Sādhanā Vol. 40, Part 2, April 2015, pp. 455–465. © Indian Academy of Sciences

Mixed convective heat transfer from a vertical plate embedded in a saturated non-Darcy porous medium with concentration and melting effect

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MS received 19 April 2014; revised 28 August 2014; accepted 6 November 2014

Abstract. The effect of melting and solute dispersion on heat and mass transfer in non-Darcy fluid flow over a vertical surface has been studied numerically in the present article. The flow is assumed to be laminar and steady state. Using similarity transformations, the governing boundary layer equations are transformed into self-similar nonlinear ordinary differential equations, which are then solved by using boundary value problem solver. A comparison with the numerical results made for different Ra/Pe values in the absence of some particular parameters. The velocity and concentration inside the boundary layer are observed to be influenced by the parameters like Ra/Pe, L, B, M. The flow heat and mass transfer coefficients are discussed through the plots.

Keywords. Porous medium; melting; double dispersion; Newtonian fluid.

1. Introduction

Melting effect with heat and mass transfer in porous media has much attention in recent years because of its applications in casting, welding and magma solidification, etc. From boundary layer theory point of view, Tien & Yen (1965) studied the effect of melting on convective heat transfer between a melting body and surrounding fluid. Sparrow *et al* (1977) studied the velocity and temperature fields, the heat transfer rate, and the melting layer thickness by means of finite-difference scheme in the melting region for natural convection. Cheng & Lin (2007) studied the melting effect on mixed convective heat transfer from a solid porous vertical plate with uniform

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wall temperature embedded in the liquid saturated porous medium, using the Runge–Kutta–Gill method and Newton's iteration for similarity solutions.

In problems dealing with porous media, the effect of melting, radiation is important in industries and technologies. The applications are found in situation such as geothermal systems, heating and cooling chamber, fossil fuel combustion, energy processes and Astro-physical flows. The effects of non-Darcy mixed convection with thermal dispersion-radiation in a saturated porous medium was studied by Prasad & Hemalatha (2010). They observed that temperature decreases with increasing melting parameter. Abbas *et al* (2008) studied numerically the combined effect of thermal dispersion and thermal radiation on the non-Darcy natural convection flow over a vertical flat plate kept at higher and constant temperature in a fluid saturated porous medium.

The effect of double dispersion on non-Darcy mixed convective flow over a vertical surface embedded in porous medium was studied by Afify & Elgazery (2013). They observed that the local heat transfer rate increases by increasing the solute dispersion parameter by both aiding and external flows. Murthy (2000) has studied the effect of double dispersion on mixed convection heat and mass transfer in non-Darcy porous medium. He presented that as Lewis number increases, the effect of solute dispersion on mass transfer coefficient is less. The effect of melting on mixed convection heat and mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium was studied by Kairi & Murthy (2012). Murthy *et al* (2005) studied the effects of mixed convection heat and mass transfer with thermal radiation in a non-Darcy porous medium. They obtained that the effect of radiation is more pronounced in the Darcy medium than non-Darcy medium. Very recently, Ahmad & Pop (2014) studied the melting effect on mixed convection boundary layer flow about a vertical surface embedded in a porous medium, Opposing flows case. In their study they found that dual solution exists in some range of the mixed convection parameter.

In this article, we investigate the effects of melting and solute dispersion on heat and mass transfer in a non-Darcy porous medium over a vertical surface. Most of the above studies in the literature have not considered the solute dispersion effect with melting in their study.

2. Mathematical formulation

Consider the steady state two-dimensional problem of non-Darcy mixed convective flow over a vertical surface embedded in a porous medium as shown in figure 1. The coordinates (x, y) are such that x-axis is aligned vertically upwards and y-axis is normal to it. It is assumed that this surface constitutes the interface between the liquid and solid phases during melting inside the porous matrix. The plate is at constant temperature T_m. The constant temperatures of the liquid phase far from the plate and of the solid phase far from the interface are denoted by T_∞ and T_0. Under these assumptions the boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial y} + \frac{C_f\sqrt{K}}{\nu}\frac{\partial}{\partial y}(u^2) = \frac{\rho_\infty g K}{\nu}\left(\beta_T\frac{\partial T}{\partial y} + \beta_C\frac{\partial C}{\partial y}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}\right) - \frac{1}{\rho_\infty C_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(D_e\frac{\partial C}{\partial y}\right), \tag{4}$$

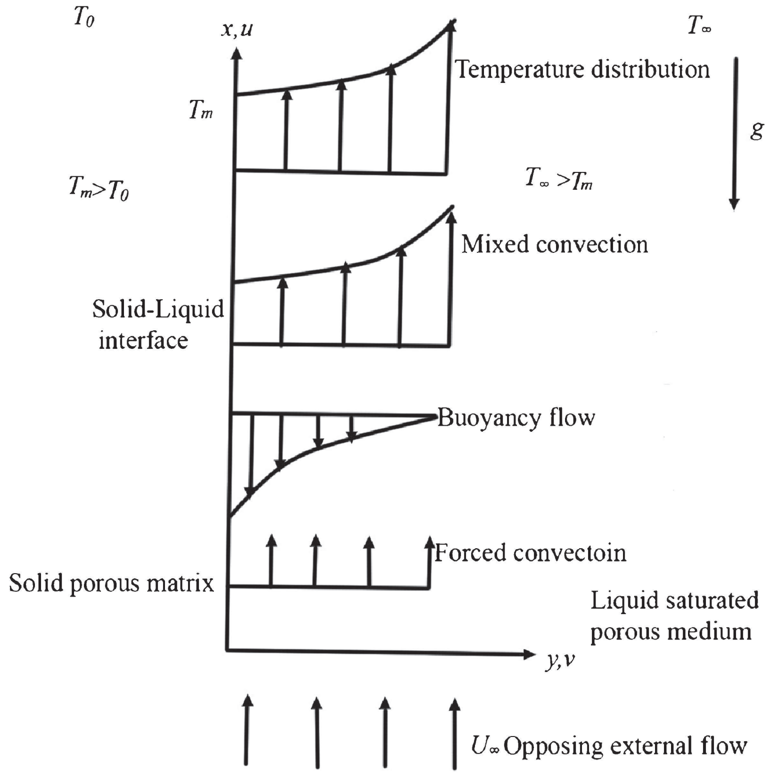


Figure 1. Schematic sketch of the problem.

where $u,\ v$ are the velocity components in the $x,\ y$ directions respectively, C_f is the Forchheimer empirical constant, ν is the kinematic viscosity, ρ is the density, g is the acceleration due to gravity, K is the permeability of the porous medium, β_T is the thermal expansion coefficient, β_C is the solute expansion coefficient, α is the thermal diffusivity, ρ is the density, C_p is the specific heat at constant pressure, q_r is the radiative heat flux, C is the concentration and D_e is the solute diffusivity.

The boundary conditions for Eqs. (1)–(4) are given in the form

$$\kappa\frac{\partial T}{\partial y} = \rho\left[h_{sf} + c_s(T_m - T_0)\right]v, \quad T = T_m, \quad C = C_w \quad \text{at} \quad y = 0,$$
$$u \to u_\infty, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty, \tag{5}$$

Following Rosseland's approximation the radiative heat flux q_r is modeled as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ^* is the Stefan– Boltzman constant, k^* is the mean absorption coefficient. Assuming that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature, then the Taylor series expansion for T^4 about T_m, after neglecting the higher order terms can be written as $T^4 \cong 4T_m^3 T - 3T_m^4$, then we have

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_m^3}{3k^*}\frac{\partial T}{\partial y}. \tag{7}$$

The continuity Eq. (1) is satisfied by introducing a stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x},$$

where $\psi = \sqrt{\alpha_m u_\infty x}\ f(\eta)$, $f(\eta)$ is the dimensionless stream function and $\eta = \frac{y}{x}\sqrt{\frac{u_\infty x}{\alpha_m}}$. The velocity components are given by

$$u = u_\infty f'(\eta) \quad \text{and} \quad v = -\frac{1}{2}\sqrt{\frac{\alpha_m\, u_\infty}{x}}\left[f(\eta) - \eta f'(\eta)\right]. \tag{8}$$

The temperature and concentrations are represented as

$$T = T_m + (T_\infty - T_m)\theta(\eta) \quad \text{and} \quad C = C_w + (C_\infty - C_w)\phi(\eta), \tag{9}$$

where $\theta(\eta)$ and $\phi(\eta)$ are the dimensionless temperature and dimensionless concentration. On using Eqs. (6) and (7), Eqs. (2)–(4) transform into the following two-point boundary value problem

$$\left(1 + Ff'\right) f'' + \frac{Ra}{Pe}(\theta' + N\phi') = 0, \tag{10}$$

$$\left(1 + Df' + \frac{4}{3}R\right)\theta'' + \frac{1}{2}f\theta' + Df''\theta' = 0, \tag{11}$$

$$\phi'' + \frac{1}{2}Lef\phi' + LeB\left(f'\phi'' + f''\phi'\right) = 0, \tag{12}$$

$$f(0) + 2M\theta'(0) = 0, \quad f'(\infty) = 1, \tag{13}$$

$$\theta(0) = 0, \quad \theta(\infty) \to 1, \tag{14}$$

$$\phi(0) = 0, \quad \phi(\infty) \to 1, \tag{15}$$

where the notation primes denote differentiation with respect to η. The non-dimensional constants in Eqs. (10)–(13) are the non-Darcy parameter F, the mixed convection parameter Ra/Pe, the buoyancy parameter N, the thermal dispersion parameter D, the radiation parameter R, the Lewis number Le, the solute dispersion parameter B and the melting parameter M, the mass diffusivity D_1. These parameters are defined as

$$F = \frac{2C_f\sqrt{K}u_\infty}{\nu}, \quad Ra = \frac{Kg\beta_T\rho_\infty(T_\infty - T_m)}{\nu\alpha_m}, \quad Pe = \frac{u_\infty}{\alpha_m}, \quad N = \frac{C_w - C_\infty}{T_m - T_\infty},$$

$$D = \frac{\gamma d u_\infty}{\alpha_m}, \quad R = \frac{4\sigma^* T_m^3}{kk^*}, \quad Le = \frac{\alpha_m}{D_1}, \quad B = \frac{\xi d u_\infty}{\alpha_m}, \quad M = \frac{c_f\,(T_\infty - T_m)}{h_{sf} + c_s\,(T_m - T_0)}.$$

3. Heat transfer coefficient

The heat transfer rate from the surface of the plate is given by

$$q_w = -k_{eff}\left[\frac{\partial T}{\partial y}\right]_{y=0} - \frac{4\sigma^*}{3k^*}\left[\frac{\partial T^4}{\partial y}\right]_{y=0}. \tag{16}$$

The local Nusselt number is defined as

$$Nu_x = \frac{xq_w}{k_{eff}\,(T_\infty - T_m)}. \tag{17}$$

The effective thermal conductivity of the porous medium is given by $k_{eff} = (1-\varepsilon)k_s + \varepsilon k_f$, ε is the porosity of the medium, k_s and k_f are the thermal conductivity of the solid and convective fluid respectively. Using Eq. (16) in (17) the dimensionless Nusselt number can be represented in terms of dimensionless temperature at the surface

$$\frac{Nu_x}{\sqrt{Pe_x}} = -\left\{1 + \frac{4}{3}R + Df'(0)\right\}\theta'(0). \tag{18}$$

4. Mass transfer coefficient

The mass transfer rate from the surface of the plate is given by

$$j_w = -D_e\left[\frac{\partial C}{\partial y}\right]_{y=0}. \tag{19}$$

The local Sherwood number is defined as

$$Sh_x = \frac{xj_w}{D_e\,(C_\infty - C_w)}. \tag{20}$$

Using Eq. (19) in (20) the dimensionless Sherwood number can be represented in terms of dimensionless concentration at the surface

$$\frac{Sh_x}{\sqrt{Pe_x}} = -\left\{1 + Bf'(0)\right\}\phi'(0). \tag{21}$$

The variables in Eqs. (19)–(21) are defined as the local mass flux j_w Sherwood number Sh_x, local peclet number Pe_x, solute dispersion parameter B.

5. Solution methodology

The set of nonlinear ordinary differential Eqs. (10)–(12) with boundary conditions (13)–(15) were solved numerically using the MATLAB `bvp4c` solver. To apply bvp4c routine to the differential Eqs. (10)–(12), these can be written as

$$f'' = -\frac{\frac{Ra}{Pe}\left(\theta' + N\phi'\right)}{1 + Ff'}, \tag{22}$$

$$\theta'' = \frac{-0.5f\theta' - Df''\theta'}{1 + Df' + \frac{4}{3}R}, \tag{23}$$

$$\phi'' = \frac{-0.5Lef\phi' - LeBf''\phi'}{1 + LeBf'}. \tag{24}$$

Defining new variables

$$f_1 = f, \quad f_2 = f', \quad f_3 = \theta, \quad f_4 = \theta', \quad f_5 = \phi, \quad f_6 = \phi',$$

Table 1. Comparison of $f'(0)$ values with Gorla *et al* (1999), Cheng & Lin (2007) for $F = N = D = R = Le = 0$ and $B = 0$.

M	$\frac{Ra_x}{Pe_x}$	$f'(0)$ (1999)	$f'(0)$ (1977)	$f'(0)$(Present)
	0.0	1.000	1.000	1.000
	1.4	2.400	2.400	2.400
	3.0	4.000	4.000	4.000
2	8.0	9.000	9.000	9.000
	10.0	11.00	11.00	11.00
	20.0	21.00	21.00	21.00

the above two second order coupled differential equations and the boundary conditions may be transformed to six first order differential equations. These can be written as

$$f_1' = f_2 \tag{25}$$

$$f_2' = -\frac{\frac{Ra}{Pe}(f_4 + Nf_6)}{1 + Ff_2} \tag{26}$$

$$f_3' = f_4 \tag{27}$$

$$f_4' = \frac{-0.5 f_1 f_4 - Df_2' f_4}{1 + Df_2 + \frac{4}{3}R} \tag{28}$$

$$f_5' = f_6 \tag{29}$$

$$f_6' = \frac{-0.5 Le f_1 f_6 - LeBf_2' f_6}{1 + LeBf_2}, \tag{30}$$

where the prime denote differentiation with respect to η. The boundary conditions are

$$f_1(0) + 2Mf_4(0) = 0, \quad f_2(\infty) \to 1, \tag{31}$$

$$f_3(0) = 0, \quad f_3(\infty) \to 1, \tag{32}$$

$$f_5(0) = 0, \quad f_5(\infty) \to 1. \tag{33}$$

When this is done in a usual way, the function `exode` can be coded. In `exbvp.m` a guess of unknown values based on linear interpolation of the boundary value specified on an initial mesh of 10 equally spaced points is made. We defined `solinit = bvpinit(linspace(0,10,10),[0 10])` and the BVP is now solved with default values using `sol = bvp4c(@exode,@exbc,solinit)`. The relative error tolerance on the residuals is `Rel Tol` $= 10^{-10}$ and the absolute error tolerance is `Abl Tol`

Table 2. Comparison of $\theta'(0)$ values with Gorla *et al* (1999), Cheng & Lin (2007) for $F = N = D = R = Le = 0$ and $B = 0$.

M	$\frac{Ra_x}{Pe_x}$	$\theta'(0)$ (1999)	$\theta'(0)$ (1977)	$\theta'(0)$ (Present)
	0.0	0.2799	0.2706	0.2706
	1.4	0.3823	0.3801	0.3800
2	3.0	0.4754	0.4745	0.4745
	8.0	0.6902	0.6902	0.6902
	10.0	0.7594	0.7594	0.7594
	20.0	1.038	1.0383	1.0383

Table 3. Comparison of $\theta'(0)$ values with Cheng (1977), Cheng & Lin (2007) for $F = N = D = R = Le = 0$ and $B = 0$.

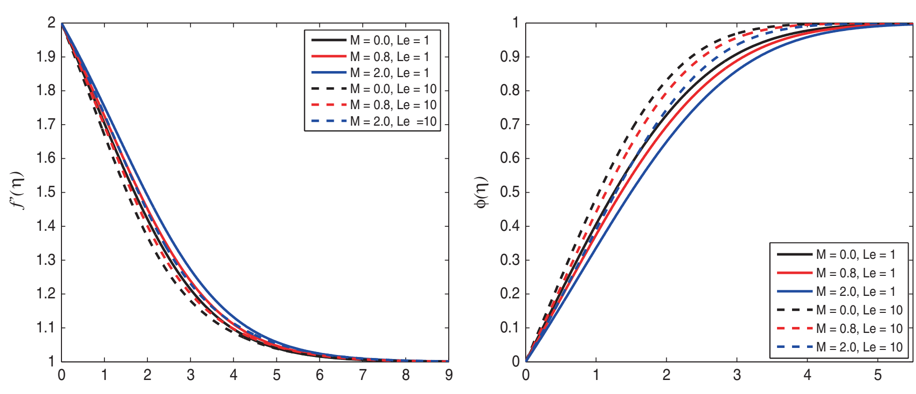
M	$\frac{Ra_x}{Pe_x}$	$\theta'(0)$ (1977)	$\theta'(0)$ (2007)	$\theta'(0)$(Present)
	-0.2	0.5269	0.5270	0.5269
0	-0.4	0.4865	0.4866	0.4866
	-0.6	0.4420	0.4421	0.4421
	-0.8	0.3917	0.3917	0.3917
	-1.0	0.3320	0.3321	0.3321

$= 10^{-10}$. The maximum value of $\eta(\eta_\infty)$ representing the ambient conditions was assumed to be 10. The accuracy of the numerical method was validated by direct comparisons with the numerical results reported earlier by Gorla *et al* (1999), Cheng (1977), Cheng & Lin (2007) for various values of mixed convection parameters in the absence of the parameters F, N, D, R, Le, B (tables 1, 2 and 3).

6. Results and discussion

In the present paper the effects of solute dispersion, mixed convection and melting on the non-dimensional velocity, concentration, heat and mass transfer rates have been studied. When $F = 0$ the effect on Darcy regime and $F \neq 0$ corresponds to non-Darcy regime. The numerical results are obtained and those are presented in tables and graphs.

The effects of melting and Lewis number on velocity and concentration profiles are shown in figure 2. It is clear from the figure that an increase in melting parameter leads to increase the velocity profile and boundary layer thickness. These results are similar to the results reported by Afify & Elgazery (2013). From a physical point of view, this result may be attributed to the fact that convection heat transfer restrain from liquid saturated porous medium to the solid porous plate. A similar kind of behavior is observed for large values of Lewis number also. But increase in velocity is less in case of large Lewis number as compared with the small values of the Lewis



(a) **(b)**

Figure 2. Effect of melting and Lewis number on (**a**) velocity, (**b**) concentration, when $F = 2$, $Ra/Pe = 2$, $N = 1$, $D = 1$, $R = 2$ and $B = 1$.

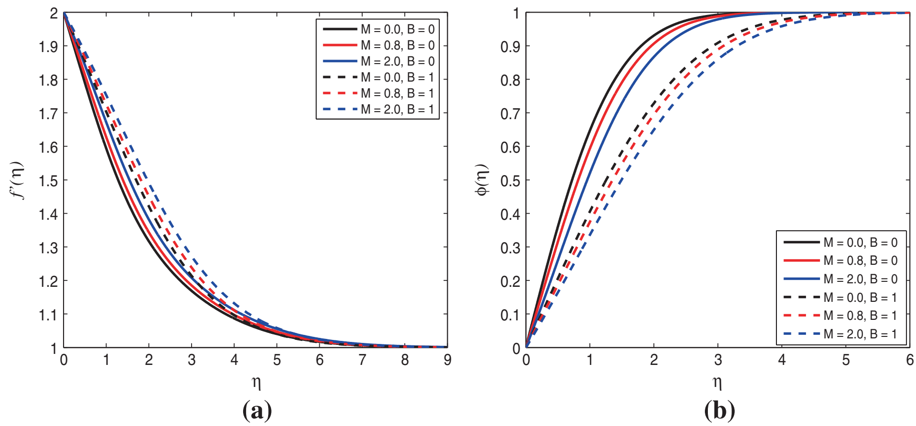


Figure 3. Effect of melting and solute dispersion on (**a**) velocity and (**b**) concentration, when $F = 2$, $Ra/Pe = 2$, $N = 1$, $D = 1$, $R = 2$ and $Le = 1$.

number. It is observed from figure 2b that concentration profile decreases with an increase in melting parameter for both large and small values of Lewis numbers. For large values of Lewis number the concentration rate is high as compared with the small values of Lewis number. This enhances the mass transfer rate.

Figure 3 shows the effects of melting parameter and solute dispersion on velocity and concentration profiles. It is observed that with an increase in melting parameter the velocity profile increases and concentration decreases. On the other hand, an increase in solute dispersion leads to increase the velocity profile and decreases the concentration profile.

The behaviour of Nusslet number and Sherwood number as an increasing function of melting parameter for various values of mixed convection and Lewis number are shown in figure 4. Mixed convection parameter is defined as the ratio of the Rayleigh number to Peclet number. Mixed convection parameter takes a positive value for aiding flow and negative value for opposing flow. Where $Ra/Pe = 0$ represents forced convection and $Ra/Pe = 1$ correspond to the

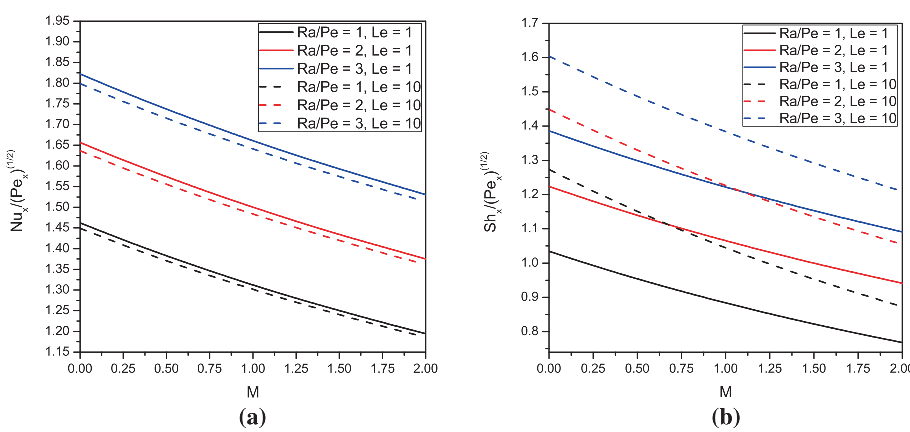


Figure 4. Variation of (**a**) Nusselt number and (**b**) Sherwood number as an increasing function of melting parameter for different values of Ra/Pe and Le, when $F = 2$, $N = 1$, $D = 1$, $R = 2$ and $B = 1$.

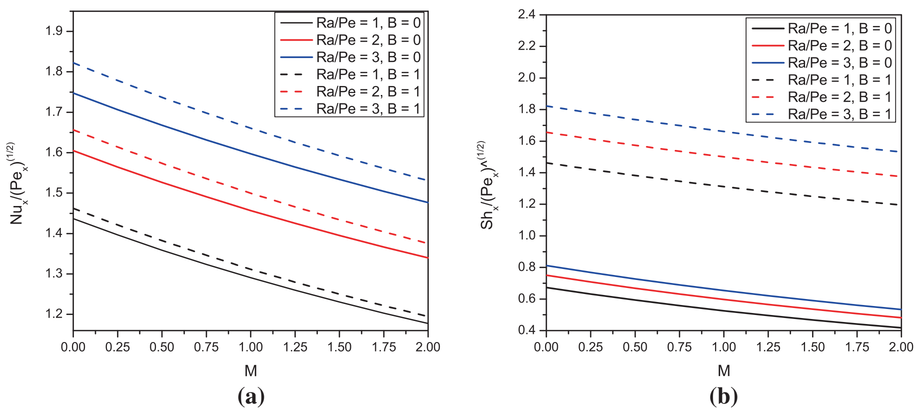


Figure 5. Variation of (**a**) Nusselt number and (**b**) Sherwood number as an increasing function of melting parameter for different values of Ra/Pe and B, when $F=2$, $N=1$, $D=1$, $R=2$ and $Le=1$.

natural convection, respectively. It is interesting to mention that heat transfer rate is more at the wall and gradually decreases far from the wall. On the other hand, heat transfer rate increases with an increase in mixed convection parameter, for small and large values of Lewis number. It can also be noticed that an increase in heat transfer rate is more for lower values of Lewis number. The physics behind this may be explained in such a way that Lewis number implies that heat dispersion is more pronounced than mass dispersion and for this particular system this results in heat and mass transfer rates are larger for large values of Lewis number as compared with the small values of Lewis number. These results are similar to the results obtained by El-Amin *et al* (2008). It is clear from figure 4b that for small values of Lewis number, mixed convection parameter enhances the mass transfer rate near that wall. Mass transfer rate decreases with an increase in mixed convection parameter from the wall, this reduction is less for small value of Lewis number as compared to the large values.

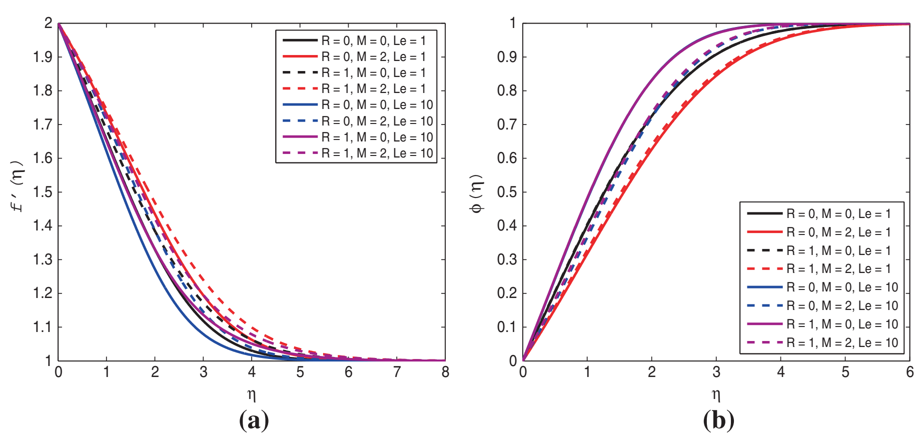


Figure 6. Radiation and Lewis number effects on (**a**) velocity and (**b**) concentration in the presence and absence of melting parameter, when $F=2$, $Ra/Pe=2$, $N=1$, $D=1$, $B=1$.

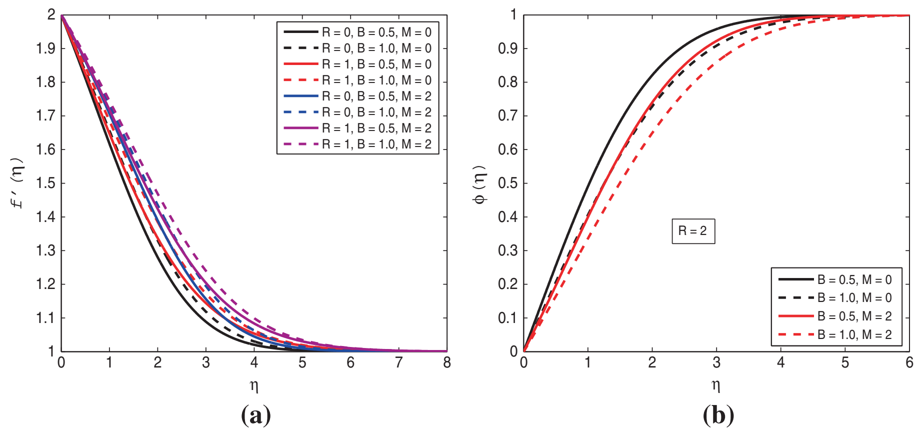


Figure 7. Radiation and solute dispersion effects on (**a**) velocity and (**b**) concentration in the presence and absence of melting parameter, when $F = 2$, $Ra/Pe = 2$, $N = 1$, $D = 1$, $Le = 1$.

Heat and mass transfer rates for various values of mixed convection and solute dispersion are shown in figure 5. It is clear from this figure that heat transfer rate increases with an increase in mixed convection parameter, this increment is more in the presence of solute dispersion. Mass transfer rate as an increasing function of melting parameter is shown in figure 5b. It is interesting to note that in the presence of solute dispersion mass transfer rate enhances farther from the wall compared to near the wall.

Sample results for velocity and concentration are given in figures 6 and 7, respectively. It is observed that the velocity profile increases with an increase in the thermal radiation parameter. And also observe that an increase in the Lewis number increases the velocity profile far from the boundary. From figure 6b, it is clear that increase in radiation parameter, the concentration increases. For a fixed values of parameters increase in melting increases the concentration for larger values of Lewis number, but the increase in concentration profile is less for the small values of Lewis number.

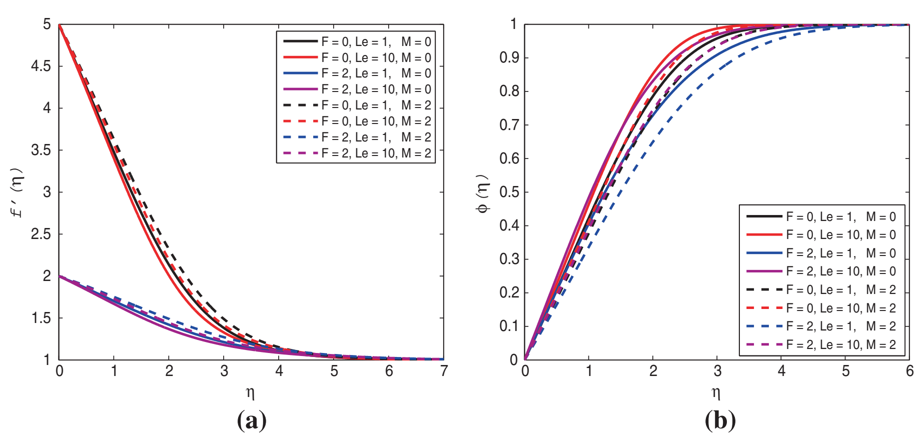


Figure 8. Non-Darcy and Lewis number effects on (**a**) velocity and (**b**) concentration in the presence and absence of melting parameter, when $Ra/Pe = 2$, $N = 1$, $D = 1$, $R = 2$, $B = 1$.

Figure 8 demonstrates that the effects of non-Darcy and Lewis number on velocity and concentration profile. It is noted that the velocity profile increases with an increase in melting parameter. It can also be observed from the figure that increases in non-Darcy parameter decrease the velocity from the wall. The concentration profile decreases with an increase in non-Darcy parameter.

7. Conclusions

The effects of melting and solute dispersion on heat and mass transfer on non-Darcy mixed convective flow over a vertical plate has been investigated in the present study. It is observed that velocity profile increases with an increase in melting parameter, whereas the concentration profile decreases. For large values of Lewis number concentration rate is high as compared with the small values of Lewis number. Mass transfer rate is more in the presence of solute dispersion far from the wall.

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