time-stamps. Overall, a time-evolving graph is described as $G = (V, E, t, \lambda)$, with t assigning time-stamps to the set of edges E.

2.2 Patterns

Consider a time-evolving graph G, as defined above. Intuitively a pattern P of G is a subgraph of G that in addition to matching edges of G also matches their time-stamps, and if present, the properties on the nodes and edges of G.

Definition 1 (Absolute-time pattern).
Let $G = (V, E, t, \lambda)$ and $P = (V_P, E_P, t_P, \lambda_P)$ be graphs, where G is the time-evolving dataset and P a pattern. We assume that P is connected. An occurrence *of P in G is a function $\varphi : V_P \mapsto V$ mapping the nodes of P to the nodes of G such that for all $u, v \in V_P$:*

$$\begin{aligned} &i)\ (u,v) \in E_P \Rightarrow (\varphi(u), \varphi(v)) \in E, \\ &ii)\ (u,v) \in E_P \Rightarrow t(\varphi(u), \varphi(v)) = t(u,v), \text{ and} \\ &iii)\ \lambda_P(v) = \lambda(\varphi(v)) \wedge \lambda_P((u,v)) = \lambda((\varphi(u), \varphi(v))) \end{aligned}$$

In case no labels are present for edges or nodes, the last condition (*iii*) is ignored. Two examples of patterns from the DBLP co-authorship network are shown in Fig. 2. Those examples motivate us to make two important decisions. First, since our goal in this paper is to study patterns of evolution we naturally focus on patterns that refer to more than one snapshots such as the examples in Fig. 2. In other terms we are not interested in patterns where all edges have the same time-stamp. The second decision is based on the following observation. Consider pattern P_1: arguably, the essence of the pattern is the fact that two distinct pairs of connected authors, one collaboration created at time 0, and one at time 1, are later (at time 2) connected by a collaboration involving one author from each pair, plus a third author. We would like to account for an occurrence of that event even if it was taking place at times, say, 16, 17 and 18. To capture this intuition we define *relative-time patterns*.

Definition 2 (Relative-time Pattern). *Let G and P be a graph and pattern as in Definition 1. We say that P* occurs *in G* at relative time *if there exists a $\Delta \in \mathbb{R}$ and a function $\varphi : V_P \mapsto V$ mapping the nodes of P to the nodes in G such that $\forall u, v \in V_P$*

$$\begin{aligned} &i)\ (u,v) \in E_P \Rightarrow (\varphi(u), \varphi(v)) \in E, \\ &ii)\ (u,v) \in E_P \Rightarrow t(\varphi(u), \varphi(v)) = t(u,v) + \Delta, \text{ and} \\ &iii)\ \lambda_P(v) = \lambda(\varphi(v)) \wedge \lambda_P((u,v)) = \lambda((\varphi(u), \varphi(v))) \end{aligned}$$

The difference between Definitions 1 and 2 is only in the second condition. As a result of Definition 2, we obtain naturally forming equivalence classes of structurally isomorphic relative time patterns that differ only by a constant on their edge time-stamps. To avoid the resulting redundancies in the search space of all relative time patterns we only pick one representative pattern for each equivalence class, namely *the one where the lowest time-stamp is zero.*

In the remainder of this paper we focus on relative time patterns, as they subsume the absolute time case: they are both more interesting and more challenging to mine.

2.3 Support

Next we discuss the support measure we use. Let $\mathcal{G}_\Sigma$ be the set of all graphs over an alphabet Σ. We define support as a function $\sigma : \mathcal{G}_\Sigma \times \mathcal{G}_\Sigma \mapsto \mathbb{N}$. Given a host-graph G and a pattern P, the value of $\sigma(P, G)$ reflects the support of the pattern in the host-graph.

Defining a concept of support for the single graph setting is a non-trivial task, which has received attention recently [14, 8, 4, 5]. The most important property that a definition of support must satisfy is anti-monotonicity, that is, for all graphs G, P and P', where P is a subgraph of P', it must hold that $\sigma(P, G) \geq \sigma(P', G)$. This property is exploited by pattern miners to prune the search space. Anti-monotonicity holds trivially in the transactional setting, but is more tricky for the single-graph setting. For instance, while the total number of occurrences

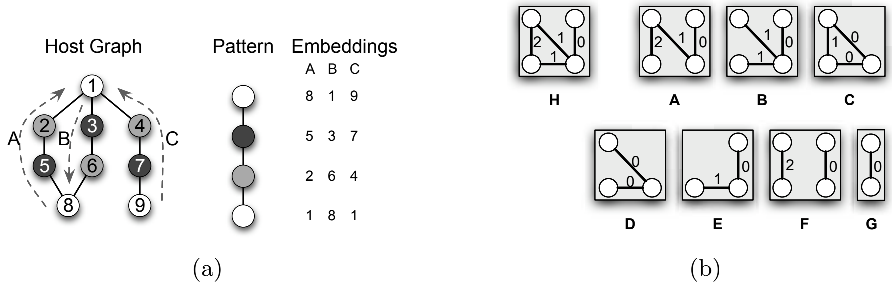


Fig. 3. (a): a graph with three different occurrences of a pattern evaluates to $\sigma = 2$. (b): a graph H with relative edge labels and all possible relative subgraphs A, B, C, D, E, F, G.

of a pattern is intuitively a meaningful measure, it is not anti-monotonic. As an example consider Fig. 4(b): the number of occurrences in the host graph Y of the pattern indicated as "body" is 1, while the number of occurrences of its supergraph indicated as "head" is 2, thus violating anti-monotonicity.

A first feasible support measure was proposed in [14] followed by a refinement published in [8]. Both measures rely on solving a maximum independent set problem *MIS* which is NP-complete. We employ the *minimum image based support* measure recently introduced in [4] which does not require solving a *MIS*. This measure is based on the number of unique nodes in the graph $G = (V_G, E_G)$ that a node of the pattern $P = (V_P, E_P)$ is mapped to, and defined as follows:

Definition 3 (Support).

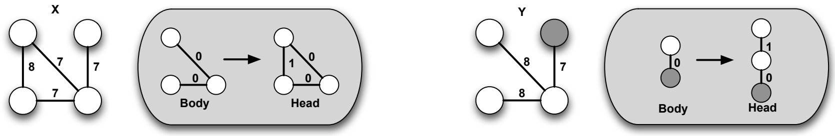
$$\sigma(P, G) = \min_{v \in V_P} \mid \{\varphi_i(v) : \varphi_i \textit{ is an occurrence of } P \textit{ in } G\} \mid$$

By taking the node in P as reference which is mapped to the least number of unique nodes in G, the anti-monotonicity of the measure is ensured. An example of minimum image based support is given in Fig. 3(a). Even if the pattern has 3 occurrences in the host graph, it has support $\sigma = 2$. In fact the lower white node of the pattern can only be mapped to nodes 1 and 8 in the host graph.

The advantage of this definition over other definitions introduced [14, 8] is twofold. From a practical point of view it is computationally easier to calculate since it does not require the computation of all possible occurrences of a pattern-graph in the host-graph. Additionally it does not require to solve a maximal independent set problem for each candidate pattern. From a theoretical perspective we know that this definition is an upper bound for the according overlap based definitions [4, 8]. Hence the support according to this definition is closer to the real number of occurrences in the graph.

2.4 Rules and Confidence Measure

The support of a pattern can provide insight into how often such an event may happen compared to other specific changes, but not how likely is a certain se-



measure	head	body	confidence
minimum image support	1	1	1.00
number of occurrences	2	6	0.33

(a)

measure	head	body	confidence
minimum image support	1	1	1.00
number of occurrences	2	1	2.00

(b)

Fig. 4. Two example host-graphs X and Y illustrating different problems with support and confidence notions.

quence of steps. To acquire this information we need to decompose a pattern into the particular steps and subsequently determine the *confidence* for each transition. Each step can be considered as a rule *body*$\rightarrow$*head* with both *body* and *head* being patterns as defined in the previous section. Unfortunately, this does not yet solve our problem, but rather introduces two important questions:

1. How to decompose a pattern into *head* and *body*?
2. What are reasonable definitions of confidence?

Regarding the decomposition consider pattern H in Fig. 3(b). An occurrence of H implies an occurrence of all its sub-patterns $A{-}G$. Similarly to the definition of association rules all $A - G$ can be considered candidate-*body* in order to form a graph evolution rule with pattern H as *head*. Fortunately, most of those possibilities can be discarded immediately. First, we are interested in evolution and hence only care about rules describing edges emerging in the future. This allows us to discard bodies A, C, D, E, and F thus only leaving B and G. Furthermore, the step should be as small as possible to allow for a high granularity wherefore we would drop candidate-body G in the example, leaving B as body for the head H. Following the same reasoning, G would be the only choice as body for B as head. Similar the other rules in the example are $E \rightarrow A, D \rightarrow C, G \rightarrow E$. The natural body thus would be the head discarding all the edges from the last time-step of the target-pattern. More formally:

Definition 4 (Graph Evolution Rule). *Given a pattern head P_H the body P_B is defined as: $E_B = \{e \in E_H \mid t(e) < \max_{e^* \in E_H}(t(e^*))\}$ and $V_B = \{v \in V_H \mid deg(v, E_B) > 0\}$, where $deg(v, E_B)$ denotes the degree of v with respect to the edges in E_B. Moreover we constrain P_B to be* connected. *Finally, the support of a graph evolution rule is the support of its head.*

This definition yields a unique body for each head and therefore a unique confidence value for each head. This allows us to represent the rules by the head only. Note that the definition disallows disconnected graphs as body due to the lack of a support-definition for disconnected graphs. As a consequence *not all frequent patterns can be decomposed into graph evolution rules.*

Consider for instance pattern P_1 in Fig. 2: after removing all edges with the highest time-stamp, and discarding disconnected nodes, the graph that remains still contains two disconnected components (the one-edge component with label 1, and the one with label 0). Since the support is not defined for such disconnected pattern, P_1 can not be decomposed to be a *GER*. On the other hand, P_2 can be decomposed: in fact after removing all edges with the maximum time-stamp, and subsequently the disconnected node, we obtain a connected graph that will become the body of the rule for which P_2 is the head. Note that a *GER* can be represented in two different ways: either explicitly as two patterns ($body \rightarrow head$), or implicitly by representing only the head as P_2 in Fig. 2. This is possible since there is a unique body for each head.

Finally, we have to choose a reasonable definition of *confidence* of a rule. Following the classic association rules framework, a first choice is to adopt the ratio of head and body supports as confidence. With the support being anti-monotonic this yields a confidence value which is guaranteed to be between zero and one. However, Fig. 4(a) shows that this definition may in some cases lack a reasonable semantic interpretation. In the upper host-graph X we find three possible ways to close a triangle given the edges from time-stamp 7. The confidence of 1 suggests that all of these will close to form triangles, while the graph shows that only one actually does. To overcome this counterintuitive result, we investigated if the ratio of number of occurrences of head and body can be employed to solve this issue. While this definition of confidence allows for more reasonable semantics for the case in Fig. 4(a), it has the clear disadvantage that, due to the lack of anti-monotonicity, it may yield confidence values larger than 1, as in Fig. 4(b). In our experiments we compare the two alternative definitions showing that the minimum-image-based support is an effective and useful concept, while the occurrence-based definition has unpredictable behavior. Moreover, while the support is already available as it is computed for extracting the frequent patterns, the occurrence based confidence needs a separate and costly computation.

3 Mining graph evolution rules

GERM is an adaptation of the algorithm in [4], which was devised to prove the feasibility of the minimum image based support measure, and which in turn, was an adaptation of gSpan [22]. Thus, *GERM* inherits the main characteristics from those algorithms. In particular, *GERM* is based on a DFS traversal of the search space, which leads to very low memory requirements. Indeed, in all the experiments that we performed the memory consumption was negligible.

We next describe in detail how to adapt gSpan to *GERM* whereas the main changes are in the *SubgraphMining* method shown as Algorithm 1. The first key point is that we mine patterns in large single graphs, while gSpan was developed to extract patterns from sets of graphs. The part most involved in adapting gSpan is the *support computation* in line 7. Thus we start from the implementation of [4], where gSpan support calculation is replaced by the minimum image based support computation, without the need for changing the core of the algorithm.

One of the key elements in gSpan is the use of the *minimum DFS code*, which is a canonical form introduced to avoid multiple generations of the same pattern.

Algorithm 1 $SubgraphMining(\mathbb{GS}, \mathbb{S}, s)$

1: **if** $s \neq min(s)$ **then** return *// using our canonical form*
2: $\mathbb{S} \leftarrow \mathbb{S} \cup s$
3: generate all s' potential children with one edge growth
4: Enumerate(s)
5: **for all** c, c is s' child **do**
6:     *// using definition 3 based on definition 1 or definition 2*
7:     **if** support(c)$\geq$ minSupp **then**
8:         $s \leftarrow c$
9:         $SubgraphMining(\mathbb{GS}, \mathbb{S}, s)$

We need to change this canonical form in order to enable *GERM* to mine patterns with relative time-stamps (cf. line 1). As explained after Definition 2, we only want one representative pattern per equivalence class, namely the one with the lowest time-stamp being zero. This is achieved by modifying the canonical form such that the first edge in the canonical form is always the one with the lowest time-stamp, as compared to gSpan where the highest label is used as a starting node for the canonical form. Any pattern grown from such a pattern by extending the canonical form will have the same lowest time-stamp, which we set to zero by a simple constraint on the first edge. Hence we guarantee to extract only one pattern per equivalence class which dramatically increases performance and eliminates redundancy in the output.

Note that when matching a pattern to the host-graph we implicitly fix a value of Δ, representing the time gap between the pattern and the host graph. In order to complete the match all remaining edges must adhere to this value of Δ. If all the edges match with the Δ set when matching the first edge, the pattern is discovered to match the host-graph with that value of Δ.

Another important issue is to be able to deal with large real-world graphs, in which several nodes have high degree (the degree distribution in our datasets follows a power law). In typical applications of frequent-subgraph mining in the transactional setting, such as biology and chemistry, the graphs are typically of small size and they are not high-degree nodes. Dealing with large graphs and high degrees give rise to increased computational complexity of the search. In particular, having nodes with large degree increases the possible combinations that have to be evaluated for each subgraph-isomorphism test. We thus equip *GERM* with a user-defined constraint specifying the maximum number of edges in a pattern. This constraint allows to deal more efficiently with the DFS strategy by reducing the search space. Our experiments confirm that the total running time is much more influenced by the maximum-edge constraint than by the minimum support threshold.

Table 1. Dataset statistics: Number of nodes and edges and resulting average degree for the total graph as well as for the largest connected component (LCC) out of all connected components (CC). Further the growth rate in terms of edges: total growth as ratio between the graph size at the final and the initial time-stamps, and average growth rate per time-stamp.

							LCC			Growth Rates	
Dataset	**Date**	**|V|**	**|E|**	**avg deg**	**T**	**#CC**	**|V|**	**|E|**	**avg deg**	**total**	**avg**
flickr-month	03-05	147	241	1.64	24	16	74	182	2.43	60347	2.832
flickr-week	02-05	149	246	1.64	76	16	76	186	2.45	246331	0.241
y!360-month	04-05	177	205	1.16	10	17	110	155	1.40	68470	5.150
y!360-week	04-05	177	205	1.16	41	17	110	155	1.40	68470	0.834
arxiv92-01	92-01	709	289	4.08	10	6	49	260	5.32	803	1.691
dblp92-02	92-02	129	277	2.15	11	13	83	220	2.63	25	0.408
dblp03-05	03-05	109	233	2.15	3	14	53	153	2.88	3	0.871
dblp05-07	05-07	135	290	2.15	3	16	72	201	2.76	3	0.749
		$\times 1000$	$\times 1000$			$\times 1000$	$\times 1000$	$\times 1000$			

4 Experimental Results

In this section, we report our experimental analysis. The *GERM* algorithm was implemented in C++. All the experiments were conducted on a Linux cluster equipped with 8 Intel Xeon processors at 1.8Ghz and 16Gb of RAM.

4.1 Datasets

We conducted experiments on four real-world datasets: two social networks (Flickr and Y!360) and two bibliographic networks (DBLP and arXiv). Table 1 reports statistics on the resulting graphs.

Flickr (`http://www.flickr.com/`): Flickr is a popular photo-sharing portal. We sampled a set of Flickr users with edges representing mutual friendship and edge time-stamp the moment when the bidirectional contact was established. We generated one graph with monthly and one with weekly granularity.

Y!360 (`http://360.yahoo.com/`): Yahoo! 360° is an online service for blogging. Again we sampled a set of users and proceed as in the Fickr dataset. In this case the monthly and weekly datasets contain exactly the same time period.

DBLP (`http://www.informatik.uni-trier.de/~ley/db/`): This dataset is based on a recent snapshot of DBLP, which has yearly time granularity. Vertices represent authors and edges represent the co-authorship relation. The edge time stamps represent the year of the first co-authorship. Three different graph snapshots were extracted, for three different years.

arXiv (`http://arxiv.org/`): Another co-authorship graph dataset, extracted from the arXiv repository. The resulting graphs *arxiv92-01* contain co-authorship relations that emerged in the years 1992 to 2001 with a yearly time granularity.

As discussed in Section 2, our framework allows to have vertex and edge labels that represent additional information. We experiment with node labels

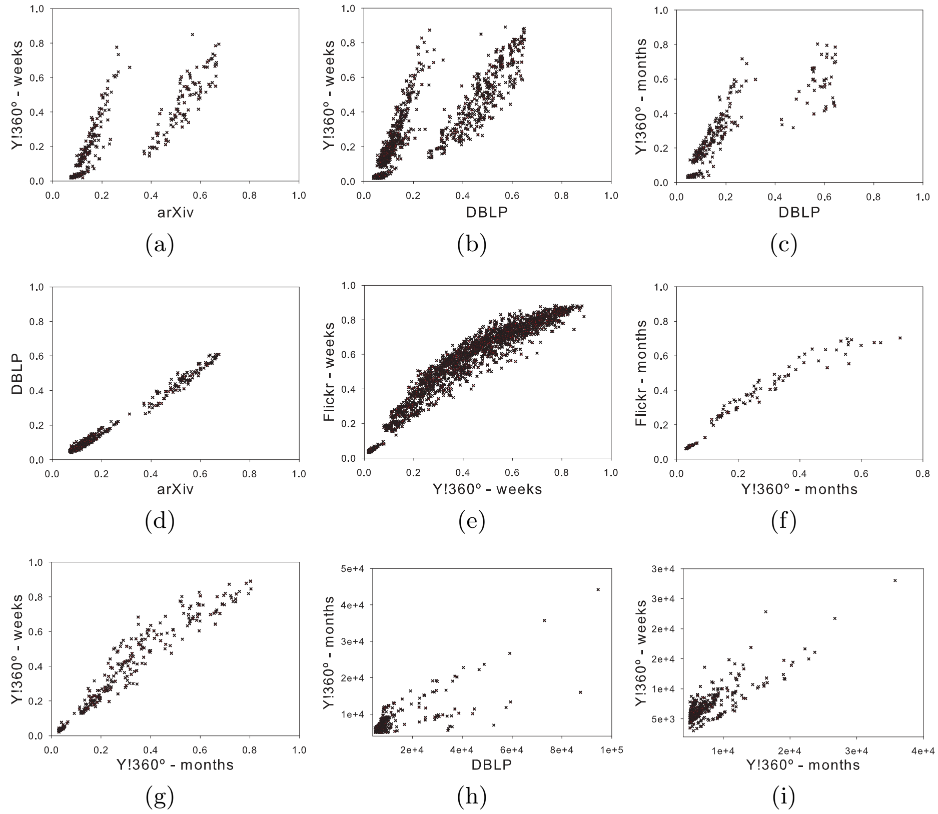


Fig. 5. (a)–(g): comparison of confidence of graph evolution rules in different networks. (h) and (i): comparison of support of patterns in different networks.

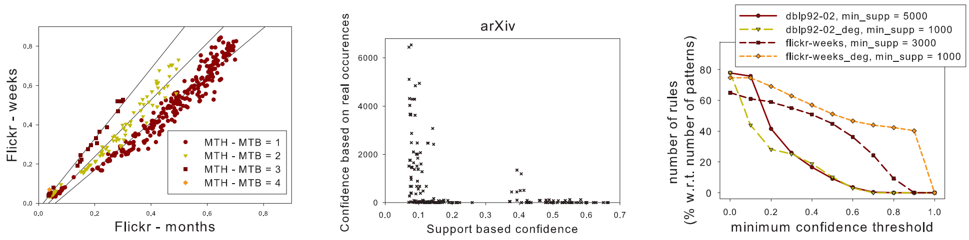
that are based on two graph-theoretic measures: the *degree* and the *closeness centrality*. These measures change as the graph evolves. To obtain static labels the measures are computed on the whole graph, corresponding to the last time stamp and then they are discretized in 5 bins.

4.2 Results

We analyze the experimental results with regard to the following questions:

Q1 Do the extracted patterns and rules characterize the studied network?
Q2 Do different time granularities influence the confidence of the rules?
Q3 How do the different confidence definitions compare?
Q4 How do the parameters and the type of dataset influence the number of derivable rules, the number of patterns obtained, and the running time?

Q1: Discriminative analysis. Fig. 5 compares different pairs of datasets in terms of the confidence of extracted rules. For each pair-wise comparison we



(a) (b) (c)

Fig. 6. (a): confidence comparison between monthly and weekly granularity. (b): scatter plot comparing the two different definitions of confidence discussed in Section 2.4. (c) number of valid rules as percentage of the number of frequent patterns, for varying confidence.

show the scatter plot of the confidence of the rules that are (*i*) most frequent in each dataset, and (*ii*) common in both datasets. The plots allow for several interesting observations. First, Fig. 5(a), (b) and (c) show that the confidence of the extracted rules are different between a co-authorship network (arXiv and DBLP) and a social network (Y!360).[4] On the other hand, the confidence of rules are similar between the two co-authorship networks (figure (d)) and the two social networks (figures (e)-(g)). Thus, the plots confirm our claim that graph evolution rules characterize the different types of networks.

Fig. 5 (h) and (i) compare the same two datasets as in figures (c) and (g), but using the measure of support instead of confidence. One notices that the measure of support cannot be used to characterize different types of networks.

Q2: Time-granularity. Fig. 6(a) is the scatter plot of confidence of rules extracted from the same network but with different time granularity. The colors/shapes in the plot correspond to the difference between maximum time stamp on an edge in the head (MTH) and maximum time stamp on an edge in the body (MTB) of the rule. We notice that the rules form three clear clusters (shown with different colors/shapes and the corresponding regression lines) that correspond to the different between the maximum time stamp in the the head and the body of each rule. In particular, we see that rules that have higher weekly confidence than monthly correspond to a larger difference MTH−MTB. This observation can be explained if we think about confidence as prediction: the difference MTH−MTB can be thought as the temporal gap that must be bridged by a prediction task, and clearly predicting further in the future is more difficult (i.e., lower confidence).

Q3: Confidence measures. Fig. 6(b) shows that the two confidence measures disagree. A more thorough investigation shows that all the rules with an occurrence-based confidence exceeding 200 have the most simple body: one single edge. Furthermore, all those rules span 3 or 4 time-steps from body to head. Given that all those rules share the same simplistic body, which can be matched

[4] Using Flickr instead of Y!360 gives similar results.

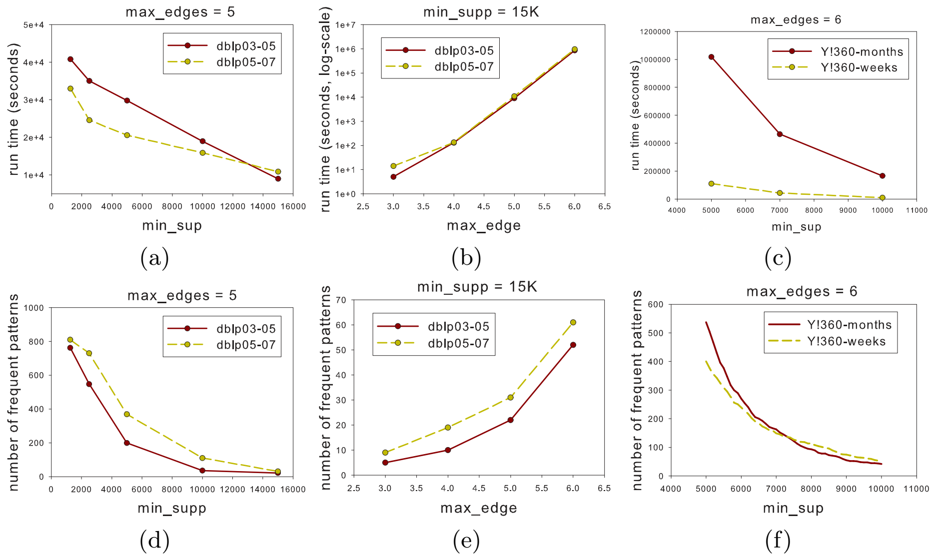


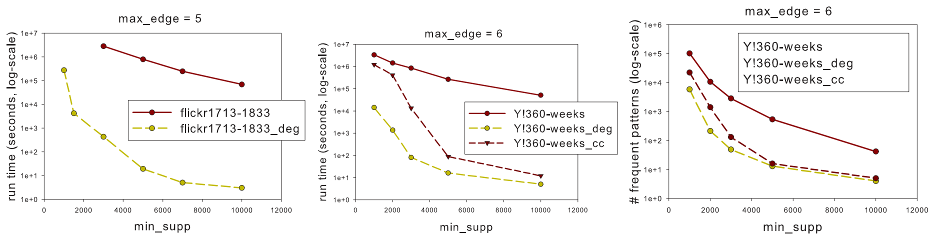
Fig. 7. Running time and number of patterns found with varying *min. support* and *max. edge* thresholds.

anywhere, a prediction task, especially at 3 or 4 steps into the future, is doomed to fail. On the other hand, the support-based confidence, assigns score below 0.2 to all rules with the simplistic body – declaring them almost meaningless – thus indicating that support-based confidence is a more appropriate measure to use.

Q4: Influence of parameters. We perform further experimentation on the number of extracted rules and the running time of the algorithm. Fig. 6(c) shows the number of valid graph evolution rules as percentage of the number of frequent patterns found for various thresholds of minimum confidence. This is done on one bibliographic and one social network, with and without node labels. In all cases, the number of rules is close to 80% of the number of frequent patterns. The results reaffirm the observation from Fig. 6(a), namely, that rules extracted from a dataset with weekly granularity have higher confidence than rules extracted from a dataset with yearly granularity.

Fig. 7(e) shows that the number of extracted patterns grows as a function of the number of edges allowed in the pattern. This behavior is expected as the number of possible graphs increases exponentially with the number of edges. Fig. 7(d) shows another typical result: lowering the support threshold allows for more complex patterns that contain more edges, and thus a similar increase in the number of extracted patterns.

Fig 7(a)-(b) show that the running time is affected more by the maximum-edge than the minimum-support constraint. While the increase is almost linear with decreasing minimum support, the running time grows exponentially with an increasing maximum edge size.



(a) (b) (c)

Fig. 8. Running time and number of patterns found on networks with labelled nodes with varying level of minimum support.

A more interesting observation can be made from Fig. 7(c) and (f), in which we compare the Y!360 graph for the two different time granularities. The weekly graph has 41 edge labels and is more *diverse* than the monthly graph, which has only 10. Comparing the two datasets, we see that while the running times are very different, the number of extracted patterns is almost the same. The explanation of this apparent discrepancy is the following: With respect to the number of patterns, notice that, on one hand, more edge labels allow for more patterns to be found, on the other hand, less edge labels allow patterns to be found more repeatedly. Thus for a fixed number of edges, there are more high-support patterns in the graph with the few edge labels, and more low-support patterns in the graph with the many edge labels. With respect to the running times, more patterns of smaller size can be found in the graph with the many edge labels, and for those small patterns the subgraph-isomorphism problem is easier to solve. Furthermore, it is easier to encounter a non-matching edge earlier, thus being able to terminate earlier the search-branch for the subgraph-isomorphism.

Similar results hold for graphs with labelled nodes, as shown in Fig. 8. However, in the case of node labels, the introduced diversity has the effect of reducing the number of patterns found, and the running time.

5 Related Work

Several papers have focused on the global evolution of networks. For instance, Backstrom et al. [2] studied the evolution of communities in social networks, and Leskovec et al. [16] discovered the shrinking diameter phenomena on time-evolving networks. On the other hand, studying network evolution at a more local level, Leskovec et al. [15] used a methodology based on the maximum-likelihood principle and they showed that edge locality plays a critical role in the evolution of networks.

Other recent papers present algorithmic tools for the analysis of evolving networks. Tantipathananandh et al. [20] focus on assessing the community affiliation of users and how it changes over time. Sun et al. [18], apply the MDL principle to the discovery of communities in dynamic networks. The main difference of the work of Sun et al. [18] from previous work such as [1, 19] is that they develop a parameter-free framework. However, as in [20], the focus lies on identifying approximate clusters of users and their temporal changes. Ferlez et al. [7] use the MDL principle for monitoring the evolution of a network.

Desikan and Srivastava [6] study the problem of mining temporally evolving web graphs. Three levels of interest are defined: single node, subgraphs and whole graph analysis, each of them requiring different techniques. Inokuchi and Washio [11] propose a fast method to mine frequent subsequences from graph-sequence data defining a formalism to represent changes of subgraphs over time. However the time in which the changes take place is not specified in the patterns. Liu et al. [17] identify subgraphs changing over time by means of vertex-importance scores and vertex-closeness changes in subsequent snapshots of the graphs. The most relevant subgraphs are hence not the most frequent, but the most significant based on the two defined measures. The paper that is most related to our work is the one by Borgwardt et al. [3] who represent the history of an edge as a sequence of 0's and 1's representing the absence and presence of the edge respectively. Then conventional graph-mining techniques are applied to mine frequent patterns. However, there are several differences to our approach. First, the employed mining algorithm GREW is not complete, but heuristic. Further, the overlap-based support measure used requires solving an maximal independent set problem for which a greedy algorithm is used. Another computational issue with their approach is the extension of an edge in the so-called inter-asynchronous FDS case. Accordingly the size of the networks analyzed in the paper is rather small.

Various proposals for mining frequent patterns in the single graph context [14, 21, 8, 4] were discussed in Section 2. A recent paper by Calders et al. [5] introduces a new measure named *minimum clique partition*, which analogous to the maximal independent set is based on the notion of an overlap graph and thus requires solving an NP-complete problem. They prove that support measures based maximal independent set and minimum clique partition are the minimal and the maximal possible meaningful overlap measures, and show that [12] introduced a function which is sandwiched between these two measures; computable in polynomial time. However, any of those measures requires computing an overlap graph for each candidate pattern, which is a costly operation in itself due to requiring enumerating all occurrences of a pattern.

6 Extensions and future work

In this section we discuss briefly how to relax some of the restrictions of our problem definition.

Consider first the pattern H in Fig. 3(b). Imagine that, for a particular dataset, it is the case that when there is a star of size 3 an edge between two peripheral nodes appear. Pattern H captures partly this phenomenon, but is too "specific" as it emphasizes that the star was formed in particular time instances before the appearance of the last edge. A more general pattern would be to replace the time-stamp of the last edge with T, and the time-stamp of all the edges in the star with the constraint "$< T$", which will have to be satisfied when tested with the time-stamps of the host graph. We plan to extend our algorithm to experiment with this idea as a continuation of our work.

For sake of presentation, in Section 2 we assumed that graphs can only grow in time. However, our approach can be easily extended to handle edge-deletions if an edge can appear and disappear at most once. The extension would consider two time-stamps t_I (time of insertion) and t_D (time of deletion) on each edge instead of the single time t. By modifying definitions 1 and 2 condition (ii) to $\forall(u, v) \in E_P$ it is $t_I(\varphi(u), \varphi(v)) = t_I(u, v) + \Delta$ and $t_D(\varphi(u), \varphi(v)) = t_D(u, v) + \Delta$.

We did not implement the above matching since two out of four datasets (arXiv and DBLP) are by definition only growing (no deletions), and deletions are rare in the other two. As future work we plan to incorporate deletions and study networks with a higher likelihood of such events.

In our approach, we have not considered node or edge relabelling so far. Considering node and edge relabeling is very interesting, as in many graphs, such as social networks, the properties of nodes and edges change over time. For example, in social-network analysis it would be interesting to study the change of leadership in communities and its effects.

Besides all the above, which are possible extensions to the type of patterns we are able to mine, we would like to go further by leveraging the concept of rule confidence, and designing a paradigm that will allow us to *predict* graph evolution, and that, together with *GERM*, will provide helpful tools analyzing datasets of dynamic graphs.

7 Conclusions

Following a frequent pattern mining approach, we defined relative time patterns and introduced introduced the problem of extracting *Graph Evolution Rules*, satisfying given constraints of minimum support and confidence, from an evolving input graph. While providing the problem definition we discussed alternative definitions of support and confidence, their merits and limits. We implemented *GERM* an effective solution to mine Graph Evolution Rules, and extensively test it on four large real-world networks (two social networks, and two co-authorship networks), using different time granularities. Our experiments confirmed the feasibility and the utility of our framework and allowed for interesting insights. In particular we showed that Graph Evolution Rules with their associated concept of confidence, indeed characterize the different types of networks.

Availability. The executable code of the GERM software is freely available at: `http://www-kdd.isti.cnr.it/~berlingerio/so/gm/`.

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