

NBER Working Paper Series

MITIGATING DEMOGRAPHIC RISK THROUGH  
SOCIAL INSURANCE

Jerry R. Green  
Harvard University

Working Paper No. 215

CENTER FOR ECONOMIC ANALYSIS OF HUMAN BEHAVIOR  
AND SOCIAL INSTITUTIONS

National Bureau of Economic Research, Inc.  
204 Junipero Serra Boulevard, Stanford, CA 94305

November 1977

Preliminary; not for quotation.

NBER working papers are distributed informally and in limited number for comments only. They should not be quoted without written permission of the author.

This report has not undergone the review accorded official NBER publications; in particular, it has not yet been submitted for approval by the Board of Directors.

This research was supported by a contract with NBER from the Office of Income Security Policy, Department of Health, Education, and Welfare (No. HEW-100-76-0170). The author is grateful to Michael Boskin and Elon Kohlberg for helpful conversations.

MITIGATING DEMOGRAPHIC RISK THROUGH  
SOCIAL INSURANCE

Jerry R. Green

Abstract

A two-period lifetime overlapping generations growth model is used to evaluate the possibility that social insurance can effectively offset economic risks associated with uncertainty about the rate of population growth. Crude measures of the seriousness of this type of risk in the current United States situation are presented. Sufficient conditions on the structure of the economy for such intergenerational risk pooling to be mutually beneficial to all members of society are derived. Although it is logically possible to satisfy them, we argue that they are unlikely to be realized empirically in an economy similar to that of the United States. Because of this failure, some more complex types of policy options are also discussed.

## 1. Introduction

Because the return to savings depends on future values of the capital-labor ratio, fluctuations in the rate of population growth induce variations in future consumption financed in this way. At the same time, wages are similarly uncertain but negatively related to interest rates. This simple fact raises the possibility that mutually advantageous social insurance schemes might be found, in which an intergenerational transfer is made contingent upon the realized rate of population growth.

Among the arguments for social insurance has always been that the government might have risk-pooling opportunities superior to those of the private market. The government's advantage might take the form of intertemporal averaging of random returns, smoothing out fluctuations that would otherwise require a perfect ability to borrow and lend. Others posit that the government might be able to pool risks at a single moment in time in ways unavailable to private investors. These claims have been seriously challenged on factual grounds, but, whether or not they are correct, they do not apply to the risks of demographic change.

By its very nature, the size of one's generation is a random variable common to all its members. There are no independent risks among which to diversify. Even intertemporally, although fertility rates display a wide range of variation, they are highly autocorrelated. Long cycles in the age profile of the population result, sometimes on the order of decades. These types of risks are not effectively dampened by the ability to borrow and lend over the life-cycle.

Therefore the prospects for mitigating demographic risks rest with the government because the necessary social insurance policies would

bridge generations not contemporaneous when the contract would have to be initiated. In this paper, the possibility of using social insurance for such a purpose is investigated. The results are not entirely positive. Even though risks to adjacent generations are negatively correlated, it may not be possible to improve the welfare of both in this way. For some economic situations, however, mutually beneficial schemes do exist.

The discussion is organized as follows: Section 2 presents some specifics of the problem of demographic risks relevant to the current situation in the United States. Section 3 discusses the criteria which can be used to evaluate alternative social insurance schemes. The case for using a Pareto improvement of per capita ex ante expected utilities is presented. This is the objective adopted throughout the later sections. Section 4 derives conditions under which a "small" change away from a laissez-faire stochastic growth equilibrium will be mutually beneficial for members of two succeeding generations. The results are largely negative. It is shown that with logarithmic preferences and Cobb-Douglas production functions there will never exist such a mutually beneficial policy, for any initial capital stock or stochastic structure of population growth rates. Either a higher degree of risk aversion, or a lower elasticity of substitution must prevail if social insurance can be useful in this regard. Both of these possibilities are then explored. With logarithmic utility, our general conditions require an elasticity of substitution under .4, throughout the range of capital-labor ratios that could arise as a result of different realizations of the growth rate. This seems too low to offer any realistic hope in this direction. Higher levels of risk aversion, within the class of those displaying constant relative risk aversion, have the unfortunate property of implying a

negative response of savings to the rate of interest. In the light of recent empirical findings that indicate a positive elasticity, such an assumption seems ill advised.

These conclusions demonstrate that although contingent inter-generational transfer schemes seem as if they could effectively mitigate risks of demographic change, and although this is a logical possibility, it requires conditions which are unlikely to be met empirically.

Two further avenues for research and, hopefully, amelioration of these results, are then presented.

## 2. The Seriousness of the Problem:

### Some Rough Calculations

To get a feeling for the kind of risk involved, we will first look at some simple estimates of potential variations in the demographic characteristics of the population. Between 1948 and 1975 the fertility rate (children per woman of child-bearing age) fluctuated between 3.77 and 1.75. A fertility rate of 2.1 represents a level consistent with zero population growth. The higher end of this variation was caused by a variety of factors, most noticeably World War II and a second-generation effect of the early 20th century immigration waves. For these reasons, demographers believe that we will not approach such a level again. A fairly conservative upper-bound estimate is 2.7. As for the lower end, the explanations are largely sociological, combined with an increasing use of contraception and abortion. Although these are imprecisely understood at present, it is perhaps not unreasonable to take 1.7 as a lower bound.

Table 1

## PROJECTION OF THE U. S. POPULATION BY BROAD AGE GROUPS

| Year          | Population (in thousands) as of July 1 |         |                | Total   | 65 and Over<br>Percent<br>of Total | Ratio to<br>Age 18-64 | Ratio of Persons<br>Under Age 18 to<br>Persons Age 18-64 |
|---------------|--|---------|----------------|---------|------------------------------------|-----------------------|--|
|               | Under<br>Age 18                        | 18-64   | 65<br>and Over |         |                                    |                       |  |
| Assumption A- |  |         |                |         |                                    |                       |  |
| 2.1 Fertility |  |         |                |         |                                    |                       |  |
| 1975          | 66,273                                 | 124,847 | 22,330         | 213,450 | 10.5                               | 0.18                  | 0.53   |
| 2000          | 71,079                                 | 160,815 | 30,600         | 262,494 | 11.7                               | 0.19                  | 0.44   |
| 2025          | 74,857                                 | 176,751 | 48,105         | 299,713 | 16.1                               | 0.27                  | 0.43   |
| 2050          | 78,701                                 | 188,448 | 51,247         | 318,396 | 16.1                               | 0.27                  | 0.42   |
| Assumption B- |  |         |                |         |                                    |                       |  |
| 2.7 Fertility |  |         |                |         |                                    |                       |  |
| 1975          | 66,273                                 | 124,847 | 22,330         | 213,450 | 10.5                               | 0.18                  | 0.53   |
| 2000          | 91,152                                 | 165,255 | 30,600         | 287,007 | 10.7                               | 0.19                  | 0.55   |
| 2025          | 121,054                                | 212,852 | 48,105         | 382,011 | 12.6                               | 0.23                  | 0.57   |
| 2050          | 158,987                                | 283,767 | 56,575         | 499,329 | 11.3                               | 0.20                  | 0.56   |
| Assumption C- |  |         |                |         |                                    |                       |  |
| 1.7 Fertility |  |         |                |         |                                    |                       |  |
| 1975          | 66,273                                 | 124,847 | 22,330         | 213,450 | 10.5                               | 0.18                  | 0.53   |
| 2000          | 57,322                                 | 157,176 | 30,600         | 245,098 | 12.5                               | 0.19                  | 0.36   |
| 2025          | 49,938                                 | 152,378 | 48,105         | 250,421 | 19.2                               | 0.32                  | 0.33   |
| 2050          | 44,146                                 | 135,532 | 47,024         | 226,702 | 20.7                               | 0.35                  | 0.33   |

Source: U. S. Bureau of Census, Current Population Report, Series P-25, No. 601, "Projections of Population of the United States: 1975-2050."

The table above gives some statistics relevant to our crude computations of variations in the return to saving. Assuming that the rate of saving is unresponsive to demographic changes occurring in the pre-retirement years, we get a first approximation to the impact of different fertility rates on the capital-labor ratio.

If the amount of capital is  $K$ , the three assumptions of the table produce capital-labor ratios of  $K/176,751$ ,  $K/212,852$  and  $K/152,378$  in the year 2025 for fertility rates of 2.1, 2.7 and 1.7, respectively. Using a Cobb-Douglas production function<sup>1</sup> with capital's share set at .25, the ratio of the marginal products of capital between the high and low estimates of fertility would be about 3:4.

This seemingly modest variation in the rental price of capital would produce very large swings in the total return to saving, accumulated over an average holding period of say 20 years. For example, assuming constant proportional taxes on the return to capital in either instance, we might have a variation in the real net rate of return to saving of between 3% under the high growth rate and 4% under the low growth rate. At the end of the 20 years, the accumulated wealth can vary by about 9% above or below its mean level, and the return to saving over a 15-year retirement horizon can vary by about 16% above or below its mean.<sup>2</sup>

The induced fluctuations in the wage rate of future generations are smaller, in relative terms, but represent a comparable real magnitude since wages form a larger share of national income. Wages net of tax vary by about 7% above and below the middle of their range.

These calculations are obviously inaccurate for several reasons. To a great extent, fluctuations in the effective working population are predictable long in advance simply because births lead labor market entry by about 20 years. On the other hand, the current age structure



of the population, including children, is taken into account in the above table. Our rough computations therefore have already incorporated this "foreknowledge" about the future size of the working population.

The rough calculation given above took savings to be independent of population. Knowledge about future births when they occur will, however, make the course of the working population somewhat predictable by a wage earner while he still has time to adjust his savings decisions before retirement. To the extent that these risks can be partially offset by the flexibility of private savings, our estimate of the induced variability of the return to capital may overstate the real risk of utility loss that may be suffered.

The two-period model developed in this paper is subject to the same qualifications. Because the working period of life is longer than the retirement period, a two-period model of an individual's lifetime does not do justice to the richness of the individual's dynamic decision process as new information accrues during his lifetime. A simulation experiment trying to capture some of these complexities is currently under way.

Further features not captured here relate to the heterogeneity of the labor force and the endogeneity of labor supply. Needless to say, the structure of wages as a function of age is not constant, nor can it be properly viewed as an exogenous function of some base wage level, especially when large swings in the age composition of the labor force are the central focus of the analysis. There are surely a variety of institutional factors at work here, some of which are themselves endogenous. We simply do not have a suitable theory to analyze this, and it is beyond the scope of this study to construct one.

These qualifications aside, however, it may still be useful to pursue the rough calculations above to estimate the present value of a potential reduction of uncertainty in future wages and interest rates. Since the results of any such calculation are going to be sensitive to our choice of a discount rate and guesses about levels of real income in the future, among other parameters, we will give an illustrative case, and then summarize the results under other assumptions in Table 2.

To give a lower bound type of estimate, we will proceed in two steps. We will estimate the present value of the stream of willingnesses to pay for a reduction in uncertainty about future returns to saving for generations retiring from 2025 to the indefinite future, and similarly for workers from that date onward. Therefore we will be neglecting uncertainty in the intervening 48 years, which could be substantial as seen in Table 1 above.

By 2025, real retirement income from savings might be on the order of \$11,000 per family, or more. Our estimates above suggest a variation of from \$9,500 to \$12,500 due to demographic factors. It is not hard to imagine that a typical family would be willing to reduce its income by \$100 per year to cut the variance of this return in half. Thus, for a typical retired family, the value of the reduction in uncertainty discounted to its date of retirement (at 3.5%, an average real net rate of return figure) over a 15-year average retirement period would be \$1152. The number of retired families in a given one-year age interval is a random variable in the long run, but is rather easy to predict for 2025. With about 30 million retired individuals, who will form about 18 million households in 2025, there would be close to 1.5 million households of age 65 in that year. Averaged over the indefinite horizon, we can safely

use a figure of about 2 million newly retiring households each year. Thus, this reduction in uncertainty is worth about \$2.3 billion to each year's retiring cohort on their 65th birthday. To compute the total social value of this stream at present, we must decide on a social rate of discount. Using the same rate that was used for the individual's own computations (a relatively high rate of return, historically, but surely much lower than the gross return to private capital), we find that this stream of \$2.3 billion per year, each year from 2025 onward, is worth today \$12.6 billion. Expressed as an annual flow of benefits, at the same discount rate this is \$442 million per year.

On the wage side, future real wages might be fluctuating in the range of \$18,000 - \$21,000, and a reduction in this uncertainty might easily be worth \$100 per family, as in the case of retirees. These working families have wage earning horizons of about 45 years each; accumulating over their working lives, one has \$2250 per family. The number of working families becomes more highly variable earlier than the number of retirees. Let us take a conservative average figure of 100 million working families aged 18-65, or roughly  $2\frac{1}{2}$  million new working families entering the 18-year old pool each year. The benefit per cohort is thus around \$5.6 billion, which means a total present value of \$30.8 billion, or an annual flow value of \$1.08 billion, at the 3.5% discount rate.

Combining the benefits of wage earners and retirees, we arrive at a flow value of about \$1.5 billion per year, for a modest reduction in the risks of population fluctuations.

Table 2  
Welfare Gain from Risk-Reduction Under Alternative Assumptions<sup>3</sup>  
(millions of dollars)

|  | Length of Savings Holding Period |       |      |          |       |      |
|--|----------------------------------|-------|------|----------|-------|------|
|  | 15 Years                         |       |      | 20 Years |       |      |
| Discount rates:  | 2.5%                             | 3.5%  | 5%   | 2.5%     | 3.5%  | 5%   |
| <u>Retirees</u>  |                                  |       |      |          |       |      |
| Value per family at age 65 (dollars)                                   | 9904                             | 921   | 829  | 1238     | 1152  | 1037 |
| Value per cohort at age 65   | 1981                             | 1843  | 1661 | 2476     | 2304  | 2076 |
| Present value of stream of retiring cohort's benefits from 2025 onward | 24219                            | 10101 | 3193 | 30274    | 12626 | 3991 |
| Annual flow equivalent value from the present                          | 605                              | 354   | 159  | 757      | 442   | 199  |
| <u>Workers</u>   |                                  |       |      |          |       |      |
| Value per family at age 18 (dollars)                                   | 2683                             | 2250  | 1777 | 2683     | 2250  | 1777 |
| Value per cohort at age 18   | 6708                             | 5625  | 4444 | 6708     | 5625  | 4444 |
| Present value for cohort's benefits from 2025 onward                   | 82020                            | 30830 | 8545 | 82020    | 30830 | 8545 |
| Annual flow equivalent value from the present                          | 2050                             | 1079  | 427  | 2050     | 1079  | 427  |
| <u>Aggregate</u>   |                                  |       |      |          |       |      |
| Annual flow equivalent   | 2655                             | 1433  | 586  | 2807     | 1521  | 626  |

Apart from its coarseness, and the other qualifying comments related to a two-period framework already discussed in this section, a few comments should be added. The method by which we computed the potential welfare gain is not a steady-state type of calculation. It is a gain which would accrue now, in the current economic situation of the United States, because of the benefits of a social policy whose impact would not be strongly felt until well into the next century. But even though we have discounted these gains to the present at a generous rate, they still are very significant. A steady-state analysis would surely show a much higher level of gain, because it would not have to be discounted over so many years.

The potential gain from mitigating risk is somewhat more hidden than efficiency gains often studied in public finance. Physically, nothing apparently happens to increase overall productivity or the efficiency of consumption as in the case of a misallocation of resources. Nevertheless, the costs are real and their intangible nature should not disguise the magnitude of the benefits at stake.

Unfortunately, the conclusions of this paper are largely pessimistic. Although large gains appear possible, we will see that it is not likely that they can be reaped while simultaneously improving the welfare of all generations. The nature of such a welfare improvement is discussed in the next section.

### 3. Intergenerational Ex Ante Risk Pooling

As discussed above, population growth fluctuations are risky both for older generations whose return to savings will be affected, and younger generations whose wages will be affected. The pre-natality of the latter is at once the source of the uncertainty and the barrier to a private market solution. A social contract is thus required which specifies a pattern of intergenerational transfer payments in advance of the realization of the demographic random variables.

How shall we measure the value of such a contract? One way would be to use a utilitarian criterion; the utility of each agent would be added in each event, and an expectation of this sum over all events would be taken. Assuming that all members of a cohort are identical, we would still have to know the joint distribution of an individual's utility and the size of his generation. Therefore the efficacy of any given policy would not be invariant to additive shifts in the utility index. That is, just as in models with an endogenous population size, some special meaning would have to attach to a utility of "zero" or else the utilitarian criterion could not consistently be employed. This is true despite the exogeneity of population growth, and quite aside from any problems of intergenerational discounting.

An alternative is to require more stringent conditions on the sequence of utility changes. Clearly no policy exists that would improve the utility of all individuals in all states, unless the capital accumulation program is inefficient. Some type of ex ante expected utility measure seems appropriate.

The utility of each generation depends on the initial circumstances into which it is born, and on the events during its lifetime. But viewed

from the date at which the policy is to be implemented, these initial circumstances are also random variables, depending upon events in the intervening period. Thus the appropriate expected utility for a given generation is as viewed from the date at which the new policy is formulated.

If a social insurance policy is Pareto improving in this sense, then all individuals in the original position would prefer to have it implemented regardless of into which generation they were (eventually) born.

Is an intergenerational transfer scheme that is mutually beneficial in this ex ante sense always feasible? If so, what are its characteristics? Before answering this question analytically in the next section, we pause to discuss the problem in more general terms. On the surface, the answer to the feasibility question should be positive. The risks to each pair of generations are inversely related. Therefore, an agreement that reduces the effective wage below the competitive level when fertility is low, increasing the yield to capital, and which reverses the direction of the compensation in the high fertility case, would apparently result in the same mean payoff to each agent while reducing its variability. Utility being concave in consumption and indirect expected utility being concave in first-period income, this arrangement is apparently sure to succeed.

Unfortunately, however, matters are not quite this simple. If we want to subsidize/tax the old people according to a function  $b(n)$  per capita, where  $n$  is the ratio of young to old, the required tax/subsidy on the young must be  $\frac{1}{n}b(n)$  in order for the government's budget to balance. The subsidy function,  $b(\cdot)$ , must be decreasing in  $n$  in order to reduce rather than increase risks. Therefore revenue to be raised through

taxes on the young would be higher when there are few of them, and per capita taxes would therefore be adversely intensified. High total taxes would be borne by fewer young workers, and high total subsidies would be spread out over many. For lack of a better term, we will call this the population-bias effect.

The population-bias effect builds in an inherent problem for such social insurance schemes. There is still one additional problem to be overcome. When the gross return to capital exceeds the rate of population growth, it is well known that social insurance on a pay-as-you-go basis reduces savings<sup>4</sup> and is therefore unambiguously harmful in the long run. If this form of risk pooling were to reduce savings, similar effects would arise.

The problem therefore is to determine when and if the potential benefits from risk pooling can outweigh the population-bias effect and the depressed savings effect, if the latter is operative. An analytical model of this is given in the next section.



#### 4. Mutual Social Insurance Between Two Adjacent Generations

##### A. Conditions for Pareto Improving "Small" Contracts

Our analysis follows the standard lines of the two-period lifetime overlapping generations model as introduced by Samuelson (1958) except that the ratio of the population sizes between adjacent generations is a random variable. A generation born at date  $t$ , called "generation  $t$ ," is assumed to consist of identical individuals with homothetic preferences for their present and future consumption. Generation  $t$  provides labor inelastically at date  $t$ , and makes a decision regarding consumption at that date. The excess of labor earnings over consumption is saved. At date  $t+1$ , the size of the new working generation is determined. The ratio of this generation's size to that of generation  $t$  is denoted  $n_t$ . After the realization of  $n_t$ , the marginal product of capital is determined, and generation  $t$ 's consumption at date  $t+1$  consists of the principal and interest on their savings. All markets are assumed to be competitive.

Let

$$(1) \quad u_t(c_{yt}, c_{rt})$$

be the utility function for members of generation  $t$  ( $y$  stands for young, and  $r$  stands for retired).

Production is specified by a neoclassical constant returns to scale technology. The function  $f(k_t)$  gives output per unit of labor as it depends on the capital labor ratio at time  $t$ .

We will consider a social insurance scheme,  $sb(n_t)$ , which represents the transfer to each retired individual of generation  $t-1$  when the realized growth rate is  $n_t$ . As discussed in the previous section, this

implies a tax of  $\frac{1}{n_t} \varepsilon b(n_t)$  on the members of generation  $t$  in order to maintain a balanced budget for the government.

The social insurance scheme is implemented at date 0, after the birth of generation zero, but before they have made their savings decisions. We assume in this section that the insurance scheme is terminated at date 1, after the realization of  $n_1$  and after the transfers have been carried out as required.

We think of  $\varepsilon$  as a small positive number, in order to represent a small change from an initial laissez-faire position. By differentiating with respect to  $\varepsilon$  we can discover whether such small systems move ex ante expected utilities in a Pareto improving direction.

The principle datum of the system from the point of view of members of generation 0 is the capital-labor ratio that they inherit,  $k_0$ . Competitive behavior in the labor market determines their wage  $w_0$ . If the agents in generation zero choose a consumption level of  $c_{y0}$  for that period, their future consumption is given by

$$(2) \quad c_{r0}(n_1) = (w_0 - c_{y0}) \left( 1 + f' \left( \frac{w_0 - c_{y0}}{n_1} \right) \right) + \varepsilon b(n_1).$$

Each agent behaves competitively with respect to his savings-consumption choice, however. That is, they each regard the argument of  $f'$ ,  $k_1(n_1)$ , as a fixed random variable, independent of their own choice of  $c_{y0}$ .

To simplify the analysis we will assume that utility is additively separable

$$(3) \quad u_t(c_{yt}, c_{rt}) = u_y(c_{yt}) + u_r(c_{rt}).$$

Let

$$(4) \quad K_1 = w_0 - c_{y0}$$

and let  $c_{y0}(K_1)$  be the optimizer of the expectation of (3) subject to

$$(5) \quad c_{r0}(n_1) = (w - c_{y0}) \left( 1 + f' \left( \frac{K_1}{n_1} \right) \right) + \varepsilon b(n_1).$$

The equilibrium  $c_{y0}^*$  at date 0 is determined by the condition,

$$(6) \quad c_{y0}(w - c_{y0}^*) = c_{y0}^*.$$

The first-order condition for an optimum of the expectation of (3) subject to (5) is

$$(7) \quad 0 = E_{n_1} u'_y(c_{y0}) - u'_r \left( (w - c_{y0}) \left( 1 + f' \left( \frac{K_1}{n_1} \right) + \varepsilon b(n_1) \right) \cdot \left( 1 + f' \left( 1 + f' \left( \frac{K_1}{n_1} \right) \right) \right) \right).$$

Let  $c_{y0}^0$  be the solution to the problem for generation 0 when the policy is followed at  $\varepsilon = 0$  — that is, when no intergenerational transfers are being made. Let  $c_{r0}^0(n_1)$  be the corresponding consumption in the retirement period as given by (2).

The expected utility of generation 0 increases with respect to  $\varepsilon$  when

$$(8) \quad E_{n_1} u'_r(c_{r0}^0(n_1)) \cdot b(n_1) > 0.$$

The welfare of generation 1 is harder to evaluate. The change in generation 0's consumption,  $\frac{dc_{y0}^*}{d\varepsilon}$ , which determines the change in the capital stock at date 1, induces only second-order changes in the welfare of generation 0. But since the marginal contribution of capital at date 1 to the expected utility of generation 1 is positive, the introduction of the

policy induces first-order welfare variations in their utility through this route as well as through the changes in risk characteristics of the attainable consumption plans.

In general it is possible that the beneficial effects of the reduction in risk may outweigh a utility loss imposed by a reduction in the stock of capital. Let us begin, however, by asking for stronger requirements, namely both that savings increase and that generation 1 should be better off even with a constant savings level due to an improved risk situation. We will return to the more general possibility at the end of this section.

To ascertain the change in savings in an initially laissez-faire situation, we totally differentiate (7) at  $\varepsilon = 0$ , obtaining

$$(9) \quad \left( \frac{dc_{y0}}{d\varepsilon} \right) = \frac{E u''_r(c_{r0}^0(n_1)) \left( 1 + f' \left( \frac{w_0 - c_{y0}^0}{n_1} \right) \right) b(n_1)}{E u''_y(c_{y0}^0) + u''_r(c_{r0}^0(n_1)) \left( 1 + f' \left( \frac{w_0 - c_{y0}^0}{n_1} \right) \right)^2 + E \frac{f'' \left( \frac{w_0 - c_{y0}^0}{n_1} \right)}{n_1} \left( u'_r(c_{r0}^0(n_1)) + c_{r0}^0(n_1) u''_r(c_{r0}^0(n_1)) \right)}$$

The denominator of (9) is negative under the hypothesis that the equilibrium attained at date 0, given by (6) and (7), is locally stable. Therefore, savings will increase only if the numerator of (9) is positive, under our assumptions.

If we were to assume that  $E b = 0$  and  $\frac{db(n_1)}{dn_1} < 0$ , a conclusion about the sign of the expression in question could be drawn from a knowledge of the sign of

$$(10) \quad \frac{d}{dn_1} \left( u''(c_{r0}^0(n_1)) \cdot \left( 1 + f' \left( \frac{w_0 - c_{y0}^0}{n_1} \right) \right) \right).$$

Expression (10) can be rewritten as

$$(11) \quad \frac{-f'' \left( \frac{w_0 - c_{y0}^0}{n_1} \right) \cdot K_1}{n_1^2} (u''' \cdot c_{r0}^0 + u'').$$

In general, (11) cannot be signed, even under the assumption of increasing relative risk aversion. However, for the family of utility functions with constant relative risk aversion we have that

$$(12) \quad u''' \cdot c_{r0}^0 + u'' > 0 \quad \text{for all } n_1.$$

Therefore (10) is positive and, together with the hypotheses

$$(13) \quad E b = 0, \quad \frac{db(n_1)}{dn_1} < 0,$$

we have that (9) is positive. Savings, unfortunately, go down in this case.

Therefore, if we are to succeed with a search for policies  $b(\cdot)$  that increase savings and are beneficial in terms of risk spreading for both generations in an economy with constant relative risk aversion, we will have to allow for  $E b < 0$ . This serves to complicate matters because (13) forms a simple sufficient condition for the positivity of (8). More generally, we will have to regard (8) as a constraint to be checked for the particular candidate  $b(\cdot)$ , a test which may depend upon the precise distribution of  $n_1$ .

The constant relative risk aversion family has an interesting property in this regard: Whenever  $b(\cdot)$  is such that savings are unchanged with respect to  $\epsilon$ , utility is unchanged as well. Moreover, this result is independent of the distribution of  $n_1$  and of the production

function  $f$ . It can be verified directly by noting that

$$(14) \quad \frac{u''(c_{r0}^0(n_1)) \left(1 + f' \left(\frac{w_0 - c_{y0}}{n_1}\right)\right)}{u'(c_{r0}^0(n_1))} = \frac{(\alpha-1)}{w_0 - c_{y0}}$$

for all  $n_1$ , where  $\alpha$  is the parameter of the utility  $\frac{1}{\alpha} c^\alpha$ ,  $\alpha < 1$ . Therefore, (8) and the numerator of (9) are identical up to a (negative) multiplicative constant.

If (8) and (9) are both non-negative, then the only remaining condition necessary to have a mutually beneficial policy is that the expected marginal utility of the transfer increase. The initial marginal utilities of income are those associated with the wage rates at the initial values of  $k_1(n_1)$ , in the absence of taxation. A member of generation 1 will face the problem

$$(15) \quad \max_{n_2} E U \left( c_{y1}, (w_1 - c_{y1}) \left(1 + f' \left(\frac{K_2}{n_2}\right)\right) \right)$$

$$\text{where } w_1 = f \left( \frac{w_0 - c_{y0}}{n_1} \right) - \left( \frac{w_0 - c_{y0}}{n_1} \right) f' \left( \frac{w_0 - c_{y0}}{n_1} \right).$$

Letting  $U_w(n_1)$  be the indirect marginal utility of income,  $w_1$ , in the problem (15), the criterion is

$$(16) \quad -E \frac{U_w(n_1) \cdot b(n_1)}{n_1} > 0$$

since the transfer to a typical member of generation 1 is  $\frac{-b(n_1)}{n_1}$ .

Thus, (8) and (16) together with the negativity of (9) assure that the policy is valuable to members of generations 0 and 1.

There is one further point to be noted, however. The policy we have considered determines transfers at date 1 between these two generations only. After that time it is assumed to be terminated. But this does not mean that the effects of the policy are non-existent beyond this date. The savings of generation 1 will surely be affected by the realization of  $n_1$  and its associated transfer. This will affect the welfare of generation 2, and all subsequent generations.

The induced change in generation 1's savings has a different character than that for generation 0. The savings of generation 0 are determined ex ante, before any realization of uncertain events, but the savings of generation 1 is a random variable, depending on the realization of  $w_1(n_1)$ . As long as the marginal propensity to save out of wage income is not zero throughout the range of  $n_1$  (an irrelevant possibility) there will be some change in the stochastic process through which the capital stock is determined over time.

Under the hypotheses with which we have been working, the stochastic process determining the capital labor ratio at each date is a Markov chain. This can be proven as follows.

In order to show that  $\langle k_t \rangle$  is Markovian, we demonstrate that  $k_{t+1}$  is functionally related to  $k_t$  and  $n_{t+1}$ . Independence of  $\langle n_t \rangle$  then assures this result. The per capita savings of each member of generation  $t$ ,  $s_t$ , depends on his labor income, which is just a function of  $k_t$ . If there are  $N_t$  members of this generation, their total savings is therefore  $K_{t+1} = N_t s_t$ . But  $k_{t+1} = \frac{K_{t+1}}{N_{t+1}} = \frac{s_t(k_t)}{n_t}$ , which is the desired result.

Since the policy we have been considering is terminated at time 1, the Markov chain  $\langle k_t \rangle_{t=2, \dots}$  evolves according to the probability laws derived above. The expected utility of members of generations 2 and later, being merely a function of  $k_t$  at the date of their birth, we can say that their ex ante expected utility depends only on the distribution of the random variable  $k_t$ . The social insurance policy affects this by changing the initial probability distribution of  $k_2$ . And, as argued above, the distribution of  $k_2$  would shift due to generation 1's transfer payment,  $\frac{-b(n_1)}{n_1}$ , as well as through changes in  $k_1$ .

Under the assumption that savings is a normal good, the distribution of  $k_2$  after the implementation of the social insurance scheme will not bear a stochastic dominance relationship to that before it. Neither, therefore, will there be an unambiguous relation for any of the  $k_t$ ,  $t > 2$ . In order to evaluate the effect of the transfer program on generations alive only after its implementation has been completed, we need to know more about individuals' risk aversion toward initial income, the production function and the underlying distribution of  $n_t$ .

Before pursuing this question in more generality, we present an example illustrating the impossibility of creating a welfare-improving policy for members of both generations 0 and 1 in an important special case. Because of this result, there is clearly no welfare-improving policy for all generations, and we need not investigate the issue of the longer run residual effects of the transfer scheme discussed above.



B. Impossibility of Successful Intergenerational Risk Pooling  
in the Logarithmic - Cobb-Douglas Case

As a benchmark, though a somewhat unfortunate one, we consider the case in which

$$(17) \quad u_t(c_{yt}, c_{rt}) = a \log c_{yt} + (1-a) \log c_{rt},$$

$$(18) \quad f(k_t) = k_t^\beta \quad 0 < \beta < 1.$$

We observed above that if  $E u'_r(c_{r0}^0) b(n_1) = 0$ , then both savings and the utility of generation zero are constant with respect to  $\varepsilon$ . Using (17) and (18) this condition is

$$(19) \quad E \frac{1}{1 + \beta \left( \frac{(1-a)w_0}{r_1} \right)^{\beta-1}} b(n_1) = 0.$$

To derive the condition for welfare improvement for members of generation 1, note that wages for this generation will be  $(1-\beta)k_1^\beta$  so that

$$(20) \quad u'_y(c_{y1}^0) = \frac{a}{c_{y1}^0(n_1)} = \frac{1}{(1-\beta) \left( \frac{(1-a)w_0}{n_1} \right)^\beta}.$$

Thus, using (14) and (20) the fact that, by the optimality of his consumption choice,  $U_w = u'_y(c_{y1}^0)$ , we have

$$(21) \quad E \frac{1}{n_1^{1-\beta}} b(n_1) < 0$$

as the condition for ex ante expected utility improvement for generation 1.

Assuming that  $b$  is continuous, it must be zero at some value of  $n_1$ , say  $\bar{n}_1$ ; for if  $b$  were one-signed, then (19) would be obviously

impossible. Moreover,  $b'(n_1) < 0$  is required to insure the second-order condition for a welfare improvement for generation 0.

Multiply (19) by the constant

$$\frac{1 + \beta(1-a)^{\beta-1} w_0^{\beta-1} n_1^{1-\beta}}{\bar{n}_1^{1-\beta}}$$

and subtract the result from (21) obtaining

$$(22) \quad E \frac{(1-a)^{1-\beta} w_0^{1-\beta} (\bar{n}_1^{1-\beta} - n_1^{1-\beta})}{n_1^{1-\beta} \bar{n}_1^{1-\beta} ((1-a)^{1-\beta} w_0^{1-\beta} \beta^{-1} + n_1^{1-\beta})} b(n_1) < 0$$

as the condition for welfare improvement. It is apparent, however, that (22) is false since by definition  $b(n_1) > 0$  if and only if  $n_1 < \bar{n}_1$ , so that the integrand will be everywhere positive. There is, therefore, no social insurance scheme capable of benefiting both generations in this example.

### C. Logarithmic Utility but More General Production

The methods of the previous section can be employed to derive sufficient conditions on the production function to admit welfare-improving policies. The zero-savings change/zero utility change for generation 0 condition, analogous to (17) is

$$(23) \quad E \frac{1}{1 + f' \left( \frac{K}{n_1} \right)} b(n_1) = 0,$$

and the welfare-improving condition for generation 1 is

$$(24) \quad E \frac{1}{n_1 f\left(\frac{K}{n_1}\right) - Kf'\left(\frac{K}{n_1}\right)} b(n_1) < 0$$

where  $K$  is the aggregate amount of capital saved by generation 0, and is independent of  $n_1$ .

In order to show that (23) and (24) are compatible with a function  $b$  satisfying  $b' < 0$ , it is sufficient to show that the elasticity of

$$(25) \quad \frac{1}{1 + f'\left(\frac{K}{n_1}\right)}$$

with respect to  $n_1$  is greater than that of

$$(26) \quad \frac{1}{n_1 f\left(\frac{K}{n_1}\right) - Kf'\left(\frac{K}{n_1}\right)}$$

Given this condition, one could multiply (23) by a factor such that these multipliers of  $b$  will be equal at  $\bar{n}_1$ , the point for which  $b(\bar{n}_1) = 0$ . If the elasticity of (25) is greater than that of (26), its slope at  $\bar{n}_1$  must be (algebraically) lower. Moreover, if this relation between elasticities held everywhere then there could be no other value of  $n_1$  at which (25), multiplied by this factor, equals (26). Thus subtracting this multiple of (25) from (26) we would find that the result is negative if and only if  $b(n_1)$  is positive, so that (24) is satisfied. Indeed, this would show that (24) would hold for any function  $b(\cdot)$  satisfying (23) and  $b' < 0$ , independent of the distribution of  $n_1$ .

Thus, the relevant condition on the production function is

$$(27) \quad \frac{\frac{d}{dn_1} \left( \frac{1}{1+f' \left( \frac{K}{n_1} \right)} \right)}{\frac{1}{1+f' \left( \frac{K}{n_1} \right)}} < \frac{\frac{d}{dn_1} \left( \frac{1}{n_1 f \left( \frac{K}{n_1} \right) - K f' \left( \frac{K}{n_1} \right)} \right)}{\frac{1}{n_1 f \left( \frac{K}{n_1} \right) - K f' \left( \frac{K}{n_1} \right)}}$$

which can be written straightforwardly as

$$(28) \quad \sigma < \frac{f'(f+k)}{(1+f')f}$$

where  $\sigma$  is the elasticity of substitution of the production function  $f$ .

As a sufficient condition for the possibility of a successful social insurance scheme, (28) expresses the fact that if the elasticity of substitution is sufficiently low, both future interest rates and future wage levels will be so variable with respect to population that each generation would willingly insure the other to some extent, even at actuarially unfair odds. It is important to keep in mind that (28) is required to hold for all values of  $k$  within the range that is induced through variations in  $n_1$ . In the Cobb-Douglas case studied above, where  $\sigma \equiv 1$ , the right-hand side of (28) can be written as

$$(29) \quad \frac{\beta k^{1-\beta} + \beta}{\beta + k^{1-\beta}}$$

which varies from 1 to  $\beta$  as  $k$  goes from 0 to  $\infty$ . This explains why mutually beneficial social insurance schemes do not exist in that case for any distribution of  $n_1$ .<sup>5</sup> With a capital output ratio of 3 and a marginal product of capital in the range of .09 to .12, the corresponding upper bound on  $\sigma$  is about 1/3. Since this is surely not relevant for the present technology, within the range of possible future values of  $k$ , there is no

realistic possibility of mutually advantageous social insurance of this type if logarithmic preferences accurately describe individuals' attitudes toward risk.

#### D. Other Forms for Utility

The essentially pessimistic conclusions of the previous sections might be different if individuals were to display a higher degree of risk aversion. If we are to maintain the strategy of keeping both the savings and utility of generation 0 constant and then ascertain whether generation 1 can be made better off by virtue of an improved risk posture alone, we must use the constant relative risk aversion form of utility as discussed in section 4A. This utility, however, has the following unfortunate property: If we consider members of this class in which risk aversion is stronger, these also imply a negative response of savings to the rate of interest.

Recent empirical work (Wright [1969]; Boskin [1976]) indicates that just the opposite is true. But the constant relative risk aversion utility functions that would be consistent with this fact are precisely those for which even more stringent conditions than (28) on the elasticity of substitution would be required.

## 5. Conclusion

The results of this paper being primarily discouraging, should not dissuade us from studying more general types of social insurance policies. It is important to investigate policies that are not of a pay-as-you-go sort. When the government can absorb part of the risk of population fluctuations by running a publicly held deficit, or by investing surplus tax receipts in capital, much better ex ante welfare improvements are possible. Stokey (1977) has studied a model like this for inter-generational risk spreading involving productivity. The great difficulty with this formalization is that the feasibility condition for the stochastic debt policy required is not obvious.

Finally, it is important to note that the results of this paper are only "local." The effects of social insurance schemes that depart drastically from the laissez-faire solution are certainly worth our attention. Non-convexities inherent in the model make this task difficult.

Footnotes

<sup>1</sup> Cross-section and time theories estimates of the aggregate elasticity of substitution differ widely. Typically, the former are much higher than the latter. The interested reader is referred to Brown (1967) and Lucas (1969).

<sup>2</sup> The difference between terminal values of wealth is further accentuated by the difference in the value of annuities at the interest rate prevailing at the date of retirement. This rate is positively correlated with the value of wealth, since interest rates vary with the (slowly changing) population structure. The computation in the text uses a 15-year annuity to cover retirement consumption, and assumes that the rate of interest is the average that prevailed during the generations working lifetime.

<sup>3</sup> The maintained assumptions underlying this table are:

1. \$100-per family annual value of uncertainty reduction, for workers and retirees in the 20-year savings horizon case and for workers in the 15-year savings horizon case. For retirees under the 15-year average holding period of savings, it was set at \$80 per family per year.
2. The retirement horizon is 15 years.
3. Every cohort enters the labor force at age 20 and works until age 65.

4. Assumptions about numbers of families per cohort are as given in the text.

The amounts shown are all in millions of dollars or millions of dollars per year, except per family values. All benefits accruing between now and 2025 are not reported.

<sup>4</sup>This point can be disputed if private intergenerational transfers are widespread. See Barro (1974) and Feldstein (1976).

<sup>5</sup>In the CES case the validity of (28) depends on the two parameters of the CES production function. For parameters giving a marginal product of capital anywhere under .10, (28) remains false for all  $\sigma \geq \frac{1}{2}$ .



References

- Barro, R. (1974), "Are Government Bonds Net Wealth?" Journal of Political Economy, 82, 1095-1117 (Nov./Dec.).
- Boskin, M. (1976), "Taxation, Saving and the Rate of Interest," National Bureau of Economic Research, Working Paper No. 135, to appear in Journal of Political Economy, April 1978.
- Brown, M. (ed.), (1967), The Theory and Empirical Analysis of Production, Studies in Income and Wealth, Vol. 31, National Bureau of Economic Research.
- Feldstein, M. (1976), "Perceived Wealth in Bonds and Social Security: A Comment," Journal of Political Economy, 84, 331-336 (April).
- Lucas, R. (1969), "Labor-Capital Substitution in U.S. Manufacturing," in A. Harberger and N. Bailey (eds.), The Taxation of Income from Capital, Brookings Institution, Washington, 223-274.
- Samuelson, P. (1958), "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money," Journal of Political Economy, 66, 467-482 (December).
- Stokey, N. (1977), "Intergenerational Risk-Pooling," Mimeographed, Harvard University, September 1977.
- Wright, C. (1969), "Saving and the Rate of Interest," in A. Harberger and M. Bailey (eds.), The Taxation of Income from Capital, Brookings Institution, Washington, 275-300.