

Minkowski 3-forms, flux string vacua, axion stability and naturalness

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ABSTRACT: We discuss the role of Minkowski 3-forms in flux string vacua. In these vacua all internal closed string fluxes are in one to one correspondence with quantized Minkowski 4-forms. By performing a dimensional reduction of the $D = 10$ Type II supergravity actions we find that the 4-forms act as auxiliary fields of the Kahler and complex structure moduli in the effective action. We show that all the RR and NS axion dependence of the flux scalar potential appears through the said 4-forms. Gauge invariance of these forms then severely restricts the structure of the axion scalar potentials. Combined with duality symmetries it suggests that all perturbative corrections to the leading axion scalar potential V_0 should appear as an expansion in powers of V_0 itself. These facts could have an important effect e.g. on the inflaton models based on F-term axion monodromy. We also suggest that the involved multi-branched structure of string vacua provides for a new way to maintain interacting scalar masses stable against perturbative corrections.

KEYWORDS: Flux compactifications, Superstring Vacua, Discrete and Finite Symmetries, Global Symmetries

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1 Introduction

Consistency of Poincare invariant field theory implies that the possibilities for Lorentz structure of massless fields is quite limited. Fermions must have spin $1/2$ or $3/2$, whereas bosons should have a Lorentz structure of any of the kinds $C^0, C^\mu, C^{\mu\nu}$ or $g^{\mu\nu}$, with g the graviton and the C-fields being antisymmetric. This list should be extended to include 3-index antisymmetric tensors $C^{\mu\nu\rho}$. At first sight this extra possibility looks irrelevant, since a Minkowski 3-form has no propagating degrees of freedom. However the presence of such fields may lead to important physical implications. A very recent example of this fact is discussed in [1], in which it is shown that the ultraviolet behaviour of pure gravity amplitudes changes if 3-form contributions are included in loops, in spite of not having propagating degrees of freedom. More well known is the fact that the corresponding field strength $F^{\mu\nu\rho\sigma}$ may be non-vanishing and permeate space-time giving rise to a constant contribution to the cosmological constant, and hence to new (quantized) degrees of freedom. Due to this fact Minkowski 4-forms have been considered in the past in trying to address the cosmological constant problem [2–6]. More specifically Brown and Teitelboim [7, 8] considered a background 4-form field strength in space-time, contributing to the vacuum energy. Membranes coupling to the $C^{(3)}$ form can nucleate and give rise to jumps in the c.c. They suggested this contribution adjusts itself dynamically to cancel the rest of the contributions to the c.c. This 4-form is assumed not to couple directly to other fields in the

theory. The main difficulty with this approach is that the 4-form steps required to cancel the c.c. should be extremely tiny and is difficult to construct a working model with the required properties.

Bousso and Polchinski [9] suggested also to consider the contribution of 4-forms to the c.c. within the context of string theory (see also [10]). They argued that in string theory plenty of Minkowski 4-forms appear upon compactification and that their values are quantized. There is then a *discretum* in which the individual (large) 4-form values could conspire to yield a detailed (almost) cancellation of the c.c. if the number of 4-forms and their possible quantized values is sufficiently large. The structure of the scalar potential has the schematic form

$$V = \sum_i F_i^2 - V_{\text{obs}} \tag{1.1}$$

where $F_i = \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}^i$ and $-V_{\text{obs}}$ denotes the remaining contributions, typically yielding a negative value. In this case the cancellation is not dynamical but is assumed to occur on the basis of anthropic arguments. A difficulty with this proposal so formulated is that within string vacua one cannot separate the issue of the c.c. from that of moduli fixing and one expects the 4-forms to couple to the moduli, making the situation far more complicated. As is well known, soon after a general approach to fix all moduli within Type IIB string theory vacua was proposed [11], in which *internal* RR and NS fluxes are turned on [12] to fix the complex structure moduli and dilaton in Type IIB orientifolds, with the Kahler moduli assumed to be fixed by non-perturbative effects. Since then a large amount of effort has been dedicated to the issue of moduli fixing, involving internal fluxes [13–17]. Still the possible role of Minkowski 4-forms has been rarely discussed.

Minkowski 4-forms were discussed in papers by Dvali [18–21] in which it was shown that the usual strong CP problem and its axion solution may be elegantly described in terms of a *composite* 3-form, the QCD Chern-Simons term $C^{(3)}$, with a dynamical 4-form proportional to $F \wedge F$. Here the PQ solution to the strong CP problem corresponds to the 3-form becoming massive via a coupling to a 2-form $B_{\mu\nu}$, the latter being the dual of a standard axion.

More recently Kaloper and Sorbo [22, 23] showed that 4-forms in field theory provide for a natural definition of quadratic chaotic inflation [24], stable under large field trips of the inflaton. Schematically, one starts from an action including not only a quadratic piece for the 4-form but a coupling to an axion-like field ϕ

$$\mathcal{L} = - F_4^2 + \mu\phi F_4 + \dots \tag{1.2}$$

with μ some mass parameter. Imposing $dC_3 = F_4$ through a Lagrange multiplier q and upon using the equations of motion for F_4 , one finds a quadratic scalar potential of the form

$$V_0 = \frac{1}{2} (q + \mu\phi)^2 \tag{1.3}$$

where q is interpreted as a F_4 vev. Membranes couple to the 3-form C_3 and induce changes $\Delta q = e$, where e is the membrane charge. The interesting point is that this is *not* a scalar

potential but rather a *family* of potentials or different branches parametrized by the value of q . The family of potentials has a discrete shift symmetry

$$\phi \rightarrow \phi + \phi_0, \quad q \rightarrow q - \mu\phi_0 \tag{1.4}$$

which is spontaneously broken when a minimum $\phi = -q/\mu$ is chosen. For each local minimum we have a quadratic potential, which can be used e.g. to induce chaotic inflation if ϕ is identified with the inflaton. The above description in terms of a 4-form is a way of gauging a shift global symmetry for a scalar field without introducing new degrees of freedom. One can formulate the same system by using the dual 2-form B_2 instead of the scalar ϕ . Here, as in [18–21], C_3 gets massive by combining with B_2 , yielding a massive degree of freedom. One then obtains an action of the schematic form [18–21, 25]

$$\mathcal{L} = -F_4^2 - \frac{\mu^2}{2}|dB_2 - C_3|^2 + \dots \tag{1.5}$$

This action is obviously invariant under a gauge transformation

$$B_2 \rightarrow B_2 + \Lambda_2, \quad C_3 \rightarrow C_3 + d\Lambda_2 \tag{1.6}$$

which corresponds to the above shift symmetry. This shift symmetry is expected to be broken in a complete theory by non-perturbative effects. However, what makes this elaborated construction of a simple quadratic potential interesting is that the symmetries will protect the potential from perturbative and Planck suppressed corrections. Indeed, gauge invariance of F_4 and the shift symmetry of ϕ force the corrections to appear in powers $(F_4^2/M^4)^n$, with M the ultraviolet cut-off of the theory, rather than arbitrary powers of ϕ . Thus corrections to an inflationary potential should appear as powers of V_0/M_p^4 , which will be very small for an inflaton potential $V_0 < (10^{16} GeV)^4$. This is crucial to get stability of large field inflation in these schemes.

This Kaloper-Sorbo Lagrangian is a 4D field theory avatar of a somewhat analogous structure found in the *monodromy inflation* models of [25–28, 45]. In those models large field inflation is attained by coupling an axion-like periodic field to an external source of energy, like e.g. a brane tension. Upon a period the field gets a shift in energy, so that the field does not come to the same point but rather perform a large trans-Planckian excursion. In the recent paper of Marchesano, Shiu and Uranga [25] it has been explicitly shown how a structure analogous to that of the KS Lagrangian appears in specific string constructions.

In the present paper we study in a systematic way the role of Minkowski 4-forms in Type II, $D = 4$, $N = 1$ orientifold vacua and discuss to what extent the above discussed 4-form avatars do appear in compactified string theory. We also study the connection between the internal RR and NS fluxes abundantly used in moduli fixing and the Minkowski 4-forms. We analyse in more detail the case of Type IIA orientifold $N = 1$ flux vacua, in which the discussion is more transparent, but also present analogous results for the Type IIB case. In the former case some of the conclusions are as follows

- RR and NS closed string fluxes through internal cycles are in one to one correspondence to Minkowski 4-forms. These 4-forms act as auxiliary fields of both Kahler and complex structure moduli as well as for the $N = 1$ supergravity multiplet.

- The full dependence of the flux scalar potential on RR and NS axions goes always through combinations of Minkowski 4-forms. As a result the scalar potentials of string flux vacua are not any random sugra potential but have a branched structure. The potential has the general form

$$V_{4\text{-forms}} = \sum_i f_{ij}(\text{Re}M_a) F_4^i F_4^j + \sum_i F_4^i \Theta_i(\text{Re}M_a, \text{Im}M_a) + V_{\text{local}}(\text{Re}M_a) . \quad (1.7)$$

Here M_a denote collectively both Kahler and complex structure moduli, and $\text{Im}M_a$ denote the RR and NS axions. The functions Θ_i come from the Type IIA Chern-Simons couplings and contain polynomials of the axion fields with coefficients involving linearly the internal fluxes. V_{local} contains the contribution of the D-branes and orientifold planes to the potential, which can be re-expressed in terms of the $\text{Re}M_a$ upon imposing RR tadpole cancellation. Upon applying the equations of motion for the 4-forms the full scalar potential is obtained, with an axion dependent part which is always positive definite.

- The above scalar potential is in some sense a string multi 4-form and multi-flux generalisation of the Kaloper-Sorbo structure in which the quadratic potential is replaced by more general (up to order six) polynomials. The role of the shift symmetry is played by the duality symmetries of the compactified theories. Under $R \leftrightarrow 1/R$ duality symmetries the different Minkowski 4-forms transform into each other. As in the KS field theory model, gauge invariance of the 4-forms combined with the duality symmetries of the compactification constrain the corrections to the potential to come suppressed by powers of V_0/M_p^4 . This shows that flux string vacua is a natural arena to construct large field inflaton models with a stable potential.

The structure in eq. (1.7) resembles the one discussed by Beasley and Witten in the context of M-theory compactified in G_2 manifolds X in the presence of G_4 flux [46]. They found that, although the superpotential W depends explicitly only on the G_4 flux supported on X , it also describes the breaking of SUSY by G_4 flux in Minkowski. The resulting scalar potential is also branched, in analogy with the Schwinger model in two dimensions [47].

This structure leads to families of scalar potentials parametrized by specific flux choices, some of which are related by orbits of duality transformations. As expected, there can be transitions from one potential to another by membrane nucleation. This has been analysed in a context similar to ours in [48, 49]. The membranes in Type IIA come from D2, D4, D6 and D8-branes wrapping even cycles (for RR 4-forms) and NS5 branes wrapping 3-cycles (for NS 4-forms). Analogous conclusions hold for Type IIA vacua with geometric fluxes. In this case the nucleating membranes will be KK5-branes wrapping 3-cycles. A similar story also applies to $N = 1$ Type IIB orientifolds with RR and NS fluxes, which we describe more briefly. We also briefly touch upon the issue of non-geometric fluxes. In the Type IIB case the natural objects which appear are complex 4-forms, involving the complex dilaton as well as both RR and NS fluxes in their definition.

We also suggest that the above structure of symmetries may provide for a new way to obtain an interacting theory of scalars in which stability against loop corrections may

be obtained. This would be a consequence of the multi-branched structure of the axion scalar fields yielding a corrected potential which is itself an expansion in powers of the uncorrected potential. We also speculate about possible applications of this idea.

The structure of this paper is as follows. In the next section we recall a few facts about Minkowski 4-forms in general. In section 3 we study the structure of Minkowski 4-forms in Type IIA orientifolds with RR and NS fluxes. We perform the dimensional reduction starting from the $D = 10$ Type IIA action and focus on the couplings of the Minkowski 4-forms. We show how they behave as moduli auxiliary fields and how they are invariant under a class of discrete symmetries involving both RR and NS axion shifts as well as internal flux transformations. We also discuss in the toroidal case the action of $R \leftrightarrow 1/R$ dualities as well as how the introduction of geometric fluxes modifies the setting. In section 4 we address the case of Type IIB orientifolds and how in this case the RR and NS 4-forms combine to yield complex auxiliary fields, but a structure otherwise analogous to that of the Type IIA case. We also discuss briefly how 4-forms may arise from the open string sector, by dimensionally reducing the duals of the F_2 gauge field strengths, and discuss in some detail the example of reference [45]. In section 5 we present a general discussion of implications of the uncovered symmetry structure for the stability of scalar potentials against perturbative corrections. We briefly discuss the case of inflation and a possible new way to obtain naturally light interacting scalars. Some conclusions are left for section 6.

2 Minkowski 3-forms

Before turning to Type II orientifold compactifications, let us recall a few facts about 3-forms (see e.g. [9, 10, 18–22, 50, 51]). The bosonic action of a 3-form includes terms

$$S = - \int d^4x \sqrt{-g} \frac{1}{48} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} + S_{\text{bound}} + S_{\text{mem}} . \quad (2.1)$$

Here S_{bound} includes some boundary terms which do not modify the equations of motion and will not play a role in our discussion, so will not be displayed here. On the other hand S_{mem} describes the possible coupling of C_3 to membranes, i.e.

$$S_{\text{mem}} = q \int_{D_3} d^3\xi \epsilon^{abc} C_{\mu\nu\rho} \left(\frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b} \frac{\partial X^\rho}{\partial \xi^c} \right) , \quad (2.2)$$

where the membrane charge q has dimensions of mass² and D_3 is the membrane world volume. Away from the membranes the equations of motion for C_3 force F_4 to be constant, i.e.

$$F_{\mu\nu\rho\sigma} = f \epsilon_{\mu\nu\rho\sigma} \quad (2.3)$$

where f is a constant. In the presence of membrane domain walls, the value of this constant varies as $\Delta f = q$ as one goes across the wall. As argued e.g. in [9] the value of the 4-form in string theory is quantized in units of the membrane charge, i.e.

$$f = nq , \quad n \in \mathbf{Z} . \quad (2.4)$$

In the case of generic string compactifications we will have multiple 4-forms, some coming directly from dimensional reduction and others upon expanding higher order antisymmetric RR or NS tensors in harmonics in the compact directions. In addition, as we will see, unlike the BT or BP scenarios, the 4-forms have couplings to the axions and moduli of the compactification, with a structure for each 4-form

$$F^2 + F\theta(\phi_i), \quad F = F_{\mu\nu\rho\sigma}\epsilon^{\mu\nu\rho\sigma} \quad (2.5)$$

with θ a function of the axions and moduli. Upon integration by parts the second piece may be written as

$$C_{\nu\rho\sigma} J^{\nu\rho\sigma} \quad ; \quad J^{\nu\rho\sigma} = \epsilon^{\mu\nu\rho\sigma} \partial_\mu \theta(\phi_i) \quad (2.6)$$

This current is conserved, i.e., $\partial_\nu J^{\nu\rho\sigma} = 0$ and the action is invariant under the gauge transformations.

$$C_{\nu\rho\sigma} \longrightarrow C_{\nu\rho\sigma} + \partial_{[\nu} \Omega_{\rho\sigma]} . \quad (2.7)$$

For 4-forms in which $\theta(\phi_i)$ is just a linear function of a RR or NS axion field, the structure of its contribution to the action is analogous to a Kaloper-Sorbo action. In this case one can dualise the axion into a Minkowski 2-form in the usual way, with

$$\partial_\mu \phi = \epsilon_{\mu\nu\rho\sigma} \partial^\nu B^{\rho\sigma} . \quad (2.8)$$

Then the ϕF_4 coupling becomes

$$C_{\nu\rho\sigma} (\partial^\nu B^{\rho\sigma}) \quad (2.9)$$

indicating how through a Higgs mechanism the 3-form gains a gauge invariant mass by swallowing the 2-form. This is the dual of the axion becoming massive in the KS setting, and is what Dvali used for his reinterpretation of the QCD axion physics [18–21]. The 3-form and 2-form have then gauge transformations

$$C_{\nu\rho\sigma} \rightarrow C_{\nu\rho\sigma} + \partial_{[\nu} \Omega_{\rho\sigma]} \quad ; \quad B_{\rho\sigma} \rightarrow B_{\rho\sigma} + \Omega_{\rho\sigma} . \quad (2.10)$$

This leads to a massive 3-form multiplet, which now contains a massive scalar degree of freedom. This structure of a massive scalar may be connected also with torsion cycles in string compactifications, as emphasized in [25].

Massless 3-forms may be embedded into $N = 1$ supersymmetric multiplets. They naturally appear as auxiliary fields in non-minimal versions of the $N = 1$ chiral multiplet [52–62]. And essentially correspond to replacing one or both of the real auxiliary fields of a chiral multiplet by corresponding 4-forms. Similarly, one can formulate non-minimal $N = 1$ sugra multiplets with one or two real scalar auxiliary fields being replaced by 4-forms. Still these type of multiplets have not been discussed much in the literature. In [58] the SUSY action of a non-minimal chiral multiplet S including one 4-form auxiliary field is discussed in detail. The corresponding superfield may be defined as

$$S = -\frac{1}{4} \overline{D}^2 V , \quad (2.11)$$

where V is a real multiplet with the same content as a standard vector multiplet, but with the vector field replaced by $\epsilon_{\mu\nu\rho\sigma}C^{\nu\rho\sigma}$. The chiral S field has then an expansion

$$S = M + i\theta\sigma^\mu\bar{\theta}\partial_\mu M + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square M + \sqrt{2}\theta\lambda + \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\theta}\sigma^\mu\partial_\mu\lambda + \theta\theta(D + iF), \quad (2.12)$$

with $F = \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}$ and D an auxiliary real scalar. This multiplet contains on-shell one complex scalar M and one Weyl fermion λ . It can combine with a linear supermultiplet L , which includes a 2-form antisymmetric field B_2 , to yield a massive 3-form multiplet. This is a SUSY generalisation of the Higgs mechanism described around eq. (2.9). In addition these non-minimal chiral super fields S can have superpotential couplings in superspace, i.e.

$$S_W = \int d^2\theta d^2\bar{\theta} S^a\bar{S}^a + \int d^2\theta W(S) + \int d^2\bar{\theta} W^*(\bar{S}) = \quad (2.13)$$

$$-|\partial M|^2 + D^a D^a + F^a F^a + W_a(D^a + iF^a) + W_a^*(D^a - iF^a) + \dots$$

where W_a denotes derivative with respect to S_a . Using the equations of motion for C_3 one gets $F^a = \text{Im}(W_a) + f_a$, with f_a a constant. Then the scalar potential has the form [54, 58, 59]

$$V_S = |W_a + if_a|^2. \quad (2.14)$$

This agrees with the result obtained for standard chiral multiplets with the replacement $W_a \rightarrow W_a + if_a$. Let us advance that this multiplet is not enough to describe the structure of 4-forms that we find in Type IIA and IIB orientifolds. In particular we find that for the Kahler(complex structure) moduli in IIA(IIB) orientifolds both auxiliary fields of a chiral multiplet are replaced by 4-forms.

3 4-forms in Type IIA orientifolds

We turn now to describe how 4-forms appear in Type IIA orientifold compactifications down to four dimensions. The compactification of ten-dimensional massive Type IIA string theory on a Calabi-Yau threefold in the presence of background fluxes has been thoroughly studied in e.g. [63–67]. Here we perform the same compactification but keeping trace of all the Minkowski 4-forms which appear upon dimensionally reducing the 10d RR and NSNS fields. This leads to a new formulation of the scalar potential in terms of Minkowski 4-forms as in eq.(1.7) and the intriguing result that the full dependence of the flux scalar potential on RR and NS axions comes only through couplings to the said 4-forms.

3.1 4-forms, RR and NS fluxes in IIA orientifolds

Let us consider Type IIA string theory compactified on a Calabi-Yau threefold Y in the presence of O6 planes. The massless ten-dimensional bosonic content of the closed string spectrum contains the metric, the dilaton and the antisymmetric two-form B_2 from the NS-NS sector and the p-form fields C_p from the RR sector. We will work in the democratic formulation [68] in which all the p-form fields C_p with $p = 1, 3, 5, 7$ are present, so we will have to impose the Hodge duality relations

$$G_6 = - *_{10} G_4 \quad , \quad G_8 = *_{10} G_2 \quad , \quad G_{10} = - *_{10} G_0 \quad (3.1)$$

at the level of the equations of motion in order to avoid overcounting of the physical degrees of freedom. The gauge invariant field strengths are defined as [65, 68]

$$G_p = dC_{p-1} - H_3 \wedge C_{p-3} + \mathcal{F}e^B \tag{3.2}$$

where $H_3 = dB_2$, $F_p = dC_{p-1}$ and \mathcal{F} is a formal sum over all the RR fluxes F_p . The background field strength G_0 may be regarded as the mass parameter (also known as Romans mass) of massive Type IIA supergravity, $G_0 = -m$. The massless 4d fields (before introducing the fluxes) are in one-to-one correspondence with the harmonic forms of the internal manifold Y , so the multiplicity is counted by the dimension of the cohomology groups $H^{(p,q)}(Y)$. To implement the orientifold projection we split the harmonic forms into forms with even or odd parity under the orientifold projection. The elements of the cohomology basis satisfy the following relations,

$$\int_Y \omega_\alpha \wedge \tilde{\omega}^\beta = \delta_\alpha^\beta, \quad \alpha, \beta \in \{1 \dots h_+^{(1,1)}\} \tag{3.3}$$

$$\int_Y \omega_a \wedge \tilde{\omega}^b = \delta_a^b, \quad a, b \in \{1 \dots h_-^{(1,1)}\} \tag{3.4}$$

$$\int_Y \alpha_K \wedge \beta^L = \delta_K^L, \quad K, L \in \{1 \dots h^{(2,1)} + 1\} \tag{3.5}$$

where $\omega, \tilde{\omega}, \alpha$ denote a 2-form, 4-form and 3-form respectively. Notice that since the volume form is odd under the orientifold projection and the Hodge star involves contraction with the volume form, the dual form of an odd 2-form ω_a is actually an even 4-form $\tilde{\omega}_a$. Therefore $\omega_a \in H_-^{(1,1)}$ and $\tilde{\omega}_a \in H_+^{(2,2)}$ while $\omega_\alpha \in H_+^{(1,1)}$ and $\tilde{\omega}_\alpha \in H_-^{(2,2)}$. Analogously $\alpha_K \in H_+^3$ and $\beta_L \in H_-^3$. The metric, the dilaton, C_3 and C_7 are even under the orientifold projection while B_2, C_1 and C_5 are odd.

We are interested in the presence of Minkowski 3-form fields in the fluxed induced scalar potential. In addition to the universal RR 3-form C_3 one can also get 3-forms by dimensionally reducing higher RR and NSNS fields, C_5, C_7, C_9 and H_7 , and considering three of the indices in Minkowski space. By allowing also for the presence of internal fluxes, the RR field strengths can be expanded as

$$\begin{aligned} F_0 &= -m, & F_2 &= \sum_i q_i \omega_i, & F_4 &= F_4^0 + \sum_i e_i \tilde{\omega}_i \\ F_6 &= \sum_i F_4^i \omega_i + e_0 dvol_6, & F_8 &= \sum_a F_4^a \tilde{\omega}_a, & F_{10} &= F_4^m dvol_6 \end{aligned} \tag{3.6}$$

where $i, a = 1, \dots, h_-^{(1,1)}$. The parameters e_0, e_i, q_i, m refer to internal RR fluxes on Y and we get $2h_-^{(1,1)} + 2$ Minkowski 4-forms labelled by F_4^0, F_4^i, F_4^a and F_4^m . Similarly the NS H_3 background is intrinsically odd under the orientifold projection so it can be expanded as

$$H_3 = \sum_{I=0}^{h_{2,1}^-} h_I \beta_I \tag{3.7}$$

while the dual H_7 can be expanded in terms of even 3-forms

$$H_7 = \sum_I H_4^I \wedge \alpha_I \tag{3.8}$$

obtaining $h_{2,1}^+ + 1$ additional Minkowski 4-forms H_4^I coming from the NSNS sector. Moreover, the fields B_2 and C_3 can be expanded as

$$B_2 = \sum_i b_i \omega_i, \quad C_3 = \sum_I c_3^I \alpha_I \quad (3.9)$$

where b_i and c_3^I are 4d scalars and correspond to the axionic part of the complex supergravity fields T, S, U as follows,

$$\text{Im}T_i = - \int B_2 \wedge \tilde{\omega}^i = -b^i; \quad i = 1, \dots, h_-^{11} \quad (3.10)$$

$$\text{Im}U_i = \int C_3 \wedge \beta^i = c_3^i; \quad i = 1, \dots, h_+^3 \quad (3.11)$$

$$\text{Im}S = - \int C_3 \wedge \beta^0 = -c_3^0. \quad (3.12)$$

The Hodge dualities of eqs.(3.1) relate the Minkowski 4-forms and the internal magnetic fluxes as we proceed to explain in the following. Separating each field strength into Minkowski and internal parts and using eq.(3.6), the duality relations given by (3.1) imply

$$\begin{aligned} *_4 F_4^0 &= \frac{1}{k} \left(e_0 + e_i b^i + \frac{1}{2} k_{ijk} q^i b^j b^k - \frac{m}{3!} k_{ijk} b^i b^j b^k - h_0 c_3^0 - h_i c_3^i \right) \\ *_4 F_4^i &= \frac{g^{ij}}{4k} \left(e_j + k_{ijk} b^j q^k - \frac{m}{2} k_{ijk} b^j b^k \right) \\ *_4 F_4^a &= 4k g^{ab} (q_b - m b_b) \\ *_4 F_4^m &= -m \end{aligned} \quad (3.13)$$

where $g_{ij} = \frac{1}{4k} \int \omega_i \wedge * \omega_j$ is the metric in the Kahler moduli space, k is the volume and k_{ijk} the topological triple intersection number.

The type IIA ten dimensional supergravity action can be divided into three terms,

$$S_{\text{IIA}} = S_{\text{RR}} + S_{\text{NS}} + S_{\text{loc}} \quad (3.14)$$

where the RR and NSNS actions are given by

$$\begin{aligned} S_{\text{RR}} &= -\frac{1}{8k_{10}^2} \int_{R^{1,3} \times Y} \sum_{p=0,2,4,6,8,10} G_p \wedge *_10 G_p + \dots, \\ S_{\text{NS}} &= -\frac{1}{4k_{10}^2} \int_{R^{1,3} \times Y} e^{-2\phi} H_3 \wedge *_10 H_3 \end{aligned} \quad (3.15)$$

and S_{loc} refers to the contribution from localized sources like D6-branes and O6-planes. Let us start analyzing the part of the action involving the RR fields. By using the duality relations (3.1), the kinetic terms for the RR fields can be written as

$$-\frac{1}{2} \sum_{p=0,2,4,6,8,10} G_p \wedge *_10 G_p = G_4 \wedge G_6 + G_2 \wedge G_8 + G_0 \wedge G_{10} \quad (3.16)$$

Plugging eqs.(3.6)–(3.9) into the above RR action and integrating over the internal dimensions we get the following effective scalar potential in four dimensions

$$V_{\text{RR}} = -\frac{1}{2} \left[F_4^0 \left(e_0 + b^i e_i + \frac{1}{2} k_{ijk} b^i b^j q_k - \frac{m}{6} k_{ijk} b^i b^j b^k \right) + F_4^i \left(e_i + k_{ijk} b^j q_k - \frac{1}{2} m k_{ijk} b^j b^k \right) + F_4^a (q_a - m b_a) - k m F_4^m \right] \quad (3.17)$$

This scalar potential can be rewritten by using eq.(3.13) in the general form

$$V_{\text{RR}} = -\frac{1}{2} \left[-k F_4^0 \wedge *F_4^0 + 2F_4^0 \rho_0 - 4k g_{ij} *F_4^i \wedge F_4^j + 2F_4^i \rho_i - \frac{1}{4k} g_{ab} F_4^a \wedge *F_4^b + 2F_4^a \rho_a + k F_4^m \wedge *F_4^m \right] \quad (3.18)$$

already discussed in the introduction, in which the relations (3.13) arise as equations of motion for the 3-forms. Since $**F_4 = -F_4$ the contribution to the potential energy is positive. Notice that although the Minkowski 3-forms have no dynamical degrees of freedom in four dimensions, the kinetic terms of these 3-forms lead to a Minkowski background which also contributes to the scalar potential of the theory. In addition we have some Chern-Simons couplings of the Minkowski 4-forms to the functions

$$\begin{aligned} \rho_0 &= e_0 + b^i e_i + k_{ijk} \frac{1}{2} q_i b^j b^k - \frac{m}{6} k_{ijk} b^i b^j b^k - h_0 c_3^0 - h_i c_3^i \\ \rho_i &= e_j + k_{jkl} b^k q^l - \frac{m}{2} k_{jkl} b^k b^l \\ \rho_a &= q_b - m b_b \\ \rho_m &= -m \end{aligned} \quad (3.19)$$

depending polinomially on the axionic fields and the internal fluxes. Analogously, the kinetic term for the NSNS field leads to the following contribution,

$$V_{\text{NS}} = \frac{1}{2} e^{-2\phi} c_{IJ} H_4^I H_4^J \quad (3.20)$$

where $c_{IJ} = \int \beta_I \wedge * \beta_J$ is the metric on the complex structure moduli space. By Hodge duality the Minkowski 4-form background is related to the NS internal flux by

$$* H_4^I = h^I \quad (3.21)$$

The contribution from the localized sources can be written as [65]

$$V_{\text{loc}} = \sum_a \int_{\Sigma} T_a \sqrt{-g} e^{-\phi} \quad (3.22)$$

where T_a is the tension of the object and Σ the worldvolume. Assuming that tadpole cancellation is satisfied, this contribution can be related to the fluxes and the real part of the moduli so that [65]

$$V_{\text{loc}} = \frac{1}{2} e^K v_i v_j v_k k_{ijk} (m h_0 s - m h_i u^i), \quad (3.23)$$

with s, u_i, v_i the real parts of the S, U_i, T_i moduli respectively. which is independent of the configuration of localized sources as long as they preserve $N = 1$ supersymmetry. Combining all the pieces and using (3.13) we get the following scalar potential

$$V = \frac{k}{2}|F_4^0|^2 + 2k \sum_{ij} g_{ij} F_4^i F_4^j + \frac{1}{8k} \sum_{ab} g_{ab} F_4^a F_4^b + k|F_4^m|^2 + \frac{1}{2s^2} \sum_{IJ} c_{IJ} H_4^I H_4^J + V_{\text{loc}} \quad (3.24)$$

which in terms of the moduli and the internal fluxes becomes

$$\begin{aligned} V = & \frac{1}{2k} \left(e_0 + e_i b^i + \frac{1}{2} q_i k_{ijk} b^j b^k - \frac{1}{6} m k_{ijk} b^i b^j b^k \right)^2 \\ & + \frac{g^{i\bar{j}}}{8k} \left(e_i + q^k k_{ikl} b^l - \frac{1}{2} m k_{ikl} b^k b^l \right) \left(e_{\bar{j}} + q^m k_{jmn} b^n - \frac{1}{2} m k_{jmn} b^m b^n \right) \\ & + 2k g_{ij} (q^i - m b^i)(q^j - m b^j) + k m^2 + \frac{1}{2s^2} \sum_{IJ} c_{IJ} h^I h^J + V_{\text{loc}} \quad (3.25) \end{aligned}$$

as has been previously obtained in the literature [64]. This potential can also be recovered from the standard Cremmer et al. supergravity description in terms of the $N = 1$ 4d effective Kahler potential and superpotential, see [63].

We would like to recall that the full axionic part of the scalar potential can be written in terms of the above couplings to Minkowski 3-form fields and it is always positive definite. This is one of the main results of the paper.

It is worth mentioning a subtlety regarding the process of integrating out the 3-form fields. By looking at (3.18) the equation of motion for the 3-form field implies

$$d(*_4 F_4 - \rho) = 0 \rightarrow *_4 F_4 - \rho = c \quad (3.26)$$

where c is a constant and ρ the function depending on the axionic moduli defined in (3.19). This would imply a shift on the 4-form background leading to a priori new terms in the scalar potential that can not be recovered from the standard Cremmer et al. supergravity description. In particular the shifts would appear as quantized spurion insertions which could have important implications for moduli fixing and the search of de Sitter vacua. These shifts agree with the results of [54, 58, 59] for which a 4-form acting as an auxiliary field implies a shift on the scalar potential with respect to the standard supergravity formula. While valid from a pure effective 4d approach, our 4-forms come actually from dimensionally reducing higher RR and NS fields which are related, at the classical level, by Hodge duality. In fact, we have seen that the Hodge dualities relate the 4-form backgrounds and the internal fluxes forcing this extra shift to vanish. However we do not discard completely the possibility of an integer quantum shift which would not be visible at the level of the classical equations of motion here considered.

The underlying well-defined structure of the scalar potential in terms of the 4-forms is also remarkable. In this description it is clear that the solution of minimum energy will correspond to have all 4-forms vanishing, and can be obtained by solving eqs. (3.13) and (3.21) in which the left side of each equation is equal to zero. We recover then the

AdS minima with $b_i = q_i/m$ previously studied in detail in [66, 67, 69]. This suggests that moduli fixing might be more intuitive in terms of 4-forms.

Note that there are in total $2h_-^{11}$ 4-forms, denoted above as F_4^i and F_4^a , which act as auxiliary fields for the h_-^{11} Kahler moduli of the compactification. This means that the SUSY multiplets associated to the Kahler moduli should contain two 4-forms acting as auxiliary fields. On the other hand there are h_+^3 4-forms H_4^I associated to the h_+^3 complex structure fields. In this case the associated SUSY multiplets would only include one 4-form auxiliary field, like the multiplets discussed in [58]. In addition there are two 4-forms F_4^0, F_4^m which seem to be associated to the $N = 1$ supergravity complex scalar auxiliary field. In this connection the relation imposed by the equations of motion between the 4-forms and the moduli of the compactification is interesting. By looking at eqs. (3.13), the Minkowski 4-forms satisfy

$$kF_4^0 - v_a F_4^a = \text{Re}W \quad (3.27)$$

$$\frac{1}{2}k_i F_4^i - kF_4^m = \text{Im}W \quad (3.28)$$

where W is the $N = 1$ type IIA RR superpotential given by

$$W = e_0 + ie_a T^a - \frac{1}{2}k_{abc}q^a T^b T^c + \frac{1}{6}imk_{abc}T^a T^b T^c \quad (3.29)$$

It would be interesting to understand if this structure is consequence of the possible identification of 4-form fields as auxiliary fields of the moduli/gravity multiplets. More generally, it would seem that non-minimal $N = 1$ supergravity formulations, with auxiliary field scalars replaced by Minkowski 4-forms, as in refs. [52–58], could be the appropriate formulation to describe the multi-branched nature of string flux vacua.

3.2 Symmetries

The above effective action features remarkable shift and duality symmetries which play an important role in constraining the structure of the scalar potential. In particular the latter is invariant under discrete group transformations acting both on the moduli and the internal fluxes. They correspond to shifts on the axionic components of the Kahler and complex structure moduli combined with the corresponding changes on the internal fluxes. In particular, a shift on the Kahler axion given by

$$b_i \rightarrow b_i + n_i \quad (3.30)$$

combined with

$$m \rightarrow \rho_m = m \quad (3.31)$$

$$q_a \rightarrow \rho_a(b_i = -n_i) = q_a + n_a m \quad (3.32)$$

$$e_i \rightarrow \rho_i(b_i = -n_i) = e_i - k_{ijk}q^j n^k - \frac{m}{2}k_{ijk}n^j n^k \quad (3.33)$$

$$e_0 \rightarrow \rho_0(b_i = -n_i) = e_0 - e_i n_i + \frac{1}{2}k_{ijk}q^i n^j n^k + \frac{m}{6}k_{ijk}n^i n^j n^k \quad (3.34)$$

leaves invariant the scalar potential and relates equivalent vacua. These transformations were first introduced in the toroidal orientifold of ref. [66]. They are however part of the duality symmetries of any CY orientifold. In the mirror Type IIB picture this corresponds to a shift on the complex structure of the torus. Notice that the above transformations leave invariant each 4-form independently, as expected by coming from higher dimensional gauge invariance. Therefore the derivation of this group of transformations is more intuitive in this formulation in terms of 4-forms than in the standard Cremmer et al. supergravity description. They also correspond to the generalization of the Kaloper-Sorbo shift symmetry underlying the axion monodromy inflationary models.

Analogously, the scalar potential is also invariant under shifts on the complex structure moduli of the form

$$c_3^I \rightarrow c_3^I + n^I \tag{3.35}$$

$$e_0 \rightarrow e_0 + h_I n_I \tag{3.36}$$

corresponding to the mirror of Type IIB $SL(2, \mathbb{Z})$ shifts. Also in this case, the 4-forms remain invariant independently.

In a different vein, the effect of performing two or more T-dualities over the system is interesting. Let us consider for simplicity a Type IIA toroidal orientifold compactification, and focus on the diagonal Kahler moduli. The results can be generalised to other geometries with non-trivial one-cycles. Given a basis of 2-forms ω_i such that the Kahler form can be written as

$$J = \sum_{i=1}^3 v^i \omega_i \tag{3.37}$$

we can perform two T-dualities along the two real directions of the Poincare-dual 2-cycle of some ω_i . In particular, if T-duality is performed along $i = 3$ we obtain again a type IIA theory in which

$$v^3 \rightarrow \frac{1}{v^3} \tag{3.38}$$

and the other two fields v^i with $i \neq 3$ remain invariant. In this case v^3 corresponds to the area of the 2-torus along which we perform the two T-dualities. Factors on α' are omitted to avoid clutter but can be easily recovered. Let us assume for simplicity an isotropic compactification such that the triple intersection number is $k_{ijk} = 1$ if all the indices are different, and zero otherwise. The volume of the overall manifold transforms as

$$k = \frac{1}{6} k_{ijk} v^i v^j v^k = v^1 v^2 v^3 \rightarrow \frac{v^1 v^2}{v^3} \tag{3.39}$$

The metric is given in general by

$$g_{ij} = -\frac{1}{4} \left(\frac{k_{ij}}{k} - \frac{1}{4} \frac{k_i k_j}{k^2} \right), \quad g^{ij} = -4k \left(k^{ij} - \frac{v^i v^j}{2k} \right) \tag{3.40}$$

and transforms under the two T-dualities as

$$\frac{g^{33}}{8k} \leftrightarrow \frac{1}{4k}, \quad \frac{g^{11}}{8k} \leftrightarrow kg_{22}, \quad \frac{g^{22}}{8k} \leftrightarrow kg_{11}, \quad 2kg_{33} \leftrightarrow \frac{k}{2} \tag{3.41}$$

The RR part of the scalar potential is invariant under this T-duality if the functions defined in (3.19) are also interchanged

$$\rho_0 \leftrightarrow \rho_i \quad \text{if } i = 3 \tag{3.42}$$

$$\rho_i \leftrightarrow \rho_a \quad \text{if } i \neq a \neq 3 \tag{3.43}$$

$$\rho_a \leftrightarrow \rho_m \quad \text{if } a = 3 \tag{3.44}$$

Therefore T-duality seems to exchange Minkowski 4-forms with each other. Recall that each 4-form comes from dimensionally reducing the field strength of the different higher dimensional RR fields. Then it can be checked that the result matches with the known transformation rules for the RR fields under T-duality,

$$C_3 \leftrightarrow C_5 \quad \text{if } C_5 \text{ propagates along the T-dual direction} \tag{3.45}$$

$$C_5 \leftrightarrow C_7 \quad \text{if } C_7 \text{ (but not } C_5) \text{ propagates along the T-dual direction} \tag{3.46}$$

$$C_7 \leftrightarrow C_9 \quad \text{if } C_9 \text{ (but not } C_7) \text{ propagates along the T-dual direction.} \tag{3.47}$$

Finally, if the internal manifold is T^6 we can perform a T-dual transformation along all the internal dimensions, obtaining

$$k \leftrightarrow \frac{1}{k}, \quad \frac{g^{ij}}{8k} \leftrightarrow kg_{ij} \tag{3.48}$$

and the potential is invariant if

$$\rho_0 \leftrightarrow \rho_m \tag{3.49}$$

$$\rho_i \leftrightarrow \rho_a \tag{3.50}$$

consistent with the transformation rules for the RR fields. Note that the fact that T-dualities relate the different 4-forms make that e.g. only the full V_{RR} combination, involving all 4-forms, will be invariant under dualities and shift symmetries, we will come back to this issue in section 5. Let us conclude by mentioning that non-vanishing values for the 4-forms will generically break SUSY (since they are auxiliary fields). However the discrete symmetries will remain unbroken, since the 4-forms are invariant under them.

3.3 4-forms and geometric fluxes in toroidal Type IIA orientifolds

It is known that beyond standard RR and NS other, less well studied NS fluxes may be present. These include the geometric fluxes in toroidal models that appear in the context of Scherk-Schwarz reductions. In this section we will just explore whether the addition of these fluxes change in any important way the above discussion. We will take an effective viewpoint on geometric fluxes and focus on the essential results. See [65, 67] and references therein for a more thorough discussion of geometric fluxes.

We are interested to see how the presence of geometric fluxes change the 4-forms described in eqs. (3.13). Geometric fluxes are easiest described on a factorized 6-torus $\otimes_{i=1}^3 T_i^2$ with O6-planes wrapping 3-cycles. In addition we assume there is a $Z_2 \times Z_2$ orbifold twist so that only diagonal moduli survive projection. In this case we are left with

3 Kahler moduli and 4 complex structure moduli (including the complex dilaton). In this setting there are 12 geometric fluxes ω_{NK}^M that are conveniently put in a 3-vector a_i and a 3×3 -matrix b_{ij} , see [67, 70] for notation.

Geometric fluxes can be used to convert a p -form into a $(p + 1)$ -form via: $(dX)_{N_1 \dots N_{p+1}} = \omega_{[N_1 N_2 \dots N_{p+1}]K}^K X$, denoted by $\omega \cdot X$. In particular we find:

$$\omega \cdot B = b^i a_i \beta_0 - b^i b_{ij} \beta^j \quad \text{and} \quad \omega \cdot C_3 = -\tilde{\omega}^i a_i c_0 + \tilde{\omega}^i b_{ij} c^j . \quad (3.51)$$

From an effective viewpoint, geometric fluxes change the field strengths of B , C_3 and C_5 as follows [65]:

$$G_4 \rightarrow F_4 + \omega \cdot C_3 - H \wedge C_1 - \omega \cdot B_2 \wedge C_1 + \mathcal{F}e^B , \quad (3.52)$$

$$G_6 \rightarrow F_6 - H \wedge C_3 - \omega \cdot B_2 \wedge C_3 + \mathcal{F}e^B , \quad (3.53)$$

$$H_3 \rightarrow H_3 + \omega \cdot B_2 . \quad (3.54)$$

Putting these field strengths in the in the IIA action and integrating over the internal dimensions as before we find an extra coupling in the NS sector,

$$- \int_Y e^{-2\phi} \omega \cdot B \wedge H_7 = \frac{e^{-2\phi_4}}{k} \left(b^i a_i H_4^0 - b^i b_{ij} H_4^j \right) , \quad (3.55)$$

and two in the RR sector,

$$- \int_Y G_4 \wedge G_6 = F_4^0 (b^i b_{ij} c^j - b^i a_i c_0) - F_4^i (b_{ij} c^j - a_i c_0) . \quad (3.56)$$

In this way the 4-forms get modified as

$$\star F_4^0 = \frac{1}{k} \left[e_0 + b^i e_i - \frac{1}{6} m k_{ijk} b^i b^j b^k + \frac{1}{2} k_{ijk} q_i b^j b^k - h_0 c_3^0 - h_i c_3^i + b^i b_{ij} c_3^j - b^i a_i c_3^0 \right]$$

$$\star F_4^i = \frac{g^{ij}}{4k} \left[e_j + k_{jkl} b^k q^l - \frac{m}{2} k_{jkl} b^k b^l + b_{jk} c_3^k - a_j c_3^0 \right]$$

$$\star H_4^0 = h^0 + b^i a_i$$

$$\star H_4^i = h^i - b^j b_{ji} .$$

The intersection numbers k_{ijk} are equal to one if all the indices are different and 0 otherwise, since we have a toroidal compactification space. It can be shown that the scalar potential that is obtained from these 4-forms and eq. (3.24) can also be obtained from the superpotential given in [65, 67].

One interesting question is how the discrete symmetries are modified in the presence of geometric fluxes. One finds that the 4-forms are still invariant under shifts of the axion in the Kahler moduli and complex structure moduli

$$b^i \rightarrow b^i + n_b^i \quad (3.57)$$

$$c^J \rightarrow c^J + n_c^J \quad (3.58)$$

in combination with

$$h^0 \rightarrow h^0 - a_i n_b^i \tag{3.59}$$

$$h^i \rightarrow h^i + n_b^j b_{ji} \tag{3.60}$$

$$e_j \rightarrow e_j + a_j n_c^0 - b_{jk} n_c^k \tag{3.61}$$

$$e_0 \rightarrow e_0 + h_i n_c^i + h_0 n_c^0 + n_b^i b_{ij} n_c^j - n_b^i a_i n_c^0 \tag{3.62}$$

in combination with the shifts of the previous section. All in all, the general structure for 4-forms we described above remains in the presence of geometric fluxes, as expected.

4 4-forms in Type IIB orientifolds

We turn now to the case of Type IIB $D = 4$ orientifolds. We concentrate on 4-forms coming from the closed string sector but we also briefly mention an example of 4-form arising from the open string sector.

4.1 4-forms and the IIB flux induced scalar potential

Compared to Type IIA the structure in IIB [71, 72] is in principle slightly simpler because the CS couplings are simpler. Only the NS H_3 and RR F_3 tensors have a role in the context of CY $N = 1$ orientifolds. It is convenient to define the complex 3-form

$$G_3 = F_3 - iSH_3 \tag{4.1}$$

where S is the complex dilaton, $S = 1/g_s + ic_0$. The relevant piece in our discussion are the kinetic terms of the 2-forms, which in this complex notation may be written as

$$S_{\text{IIB}} = -\frac{1}{2k_{10}^2} \int_{\mathbb{R}^{1,3} \times Y} \frac{1}{3!} \frac{1}{S + S^*} G_3 \wedge {}^* \bar{G}_3 \tag{4.2}$$

where ${}^* \bar{G}_3 = \bar{G}_7$. As we did in the Type IIA case we can now expand G_7 in terms of internal harmonics with coefficients given by Minkowski 4-forms. We will consider here only IASD G_3 fluxes of class $(3, 0)$ and $(2, 1)$, which can induce SUSY-breaking. The contribution from ISD G_3 fluxes does not depend on the moduli and it is proportional to the topological number giving the flux contribution to the D3 RR charge [12, 73], so it appears (combined with the contribution from localised sources) in the tadpole cancellation conditions. Then the relevant expansion is given by

$$\bar{G}_7 = \bar{G}_4^0 \wedge \bar{\Omega} + \bar{G}_4^a \wedge \chi_a, \quad a = 1, \dots, h_{21}, \tag{4.3}$$

where Ω is the holomorphic $(3, 0)$ form, and the χ_a form a basis of the h_{21} 3-forms in the CY X . Here G_4^0 and G_4^a are complex Minkowski 4-forms which may be written in terms of NS and RR pieces F_4, H_4 as

$$G_4^0 = F_4^0 - iSH_4^0, \quad G_4^a = F_4^a - iSH_4^a. \tag{4.4}$$

The basis of (2, 1) forms may be expressed in terms of the holomorphic 3-form Ω and the complex structure Kahler potential K as [74]

$$\chi_a = \frac{\partial\Omega}{\partial U_a} + K_{U_a}\Omega. \tag{4.5}$$

From the term $G_3 \wedge \tilde{G}_7$ we then get the kinetic terms for the Minkowski 3-forms and some couplings to the dilaton and complex structure moduli. For G_4^a the coupling is given by

$$\begin{aligned} \frac{1}{S+S^*} \sum_a \bar{G}_4^a \int_X G_3 \wedge \left(\frac{\partial\Omega}{\partial U_a} + K_{U_a}\Omega \right) &= \frac{1}{S+S^*} \sum_a \bar{G}_4^a D^a \int_X G_3 \wedge \Omega \\ &= \frac{1}{S+S^*} \sum_a \bar{G}_4^a D^a W_{\text{GVW}}, \end{aligned} \tag{4.6}$$

where W_{GVW} is the Gukov-Vafa-Witten superpotential and D^a are the Kahler covariant derivatives with respect to the complex structure fields U_a . For the remaining 4-form G_4^0 one gets the coupling

$$\frac{1}{S+S^*} \bar{G}_4^0 \int_X G_3 \wedge \bar{\Omega} = -\bar{G}_4^0 (\overline{D_S W_{\text{GVW}}}). \tag{4.7}$$

The kinetic terms of the 7-form yield the quadratic pieces

$$\frac{\kappa}{S+S^*} (|G_4^0|^2 - G_4^a \bar{G}_4^b G_{a\bar{b}}), \tag{4.8}$$

where $G_{a\bar{b}}$ is the metric of the complex structure fields and

$$\kappa = \int_X \Omega \wedge \bar{\Omega} = ie^{-K_{\text{c.s.}}(U_a)}, \quad G_{a\bar{b}} = -\frac{\int_X \chi_a \wedge \bar{\chi}_b}{\int_X \Omega \wedge \bar{\Omega}}, \tag{4.9}$$

where $K_{\text{c.s.}}(U_a)$ is the Kahler potential of the complex structure moduli. Collecting all the pieces, the ten dimensional action (4.2) reduces to the following four dimensional effective lagrangian in terms of the Minkowski 4-forms,

$$\mathcal{L}_{\text{IIB}} = \frac{1}{S+S^*} \left(\kappa (|G_4^0|^2 - G_4^a \bar{G}_4^b G_{a\bar{b}}) - \bar{G}_4^0 (S+S^*) \overline{D_S W_{\text{GVW}}} + \sum_a \bar{G}_4^a D^a W_{\text{GVW}} \right) \tag{4.10}$$

Notice that, in analogy to Type IIA, the full scalar potential, excluding the contribution from localised sources, can be written in terms of the Minkowski 3-form fields. One can now introduce Lagrange multipliers enforcing $dC_3 = F_4$ for each of the Minkowski 3-forms. Upon using the equations of motion for the 4-forms one gets

$$\begin{aligned} *G_4^{\bar{b}} &= -ie^{K_{\text{c.s.}}} G^{a\bar{b}} (D_a W_{\text{GVW}} + (f_4 - iSh_4)_a), \\ *G_4^0 &= -ie^{K_{\text{c.s.}}} ((S+S^*) \overline{D_S W_{\text{GVW}}} + (f_4 - iSh_4)_0) \end{aligned} \tag{4.11}$$

where $f_4^{a,0}, h_4^{a,0}$ are RR and NS constants, from the Lagrange multipliers. We thus see that the complex 4-forms $G_4^{a,0}$ are associated to the auxiliary fields of the complex structure and dilaton, but include also a shift associated to the Minkowski 4-form backgrounds. This

result is very similar to the one discussed around eq. (2.14) and suggested in [54, 58, 59]. The main difference is that here the two $N = 1$ auxiliary fields are replaced by 4-forms and also here it is the *supergravity* auxiliary fields, with the covariant derivatives D_a, D_S , which suffer the shift. Also the shift depends on the complex dilaton. In Type IIA we argued that the classical Hodge dualities forced this shift to vanish, identifying the constant background terms of the Minkowski 4-forms with the internal fluxes of the magnetic duals. The analogy here would be to set $f_4, h_4 = 0$ with the argument that the internal fluxes parametrizing the G_3 background are enough to account for all the degrees of freedom (there should not be extra parameters). However we do not discard completely the possibility of an integer quantum shift, not visible in the classical expressions here considered.

By inserting eqs.(4.11) in the Lagrangian (4.10) we get the following scalar potential

$$V = e^{K_S + K_{c.s.}} \left(|(S + S^*) \overline{D_S W} + g_0|^2 + K^{a\bar{b}} |D_a W - g_a|^2 \right) \quad (4.12)$$

where we have used that $K_S = -\log(S + S^*)$ and redefined $g_{0,a} \equiv (f_4 - iSh_4)_{0,a}$. If the shifts vanish, we recover the standard formula for the $N = 1$ supergravity scalar potential. Note that, due to the no-scale structure (there is no dependence of the superpotential on the Kahler moduli), after using the equations of motion for the 4-forms one obtains a positive definite scalar potential of the standard no-scale form.

Finally, the same web of transformations studied in section 3.2 relating different vacua of Type IIA compactified in a *CY* threefold, are also present in Type IIB compactified in the mirror $\tilde{C}Y$. The discrete shift given by (3.30) acting on a Kahler modulus of Type IIA corresponds to a shift on the complex structure of the mirror Type IIB. Notice that since the supergravity description to leading order in α' is reliable at large volume, this shift symmetry will arise at the large complex structure limit of the mirror $\tilde{C}Y$. If we are dealing with a toroidal compactification instead, then the shift symmetry will correspond to the usual complex structure reparametrizations of the torus. Recall that this shift on the complex structure (in Type IIB) or in the Kahler modulus (in Type IIA) leaves invariant the effective theory only if it is combined with the corresponding transformations on the internal fluxes, studied in section 3.2. Analogously a shift on the complex structure (3.35) in Type IIA maps to a shift on the axionic component of the Kahler moduli in Type IIB. This latter shift symmetry is expected from the fact that the imaginary part of the a Kahler modulus in IIB is actually an axion coming from dimensionally reducing the RR field C_4 .

While in Type IIA the current description in terms of 4-forms offered a very intuitive picture about these transformation (leaving each 4-form invariant), the situation in Type IIB is less transparent. Since we only have the 4-forms coming from G_7 we can not decompose the scalar potential into different smaller invariant pieces. Hence in the end the exercise of finding the transformation rules in this description is not easier than just studying the symmetries of the full scalar potential (or of the auxiliary fields). We would like to remark though that this set of transformations leave each 4-form invariant and are the generalization of the shift symmetry of axion monodromy models and the Kaloper-Sorbo Lagrangian. In other words, string theory provides a rich and more intriguing web of duality symmetries which are the generalization of the aforementioned shift symmetry.

Besides, the full appearance of the ‘axionic’ moduli in terms of couplings to the Minkowski 3-form fields highly constrains the form of α' and perturbative corrections. As we will see in section 5 this structure acts as a sort of ‘chiral symmetry’ protecting the scalar potential from dangerous higher order corrections, apparently independently whether the field appears or not in the 4d Kahler potential.

Before closing this section let us make a few remarks about non-geometric fluxes [69, 75] in toroidal Type IIB orientifolds, see e.g.([70]) for a brief description of these fluxes. Non-geometric fluxes are still poorly understood although their existence is implied by T-dualities. They are known to induce additional terms in the superpotential. Type IIB orientifolds allow only for so called Q -type non-geometric fluxes (we use notation in [69]). The fluxes have index structure Q_M^{NP} with antisymmetric upper indices and they are odd under the $O(3)$ orientifold involution. In IIB there are no geometric ω_{NP}^M nor non-geometric R^{MNP} fluxes which are even. The effect of the Q -fluxes on the Gukov-Vafa superpotential is captured by the replacement

$$G_3 = (F_3 - iSH_3) \longrightarrow G_3 + \mathcal{Q}\mathcal{J}_c \tag{4.13}$$

where the 4-form

$$\mathcal{J}_c = i \sum_{i=1}^3 T_i \tilde{\omega}_i, \tag{4.14}$$

with T_i the three diagonal Kahler moduli and

$$(\mathcal{Q}\mathcal{J}_c)_{MNP} = \frac{1}{2} Q_{[M}^{AB} (\mathcal{J}_c)_{NP]AB}. \tag{4.15}$$

Going back now to the 4-forms in IIB, eq. (4.6) gets modified as

$$\frac{1}{S + S^*} \sum_a \bar{G}_4^a D^a \int_X (G_3 + \mathcal{Q}\mathcal{J}_c) \wedge \Omega. \tag{4.16}$$

The right-hand side is nothing but the Kahler derivative (with respect to the complex structure) of the extension of the GVW super potential to include non-geometric fluxes. So it seems that also in the presence of this class of non-geometric fluxes the structure of the Minkowski 4-forms acting as auxiliary fields in the effective action persists.

4.2 Minkowski 4-forms and open string moduli

Up to here we have discussed the role of Minkowski 3-form fields in the closed string sector of Type II. We have seen that the full RR and NSNS axion dependence of the flux scalar potential can be written in terms of these 3-forms. A similar question arises for the open string sector of the theory. Can also the scalar potential of the open (periodic) string moduli be written in terms of 3-form fields? In this section we address the issue for the D7-brane moduli sector of a Type IIB orientifold compactification. In particular, we review the computation done in [45], for which the flux induced scalar potential of a D7 position modulus can be written in terms of a Kaloper-Sorbo coupling of the scalar with a Minkowski 3-form field arising from the magnetic open string field strength. In [45] the

goal was to derive the Kaloper-Sorbo Lagrangian of a concrete inflationary model, dubbed Higgs-otic inflation, in which the inflaton is the position modulus of a wandering D7-brane in a transverse torus. This way one can use the Kaloper-Sorbo symmetry properties to argue from an effective approach that the higher order corrections are under control and do not spoil inflation. Here we derive the effective theory and discuss the result with the new insight gained from previous sections.

In the open string sector of Type II string theory, Minkowski 3-forms may arise from the dual magnetic potentials of the worldvolume gauge fields of the D-branes. In particular, for a D7-brane the magnetic gauge potential is a 5-form A_5 , whose field strength can be expanded as

$$F_6 = iF_4 \wedge \bar{\omega}_2 - i\bar{F}_4 \wedge \omega_2 \tag{4.17}$$

where ω_2 is a (2,0)-form associated to the position modulus Φ of the D7. This field can be expanded as $\Phi = \phi\omega_2$ where ϕ is a 4d complex scalar. Notice that unlike the 4-forms coming from the closed string sector, now F_4 is a complex Minkowski 4-form. We are going to focus on the Abelian case, but a priori it could be generalised to non-Abelian gauge groups.

Consider ISD G_3 bulk fluxes inducing a B-field on the brane given by [45, 76–80]

$$B_2 = \frac{g_s\sigma}{2i}(G^*\phi - S\bar{\phi})\omega_2 + cc. \ , \tag{4.18}$$

where we have denoted the non-supersymmetric ISD (0,3)-flux as $G \equiv G_{\bar{1}\bar{2}\bar{3}}$ and the supersymmetric (2,1)-flux as $S \equiv \epsilon_{3jk}G_{3\bar{j}\bar{k}}$ (see [80] for notation). The relevant part of the DBI action to leading order in α' , ie. in the Yang-Mills approximation, is given by [45, 76, 80]

$$\mathcal{S}_{\text{DBI}} = \mu_7\sigma \int_{\mathbb{R}^{1,3} \times S_4} \frac{1}{2}(B_2 + \sigma F_2) \wedge *_8(B_2 + \sigma F_2) + \dots \tag{4.19}$$

where $\sigma = 2\pi\alpha'$. Plugging the decomposition (4.17) into the above Lagrangian and performing dimensional reduction we obtain

$$\int_{S_4} F_6 \wedge *_8 F_6 = |F_4|^2 2 \int_{S_4} \omega_2 \wedge *_4 \bar{\omega}_2, \tag{4.20}$$

$$\int_{S_4} B_2 \wedge F_6 = \frac{1}{2}g_s\sigma (F_4(G^*\phi - S\phi^*) + \bar{F}_4(G\phi^* - S^*\phi)) \int_{S_4} \omega_2 \wedge \bar{\omega}_2, \tag{4.21}$$

leading to the following effective four dimensional Lagrangian

$$\mathcal{L}_4 = \mu_7\sigma\rho \left(|F_4|^2 - \frac{1}{2}g_s\sigma (F_4(G^*\phi - S\phi^*) + \bar{F}_4(G\phi^* - S^*\phi)) \right) + \dots \tag{4.22}$$

Here $\rho = \int_{S_4} \omega_2 \wedge *_4 \bar{\omega}_2$ and we have used that $*_4\omega_2 = -\omega_2$. Upon integrating out the 3-form field we get

$$V_4 = \mu_7\sigma\rho \left| f - \frac{1}{2}g_s\sigma(G^*\phi - S\phi^*) \right|^2, \tag{4.23}$$

where f is an integration constant which can be identified with the magnetic flux F_2 . Note however that, by an appropriate choice for the B_2 gauge, the constant term f may be

reabsorbed into the definition of what the origin of the wandering D7-brane is. In fact it was took equal to zero in [45]. The above expression reflects the branched properties of the Higgs vev as the D7-moves along a cycle in the torus. The potential is invariant under shifts on the position modulus if they are combined with the corresponding shift on F_2 flux. This shift symmetry underlies the typical multi-branch structure of a Kaloper-Sorbo Lagrangian (or an F-term axion monodromy model), its presence being important to keep the potential under control in large field inflationary models. The idea again is that the underlying shift symmetry and the gauge invariance of the 3-form field protects the potential from dangerous higher order corrections, as we will discuss in section 5. Once a specific branch is chosen, ie. the flux background is fixed, we can inflate with the position modulus inducing the monodromy and allowing for large field excursions. Let us finally note that we recover only half of the complete scalar potential because we are missing the Chern-Simons part of the action, which because of supersymmetry will give the same contribution as in eq.(4.23) (see [81] for a similar computation of the flux-induced scalar potential obtained from the Chern-Simons action of a D5-brane, in which the authors also keep explicitly the presence of the 4-forms coming from the RR fields).

In addition to the above quadratic piece, the position modulus can also have couplings with other matter fields. Yukawa couplings with the D7 Wilson lines A_I can be obtained by considering the non-abelian part of F_2 in eq.(4.20) and can also be written in terms of the Minkowski 3-form field.

One can think of exploring a similar structure within Type IIA orientifolds. The magnetic gauge field is a 4-form A_4 , which has to be expanded in a basis of 1-forms on the $D6$ -brane 3-cycles in order to get a Minkowski 3-form field. This can be done e.g. in toroidal models, recovering the T-dual picture of the D7-brane models discussed above, and also CY's with appropriate 3-cycle topology. One can expand also the magnetic gauge field in a basis of torsion cycles obtaining the models of Massive Wilson lines discussed in [25]. We leave the study of this IIA case for future work.

5 4-forms, inflation and stability of scalar potentials

5.1 Stability of axions in flux vacua

In this section we discuss possible physical consequences of the structure of flux vacua described in terms of minkowski 4-form fluxes as discussed above. For these applications a crucial point is that we have found that all the RR and NS axion dependence of the flux scalar potential goes always through Minkowski 3-forms. And by gauge invariance of the latter, the flux potential, even after α' and perturbative corrections are considered, should admit an expansion in powers of the gauge invariant Minkowski 4-forms, i.e. for V_{RR} in Type IIA

$$V(b_i, c_a) = \sum_{r, s_i, t_a, u} \frac{c_{rs_i t_a u}}{m_p^{4(w-1)}} (F_0^{2r})(F_m^{2u})(\Pi_i(g_{kl}F^k F^l)^{s_i})(\Pi_a(g_{bc}F^b F^c)^{t_a}) - V_{\text{non-axionic}}, \tag{5.1}$$

where r, u, s_i, t_a are integers and the c 's are coefficients depending on the non-axionic components of the Kahler and complex structure moduli. F_0, F_i, F_a, F_m are the 4-forms

discussed in section 3 (contracted with the Levi-Civita tensor), and

$$w = r + u + \sum s_i + \sum t_a - 4 . \tag{5.2}$$

$V_{\text{non-axionic}}$ collects pieces of the potential which will not depend on the axion fields, like the local contributions in V_{local} discussed in section 3. Note that for a single factor, e.g $t_1 \neq 0$, $g_{11} = 1$ with the rest of the integers vanishing, one gets the familiar Kaloper-Sorbo structure of the type

$$V(b_1) = \sum_{n \geq 1} \frac{c_n}{m_p^{4(n-1)}} V_0^n = \sum_{n \geq 1} \frac{c_n}{m_p^{4(n-1)}} (F_1)^{2n} = \sum_{n \geq 1} \frac{c_n}{m_p^{4(n-1)}} (q_1 + mb_1)^{2n} , \tag{5.3}$$

and a discrete symmetry $q_1 \rightarrow q_1 - m$, $b_1 \rightarrow b_1 + 1$. As claimed in [22], due to this symmetry and the gauge invariance of F_1 , all corrections to the leading quadratic potential are suppressed. Indeed, if applied to inflation, the axion/inflaton b_1 can have large field trans-Planckian excursions since, with a Hubble parameter at inflation $H_I \simeq 10^{16}$ GeV, the possible corrections will be suppressed by powers V_0/m_p^4 and there will not be isolated b_1^n/m_p^n terms in the potential.

As we have seen, in string theory the story is slightly more complicated, there are many 4-forms in the game and a complicated moduli fixing potential. Still the message we have found is similar. All axion dependence comes through 4-forms, which are gauge invariant, and each 4-form is invariant under discrete transformations under which the axions shift while internal fluxes also shift. These transformations are a subset of the duality symmetries present in a given CY compactification. The axions are not real axions, in the sense that they may have masses and Yukawa couplings. They could be called *multi-branched axions* since they feature a discrete shift symmetry as long as internal fluxes are also shifted. This is the branched structure which has appeared in the past in the context of F-term monodromy inflation [25]. Moreover, the quadratic potential of Kaloper-Sorbo is replaced by more general polynomials up to order six.

In fact the situation in string theory is often much simpler than what eq. (5.1) seems to indicate. Indeed, as we have shown in the Type IIA toroidal orbifold example, although the 4-forms are invariant under axion shifts, they transform into each other under duality transformations, T-dualities in this example. Due to the transformations in eq. (3.41) the different RR 4-forms appear in the particular combination V_{RR} in eq. (3.24) so that actually the corrections to the RR potential will appear as an expansion in powers of V_{RR} itself. More generally, the full duality group of a specific Type II orientifold would often force the corrections to the original potential to be an expansion in powers of the leading potential V_0 , i.e. one expects in these cases

$$V = \sum_n c_n V_0^n , \tag{5.4}$$

where V_0 is the tree level, leading order in α' , flux potential.

In the applications of this setting to inflation, the higher order corrections in V_0 , although under control, may be non-negligible in the particular case of large field inflation. In fact in that case they may lead to a flattening effect [82] so that, although for small

field the uncorrected V_0 gives an appropriate description, the asymptotic behaviour of the potential for large field gets flattened. This has been observed in the context of certain monodromy inflation models. It may also appear as an effect of the interaction of the inflaton with heavy modes. This flattening effect is important, since e.g. it makes standard quadratic chaotic inflation become e.g. linear and be consistent with Planck/BICEP limits. Note however that this flattening effect does not modify the value of masses at the origin.

In models of inflation, in which the inflaton appears in the Dirac-Born-Infeld action of a D-brane, the α' corrections to the scalar potential do appear as a series expansion in the leading scalar potential V_0 , see e.g. [25–27, 45, 83], in agreement with (5.4). It arises upon expansion of the square root in the DBI action or, in the case in which the scalar is an open string mode, due to a non-canonical redefinition of the scalar kinetic term. An example of this effect is discussed in [45] in which the inflaton (which in this case is identified with a MSSM Higgs) has an action with terms of the general form

$$\mathcal{L} = -(1 + \xi V_0)(D_\mu \phi)^2 - V_0 + \dots, \tag{5.5}$$

where in this case V_0 is just quadratic and ξ is a constant factor proportional to α' . After setting the kinetic term in canonical form, α' corrections to the potential appear as a power series in V_0 , giving rise to a linear behaviour for large ϕ . In this case ϕ parametrizes the position of a D-brane on a torus, which is T-dual to a continuous Wilson line. Although here ϕ is not a closed string monodromy axion, the model is an example of monodromy inflation since there is a shift symmetry corresponding to discrete translation of the D-brane of the torus and a non-trivial scalar potential arising from fluxes. As discussed in section 4.2 the scalar potential V_0 admits a description in terms of a complex open string Minkowski 4-form.

Let us also note in closing that kinetic term redefinitions like that appearing in eq. (5.5) also appear in computing the higher derivative corrections to general $N = 1$ supergravity Lagrangians, see [84, 85] and references therein.

5.2 Multi-branched axions, scalar stability and naturalness

There are essentially a two well established ideas in order to make stable scalar masses against loop corrections and get naturally light scalars, i.e. naturalness. One of them is supersymmetry and the other is Goldstone bosons. To the latter case belong the (continuous) Peccei-Quinn shift symmetry of axions. This symmetry is only broken at the non-perturbative level by instanton effects. It has however the drawback that axions have only derivative couplings (e.g., no Yukawas). Analogously, there are BSM models in which the Higgs fields are Goldstones bosons of a spontaneously broken global symmetry but again Yukawa couplings are forbidden to leading order.

The class of flux potentials discussed in this paper seem to show the existence of a third alternative to achieve scalar masses stable against loop corrections. These are fields analogous to the *multi-branched axions* discussed in the previous sections. These fields present a discrete shift invariance when accompanied by adequate transformations of de Lagrangian parameters, fluxes in the case at hand. So it is important to realise that these

are not symmetries of the field action. Rather, this is a symmetry of a Landscape of Lagrangians differing by the different values of the parameters, fluxes in the string theory case. But this Landscape structure, with different values for fluxes, is what one really finds in string theory. Membrane domain walls, coupled to 3-forms are able to interpolate between potentials corresponding to different Lagrangian parameters (fluxes).

A very important difference with the axion or, in general, Goldstone boson symmetry is that the symmetry is *consistent with interactions*. Consider as an explicit model that provided by the RR potential V_{RR} of Type IIA string compactifications in section 3. The potential presents polynomial interactions of the scalars b_i , which are still consistent with shifts $b_i \rightarrow b_i + n_i$ as long as the e_0, e_i, q_a parameters are transformed as in eq. (3.34). There is in fact a landscape of potentials corresponding to different choices for these parameters, and the symmetries relate different potentials and vacua. In view of these symmetries and the landscape structure, the perturbative corrections to this scalar potential should appear as a power series in the tree potential V_{RR} . One expects for the corrected potential to be a function $V = f(V_{RR})$ of the classical potential. Assume that the uncorrected potential $V_{RR}^0(\phi_i)$ has a minimum at some values $\phi_i = \phi_i^0$, Then at that minimum

$$\frac{dV_{cor}}{d\phi_i} = \left(\frac{df}{dV_{RR}} \right) \left(\frac{dV_{RR}}{d\phi_i} \right) = 0 \tag{5.6}$$

so that the corrected potential has also in general a extremum there. For the masses at that point one then has

$$\frac{\partial^2 V_{cor}}{\partial \phi_i \partial \phi_j} = \left(\frac{\partial f}{\partial V_{RR}} \right) \left(\frac{\partial^2 V_{RR}}{\partial \phi_i \partial \phi_j} \right) \tag{5.7}$$

and for $\frac{\partial f}{\partial V_{RR}} > 0$ the corrected potential has also a minimum there.

The obvious observation is that both mass matrices are proportional and hence, for an analytic function f , if the uncorrected potential has a massless scalar, the corrected potential will also have a massless scalar. There exists a sort of *scalar chirality* for the scalar potential if the structure (5.4) is true. In particular it would seem that modifications like e.g. a quadratic loop divergence for the masses of scalars should be forbidden, since they would violate explicitly this property. One way to understand this is to think that, from the effective field theory point of view, upon renormalisation we should have to use a regulator which is consistent with 4-form gauge invariance and the shift and duality symmetries, and that such a regulator should not allow for a quadratic cut-off. While working on a given branch, a perturbative calculation would yield e.g. a quadratic scalar divergence for interacting scalars like these. However that would violate the branched structure of the theory. Once taking the latter into account such divergences should be forbidden by the symmetries.

It is tempting to speculate about the application of this stability property to the SM hierarchy problem and the Higgs. Although the Higgs field is not directly an axion, there are string constructions in which the Higgs degrees of freedom may be identified with complex Wilson lines (see e.g. [45] and references there in) or their T-dual, D-brane position moduli. In such a case the scalar Higgs may possess a multi-branch structure and one could conceive such an scenario. The symmetries would forbid quadratic divergences for the

Higgs mass. Still the scalar potentials considered in this paper contain only moduli and no gauge interactions with charged fields. Furthermore, in order to be useful, one should be able to obtain a SUSY breaking scale much larger than the discrete symmetry breaking scale, so that it is the latter which is making scalars light rather than SUSY. In this respect one should play with the different flux degrees of freedom and compact volumes, which requires a complete scheme of moduli fixing in De Sitter. It would be interesting to see whether models with these characteristics and the property (5.4) can be built. We leave the study of that possibility to future research.

6 Conclusions and outlook

In this paper we have studied the role of Minkowski 3-forms in (orientifold) flux compactifications of Type IIA and Type IIB theory. To this aim we have performed an explicit dimensional reduction of the D=10 Type II actions in the presence of RR and NS internal fluxes, keeping track of the resulting Minkowski 4-forms and their couplings. These external fluxes are in one to one correspondence with the more familiar internal fluxes. We find that the Minkowski 4-forms act as auxiliary fields of the Kahler and complex structure moduli of the compactifications. This is consistent with the fact that 3-forms in Minkowski have no propagating degrees of freedom, but the corresponding field strength 4-forms may contribute to the cosmological constant.

We find that the dependence of the flux scalar potential on the RR and NS axions always goes through the corresponding Minkowski 4-forms. In this context it is then important to realise that in any Type II orientifold compactifications there are symmetries under which these RR and NS axions suffer discrete shifts as long as appropriate discrete translations of the internal fluxes are performed. Interestingly, the 4-forms are invariant under these transformations. This, combined with the (3-form) gauge invariance, forces the axion scalar potentials, even after perturbative corrections are included, to be expressible as an expansion in powers of the 4-forms. We also argue, and exemplify in a Type IIA toroidal case, that additional duality symmetries will typically force a more restrictive structure with a perturbatively corrected scalar potential being a power series of the leading order potential V_0 , as in eq. (5.4). This would be both a multifield generalisation with higher order couplings and a string realisation of the field theory idea of Kaloper and Sorbo. An important difference though is that the axions in this paper possess non-trivial polynomial interactions.

The use of 4-forms, acting as auxiliary fields, to describe string flux vacua is most appropriate to reveal the multi branched structure of flux scalar potentials. We have found that in both Type IIA and Type IIB orientifolds keeping track of the 4-forms appearing upon compactification allow us to identify in a simpler way the underlying symmetries of the flux multi branched vacua. The corresponding 3-forms couple to membranes which can break these symmetries but only at the non-perturbative level, inducing vacuum transitions through domain walls.

The discrete symmetries which are preserved by the 4-forms are not standard single Lagrangian symmetries but rather symmetries of a landscape of Lagrangians parametrized

by the different internal fluxes. Not only fields transform, but masses and couplings (fluxes) as well. The RR and NS fields are not axions in the usual sense (since their flux couplings break the Peccei-Quinn symmetry explicitly) but *multi-branched axions*, which are only invariant under discrete symmetries accompanied by flux transformations. The property in eq. (5.4), preserved in an α' expansion, would be of interest for large field models of inflation in the string theory context, particularly in models of F-term monodromy [25], which directly contain this Kaloper-Sorbo structure. The symmetries will protect the corresponding axion/inflaton when featuring trans-Planckian trips. Furthermore one expects the loop corrections to preserve also the structure in (5.4). In particular we have argued that quadratic divergences are not expected to appear for these *multi-branched axions* even though they can have non-trivial couplings and masses and no supersymmetry. This would provide for a new mechanism to maintain interacting scalars stable against quadratic loop corrections. We hope to report on possible applications of these ideas in future work.

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References

- [1] Z. Bern, C. Cheung, H.-H. Chi, S. Davies, L. Dixon and J. Nohle, *Evanescent effects can alter ultraviolet divergences in quantum gravity without physical consequences*, *Phys. Rev. Lett.* **115** (2015) 211301 [[arXiv:1507.06118](https://arxiv.org/abs/1507.06118)] [[INSPIRE](#)].
- [2] M.J. Duff and P. van Nieuwenhuizen, *Quantum inequivalence of different field representations*, *Phys. Lett.* **B 94** (1980) 179 [[INSPIRE](#)].
- [3] S.W. Hawking, *The cosmological constant is probably zero*, *Phys. Lett.* **B 134** (1984) 403 [[INSPIRE](#)].
- [4] M.J. Duff, *The cosmological constant is possibly zero, but the proof is probably wrong*, *Phys. Lett.* **B 226** (1989) 36 [[INSPIRE](#)].
- [5] Z.C. Wu, *The cosmological constant is probably zero and a proof is possibly right*, *Phys. Lett.* **B 659** (2008) 891 [[arXiv:0709.3314](https://arxiv.org/abs/0709.3314)] [[INSPIRE](#)].
- [6] M.J. Duncan and L.G. Jensen, *Four forms and the vanishing of the cosmological constant*, *Nucl. Phys.* **B 336** (1990) 100.
- [7] J.D. Brown and C. Teitelboim, *Neutralization of the cosmological constant by membrane creation*, *Nucl. Phys.* **B 297** (1988) 787 [[INSPIRE](#)].

- [8] J.D. Brown and C. Teitelboim, *Dynamical neutralization of the cosmological constant*, *Phys. Lett. B* **195** (1987) 177 [[INSPIRE](#)].
- [9] R. Bousso and J. Polchinski, *Quantization of four form fluxes and dynamical neutralization of the cosmological constant*, *JHEP* **06** (2000) 006 [[hep-th/0004134](#)] [[INSPIRE](#)].
- [10] J.L. Feng, J. March-Russell, S. Sethi and F. Wilczek, *Saltatory relaxation of the cosmological constant*, *Nucl. Phys. B* **602** (2001) 307 [[hep-th/0005276](#)] [[INSPIRE](#)].
- [11] S. Kachru, R. Kallosh, A.D. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev. D* **68** (2003) 046005 [[hep-th/0301240](#)] [[INSPIRE](#)].
- [12] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev. D* **66** (2002) 106006 [[hep-th/0105097](#)] [[INSPIRE](#)].
- [13] J.P. Conlon, F. Quevedo and K. Suruliz, *Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking*, *JHEP* **08** (2005) 007 [[hep-th/0505076](#)] [[INSPIRE](#)].
- [14] V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **03** (2005) 007 [[hep-th/0502058](#)] [[INSPIRE](#)].
- [15] M. Graña, *Flux compactifications in string theory: a comprehensive review*, *Phys. Rept.* **423** (2006) 91 [[hep-th/0509003](#)] [[INSPIRE](#)].
- [16] M.R. Douglas and S. Kachru, *Flux compactification*, *Rev. Mod. Phys.* **79** (2007) 733 [[hep-th/0610102](#)] [[INSPIRE](#)].
- [17] F. Denef, *Les Houches lectures on constructing string vacua*, [arXiv:0803.1194](#) [[INSPIRE](#)].
- [18] G. Dvali, *Large hierarchies from attractor vacua*, *Phys. Rev. D* **74** (2006) 025018 [[hep-th/0410286](#)] [[INSPIRE](#)].
- [19] G. Dvali, *Three-form gauging of axion symmetries and gravity*, [hep-th/0507215](#) [[INSPIRE](#)].
- [20] G. Dvali, *A vacuum accumulation solution to the strong CP problem*, *Phys. Rev. D* **74** (2006) 025019 [[hep-th/0510053](#)] [[INSPIRE](#)].
- [21] G. Dvali, S. Folkerts and A. Franca, *How neutrino protects the axion*, *Phys. Rev. D* **89** (2014) 105025 [[arXiv:1312.7273](#)] [[INSPIRE](#)].
- [22] N. Kaloper and L. Sorbo, *A natural framework for chaotic inflation*, *Phys. Rev. Lett.* **102** (2009) 121301 [[arXiv:0811.1989](#)] [[INSPIRE](#)].
- [23] N. Kaloper, A. Lawrence and L. Sorbo, *An ignoble approach to large field inflation*, *JCAP* **03** (2011) 023 [[arXiv:1101.0026](#)] [[INSPIRE](#)].
- [24] A.D. Linde, *Chaotic inflation*, *Phys. Lett. B* **129** (1983) 177 [[INSPIRE](#)].
- [25] F. Marchesano, G. Shiu and A.M. Uranga, *F-term axion monodromy inflation*, *JHEP* **09** (2014) 184 [[arXiv:1404.3040](#)] [[INSPIRE](#)].
- [26] E. Silverstein and A. Westphal, *Monodromy in the CMB: gravity waves and string inflation*, *Phys. Rev. D* **78** (2008) 106003 [[arXiv:0803.3085](#)] [[INSPIRE](#)].
- [27] L. McAllister, E. Silverstein and A. Westphal, *Gravity waves and linear inflation from axion monodromy*, *Phys. Rev. D* **82** (2010) 046003 [[arXiv:0808.0706](#)] [[INSPIRE](#)].
- [28] M. Berg, E. Pajer and S. Sjörs, *Dante's inferno*, *Phys. Rev. D* **81** (2010) 103535 [[arXiv:0912.1341](#)] [[INSPIRE](#)].

- [29] L.E. Ibáñez and I. Valenzuela, *BICEP2, the Higgs mass and the SUSY-breaking scale*, *Phys. Lett. B* **734** (2014) 354 [[arXiv:1403.6081](#)] [[INSPIRE](#)].
- [30] E. Palti and T. Weigand, *Towards large r from $[p, q]$ -inflation*, *JHEP* **04** (2014) 155 [[arXiv:1403.7507](#)] [[INSPIRE](#)].
- [31] R. Blumenhagen and E. Plauschinn, *Towards universal axion inflation and reheating in string theory*, *Phys. Lett. B* **736** (2014) 482 [[arXiv:1404.3542](#)] [[INSPIRE](#)].
- [32] N. Kaloper and A. Lawrence, *Natural chaotic inflation and ultraviolet sensitivity*, *Phys. Rev. D* **90** (2014) 023506 [[arXiv:1404.2912](#)] [[INSPIRE](#)].
- [33] L.E. Ibáñez and I. Valenzuela, *The inflaton as an MSSM Higgs and open string modulus monodromy inflation*, *Phys. Lett. B* **736** (2014) 226 [[arXiv:1404.5235](#)] [[INSPIRE](#)].
- [34] A. Hebecker, S.C. Kraus and L.T. Witkowski, *D7-brane chaotic inflation*, *Phys. Lett. B* **737** (2014) 16 [[arXiv:1404.3711](#)] [[INSPIRE](#)].
- [35] M. Arends et al., *D7-brane moduli space in axion monodromy and fluxbrane inflation*, *Fortsch. Phys.* **62** (2014) 647 [[arXiv:1405.0283](#)] [[INSPIRE](#)].
- [36] L. McAllister, E. Silverstein, A. Westphal and T. Wrase, *The powers of monodromy*, *JHEP* **09** (2014) 123 [[arXiv:1405.3652](#)] [[INSPIRE](#)].
- [37] S. Franco, D. Galloni, A. Retolaza and A. Uranga, *On axion monodromy inflation in warped throats*, *JHEP* **02** (2015) 086 [[arXiv:1405.7044](#)] [[INSPIRE](#)].
- [38] R. Blumenhagen, D. Herschmann and E. Plauschinn, *The challenge of realizing F-term axion monodromy inflation in string theory*, *JHEP* **01** (2015) 007 [[arXiv:1409.7075](#)] [[INSPIRE](#)].
- [39] H. Hayashi, R. Matsuda and T. Watari, *Issues in complex structure moduli inflation*, [arXiv:1410.7522](#) [[INSPIRE](#)].
- [40] A. Hebecker, P. Mangat, F. Rompineve and L.T. Witkowski, *Tuning and backreaction in F-term axion monodromy inflation*, *Nucl. Phys. B* **894** (2015) 456 [[arXiv:1411.2032](#)] [[INSPIRE](#)].
- [41] I. García-Etxebarria, T.W. Grimm and I. Valenzuela, *Special points of inflation in flux compactifications*, *Nucl. Phys. B* **899** (2015) 414 [[arXiv:1412.5537](#)] [[INSPIRE](#)].
- [42] A. Retolaza, A.M. Uranga and A. Westphal, *Bifid throats for axion monodromy inflation*, *JHEP* **07** (2015) 099 [[arXiv:1504.02103](#)] [[INSPIRE](#)].
- [43] D. Escobar, A. Landete, F. Marchesano and D. Regalado, *Large field inflation from D-branes*, [arXiv:1505.07871](#) [[INSPIRE](#)].
- [44] S. Bielleman, L.E. Ibáñez, F.G. Pedro and I. Valenzuela, *Multifield dynamics in Higgs-otic inflation*, [arXiv:1505.00221](#) [[INSPIRE](#)].
- [45] L.E. Ibáñez, F. Marchesano and I. Valenzuela, *Higgs-otic inflation and string theory*, *JHEP* **01** (2015) 128 [[arXiv:1411.5380](#)] [[INSPIRE](#)].
- [46] C. Beasley and E. Witten, *A note on fluxes and superpotentials in M-theory compactifications on manifolds of G_2 holonomy*, *JHEP* **07** (2002) 046 [[hep-th/0203061](#)] [[INSPIRE](#)].
- [47] S.R. Coleman, *More about the massive Schwinger model*, *Annals Phys.* **101** (1976) 239 [[INSPIRE](#)].
- [48] M. Berasaluce-Gonzalez, P.G. Camara, F. Marchesano and A.M. Uranga, *Z_p charged branes in flux compactifications*, *JHEP* **04** (2013) 138 [[arXiv:1211.5317](#)] [[INSPIRE](#)].

- [49] M. Berasaluce-González, G. Ramírez and A.M. Uranga, *Antisymmetric tensor Z_p gauge symmetries in field theory and string theory*, *JHEP* **01** (2014) 059 [[arXiv:1310.5582](#)] [[INSPIRE](#)].
- [50] E. Dudas, *Three-form multiplet and inflation*, *JHEP* **12** (2014) 014 [[arXiv:1407.5688](#)] [[INSPIRE](#)].
- [51] F. Quevedo and C.A. Trugenberger, *Phases of antisymmetric tensor field theories*, *Nucl. Phys. B* **501** (1997) 143 [[hep-th/9604196](#)] [[INSPIRE](#)].
- [52] S.J. Gates, Jr., *Super p form gauge superfields*, *Nucl. Phys. B* **184** (1981) 381 [[INSPIRE](#)].
- [53] S.J. Gates, Jr. and W. Siegel, *Variant superfield representations*, *Nucl. Phys. B* **187** (1981) 389 [[INSPIRE](#)].
- [54] P. Binetruiy, F. Pillon, G. Girardi and R. Grimm, *The three form multiplet in supergravity*, *Nucl. Phys. B* **477** (1996) 175 [[hep-th/9603181](#)] [[INSPIRE](#)].
- [55] B.A. Ovrut and D. Waldram, *Membranes and three form supergravity*, *Nucl. Phys. B* **506** (1997) 236 [[hep-th/9704045](#)] [[INSPIRE](#)].
- [56] P. Binetruiy, G. Girardi and R. Grimm, *Supergravity couplings: a geometric formulation*, *Phys. Rept.* **343** (2001) 255 [[hep-th/0005225](#)] [[INSPIRE](#)].
- [57] G. Girardi, R. Grimm, B. Labonne and J. Orloff, *Correspondence between 3-form and non-minimal multiplet in supersymmetry*, *Eur. Phys. J. C* **55** (2008) 95 [[arXiv:0712.1923](#)] [[INSPIRE](#)].
- [58] K. Groh, J. Louis and J. Sommerfeld, *Duality and couplings of 3-form-multiplets in $N = 1$ supersymmetry*, *JHEP* **05** (2013) 001 [[arXiv:1212.4639](#)] [[INSPIRE](#)].
- [59] D.G. Cerdeno, A. Knauf and J. Louis, *A note on effective $N = 1$ super Yang-Mills theories versus lattice results*, *Eur. Phys. J. C* **31** (2003) 415 [[hep-th/0307198](#)] [[INSPIRE](#)].
- [60] B.B. Deo and S.J. Gates, *Comments on nonminimal $N = 1$ scalar multiplets*, *Nucl. Phys. B* **254** (1985) 187 [[INSPIRE](#)].
- [61] I.A. Bandos and C. Meliveo, *Three form potential in (special) minimal supergravity superspace and supermembrane supercurrent*, *J. Phys. Conf. Ser.* **343** (2012) 012012 [[arXiv:1107.3232](#)] [[INSPIRE](#)].
- [62] H. Nishino and S. Rajpoot, *Alternative auxiliary fields for chiral multiplets*, *Phys. Rev. D* **80** (2009) 127701 [[INSPIRE](#)].
- [63] T.W. Grimm and J. Louis, *The effective action of type IIA Calabi-Yau orientifolds*, *Nucl. Phys. B* **718** (2005) 153 [[hep-th/0412277](#)] [[INSPIRE](#)].
- [64] J. Louis and A. Micu, *Type 2 theories compactified on Calabi-Yau threefolds in the presence of background fluxes*, *Nucl. Phys. B* **635** (2002) 395 [[hep-th/0202168](#)] [[INSPIRE](#)].
- [65] G. Villadoro and F. Zwirner, *$N = 1$ effective potential from dual type-IIA $D6/O6$ orientifolds with general fluxes*, *JHEP* **06** (2005) 047 [[hep-th/0503169](#)] [[INSPIRE](#)].
- [66] O. DeWolfe, A. Giriyavets, S. Kachru and W. Taylor, *Type IIA moduli stabilization*, *JHEP* **07** (2005) 066 [[hep-th/0505160](#)] [[INSPIRE](#)].
- [67] P.G. Camara, A. Font and L.E. Ibáñez, *Fluxes, moduli fixing and MSSM-like vacua in a simple IIA orientifold*, *JHEP* **09** (2005) 013 [[hep-th/0506066](#)] [[INSPIRE](#)].

- [68] E. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. Van Proeyen, *New formulations of $D = 10$ supersymmetry and D8-O8 domain walls*, *Class. Quant. Grav.* **18** (2001) 3359 [[hep-th/0103233](#)] [[INSPIRE](#)].
- [69] G. Aldazabal, P.G. Camara, A. Font and L.E. Ibáñez, *More dual fluxes and moduli fixing*, *JHEP* **05** (2006) 070 [[hep-th/0602089](#)] [[INSPIRE](#)].
- [70] L.E. Ibáñez and A.M. Uranga, *String theory and particle physics: an introduction to string phenomenology*, Cambridge University Press, Cambridge U.K. (2012).
- [71] T.W. Grimm and J. Louis, *The effective action of $N = 1$ Calabi-Yau orientifolds*, *Nucl. Phys. B* **699** (2004) 387 [[hep-th/0403067](#)] [[INSPIRE](#)].
- [72] H. Jockers and J. Louis, *The effective action of D7-branes in $N = 1$ Calabi-Yau orientifolds*, *Nucl. Phys. B* **705** (2005) 167 [[hep-th/0409098](#)] [[INSPIRE](#)].
- [73] S. Kachru, M.B. Schulz and S. Trivedi, *Moduli stabilization from fluxes in a simple IIB orientifold*, *JHEP* **10** (2003) 007 [[hep-th/0201028](#)] [[INSPIRE](#)].
- [74] P. Candelas and X. de la Ossa, *Moduli space of Calabi-Yau manifolds*, *Nucl. Phys. B* **355** (1991) 455 [[INSPIRE](#)].
- [75] J. Shelton, W. Taylor and B. Wecht, *Nongeometric flux compactifications*, *JHEP* **10** (2005) 085 [[hep-th/0508133](#)] [[INSPIRE](#)].
- [76] P.G. Cámara, L.E. Ibáñez and I. Valenzuela, *Flux-induced soft terms on type IIB/F-theory matter curves and hypercharge dependent scalar masses*, *JHEP* **06** (2014) 119 [[arXiv:1404.0817](#)] [[INSPIRE](#)].
- [77] P.G. Camara, L.E. Ibáñez and A.M. Uranga, *Flux induced SUSY breaking soft terms*, *Nucl. Phys. B* **689** (2004) 195 [[hep-th/0311241](#)] [[INSPIRE](#)].
- [78] M. Graña, T.W. Grimm, H. Jockers and J. Louis, *Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes*, *Nucl. Phys. B* **690** (2004) 21 [[hep-th/0312232](#)] [[INSPIRE](#)].
- [79] D. Lüüst, S. Reffert and S. Stieberger, *Flux-induced soft supersymmetry breaking in chiral type IIB orientifolds with D3/D7-branes*, *Nucl. Phys. B* **706** (2005) 3 [[hep-th/0406092](#)] [[INSPIRE](#)].
- [80] P.G. Camara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms on D7-D3 brane systems*, *Nucl. Phys. B* **708** (2005) 268 [[hep-th/0408036](#)] [[INSPIRE](#)].
- [81] T.W. Grimm, T.-W. Ha, A. Klemm and D. Klevers, *The D5-brane effective action and superpotential in $N = 1$ compactifications*, *Nucl. Phys. B* **816** (2009) 139 [[arXiv:0811.2996](#)] [[INSPIRE](#)].
- [82] X. Dong, B. Horn, E. Silverstein and A. Westphal, *Simple exercises to flatten your potential*, *Phys. Rev. D* **84** (2011) 026011 [[arXiv:1011.4521](#)] [[INSPIRE](#)].
- [83] G. Gur-Ari, *Brane inflation and moduli stabilization on twisted tori*, *JHEP* **01** (2014) 179 [[arXiv:1310.6787](#)] [[INSPIRE](#)].
- [84] S. Aoki and Y. Yamada, *Impacts of supersymmetric higher derivative terms on inflation models in supergravity*, *JCAP* **07** (2015) 020 [[arXiv:1504.07023](#)] [[INSPIRE](#)].
- [85] D. Ciupke, J. Louis and A. Westphal, *Higher-derivative supergravity and moduli stabilization*, *JHEP* **10** (2015) 094 [[arXiv:1505.03092](#)] [[INSPIRE](#)].