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ABSTRACT

This paper studies the consensus problem of a second-order nonlinear multi-agent system with directed topologies. A distributed control protocol is proposed for each agent using the relative states among neighboring agents. A mixed H_∞ and passivity-based control is maneuvered to deal the bounded disturbances enduring in the system. Based on the theory of the sampled-data control technique and Lyapunov stability theory, some novel conditions are given to realize the consensus of a class of second-order multi-agent nonlinear systems. A new set of delay dependent sufficient conditions is derived in terms of linear matrix inequalities, which guarantees that all agents asymptotically converge to the convex hull with the prescribed H_∞ and passive performance. Finally, an example with simulation results is given to verify the theoretical results.

I. INTRODUCTION

The multi-agent system, which is composed of multiple interacting agents, is becoming one of the most discussed control system models, where the main attribute is communication among individual agents.[1–7] While the first-order multi-agent system is receiving more focus due to its applications in a wider area such as voltage control,[8] biological networking,[9] cybersecurity,[10] modeling business processes,[11] multi-face tracking,[12] energy control,[13,14] power system,[15] and robotics,[16] the researchers have realized that the second-order multi-agent system is equally important.[17–20]

We know that the system's performance declines if nonlinear dynamics prevail in the system. Despite the other challenges, nonlinear dynamics have to be contemplated while designing the multi-agent system, as they are inherent in any practical system. The consensus problem for a class of second-order nonlinear multi-agent systems is investigated in Refs. 21–23. When we consider a multi-agent system, there may arise various sources that cause disturbances as well as uncertainties in the dynamics of the agents, which are additional reasons for the system's performance declination. From a detailed study on the research treatises, H_∞-based and passivity-based controls have been proven to be two appropriate controllers in preparing the system to be robust against exogenous disturbances and uncertainties. Recently, a control strategy called mixed H_∞ and passivity-based control[24–28] has been used for minimizing the effects of disturbances in the system. Using this strategy, we can also execute the aforementioned control schemes individually by switching the value of a particular parameter, whenever required. This reduces the cost and time of implementing and designing separate control schemes for a considered system.

In systems science, a sampled-data system is a control system[29] in which a continuous-time plant is controlled with a digital device. Under periodic sampling, the sampled-data system is time-varying but also periodic. Thus, it may be modeled by a simplified discrete-time system obtained by discretizing the plant.[30–35] The sampling issues and mixed H_∞ and passivity with a nonlinear second order multi-agent consensus control are not yet considered in the literature.

Although the dynamics of the multi-agent system[32] have been investigated by many researchers, few of them have addressed the problem of sampled-data driven mixed H_∞ and passivity-based control for second-order multi-agent systems. Motivated by these considerations, in this paper, we investigate the mixed H_∞ and passivity analysis using a sampled-data control for second-order multi-agent systems. More precisely, a second-order multi-agent consensus control is designed based on the linear matrix inequalities (LMI) based conditions, which guarantees the stability of the nonlinear system with mixed H_∞ and passivity performance. The main contributions of this paper can be summarized as follows: The analysis of the mixed H_∞ and passivity performance index level γ to deal the external disturbance is proposed for the first time to nonlinear second-order multi-agent systems. A novel idea of the sampled-data controller is designed for the nonlinear second-order multi-agent system using the time delay concept, which takes into account the effects of inter-sampling behavior and has no degradation of closed-loop performance. It has been found that both the real and imaginary parts of the eigenvalues of the Laplacian matrix of the network play key roles in reaching second-order consensus in general.

Notations. Throughout this paper, the following notations are used: M^T and M^{-1} represent the transpose and inverse of matrix M, respectively; $\mathbb{R}^n$ represents the n-dimensional Euclidean space; $\mathbb{Z}_+$ denotes the set of positive integers; $\mathbb{R}^{n\times n}$ represents the set of all $n \times n$ real matrices; $P > 0$ ($P < 0$) means that P is a positive (negative) definite matrix; I and 0 represent identity matrix and zero matrix with appropriate dimensions; diag$\{\cdot\}$ stands for a block-diagonal matrix; and sym (A) is defined as $A + A^T$. In the symmetric block matrices or long matrix expressions, we use an asterisk ($*$) to represent a term that is induced by symmetry. $\|\cdot\|$ and $\otimes$ represent the Euclidean norm and the Kronecker product, respectively.

II. PROBLEM FORMULATION AND PRELIMINARIES

We denote a weighted digraph by $G = (V, E, A)$, where $V = 1, 2, \ldots, n$ is the set of nodes or vertices with $n \geq 2$, node i represents the ith agent; $E \subseteq V \times V$ is the set of edges, and an edge of G is denoted by an order pair (i, j); and $A = [a_{ij}]$ is an $n \times n$-dimensional weighted adjacency matrix with $a_{ii} = 0$. Say, $(i, j) \in E$ if $a_{ji} > 0$. The set of neighbors of the ith agent is denoted by $N_i = j \in V$: $(j, i) \in E$. If (i, j) is an edge of G, node i is called the parent of node j. A directed tree is a directed graph, where every node, except one special node without any parent, which is called the root, has exactly one parent, and the root can be connected to any other nodes through paths. The $n \times n$-dimensional Laplacian matrix $L(G) = [l_{ij}]$ of digraph G is defined by $l_{ii} = \sum_{k=1}^{n} a_{ik}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. It is easy to see that $L(G)$ has at least one zero eigenvalue and $L(G)1 = 0$.

Consider a system composed of n agents. The information exchange between agents is modeled by a weighted digraph G. Each agent is regarded as a node and the (i, j), $i, j = 1, \ldots, n$, element a_{ij} of the adjacent matrix denotes the weight on information link (j, i). The dynamics of the ith agent are described by

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t) + f_i(x_i, v_i) + d_i(t), \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}, v_i \in \mathbb{R}$, and $u_i \in \mathbb{R}$ represent the position, velocity, and control input of the ith agent, respectively. $f_i \in \mathbb{R}$ is a smooth function, and $d_i(t) \in \mathbb{R}$ represents the unknown external disturbance.

We consider that both information on the relative position and velocity of agents at the discrete sampling instants can be obtained. The protocol can be proposed as

$$\begin{aligned} u_i(t) = &-\alpha \sum_{j=1, j\neq i}^{N} a_{ij}(x_i(t_k) - x_j(t_k)) \\ &-\beta \sum_{j=1, j\neq i}^{N} a_{ij}(v_i(t_k) - v_j(t_k)), \quad t \in [t_k, t_{k+1}), \end{aligned} \tag{2}$$

where α and β are the coupling strengths, $\tau = t_{k+1} - t_k$ is the sampling interval, and t_k's are the sampling instants satisfying $0 = t_0 < t_1 < \cdots < t_k < \cdots$. We present a time-varying piecewise-continuous delay $\tau(t) = t - t_k$, $t \in (t_k, t_{k+1})$ and $\dot{\tau}(t) = 1$ for $t \neq t_k$ and $\tau(t) \leq \tau$.

Remark II.1. In engineering practice, the usage of computer control is in peak, which makes the closed-loop system a sampled-data one that contains both discrete and continuous time signals. This aroused our interest to propose a sampled-data control for a nonlinear second-order multi-agent system, which takes into account the effects of inter-sampling behavior and has no degradation of closed-loop performance.

Then, systems (1) with (2) can be framed as

$$\dot{x}_i(t) = v_i(t), \tag{3}$$

$$\begin{aligned} \dot{v}_i(t) = &-\alpha \sum_{j=1}^{N} l_{ij}(x_j(t - \tau(t)) - \beta \sum_{j=1}^{N} l_{ij}(v_j(t - \tau(t)) \\ &+ f_i(x_i, v_i) + d_i(t), \quad t \in [t_k, t_{k+1}). \end{aligned} \tag{4}$$

Let $\Phi_i(t) = [x_i^T, v_i^T]^T$, $\Phi(t) = [\Phi_1^T(t), \Phi_2^T(t), \ldots, \Phi_N^T(t)]^T$, $x(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T$, and $v(t) = [v_1^T(t), v_2^T(t), \ldots, v_N^T(t)]^T$. Now by employing the concept of the Kronecker product, Eq. (3) can be obtained as

$$\begin{aligned} \dot{\Phi}(t) = &(I_N \otimes A)\Phi(t) - (L \otimes B)\Phi(t - \tau(t)) \\ &+ (I_N \otimes C)\Phi(t, f(t)) + (I_N \otimes C)d(t), \end{aligned} \tag{5}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \alpha & \beta \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(A1) The nonlinear wave force $\Phi(t, f(t))$ in (5) is uniformly bounded and satisfies the following constraint $\|\Phi(t, f(t))\| \leq \alpha_1 \|\Phi(t)\|$, where α_1 is a positive scalar.

Lemma II.2. The Laplacian matrix L has a simple eigenvalue 0, and all the other eigenvalues have positive real parts if and only if the directed network has a directed spanning tree. In addition, there exist $p = [p_1 \ldots p_N]^T \in \mathbb{R}^N$ satisfying $p^T 1_N = 1$.

Lemma II.3. Suppose that the network G contains a directed spanning tree. There exist nonsingular matrix Q such that $L = Q\bar{J}Q^T$, with $\bar{J}$ being the upper triangular real Jordan canonical form of L, and the Jordan blocks associated with the real eigenvalues of L are consistent with the normal forms. In addition, Q satisfies $P^T Q = C[1, 0, \ldots, 0]$, $C \neq 0$.

Lemma II.4 (Ref. 36). *For any constant matrix $M > 0$, the following inequality holds for all continuously differentiable function φ in $[a, b] \to \mathbb{R}^n$:*

$$(b-a)\int_a^b \varphi^T(s)(I_N \otimes M)\varphi(s)ds \geq \left(\int_a^b \varphi(s)ds\right)^T (I_N \otimes M)\left(\int_a^b \varphi(s)ds\right) + 3\theta^T(I_N \otimes M)\theta,$$

$$\frac{(b-a)^2}{2}\int_a^b \int_s^b \varphi^T(s)(I_N \otimes M)\varphi(s)ds \geq \left(\int_a^b \int_s^b \varphi(s)ds\right)^T (I_N \otimes M)\left(\int_a^b \int_s^b \varphi(s)ds\right) + 2\theta_1^T(I_N \otimes M)\theta_1,$$

where $\theta = \int_a^b \varphi(s)ds - \frac{2}{b-a}\int_a^b\int_s^b \varphi(u)duds$ and $\theta_1 = -\int_a^b\int_s^b \varphi(u)duds + \frac{3}{b-a}\int_a^b\int_s^b\int_u^b \varphi(v)dvduds$.

Lemma II.5 (Ref. 37). *For any constant matrix $R > 0$, the following inequality holds for all continuously differentiable function ω in $[a, b] \to \mathbb{R}^n$:*

$$\int_a^b \dot{\omega}^T(s)(I_N \otimes R)\dot{\omega}(s)ds \geq \frac{1}{b-a}(\omega(b) - \omega(a))^T(I_N \otimes R) \times (\omega(b) - \omega(a)) + \frac{3}{b-a}\Theta^T(I_N \otimes R)\Theta,$$

where $\Theta = \omega(b) + \omega(a) - \frac{2}{b-a}\int_a^b \omega(s)ds$.

Definition II.6 (Ref. 24). *Second-order nonlinear multi-agent time-delay system (8) is said to be asymptotically stable with mixed $\mathscr{H}_\infty$ and passivity-performance γ if the system with $d(t) \equiv 0$ is stable and under zero initial condition, there exists a scalar $\gamma > 0$ such that*

$$\left\{\int_0^{t^*} (-\gamma^{-1}\theta z(\alpha)^T z(\alpha) + 2(1-\theta)z(\alpha)^T d(\alpha))d\alpha\right\} \geq -\gamma\int_0^{t^*} d(\alpha)^T d(\alpha)d\alpha \tag{6}$$

for all $t^* > 0$ and any non-zero $d(t) \in \mathscr{L}_2[0, \infty)$, where $\theta \in [0, 1]$ denotes a weighting parameter that defines the trade-off between $\mathscr{H}_\infty$ and passivity-performance.

Note II.7. It has to be stated that when $\theta = 1$, system (8) reduces to the $\mathscr{H}_\infty$ norm constraint and when $\theta = 0$, system (8) reduces to the passivity condition.

III. MAIN RESULT

From Lemma II.2, let $\bar{\xi} = (Q^{-1} \otimes I_2)\xi$, where Q satisfies $Q\bar{J}Q^T = L$, with $\bar{J}$ being the upper triangular real Jordan canonical form of L. Obviously, it can be verified from Lemmas II.2 and II.3 that $\bar{J} = diag\{0, \hat{J}\}$. Then, system (5) can be written as

$$\dot{\varphi}(t) = A\varphi(t), \tag{7}$$

$$\dot{\Phi}(t) = (I_{N-1} \otimes A)\Phi(t) - (\hat{J} \otimes B)\Phi(t - \tau(t)) + (I_{N-1} \otimes C)(\Phi(t, f(t)) + d(t)). \tag{8}$$

In this paper, the controlled output function $z(t)$ can be defined as

$$z(t) = \Phi(t). \tag{9}$$

It can be easily followed from Lemmas II.2 and II.3 that system (7) has zero solution. Therefore, the consensus problem of system (5) can be transformed into the asymptotic stability problem of system (8).

Theorem III.1. *Suppose that the network G contains a directed spanning tree, then (8) reaches consensus, if α and β satisfy $\frac{\beta^2}{\alpha} > \max_{i\in\mathscr{F}} \frac{lm^2(\lambda_i)}{Re(\lambda_i)\|\lambda_i\|^2}$, and there exist matrices $P > 0$, $R > 0$, $Q > 0$, $M > 0$, and $S > 0$ such that the following LMIs hold:*

$$\begin{bmatrix} \Xi_{11\times 11} & \hat{\Xi}_1 & \hat{\Xi}_2 \\ * & -(I_{N-1} \otimes R) & 0 \\ * & * & -(I_{N-1} \otimes M) \end{bmatrix} < 0, \tag{10}$$

where $\Xi_{1,1} = 2((I_{N-1} \otimes P)(I_{N-1} \otimes A)) - \frac{3}{\tau}(I_{N-1} \otimes M) - \frac{3}{\tau}(I_{N-1} \otimes R) + \tau(I_{N-1} \otimes Q) + \frac{\tau^2}{2}(I_{N-1} \otimes S) - \frac{1}{\tau}(I_{N-1} \otimes M) - \frac{1}{\tau}(I_{N-1} \otimes R) + \rho\alpha_1^2$, $\Xi_{1,3} = (I_{N-1} \otimes P)(\hat{J} \otimes B) - \frac{3}{\tau}(I_{N-1} \otimes M) - \frac{3}{\tau}(I_{N-1} \otimes R) + \frac{1}{\tau}(I_{N-1} \otimes M) + \frac{1}{\tau}(I_{N-1} \otimes R)$, $\Xi_{1,4} = \frac{6}{\tau^2}(I_{N-1} \otimes M) + \frac{6}{\tau^2}(I_{N-1} \otimes R)$, $\Xi_{1,9} = (I_{N-1} \otimes P)(I_{N-1} \otimes C)$, $\Xi_{1,10} = (I_{N-1} \otimes P)(I_{N-1} \otimes C) - (1 - \theta)(I_{N-1} \otimes C)^T$, $\Xi_{1,11} = \sqrt{\theta}(I_{N-1} \otimes C)^T$, $\Xi_{2,2} = -\frac{3}{\tau}(I_{N-1} \otimes M) - \frac{1}{\tau}(I_{N-1} \otimes M)$, $\Xi_{2,3} = -\frac{3}{\tau}(I_{N-1} \otimes M) + \frac{1}{\tau}(I_{N-1} \otimes M)^T$, $\Xi_{2,5} = \frac{6}{\tau^2}(I_{N-1} \otimes M)$, $\Xi_{3,3} = -\frac{3}{\tau}(I_{N-1} \otimes M) - \frac{3}{\tau}(I_{N-1} \otimes M)^T - \frac{3}{\tau}(I_{N-1} \otimes R) - \frac{1}{\tau}(I_{N-1} \otimes M) - \frac{1}{\tau}(I_{N-1} \otimes M) - \frac{1}{\tau}(I_{N-1} \otimes R)$, $\Xi_{3,4} = \frac{6}{\tau^2}(I_{N-1} \otimes M) + \frac{6}{\tau^2}(I_{N-1} \otimes R)$, $\Xi_{3,5} = \frac{6}{\tau^2}(I_{N-1} \otimes M)$, $\Xi_{4,4} = -\frac{3}{\tau}(I_{N-1} \otimes Q) - \frac{12}{\tau^3}(I_{N-1} \otimes M) - \frac{12}{\tau^3}(I_{N-1} \otimes R) - \frac{1}{\tau}(I_{N-1} \otimes Q)$, $\Xi_{4,7} = \frac{6}{\tau^2}(I_{N-1} \otimes Q)$, $\Xi_{5,5} = -\frac{3}{\tau}(I_{N-1} \otimes Q) - \frac{12}{\tau^3}(I_{N-1} \otimes M) - \frac{1}{\tau}(I_{N-1} \otimes Q)$, $\Xi_{5,6} = \frac{6}{\tau^2}(I_{N-1} \otimes Q)$, $\Xi_{6,6} = -\frac{12}{\tau^3}(I_{N-1} \otimes Q) - 2(I_{N-1} \otimes S)$, $\Xi_{6,8} = \frac{6}{\tau^2}(I_{N-1} \otimes S)$, $\Xi_{7,7} = -\frac{12}{\tau^3}(I_{N-1} \otimes Q)$, $\Xi_{8,8} = -\frac{18}{\tau^2}(I_{N-1} \otimes S)$, $\Xi_{9,9} = -\rho I$, $\Xi_{10,10} = \Xi_{11,11} = -\gamma I$, $\Xi_1 = [(I_{N-1} \otimes A)^T(I_{N-1} \otimes R)\ \ 0_n\ \ (\hat{J} \otimes B)^T(I_{N-1} \otimes R)\ \ 0_{5n}\ \ (I_{N-1} \otimes C)^T(I_{N-1} \otimes R)\ \ (I_{N-1} \otimes C)^T(I_{N-1} \otimes R)]^T$, $\Xi_2 = [(I_{N-1} \otimes A)^T(I_{N-1} \otimes M)\ \ 0_n\ \ (\hat{J} \otimes B)^T(I_{N-1} \otimes M)\ \ 0_{5n}\ \ (I_{N-1} \otimes C)^T(I_{N-1} \otimes M)\ \ (I_{N-1} \otimes C)^T(I_{N-1} \otimes M)]^T$.

Proof: By Theorem 1 in Ref. 22, we can conclude $(I_{N-1} \otimes A) - (\hat{J} \otimes B)$ is Hurwitz stable if and only if $\frac{\beta^2}{\alpha} > \max_{i\in\mathscr{F}} \frac{lm^2(\lambda_i)}{Re(\lambda_i)\|\lambda_i\|^2}$. In order to prove that the LMI (10) is stable, we construct the Lyapunov–Krasovskii functional (LKF) candidate for system (8) in the following form:

$$\begin{aligned} V(t) = {} & \Phi^T(t)(I_{N-1} \otimes P)\Phi(t) + (t - (t - t_k)) \times \int_{t_k}^t \dot{\Phi}^T(s)(I_{N-1} \otimes R)\dot{\Phi}(s)ds \\ & + \int_{-\tau}^0 \int_{t+\beta}^t \Phi^T(s)(I_{N-1} \otimes Q)\Phi(s)dsd\beta \\ & + \int_{-\tau}^0 \int_{t+\beta}^t \dot{\Phi}^T(s)(I_{N-1} \otimes M)\dot{\Phi}(s)dsd\beta \\ & + \int_{-\tau}^0 \int_{\theta}^0 \int_{t+\beta}^t \Phi^T(s)(I_{N-1} \otimes S)\Phi(s)dsd\beta d\theta. \end{aligned} \tag{11}$$

Calculating the derivatives $\dot{V}(t)$ along the trajectories of system (8), we have

$$\dot{V}(t) = \Phi^T(t)(I_{N-1}\otimes P)\dot{\Phi}(t) + (t-(t-t_k))\dot{\Phi}^T(s)(I_{N-1}\otimes R)\dot{\Phi}(t) - \int_{t_k}^{t}\dot{\Phi}^T(s)(I_{N-1}\otimes R)\dot{\Phi}(s)ds + \tau\Phi^T(t)(I_{N-1}\otimes Q)\Phi(t) - \int_{t-\tau}^{t}\Phi^T(s)(I_{N-1}\otimes Q)\Phi(s)ds + \tau\dot{\Phi}^T(t)(I_{N-1}\otimes M)\dot{\Phi}(t) - \int_{t-\tau}^{t}\dot{\Phi}^T(s)(I_{N-1}\otimes M)\dot{\Phi}(s)ds + \frac{\tau^2}{2}\Phi^T(t)(I_{N-1}\otimes S)\Phi(t) - \int_{-\tau}^{0}\int_{t+\theta}^{t}\Phi^T(s)(I_{N-1}\otimes S)\Phi(s)dsd\theta. \tag{12}$$

Using the notion of time delay, the integral terms in (12) can be written as

$$-\int_{t-\tau}^{t}\Phi^T(s)(I_{N-1}\otimes Q)\Phi(s)ds = -\int_{t-\tau}^{t-\tau(t)}\Phi^T(s)(I_{N-1}\otimes Q)\Phi(s)ds - \int_{t-\tau(t)}^{t}\Phi^T(s)(I_{N-1}\otimes Q)\Phi(s)ds, \tag{13}$$

$$-\int_{t-\tau}^{t}\dot{\Phi}^T(s)(I_{N-1}\otimes M)\dot{\Phi}(s)ds = -\int_{t-\tau}^{t-\tau(t)}\dot{\Phi}^T(s)(I_{N-1}\otimes M)\dot{\Phi}(s)ds - \int_{t-\tau(t)}^{t}\dot{\Phi}^T(s)(I_{N-1}\otimes M)\dot{\Phi}(s)ds. \tag{14}$$

Applying Lemma II.4 to the right hand to the integral terms in (13), we can obtain

$$-\int_{t-\tau}^{t-\tau(t)}\Phi^T(s)(I_{N-1}\otimes Q)\Phi(s)ds \le -\frac{1}{\tau}\left(\int_{t-\tau}^{t-\tau(t)}\Phi(s)ds\right)^T(I_{N-1}\otimes Q)\left(\int_{t-\tau}^{t-\tau(t)}\Phi(s)ds\right) - \frac{3}{\tau}\Theta_2^T(I_{N-1}\otimes Q)\Theta_2,$$

$$-\int_{t-\tau(t)}^{t}\Phi^T(s)(I_{N-1}\otimes Q)\Phi(s)ds \le -\frac{1}{\tau}\left(\int_{t-\tau(t)}^{t}\Phi(s)ds\right)^T(I_{N-1}\otimes Q)\left(\int_{t-\tau(t)}^{t}\Phi(s)ds\right) - \frac{3}{\tau}\Theta_1^T(I_{N-1}\otimes Q)\Theta_1,$$

where $\Theta_1 = \int_{t-\tau(t)}^{t}\Phi(s)ds - \frac{2}{\tau}\int_{-\tau(t)}^{0}\int_{t+\theta}^{t}\Phi(s)dsd\theta$ and $\Theta_2 = \int_{t-\tau}^{t-\tau(t)}\Phi(s))ds - \frac{2}{\tau}\int_{-\tau}^{-\tau(t)}\int_{t+\theta}^{t}\Phi(s)dsd\theta$.

By using Lemma II.5 to the right hand side of integral terms in (14), we have

$$-\int_{t-\tau}^{t-\tau(t)}\dot{\Phi}^T(s)(I_{N-1}\otimes M)\dot{\Phi}(s)ds \le -\frac{1}{\tau}\left(\int_{t-\tau}^{t-\tau(t)}\dot{\Phi}(s)ds\right)^T(I_{N-1}\otimes M)\left(\int_{t-\tau}^{t-\tau(t)}\dot{\Phi}(s)ds\right) - \frac{3}{\tau}\Theta_4^T(I_{N-1}\otimes M)\Theta_4,$$

$$-\int_{t-\tau(t)}^{t}\dot{\Phi}^T(s)(I_{N-1}\otimes M)\dot{\Phi}(s)ds \le -\frac{1}{\tau}\left(\int_{t-\tau(t)}^{t}\dot{\Phi}(s)ds\right)^T(I_{N-1}\otimes M)\left(\int_{t-\tau(t)}^{t}\dot{\Phi}(s)ds\right) - \frac{3}{\tau}\Theta_3^T(I_{N-1}\otimes M)\Theta_3,$$

where $\Theta_3 = \Phi(t) + \Phi(t-\tau(t)) - \frac{2}{\tau}\int_{t-\tau(t)}^{t}\Phi(s)ds$ and $\Theta_4 = \Phi(t-\tau(t)) + \Phi(t-\tau) - \frac{2}{\tau}\int_{t-\tau}^{t-\tau(t)}\Phi(s)ds$.

Furthermore, applying Lemma II.5 to the following integral present in (12), we can obtain

$$-\int_{t_k}^{t}\dot{\Phi}^T(s)(I_{N-1}\otimes R)\dot{\Phi}(s)ds \le -\frac{1}{\tau}(\Phi(t)-\Phi(t_k))^T \times (I_{N-1}\otimes R)(\Phi(t)\Phi(t_k)) - \frac{3}{\tau}\Theta_5^T(I_{N-1}\otimes R)\Theta_5, \tag{15}$$

where $\Theta_5 = \Phi(t) + \Phi(t_k) - \frac{2}{\tau}\int_{t_k}^{t}\Phi(s)ds$.

Also,

$$-\int_{-\tau}^{0}\int_{t+\theta}^{t}\Phi^T(s)(I_{N-1}\otimes S)\Phi(s)dsd\theta \le -\frac{2}{\tau^2}\left(\int_{-\tau}^{0}\int_{t+\theta}^{t}\Phi(s)ds\right)^T(I_{N-1}\otimes S)\left(\int_{-\tau}^{0}\int_{t+\theta}^{t}\Phi(s)ds\right) + 2\Theta_6^T(I_{N-1}\otimes S)\Theta_6,$$

where $\Theta_6 = -\int_{-\tau}^{0}\int_{t+\theta}^{t}\varphi(u)duds + \frac{3}{\tau}\int_{-\tau}^{0}\int_{\theta}^{t}\int_{t+\beta}^{t}\varphi(v)dvd\beta d\theta$.

From assumption **(A1)**, for any scalar $\rho > 0$, we can obtain

$$\rho(\alpha_1^2\Phi^T(t)\Phi(t) - \Phi^T(t,f(t))\Phi(t,f(t))) \ge 0. \tag{16}$$

Then, we have $\dot{V}(t) \le \xi^T(t)\Omega\xi(t)$, where

$$\xi^T(t) = \Big[\Phi^T(t)\Phi^T(t-\tau)\ \Phi^T(t-\tau(t))\ \int_{t-\tau(t)}^{t}\Phi^T(s)ds \times \int_{t-\tau}^{t-\tau(t)}\Phi^T(s)ds\int_{-\tau}^{-\tau(t)}\int_{t+\theta}^{t}\Phi^T(s)dsd\theta \times \int_{-\tau(t)}^{0}\int_{t+\theta}^{t}\Phi^T(s)dsd\theta \times \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t}\Phi^T(s)dsd\beta d\theta\ \Phi^T(t,f(t))\Big],$$

with

$$\Omega = \begin{bmatrix} \Xi_{9\times 9} & \Xi_1 & \Xi_2 \\ * & -(I_{N-1}\otimes R) & 0 \\ * & * & -(I_{N-1}\otimes M) \end{bmatrix}, \tag{17}$$

where $\Xi_{1,1} = 2((I_{N-1} \otimes P)(I_{N-1} \otimes A)) - \frac{3}{\tau}(I_{N-1} \otimes M) - \frac{3}{\tau}(I_{N-1} \otimes R) + \tau(I_{N-1} \otimes Q) + \frac{\tau^2}{2}(I_{N-1} \otimes S) - \frac{1}{\tau}(I_{N-1} \otimes M) - \frac{1}{\tau}(I_{N-1} \otimes R) + \rho\alpha_1^2$, $\Xi_{1,3} = (I_{N-1} \otimes P)(\hat{J} \otimes B) - \frac{3}{\tau}(I_{N-1} \otimes M) - \frac{3}{\tau}(I_{N-1} \otimes R) + \frac{1}{\tau}(I_{N-1} \otimes M) + \frac{1}{\tau}(I_{N-1} \otimes R)$, $\Xi_{1,4} = \frac{6}{\tau^2}(I_{N-1} \otimes M) + \frac{6}{\tau^2}(I_{N-1} \otimes R)$, $\Xi_{1,9} = (I_{N-1} \otimes P)(I_{N-1} \otimes C)$, $\Xi_{2,2} = -\frac{3}{\tau}(I_{N-1} \otimes M) - \frac{1}{\tau}(I_{N-1} \otimes M)$, $\Xi_{2,3} = -\frac{3}{\tau}(I_{N-1} \otimes M) + \frac{1}{\tau}(I_{N-1} \otimes M)^T$, $\Xi_{2,5} = \frac{6}{\tau^2}(I_{N-1} \otimes M)$, $\Xi_{3,3} = -\frac{3}{\tau}(I_{N-1} \otimes M) - \frac{3}{\tau}(I_{N-1} \otimes M)^T - \frac{3}{\tau}(I_{N-1} \otimes R) - \frac{1}{\tau}(I_{N-1} \otimes M) - \frac{1}{\tau}(I_{N-1} \otimes M) - \frac{1}{\tau}(I_{N-1} \otimes R)$, $\Xi_{3,4} = \frac{6}{\tau^2}(I_{N-1} \otimes M) + \frac{6}{\tau^2}(I_{N-1} \otimes R)$, $\Xi_{3,5} = \frac{6}{\tau^2}(I_{N-1} \otimes M)$, $\Xi_{4,4} = -\frac{3}{\tau}(I_{N-1} \otimes Q) - \frac{12}{\tau^3}(I_{N-1} \otimes M) - \frac{12}{\tau^3}(I_{N-1} \otimes R) - \frac{1}{\tau}(I_{N-1} \otimes Q)$, $\Xi_{4,7} = \frac{6}{\tau^2}(I_{N-1} \otimes Q)$, $\Xi_{5,5} = -\frac{3}{\tau}(I_{N-1} \otimes Q) - \frac{12}{\tau^3}(I_{N-1} \otimes M) - \frac{1}{\tau}(I_{N-1} \otimes Q)$, $\Xi_{5,6} = \frac{6}{\tau^2}(I_{N-1} \otimes Q)$, $\Xi_{6,6} = -\frac{12}{\tau^3}(I_{N-1} \otimes Q) - 2(I_{N-1} \otimes S)$, $\Xi_{6,8} = \frac{6}{\tau^2}(I_{N-1} \otimes S)$, $\Xi_{7,7} = -\frac{12}{\tau^3}(I_{N-1} \otimes Q)$, $\Xi_{8,8} = -\frac{18}{\tau^2}(I_{N-1} \otimes S)$, $\Xi_{9,9} = -\rho I$, $\Xi_1 = [(I_{N-1} \otimes A)^T (I_{N-1} \otimes R)\ \ 0_n\ \ (\hat{J} \otimes B)^T (I_{N-1} \otimes R)\ \ 0_{5n}\ \ (I_{N-1} \otimes C)^T (I_{N-1} \otimes R)]^T$, $\Xi_2 = [(I_{N-1} \otimes A)^T (I_{N-1} \otimes M)\ \ 0_n\ \ (\hat{J} \otimes B)^T (I_{N-1} \otimes M)\ \ 0_{5n}\ \ (I_{N-1} \otimes C)^T (I_{N-1} \otimes M)]^T$.

Clearly, a sufficient condition for $\dot{V}(t) < 0$ is $\Omega < 0$, which can be guaranteed by the condition (10). Therefore, system (8) is asymptotically stable when $d(t) = 0$, i.e., the states of agents asymptotically converge to the convex hull.

In the upcoming part of this paper, we will be discussing the mixed $\mathscr{H}_\infty$ and passive performance of the closed-loop system (8) with nonzero disturbance $d(t)$. For this, a similar method as in stability analysis will be followed with the LKF (11) and by considering an index $J_{zw}(t)$ for system (8) as

$$J_{zw}(t) = \left\{ \int_0^{t^*} \left[\gamma^{-1}\theta z(\alpha)^T z(\alpha) - 2(1-\theta) z(\alpha)^T d(\alpha)) d\alpha \right.\right.$$
$$\left.\left. - \gamma d(\alpha)^T d(\alpha) d\alpha \right] \right\}, \quad t^* \geq 0.$$

Under zero initial condition, it is easy to see that

$$J_{zw}(t) = \left\{ \int_0^{t^*} \left[\gamma^{-1}\theta z(\alpha)^T z(\alpha) - 2(1-\theta) z(\alpha)^T d(\alpha)) d\alpha \right.\right.$$
$$\left.\left. - \gamma d(\alpha)^T d(\alpha) d\alpha + V(s) \right] ds \right\}$$
$$\leq \left\{ \int_0^t \xi_1^T(s) \Phi \xi_1(s) ds \right\},$$

where $\xi_1^T(t) = [\xi^T(t)\ \ d^T(t)]^T$ and

$$\Phi = \begin{bmatrix} \Omega + \gamma^{-1}\theta (I_{N-1} \otimes C)^T (I_{N-1} \otimes C) & \Xi_{1,10} \\ * & -\gamma I \end{bmatrix} < 0,$$

with $\Xi_{1,10} = (I_{N-1} \otimes P)(I_{N-1} \otimes C) - (1-\theta)(I_{N-1} \otimes C)^T$. Furthermore, from Schur complement and (10), consequently $J_{zw}(t) < 0$ for all $t > 0$. Therefore, for any non-zero $d(t) \in \mathscr{L}_2[0, \infty)$, (6) holds for all $t^* > 0$. Therefore by Definition II.6, it is concluded that the multi-agent system (8) attains consensus with mixed $\mathscr{H}_\infty$ and passivity-based performance γ.

Remark III.2. In Theorem III.1, we have considered the Lyapunov–Krasovskii functional (11) which is a time dependent functional, wherein the complete information about the actual sampling pattern is used in its construction. An interesting feature of this Lyapunov–Krasovskii functional is that it can effectively deal the second-order multi-agent system with a sampled-data controller. Theorems III.1 imply that our sampled-data control design problem can be reduced to a simple LMI problem, which can be solved very efficiently via *various powerful LMI optimization algorithms.*

Remark III.3. It should be pointed out that one of the major contributions of this paper is the mixed H_∞ and passivity based sampled-data consensus controller design for nonlinear second-order multi-agent systems. In conjunction with the consensus of the second-order nonlinear multi-agent system with a sampled-data controller, exogenous disturbance attenuation is carried out with mixed H_∞ and passivity analysis, where the H_∞ and passivity performances are combined in a unified framework, which will cut off the cost and time during analysis. When in need, the two individual cases can also be examined with the proposed algorithm.

Remark III.4. It is well known that the computational complexity of LMI based conditions is decided by the number of decision variables used in the Lyapunov–Krasovskii functional and the slack variables arising in the mathematical derivation of the results. As we know that more number of decision variables will increase the computational burdens, but the net result will be less conservative. It should be mentioned that it is possible to get more less conservative results by using some integral zero inequality approach, which will be addressed in the future work.

IV. NUMERICAL SIMULATION

In this section, we provide an example to demonstrate the effectiveness of the proposed method.

Example IV.1. Let us consider a multi-agent system with four agents, where the interaction topology is shown in Fig. 1. Furthermore, the Laplacian matrix is

$$L = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & -1 & 0 & 2 \end{bmatrix}.$$

Let $\alpha = 3$, $\beta = 2$, $\theta = 0.3$, and the sampling interval bound be $\tau = 0.3$. By solving the LMI in Theorem III.1 via *Matlab LMI toolbox,*

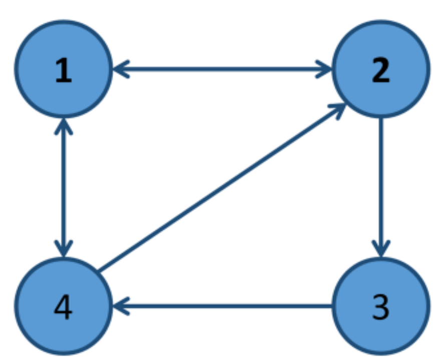
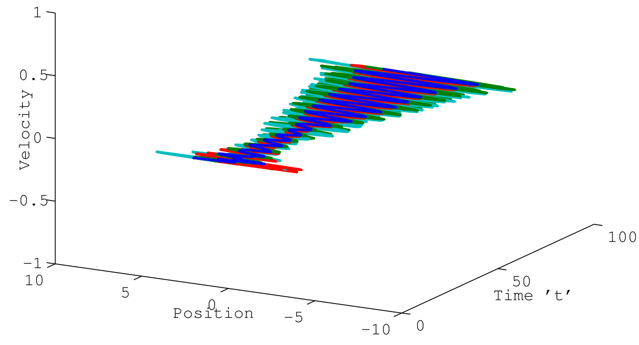


FIG. 1. Topology structure of systems with a directed spanning tree.



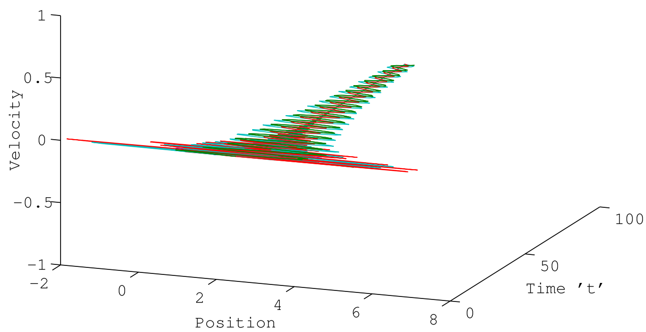


FIG. 2. Responses of position and velocity states of agents, when h = 0.3, α = 3, and β = 2. (a) Unforced system and (b) forced system.

the minimum mixed H_∞ and passivity-performance is obtained as γ_{min} = 0.7630 and the condition in Theorem III.1 of $\frac{\beta^2}{\alpha} > \frac{l^2(\mu_i)}{\beta(\mu_i)|\mu_i|^2}$, $i = 2, 3, 4$ is satisfied. The maximal allowable sampling interval bound can be obtained as 0.3. The initial conditions for the agents are set as $x(0) = (2, -1, 1.5, 1.8)^T$ and $v(0) = (1.8, 2, 1.2, 1.5)^T$ and the

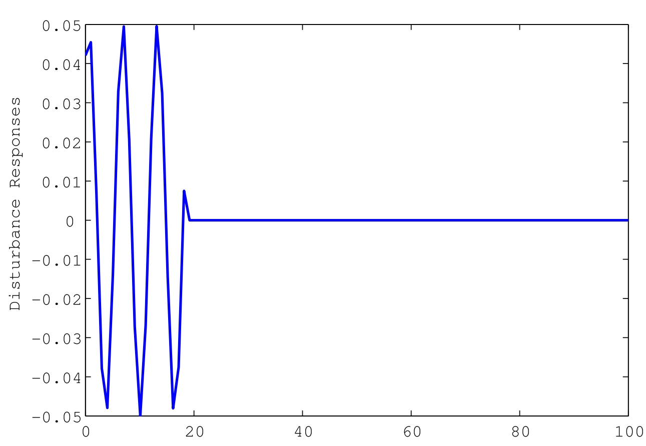


FIG. 3. Disturbance $d(t)$.

disturbance signal as

$$d(t) = \begin{cases} 0.05\sin(t), & 0 \le t \le 20 \\ 0, & \text{otherwise.} \end{cases}$$

Figures 2(a) and 2(b) show the plots of position and velocity states of 4 agents for an unforced and forced system. The response of the disturbance signal is depicted in Fig. 3. When we observe Fig. 2(b), we can comprehend easily that the agents are able to reach consensus as proved in Theorem III.1, which manifests that the consensus of the second-order nonlinear multi-agent system is achieved. Comparing Figs. 2(a) and *2(b), it is obvious that the consensus problem for a second-order multi-agent system with nonlinear dynamics and external disturbance can be solved effectively using the proposed mixed H_∞ and passivity-based sampled-data control.*

V. CONCLUSIONS

In this paper, the communication predicaments of agents of a second-order multi-agent system such as disturbances and nonlinear dynamics are considered. Following the concept of time-varying delay, a sampled-data control is proposed which is effective in the solvability of consensus problem. Furthermore, we exploit a mixed H_∞ and passivity-based control for a second-order nonlinear multi-agent system. The specified performance index is guaranteed when the system experiences bounded energy disturbances. Correspondingly, an example is given to prove the effectiveness of the proposed control method. In the future work, a robust adaptive output feedback scheme is expected to be developed for the nonlinear second-order multi-agent system with unknown model dynamics and unknown disturbances under consensus sampled-data control.

REFERENCES

[1] X. Liu, K. Zhang, and W.-C. Xie, "Consensus of multi-agent systems via hybrid impulsive protocols with time-delay," Nonlinear Anal.: Hybrid Syst. **30**, 134–146 (2018).

[2] J. Xi, Y. Yu, G. Liu, and Y. Zhong, "Guaranteed-cost consensus for singular multi-agent systems with switching topologies," IEEE Trans. Circuits Syst. I **61**, 1531–1542 (2014).

[3] C.-Q. Ma and J.-F. Zhang, "Necessary and sufficient conditions for consensusability of linear multi-agent systems," IEEE Trans. Autom. Control **55**, 1263–1268 (2010).

[4] M. S. Mahmoud and G. D. Khan, "LMI consensus condition for discrete-time multi-agent systems," IEEE/CAA J. Autom. Sin. **5**, 509–513 (2016).

[5] Y. Shang, "Resilient consensus of switched multi-agent systems," Syst. Control Lett. **122**, 12–18 (2018).

[6] J. Wang, X. Zhao, B. Xu, W. Wang, and Z. Niu, "Immune multi-agent model using vaccine for cooperative air-defense system of systems for surface warship formation based on danger theory," J. Syst. Eng. Electron. **24**, 946–953 (2013).

[7] Z. Wang, J. Xu, X. Song, and H. Zhang, "Consensus problem in multi-agent systems under delayed information," Neurocomputing **316**, 277–283 (2018).

[8] A. Ashrafi and S. Shahrtash, "Dynamic wide area voltage control strategy based on organized multi-agent system," IEEE Trans. Power Syst. **29**, 2590–2601 (2014).

[9] M. Jafari and H. Xu, "A biologically-inspired distributed fault tolerant flocking control for multi-agent system in presence of uncertain dynamics and unknown disturbance," Eng. Appl. Artif. Intell. **79**, 1–12 (2019).

[10]P. Kendrick, N. Criado, A. Hussain, and M. Randles, "A self-organising multi-agent system for decentralised forensic investigations," Expert Syst. Appl. **102**, 12–26 (2018).

[11]F.-r. Lin and Y.-H. Pai, "Using multi-agent simulation and learning to design new business processes," IEEE Trans. Syst., Man, Cybern. Part A **30**, 380–384 (2000).

[12]L. Maleš, D. Marčetić, and S. Ribarić, "A multi-agent dynamic system for robust multi-face tracking," Expert Syst. Appl. **126**, 246–264 (2019).

[13]K. Manickavasagam, "Intelligent energy control center for distributed generators using multi-agent system," IEEE Trans. Power Syst. **30**, 2442–2449 (2014).

[14]Z. Yan, Y. Liu, X. Huang, J. Zhou, and H. Shen, "Mixed H_∞ and $L_2 - L_\infty$ anti-synchronization control for chaotic delayed recurrent neural networks," Int. J. Control, Autom. Syst. **17**, 3158–3169 (2019).

[15]T. Nagata and H. Sasaki, "A multi-agent approach to power system restoration," IEEE Trans. Power Syst. **17**, 457–462 (2002).

[16]Y. Tang, X. Xing, H. R. Karimi, L. Kocarev, and J. Kurths, "Tracking control of networked multi-agent systems under new characterizations of impulses and its applications in robotic systems," IEEE Trans. Ind. Electron. **63**, 1299–1307 (2015).

[17]H. Li, X. Liao, T. Huang, and W. Zhu, "Event-triggering sampling based leader-following consensus in second-order multi-agent systems," IEEE Trans. Autom. Control **60**, 1998–2003 (2014).

[18]Z. Yu, H. Jiang, X. Mei, and C. Hu, "Guaranteed cost consensus for second-order multi-agent systems with heterogeneous inertias," Appl. Math. Comput. **338**, 739–757 (2018).

[19]C.-E. Ren, Z. Shi, and T. Du, "Distributed observer-based leader-following consensus control for second-order stochastic multi-agent systems," IEEE Access **6**, 20077–20084 (2018).

[20]Y. Su, "Cooperative global output regulation of second-order nonlinear multi-agent systems with unknown control direction," IEEE Trans. Autom. Control **60**, 3275–3280 (2015).

[21]H. Du, Y. He, and Y. Cheng, "Finite-time synchronization of a class of second-order nonlinear multi-agent systems using output feedback control," IEEE Trans. Circuits Syst. I **61**, 1778–1788 (2014).

[22]W. Yu, Y. Li, G. Wen, X. Yu, and J. Cao, "Observer design for tracking consensus in second-order multi-agent systems: Fractional order less than two," IEEE Trans. Autom. Control **62**, 894–900 (2016).

[23]W. Zou, C. K. Ahn, and Z. Xiang, "Leader-following consensus of second-order nonlinear multi-agent systems with unmodeled dynamics," Neurocomputing **322**, 120–129 (2018).

[24]R. Sakthivel, M. Joby, K. Mathiyalagan, and S. Santra, "Mixed H_∞ and passive control for singular Markovian jump systems with time delays," J. Franklin Inst. **352**, 4446–4466 (2015).

[25]M. Sathishkumar, R. Sakthivel, F. Alzahrani, B. Kaviarasan, and Y. Ren, "Mixed H_∞ and passivity-based resilient controller for nonhomogeneous Markov jump systems," Nonlinear Anal.: Hybrid Syst. **31**, 86–99 (2019).

[26]M. Sathishkumar, R. Sakthivel, O. Kwon, and B. Kaviarasan, "Finite-time mixed H_∞ and passive filtering for Takagi–Sugeno fuzzy nonhomogeneous Markovian jump systems," Int. J. Syst. Sci. **48**, 1416–1427 (2017).

[27]L. Su and H. Shen, "Mixed H_∞/passive synchronization for complex dynamical networks with sampled-data control," Appl. Math. Comput. **259**, 931–942 (2015).

[28]Z.-G. Wu, J. H. Park, H. Su, B. Song, and J. Chu, "Mixed H_∞ and passive filtering for singular systems with time delays," Signal Process. **93**, 1705–1711 (2013).

[29]Y. Zhilian, H. Xia, and C. JinDe, "Variable-sampling-period dependent global stabilization of delayed memristive neural networks via refined switching event-triggered control," Sci. China Inf. Sci. (published online).

[30]A. Amini, A. Asif, and A. Mohammadi, "A performance guaranteed sampled-data event-triggered consensus approach for linear multi-agent systems," Inf. Sci. **484**, 338–349 (2019).

[31]G. Guo, L. Ding, and Q.-L. Han, "A distributed event-triggered transmission strategy for sampled-data consensus of multi-agent systems," Automatica **50**, 1489–1496 (2014).

[32]J. Zhou, C. Sang, X. Li, M. Fang, and Z. Wang, "H_∞ consensus for nonlinear stochastic multi-agent systems with time delay," Appl. Math. Comput. **325**, 41–58 (2018).

[33]K. Liu, H. Zhu, and J. Lü, "Bridging the gap between transmission noise and sampled data for robust consensus of multi-agent systems," IEEE Trans. Circuits Syst. I **62**, 1836–1844 (2015).

[34]W. Zhang, Y. Tang, T. Huang, and J. Kurths, "Sampled-data consensus of linear multi-agent systems with packet losses," IEEE Trans. Neural Networks Learn. Syst. **28**, 2516–2527 (2016).

[35]X.-Y. Zhang and J. Zhang, "Sampled-data consensus of multi-agent systems with general linear dynamics based on a continuous-time model," Acta Autom. Sin. **40**, 2549–2555 (2014).

[36]M. Park, O. Kwon, J. H. Park, S. Lee, and E. Cha, "Stability of time-delay systems via Wirtinger-based double integral inequality," Automatica **55**, 204–208 (2015).

[37]A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: Application to time-delay systems," Automatica **49**, 2860–2866 (2013).