# Mixed finite element formulation for non-isothermal porous media in dynamics

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## ABSTRACT

We present a mixed finite element formulation for the spatial discretization in dynamic analysis of non-isothermal variably saturated porous media using different order of approximating functions for solid displacements and fluid pressures/temperature. It is known in fact that there are limitations on the approximating functions $\mathbf{N}^u$ and $\mathbf{N}^p$ for displacements and pressures if the Babuska–Brezzi convergence conditions or their equivalent [5] are to be satisfied. Although this formulation complicates the numerical implementation compared to equal order interpolation, it provides competitive advantages e.g. in speed of computation, accuracy and convergence.

A fully coupled mathematical [1] and numerical model for the analysis of the thermo-hydro-mechanical dynamic behaviour of multiphase geomaterials is reduced to a computationally efficient formulation by neglecting the relative acceleration of the fluid phases and the convective terms [2], [3]. The resulting mathematical model is based on an average procedure following Lewis and Schrefler [1] within the Hybrid Mixture theory. Small strains and dynamic loading conditions are assumed. The porous medium is treated as a multiphase system composed of a solid skeleton with open pores, filled with liquid water and gas. All the fluids are in contact with the solid phase. The constituents are assumed to be isotropic, homogeneous, immiscible, except for dry air and water vapour and chemically non-reacting. Local thermal equilibrium between the solid matrix, gas and liquid phases is assumed. Heat conduction, vapour diffusion, heat convection, and liquid water flow due to pressure gradients or capillary effects and water phase change (evaporation and condensation) inside pores are all taken into account.

The model has been implemented in the finite element code COMES-GEO, [1], [4]. The numerical examples will show the effectiveness of the implementation by comparison with analytical or finite element solutions for quasi-static and dynamic problems.

**Acknowledgement**. The authors would like to thank the 7th Framework Programme of the European Union (ITN MuMoLaDe project 289911) for the financial support.

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